

Extracting a New Value of the Slope of the Neutron Form Factor Compatible with Deuteron Properties

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Abstract: We extract a new value $b = 0.0205 \pm 0.0017 \text{ fm}^2$ of the slope of the neutron form factor $G_{En}(q^2)$ at $q^2 = 0$ compatible with deuteron properties by using a linear relation between b and $A_S^2(1 + \eta^2)$ we found for a class of nonlocal potential models having the experimental values of both r_D and Q . Another model dependent value $b^{\text{MHKZ}} = 0.0206 \pm 0.0014 \text{ fm}^2$, which is also compatible with deuteron properties, has been determined by applying "constrained" unitary transformations to the local MHKZ potential model. The sensitivity of a small changes in the experimental values used for r_D and Q on the value obtained for b is also investigated.

Keywords: Electromagnetic processes and properties, Elastic scattering, Electromagnetic form factors, Nucleon-nucleon interactions

1 Introduction

Measurements of the elastic scattering of electrons by deuterons lead in a direct way to the deuteron form factors as functions of the momentum transfer q^2 . These form factors are of interest in connection with the study of the structure of the deuteron and the determination of the electromagnetic form factors of the neutron [1].

This paper deals with the neutron form factor which is experimentally not very well known but new precision measurements of this quantity are planned or presently being performed [2,3,4,5,6,7,8,9,10,11,12,13,14]. Because there are no free neutron targets the neutron form factor is determined from electron scattering on the deuteron. In order to extract the neutron form factor from the experimental deuteron structure function, the deuteron wave functions must be calculated using some nucleon-nucleon potential models. Therefore, the determination of the neutron form factor is not model independent but depends on the underlying nucleon-nucleon potential that is used to calculate the deuteron wave function.

Correlations between the slope b of the neutron form factor $G_{En}(q^2)$ at $q^2 = 0$ and the deuteron root mean square (rms) matter radius r_D for different NN-potential models have been used to determine r_D [15,16,17].

Berard *et al.* [15] found a linear relation with a positive slope between b and r_D^2 for a class of potential models. They used their experimental data for the ratio $R(q^2)$ of the deuteron to the proton electric form factors in the range of momentum transfers $0.05 \leq q^2 \leq 0.50 \text{ fm}^{-2}$ and the experimental value $b = 0.0189 \pm 0.0004 \text{ fm}^2$ given by Krohn and Ringo [18] to extract the value $r_D = 1.9635 \pm 0.0045 \text{ fm}$. Allen *et al.* [16] found a linear relation with a positive slope between b and r_D^2 for the radial wave functions for a single local potential model and its family of the wave functions produced by unitary transformations. The experimental data of $R(q^2)$ of Berard *et al.* [15] and the experimental value $b = 0.0199 \pm 0.0003 \text{ fm}^2$ given by Koester *et al.* [19] have been used to determine the model dependent value $r_D = 1.952 \pm 0.004 \text{ fm}$ and the model independent one $r_D = 1.948 \pm 0.023 \text{ fm}$ indirectly from the plotted straight line. They used also the experimental data of $R(q^2)$ of Akimov *et al.* [1] in the range $0.05 \leq q^2 \leq 0.50 \text{ fm}^{-2}$ to determine $r_D = 2.005 \pm 0.118 \text{ fm}$. In an analogous procedure Mustafa *et al.* [17] found a linear relation between b and r_D^2 for the radial wave functions for the MHKZ local potential model [17] and its family of the wave functions produced by unitary transformations. They used also the experimental data of $R(q^2)$ of Berard

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et al. [15] and the experimental value $b = 0.0199 \pm 0.0003 \text{ fm}^2$ given by Koester *et al.* [19] to determine $r_D = 1.9546 \pm 0.0021 \text{ fm}$.

In this work, correlations between b and deuteron properties, i.e. the asymptotic S -state amplitude A_S , the asymptotic D/S ratio η , the rms radius r_D , the quadrupole moment Q and the binding energy E_b , have been used to determine b . Two linear relations between b and $A_S^2(1 + \eta^2)$ have been found for deuteron potential models, one with a positive slope and the other with a negative one.

