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Core of Nucleon-Nucleon Interaction and Deuteron Properties

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Abstract: We investigated the argument of recent publications that the good description of the nucleon-nucleon interaction at short distances by quark models is at present only qualitative. The quark model is simulated at short distances by a repulsive short range nonlocality. For larger radii, the one pion exchange potential is used. We found that the quark model alone at the core region can only reproduce the deuteron binding energy, but fitting other nucleon-nucleon properties needs high local repulsion.

Keywords: Hard core, Nonlocal potential, Nucleon-nucleon interactions, Deuteron properties

1 Introduction

There is an extensive interest in the literature in investigating the nucleon-nucleon (NN) at small radii using the quark model, because it is believed that one gluon exchange between quarks plays the most important role in the nucleon-nucleon interaction, Shimizu [1]. Cvetic [2] argued that the interaction between clusters of quarks representing nucleons is expected to manifest itself in a nonrelativistic potential to a good approximation.

Oka and Yazaki [3] showed that the short-range repulsion of the nuclear force can be explained as a quark-exchange force with appropriate quark-quark interaction and that the quark-exchange force between two baryons is repulsive, short-range and nonlocal. The quark model fails to describe the intermediate-range and the long-range part of the interaction which can be explained by introducing the one-pion and two-pion contributions to the quark model. This step improves the results of the NN interaction but does not provide a rigorous test of the validity of the quark model.

Lacombe [4] and Vinh Mau [5] showed that the quark model should be supplemented by an accurate and well founded model for the long- and medium-range (LR+MR) forces. They studied the proton-proton pp and neutron-proton np interactions using the quark model for the core region supplemented by the Paris potential [6] for LR+MR distances. The results for the fit to the proton-proton (pp) and neutron-proton (np) data which they obtained is not good. They concluded that the ability of the quark model to describe the short-range of NN forces clearly differs whether one adopts the quark model with adjustable LR+MR forces or one chooses to associate the quark model with realistic and well founded LR+MR forces. We investigated this argument, where we study the neutron-proton np interaction in the $^3S_1-{}^3D_1$ coupled state as an example of the NN interaction.

Sprung *et al.*[7] studied deuteron properties using the one-pion exchange potential (OPEP)[8] truncated at a cut-off radius R_{cut} , with a constant interior potential, i.e. at short-distances. The cut-off radius R_{cut} and the potential depth were adjusted in all cases to give a bound state at the deuteron binding energy. While this is not a realistic model for the NN interaction, it does incorporate its most important features.

In this work, we discuss deuteron properties and low energy parameters of a particular potential model. In this model, the short-range SR part of the interaction ,i.e., the core region of the nucleon-nucleon interaction, is given by a nonlocal potential [9,10] and the long- and medium-range LR+MR part is described by the one-pion exchange potential OPEP. The nonlocal potential with nonlocality strength parameter λ is chosen to be repulsive and short ranged in agreement with the properties of the quark model, in order to simulate the quark model.

In this model, we make a sharp separation between the short-range and the long- and medium-range LR+MR parts of the interaction. The potential is nonlocal in the

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region $r_C < r \leq R_{cut}$ and is OPEP in the region $r > R_{cut}$, where R_{cut} is the cut-off radius and r_C is the hard-core radius if a hard-core is assumed. R_{cut} is the radius after which the nonlocal potential begins to be zero. Both the nonlocality strength parameter λ and the cut-off radius R_{cut} are adjusted to reproduce the deuteron binding energy $E_b = -2.224644 MeV$.

First, no hard-core is assumed in the core region, therefore the SR part is completely described by the quark model. We calculate the nucleon-nucleon NN properties in order to investigate the argument of Vinh Mau *et al.*[5] and Lacombe *et al.*[4].

We introduce a hard-core in the interaction which causes an infinite local short-range repulsion in the inner region in the presence of the quark model. Both the nonlocality strength parameter λ and the cut-off radius R_{cut} are adjusted to reproduce the binding energy of deuteron $E_b = -2.224644\ MeV$ while keeping the value of the hard-core radius r_C constant. Also, the cut-off radius R_{cut} is assumed to be constant, and in this case, both λ and r_C are adjusted to reproduce that value of the binding energy. The fixing of R_{cut} would make it possible to isolate the core region from the effect of the variation of the LR+MR attraction.