At large distances, outside the nuclear potential range, the 3S_1 - and the 3D_1 -state radial wave functions (u and w , respectively) behave asymptotically as [20]

$$u(r) = A_S e^{-\gamma r}, \quad (1)$$

and

$$w(r) = A_D \left(1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2} \right) e^{-\gamma r}, \quad (2)$$

where $\gamma^2 = -\frac{2m}{\hbar^2} E_b$ [21] and m is the reduced neutron-proton mass. The radial wave functions are properly normalized to unity,

$$\int_0^\infty [u^2(r) + w^2(r)] dr = 1. \quad (3)$$

If the proton and the neutron are at a distance r apart, the deuteron rms radius r_D may be written as [20]

$$r_D^2 = \frac{1}{4} \int_0^\infty r^2 (u^2 + w^2) dr. \quad (4)$$

The deuteron quadrupole moment Q may be also given by [20]

$$Q = \frac{1}{\sqrt{50}} \int_0^\infty r^2 u w dr - \frac{1}{20} \int_0^\infty r^2 w^2 dr. \quad (5)$$

The correlation with $A_S^2(1 + \eta^2)$ has been used because the term $(u^2 + w^2)$ which has been involved in the definition of r_D^2 proportional to $A_S^2(1 + \eta^2)$ if we use the asymptotic forms of the radial wave functions u and w of Eqs. (1) and (2) and ignore the centrifugal term $[1 + 3/(\gamma r) + 3/(\gamma^2 r^2)]$.

The aim of the present work is to extract a new value of b which is compatible with deuteron properties. This value is extracted indirectly by using the linear correlation between b and $A_S^2(1 + \eta^2)$ of nonlocal potential models (having the experimental values of both r_D and Q) produced by applying unitary transformations to standard nonrelativistic potential models.

The paper is organized as follows: In the next section we outline the method which has been used to calculate a model value of the slope b of the neutron form factor $G_{En}(q^2)$ at $q^2 = 0$. The new determinations of this parameter using the local and nonlocal potential models will be distinguished in Section 3. In Section 4 a simple method to calculate the parameter b using one single potential model will be obtained. The sensitivity of a

small changes in the experimental values used for r_D and Q on the extracted value of b , i.e. $\frac{\delta b}{\delta r_D}$ and $\frac{\delta b}{\delta Q}$, will be given in Section 5. Finally, we close with a conclusion in Section 6.

2 Calculating model values of the slope of the neutron form factor

In order to extract the slope b of the neutron form factor $G_{En}(q^2)$ at $q^2 = 0$ from elastic electron-deuteron scattering, we start from the expression of the deuteron electric form factor in the nonrelativistic impulse approximation, neglecting two-body contributions [20]

$$G_{ED}(q^2) = \left[G_{Ep}(q^2) + G_{En}(q^2) \right] \left[C_E^2(q^2) + C_Q^2(q^2) \right]^{1/2} \times \left(1 + \tau \right)^{-1/2} \quad (6)$$

$$\simeq \left[G_{Ep}(q^2) + G_{En}(q^2) \right] C_E(q^2) \left(1 + \tau \right)^{-1/2}, \quad (7)$$

where $G_{Ep}(q^2)$ is the proton electric form factor, $C_E(q^2)$ and $C_Q(q^2)$ are electric charge and quadrupole form factors, respectively, which are given in terms of the nonrelativistic S - and D -wave functions by [20]

$$C_E(q^2) = \int_0^\infty (u^2 + w^2) j_0\left(\frac{qr}{2}\right) dr \quad (8)$$

and

$$C_Q(q^2) = 2 \int_0^\infty \left(uw - \frac{w^2}{\sqrt{8}} \right) j_2\left(\frac{qr}{2}\right) dr, \quad (9)$$

where j_0 and j_2 are the spherical Bessel functions of order zero and two, respectively. The factor $\tau = (q^2/4m_p^2)$ is called the Darwin-Foldy correction to the nucleon form factors [22], where $m_p = 4.5749098 \text{ fm}^{-1}$ is the proton mass. Note that, Eq. (7) is usually used because $C_Q \ll C_E$ for small q^2 .

The value of b for a particular potential model is insensitive to the choice of the proton electric form factor $G_{Ep}(q^2)$. In this work we take the parametrization of Simon *et al.* [23] for q^2 in fm^{-2}

$$G_{Ep}(q^2) = \frac{0.312}{\left(1 + \frac{q^2}{6} \right)} + \frac{1.312}{\left(1 + \frac{q^2}{15.02} \right)} - \frac{0.709}{\left(1 + \frac{q^2}{44.08} \right)} + \frac{0.085}{\left(1 + \frac{q^2}{154.2} \right)}. \quad (10)$$

For the evaluation of the electric charge form factor $C_E(q^2)$ given in Eq. (8) we split the integration according to

$$C_E(q^2) = \int_0^R (u^2 + w^2) j_0\left(\frac{qr}{2}\right) dr + \Delta C_E(q^2), \quad (11)$$

where $\Delta C_E(q^2)$ is the asymptotic analytic correction to $C_E(q^2)$ given by

$$\Delta C_E(q^2) = \int_R^\infty (u^2 + w^2) j_0\left(\frac{qr}{2}\right) dr. \quad (12)$$

The first integration of Eq. (11) is calculated numerically from $r = r_c$ to $r = R$, where r_c is the hard-core radius - if any - and R is chosen to be $16+r_c$ fm which is beyond the range of the nuclear potential. The integral in Eq. (12) is carried out analytically from $r = R$ to $r = \infty$ by using the asymptotic forms of the radial wave functions u and w of Eqs. (1) and (2). The analytic formula of the $\Delta C_E(q^2)$ is given in Ref.[17], for which the values of $C_E(q^2)$ are correct in the momentum transferred range $0 \leq q^2 \leq 25 \text{ fm}^{-2}$. In an analogous procedure the deuteron rms radius r_D given in Eq. (4) and the deuteron quadrupole moment Q given in Eq. (5) are evaluated.

Given $C_E(q^2)$ for a particular potential model, then the experimental values of the ratio $R(q^2)$ of the deuteron form factor $G_{ED}(q^2)$ to the proton form factor $G_{EP}(q^2)$ can be used to determine the neutron form factor [15] from

$$G_{En}(q^2) = \left[\frac{R(q^2)\sqrt{1+\tau}}{C_E(q^2)} - 1 \right] G_{Ep}(q^2) \quad (13)$$

and hence, the slope b of $G_{En}(q^2)$ at $q^2 = 0$,

$$b = \left. \frac{dG_{En}(q^2)}{dq^2} \right|_{q^2=0}. \quad (14)$$

The experimental values of $R(q^2)$ of Simon *et al.* [24] are used in this work. This set of data covers the range of values $0.044 \leq q^2 \leq 4 \text{ fm}^{-2}$.

Various parametrizations have been used for $G_{En}(q^2)$ [25,26,27,28,29,30,31]. In order to calculate b in this work we parametrize $G_{En}(q^2)$ by a polynomial of order three in q^2 as

$$G_{En}(q^2) = bq^2 + cq^4 + dq^6 \quad (15)$$

which agrees well for $0.044 \leq q^2 \leq 4 \text{ fm}^{-2}$. The model dependency of b , here, is a result of the model dependence of the electric charge form factor $C_E(q^2)$ via the radial wave functions.

3 The new determinations of the slope parameter

In order to extract a new value of the slope b of the neutron form factor $G_{En}(q^2)$ at $q^2 = 0$, a linear relation between b and $A_S^2(1 + \eta^2)$ with a positive slope has been found for forty-nine standard nonrelativistic potential models which all fit the experimental deuteron binding energy but have in general different values for the deuteron rms radius and the quadrupole moment as shown in Fig. 1. These models and the "names" given to

Table 1: The experimental values used in the analysis of this work.

Value	Ref.
$r_D=1.9547 \pm 0.0019 \text{ fm}$	[22]
$Q=0.2859 \pm 0.0003 \text{ fm}^2$	[45]
$A_S=0.8838 \pm 0.0004 \text{ fm}^{-1/2}$	[46]
$\eta=0.02713 \pm 0.00006$	[43]
$E_b=-2.224575 \pm 0.000009 \text{ MeV}$	[47]

them are the potentials of Glendenning and Kramer "GK1,, GK9" [32], Lacombe *et al.* "PARIS" [33], Mustafa *et al.* "MHKZ" [17], Reid "RHC, RSC, RSCA" [34], Machleidt *et al.* "MACH-A, -B, -C" [35], Machleidt *et al.* "Bonn-F, -Q" [36], de Tourreil and Sprung "TS-A, -B, -C" [37], de Tourreil *et al.* "TRS" [38], Hamada and Johnston "HJ" [39], Mustafa and Zahran "MZ" [40], Mustafa "A, B" [41], Mustafa *et al.* "r1, r3,, r7" [42], Mustafa "L1, L2, 1, 2,, 6" [43] and Mustafa "a, b,, i" [44]. The value

$$b = 0.0296 \pm 0.0118 \text{ fm}^2 \quad (16)$$

which is compatible with the experimental values of A_S , η and E_b but not with the experimental values of r_D and Q is extracted from this correlation. It is the value of b corresponding to the experimental value $A_S^2(1 + \eta^2) = 0.7817 \text{ fm}^{-1}$. This value of b is greater than the previously extracted value $b = 0.0199 \pm 0.0003 \text{ fm}^2$ by Koester *et al.* [19] and its error Δb is relatively large. Therefore, the result obtained for b using the local potential models is not very reliable.

Moreover, it is desirable to obtain a value of b which is not only compatible with the experimental values of A_S , η and E_b , but also with the experimental values of Q and r_D , which have dependencies on the interior parts of the radial wave functions. The experimental values of these quantities are listed in Table 1. Contributions to the value of r_D and Q from meson exchange currents and other relativistic corrections $\Delta r_D = 0.0034 \text{ fm}$ and $\Delta Q = 0.0063 \text{ fm}^2$ of Kohno [48] are neglected in the present work. Kermode *et al.* [49] and Mustafa and Hassan [50] have proven that Q and r_D are not purely asymptotic quantities by showing that phase-equivalent potentials can have different quadrupole moments and radii. Therefore, the linear relation between b and $A_S^2(1 + \eta^2)$ obtained in Fig. 1 has been re-drawn using only potential models which reproduce the experimental values of both Q and r_D . These phase-equivalent potential models are produced by using "constrained" short-range unitary transformations like those used previously by Kermode *et al.* [49] for the radial wave functions (u_i, w_i) of a local potential models. The transformed wave functions (\bar{u}_i, \bar{w}_i) of a nonlocal potential model are given

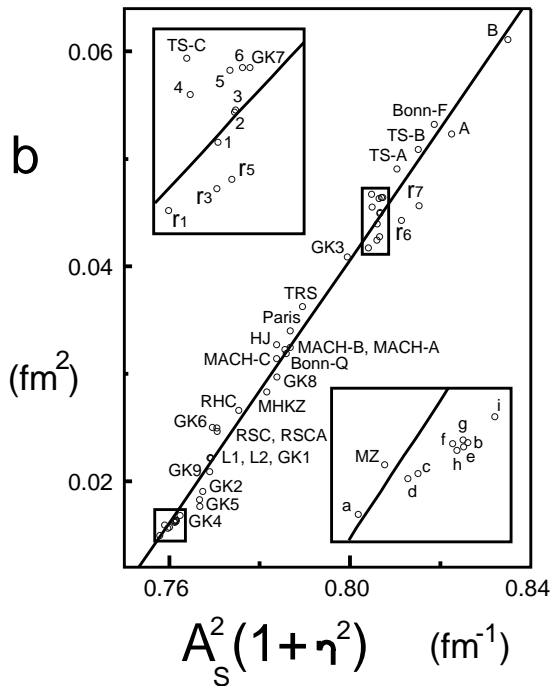


Fig. 1: The variation of the slope b versus $A_S^2(1+\eta^2)$ of standard local potential models. The middle (lower) part of the graph is magnified in the upper (lower) inner frame. The value of b of Eq. (16) is extracted from this straight line corresponding to $A_S^2(1+\eta^2)=0.7817 \text{ fm}^{-1}$.

by

$$\bar{u}_i = u_i - 2g(r) \int_0^\infty g(s)u_i(s)ds, \quad (17)$$

$$\bar{w}_i = w_i - 2g(r) \int_0^\infty g(s)w_i(s)ds. \quad (18)$$

Various parametrizations have been used for the function $g(s)$ [50,51,52,53,54]. We have chosen the parametrization $g(s) = Cs(1-\beta s)e^{-\alpha s}$, where $s = r - r_c$ and $C = [4\alpha^5/(\alpha^2 - 3\beta\alpha + 3\beta^2)]^{1/2}$ is a normalizing factor such that $\int_0^\infty g^2(r)dr = 1$. The parameters α and β are adjustable parameters. The parameter α can be regarded as the "range" and β as the "strength" of the unitary transformation. The values of $\alpha < 1.5 \text{ fm}^{-1}$ produce wave functions of different asymptotic behaviour [50]. In this work, the parameter α is assumed to changed from $\alpha = 1.5 \text{ fm}^{-1}$ to $\alpha = 7 \text{ fm}^{-1}$ in steps of 0.0001 fm^{-1} . For each value of α , the parameter β is changed from $\beta = 0 \text{ fm}^{-1}$ to $\beta = 4 \text{ fm}^{-1}$ in steps of 0.0001 fm^{-1} . The function $g(r)$ is of short range, hence, the wave functions u and w of a local potential model and \bar{u} and \bar{w} of the corresponding nonlocal potential model are the same in the asymptotic region. This implies that the asymptotic quantities A_S and A_D and hence, $\eta = A_D/A_S$,

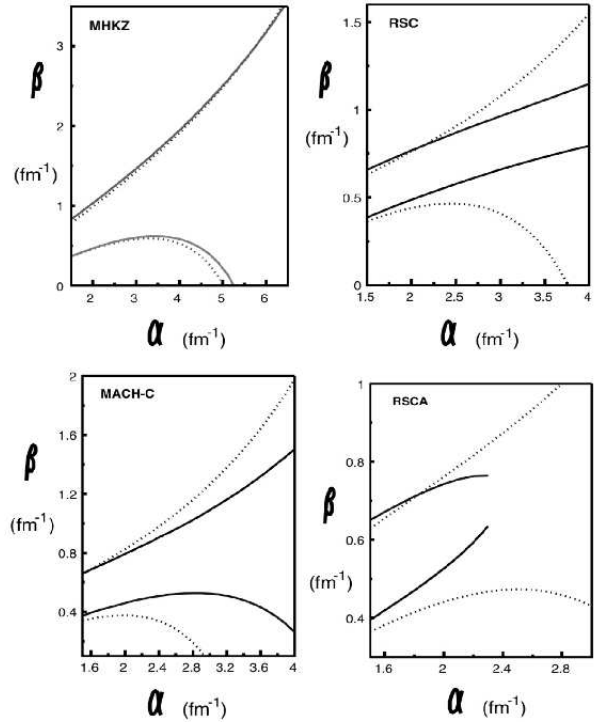


Fig. 2: The variation of the "strength" parameter β versus the "range" parameter α for some potential models which have intersection points, i.e., have a pair (α, β) which give the experimental values of both Q and r_D . The upper left, upper right, lower left, and lower right panels represent the results for the MHKZ, RSC, MACH-C, and RSCA NN potentials, respectively. The solid (dotted) lines represent the values of α and β which give the experimental values of Q (r_D).

are the same for both the local and nonlocal potential models, but r_D and Q could be different.

The values of the nonlocality parameters α and β are adjusted to produce transformed wave functions having the experimental values of r_D and Q . This could not be achieved for all the local potentials considered. It is always possible to find for a given value of α a value of β which corresponds either to the experimental value of r_D or to the experimental value of Q . In fact, for a given α one often finds two solutions for β as is shown by the two branches in Fig. 2. But it was not always possible for some potential models to find pairs (α, β) such that both r_D and Q could be reproduced. In this case we fix the nonlocality parameters (α, β) which give the experimental value of r_D by searching for the closest value of Q with respect to the experimental value or vice versa. If both values would be fitted, the two type of curves possess an intersection point as is demonstrated in Fig. 2 for selected potential models. The transformed wave functions having the experimental values of both r_D

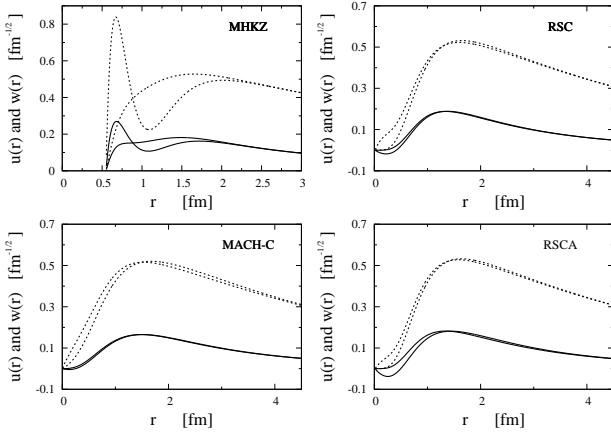


Fig. 3: Transformed wave functions $u'(r)$ and $w'(r)$ having the experimental values of both r_D and Q produced by the unitary transformation (solid line) are compared to the deuteron wave functions of local potentials $u(r)$ and $w(r)$ which are represented by the dotted line. The upper (lower) curves are the u and u' (w and w') wave functions. The upper left, upper right, lower left, and lower right panels represent the results for the MHKZ, RSC, MACH-C, and RSCA NN potentials, respectively.

and Q are compared to the radial wave functions in Fig. 3 for selected potential models.

In contrast to the neutron form factor at finite momentum transfers the slope of this form factor at zero momentum transfer is experimentally much better determined. The slope of the neutron form factor at zero momentum transfers is closely related to the neutron radius. This observable is used as an additional test for various nucleon-nucleon potential models. Altogether, we have studied forty-nine potential models. Out of these, we only found seventeen transformed models giving the experimental values for both r_D and Q , i.e. are able to simultaneously describe the experimental rms radius and quadrupole moment. The pairs of the nonlocality parameters (α, β) having the experimental values of both r_D and Q are listed in Table 2.

In this case of applying “constrained” unitary transformations, a linear relation with a negative slope between b and $A_S^2(1 + \eta^2)$ is found as shown in Fig. 4. The points representing the transformed potential models lie on or closely scattered around the straight line. We would like to mention that the five points representing the nonlocal potentials which are phase equivalent to the family of the Glendenning and Kramer potential models (GK2, GK3, GK5, GK7 and GK8) [32] lie on a separate line with a different negative slope (see also, Fig. 1 in Klarsfeld *et al.* [55] in which they used deuteron potential models in an empirical relation and found that the point representing the Glendenning and Kramer potentials also not lie on the empirical line, but lie on a separate line with a different slope). However, these old models predict a relatively larger deuteron binding energy E_b in agreement

Table 2: The pairs of the nonlocality parameters (α, β) of the unitary transformations producing transformed wave functions having the experimental values of both r_D and Q .

local NN potential	Ref.	α fm ⁻¹	β fm ⁻¹
MHKZ	[17]	1.9852	0.4624860
GK2	[32]	1.3793	0.3519344
GK3	[32]	2.1190	1.018222
GK5	[32]	1.3182	0.3375304
GK6	[32]	3.8880	0.7406667
GK7	[32]	2.0998	1.033014
GK8	[32]	2.2207	1.054492
RSC	[34]	2.3926	0.8450504
RHC	[34]	2.9143	1.332711
RSCA	[34]	1.9280	0.7318227
MACH-C	[35]	1.6018	0.6853457
TS-B	[37]	1.5767	0.6945233
TS-C	[37]	1.2781	0.5850970
r1	[42]	2.3327	1.010562
r3	[42]	2.5710	1.198746
r5	[42]	3.2163	1.637167
r6	[42]	3.8893	2.111205
r7	[42]	3.2682	1.767236
TRS	[38]	1.5228	0.6574996
HJ	[39]	3.4835	1.623580
PARIS	[33]	1.5030	0.6452400

with the then experimental value $E_b = -2.226 \pm 0.003$ MeV [56]. Therefore, these five potential models are not considered in Fig. 4. The new value

$$b = 0.0205 \pm 0.0017 \text{ fm}^2 \quad (19)$$

is extracted from the straight line of Fig. 4 using only the remaining twelve nonlocal potential models. This new value of b is compatible with the experimental values of A_S , η , r_D , Q and E_b . It also agrees very well with the previous experimental determination of this quantity by thermal neutron electron scattering [19]. The standard error in Eq. (19) which is caused by the experimental uncertainty in the deuteron structure function is much less than that of Eq. (16), but is greater than those of the previous measurements.

4 Determination of the slope parameter using MHKZ potential model

So far, we have used two methods to extract the slope parameter b . The first uses the linear relation between b and $A_S^2(1 + \eta^2)$ of the local potential models as shown in Fig. 1. The second is based on Fig. 4 using the nonlocal potential models having the experimental values of both

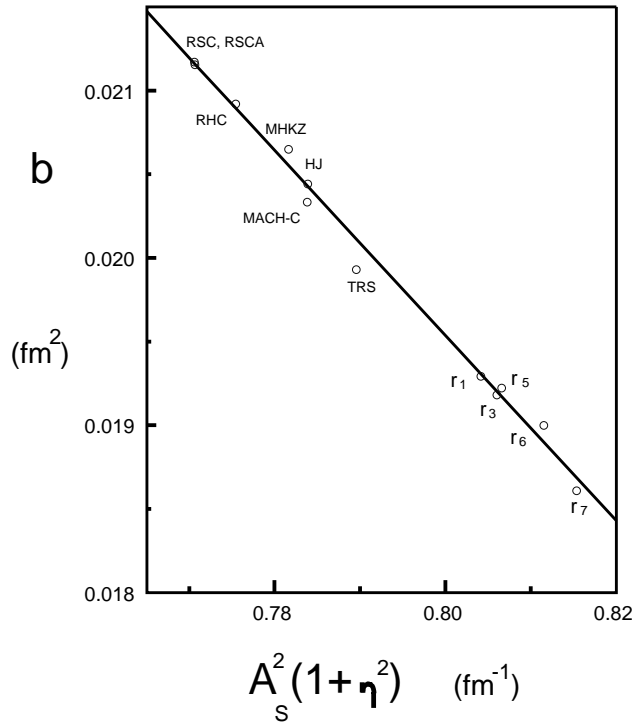


Fig. 4: The correlation of the slope b versus $A_S^2(1 + \eta^2)$ of the twelve nonlocal "transformed" potential models having the experimental values of r_D and Q . The new value of b of Eq. (19) is extracted from this line. It is the value of b corresponding to the experimental value of $A_S^2(1 + \eta^2) = 0.7817 \text{ fm}^{-1}$.

r_D and Q . The two extracted values in Eqs. (16) and (19) differ substantially.

A third method to extract b does not use the linear relation between b and $A_S^2(1 + \eta^2)$ and a large number of potential models. It uses only one *single* potential model which is the potential model of Mustafa *et al.* [17]. The importance of the MHKZ potential model comes from the fact that it reproduces the experimental values of A_S and η . The unitary transformation applied to the MHKZ wave functions are constrained to produce a phase-equivalent transformed wave functions having the experimental values of both r_D and Q . The *model* value,

$$b^{\text{MHKZ}} = 0.0206 \pm 0.0014 \text{ fm}^2 \quad (20)$$

is extracted by using the phase-equivalent potential of the MHKZ potential model [17]. It is also compatible with the experimental values of A_S , η , E_b , r_D and Q . This value is in very good agreement with the value obtained in Eq. (19) of this work.

5 The sensitivity of a small changes in the used experimental values on the extracted value of the slope parameter

The dependence of the determined value of b on the experimental values used for r_D and Q has been investigated. The whole procedure has been repeated twice, first by allowing for a small change δr_D in the value assumed as an experimental value of r_D and fixing the experimental values of A_S , η , E_b and Q to find the corresponding change δb . The published experimental values of r_D are in the range between $r_D = 1.947 \pm 0.029 \text{ fm}$ of Akimov *et al.* [1] and $r_D = 1.9635 \pm 0.0045 \text{ fm}$ of Berard *et al.* [15]. The value of r_D of Akimov *et al.* [1], which is the lower limit of the experimental determination of r_D , has been used in this work to study the correlation between the determined value of b , Eq. (19), and the value used for r_D as an experimental value. The value

$$b = 0.0176 \pm 0.0013 \text{ fm}^2 \quad (21)$$

is extracted in this case by using the transformed wave functions of potential models having the values of both r_D and Q . We show that the decrease in the value of r_D leads to a corresponding decrease in the value of b . Therefore, we find

$$\frac{\delta b}{\delta r_D} = 0.6744 \text{ fm}. \quad (22)$$

In the second case, only the experimental value used for Q is changed to obtain $\delta b/\delta Q$. The latest two published measurements of the experimental value of the deuteron quadrupole moment Q are the value of Reid and Vaida [57], $Q = 0.2860 \pm 0.0015 \text{ fm}^2$, and the value of Bishop and Cheung [45], $Q = 0.2859 \pm 0.0003 \text{ fm}^2$. To study the correlation between the extracted value of b and Q we used the experimental value $Q = 0.2860 \pm 0.0015 \text{ fm}^2$ [57]. The result of the analysis is

$$b = 0.0205 \pm 0.0011 \text{ fm}^2. \quad (23)$$

From these calculations we show that the increase in the value of Q leads to a small decrease in the value of b resulting in

$$\frac{\delta b}{\delta Q} = -0.0052. \quad (24)$$

Similarly, we found

$$\frac{\delta b}{\delta A_S} = -0.098 \text{ fm}^{3/2}, \quad (25)$$

and

$$\frac{\delta b}{\delta \eta} = -0.002 \text{ fm}^2. \quad (26)$$

From these results we conclude that the correlation between b and r_D is the dominant one.

Now, we would like to point out the reason for the changing slope of the correlation between b and

$A_S^2(1 + \eta^2)$ from being positive in the case of the local potential models of Fig. 1 to a negative value in the case of the nonlocal potential models of Fig. 4. Since b is proportional to r_D and r_D in turn proportional to $A_S^2(1 + \eta^2)$, the value of b is proportional to $A_S^2(1 + \eta^2)$. The values of A_S and η and then $A_S^2(1 + \eta^2)$ are the same for the local and the corresponding nonlocal model, but b and r_D are different. We see that the value of b in the case of using the radial wave functions of the local model is considerably greater than that in the case of using the transformed wave functions of the corresponding nonlocal potential model. Therefore, the slope will change sign in the case of using the transformed wave functions of the nonlocal potential models.

6 Summary

In this paper, we have found a linear relation between b and $A_S^2(1 + \eta^2)$ using forty-nine standard nonrelativistic NN potentials which all fit the experimental deuteron binding energy but have in general different values for r_D and Q . These are the potentials of Glendenning and Kramer "GK1,, GK9" [32], Lacombe *et al.* "PARIS" [33], Mustafa *et al.* "MHKZ" [17], Reid "RHC, RSC, RSCA" [34], Machleidt *et al.* "MACH-A, -B, -C" [35], Machleidt *et al.* "Bonn-F, -Q" [36], de Tourreil and Sprung "TS-A, -B, -C" [37], de Tourreil *et al.* "TRS" [38], Hamada and Johnston "HJ" [39], Mustafa and Zahran "MZ" [40], Mustafa "A, B" [41], Mustafa *et al.* "r1, r3,, r7" [42], Mustafa "L1, L2, 1, 2,, 6" [43] and Mustafa "a, b,, i" [44].

We have extracted a new value $b = 0.0205 \pm 0.0017$ fm² of the slope of the neutron form factor $G_{En}(q^2)$ at $q^2 = 0$ compatible with deuteron properties by using a linear relation between b and $A_S^2(1 + \eta^2)$ we found for a class of nonlocal potential models having the experimental values of both r_D and Q . Another model dependent value $b^{\text{MHKZ}} = 0.0206 \pm 0.0014$ fm², which is also compatible with deuteron properties, has been determined by applying "constrained" unitary transformations to the local MHKZ potential model. The sensitivity of a small changes in the experimental values used for r_D and Q on the value obtained for b is also investigated.

We found that the new value of the slope of the neutron form factor b of the correlation between b and $A_S^2(1 + \eta^2)$ is changed from being positive for local potentials to be negative for nonlocal potentials. The reason for that is explained as follows: since b is proportional to r_D and r_D in turn proportional to $A_S^2(1 + \eta^2)$, then the value of b is proportional to $A_S^2(1 + \eta^2)$. The values of A_S and η and then $A_S^2(1 + \eta^2)$ are the same for the local and nonlocal models, but b and r_D are different. Therefore, we see that the value of b using the radial wave functions of local potential is considerably greater than that of using the transformed

wave functions of nonlocal potential. Therefore, the slope is changed sign in the case of using the transformed wave functions of the nonlocal potentials.

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