Hence, we can see more clearly the effect of the variation of the SR repulsion on other observables of the NN interaction. These observables are deuteron properties such as $D-state$ probability P_D, the quadrupole moment Q_d, the *rms* radius r_d, the asymptotic $S-state$ amplitude A_S, the asymptotic D- to S-state ratio η, and the low energy parameters such as the triplet scattering length a_t, the effective range parameter r_t, the shape parameter P. It is shown that adding a hard-core to the quark model in the core region would largely improve agreement with experimental values of the observables.

2 The potential model

The potential model has the form

$$V = \begin{cases} V^N & for\ r_C < r \leq R_{cut} \\ V^{OPEP} & for\ r > R_{cut}. \end{cases} \quad (1)$$

where, R_{cut} and r_C are the cut-off and the hard-core radii, respectively.

2.1 The nonlocal potential $\mathbf{V^N}$

The nonlocal potential used in this work is given by Mustafa and Kermode [9]. This potential is a nonlocal tensor contribution with nonlocality strength parameter λ and, it satisfies the time reversal invariance properties. Also, this potential is repulsive and short ranged to be consistent with the quark model. The expression for this potential can be written as

$$V^N(r,r') = S_{12}^N F(r,r') = \lambda g(r) g(r') S_{12}^N \quad (2)$$

where, the radial function $F(r,r')$ has a separable form. S_{12}^N is the nonlocal tensor operator and has the expression, [10,9]

$$S_{12}^N = \frac{1}{2}\left[9(\rho.\rho')[(\sigma_1.\rho)(\sigma_2.\rho') + (\sigma_1.\rho')(\sigma_2.\rho)] - 2(\sigma_1.\sigma_2)\right] \quad (3)$$

where, $\rho = \frac{\mathbf{r}}{r}$, $\rho' = \frac{\mathbf{r}'}{r'}$ and σ_1, σ_2 are the Pauli spin matrices for the two nucleons and the nonlocality strength parameter λ is measured in units of fm^{-3}.

2.2 The one-pion exchange potential V^{OPEP}

The one-pion exchange potential [8] consists of two parts

$$V^{OPEP} = V_C + S_{12} V_T \quad (4)$$

where, V_C is the central component, V_T is the tensor component and S_{12} is the usual local tensor operator. The radial dependence of these components can be written in the form

$$V_C(r) = V_o \frac{e^{-\mu r}}{\mu r} \quad (5)$$

$$V_T(r) = V_o \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right) \frac{e^{-\mu r}}{\mu r} \quad (6)$$

$$S_{12} = 3(\sigma_1.\rho)(\sigma_2.\rho) - 2(\sigma_1.\sigma_2) \quad (7)$$

where

$$V_o = \frac{g^2}{12\mu}\left(\frac{m_\pi}{M}\right)^2 m_\pi c^2 (\tau^{(1)}.\tau^{(2)})$$
$$= -10.4410938 MeV \quad (8)$$

μ is the inverse of the pion Compton wave length and the numerical value used is $0.699504\ fm^{-1}$, m_π is the pion mass, g^2 is the pion-nucleon coupling constant and $\tau^{(1)}.\tau^{(2)}$ is the isospin operator of the two nucleons.

Then, the coupled radial Schrödinger equations for the $^3S_1 - {}^3D_1$ coupled state, in the cases of the scattering and the bound states, can have the form,

$$\frac{d^2u}{dr^2} = (P - k^2)u + Sw + 2\sqrt{2}\lambda g(r) \int_{r_C}^{\infty} g(r') w(r') dr' \quad (9)$$

$$\frac{d^2w}{dr^2} = (Q - k^2)w + Su + 2\sqrt{2}\lambda g(r) \int_{r_C}^{\infty} g(r') u(r') dr'$$
$$-\lambda g(r) \int_{r_C}^{\infty} g(r') w(r') dr' \quad (10)$$

where,

$$g(r) = e^{-\alpha r} \text{ and } \alpha = 2.1\ fm^{-1} \quad (11)$$

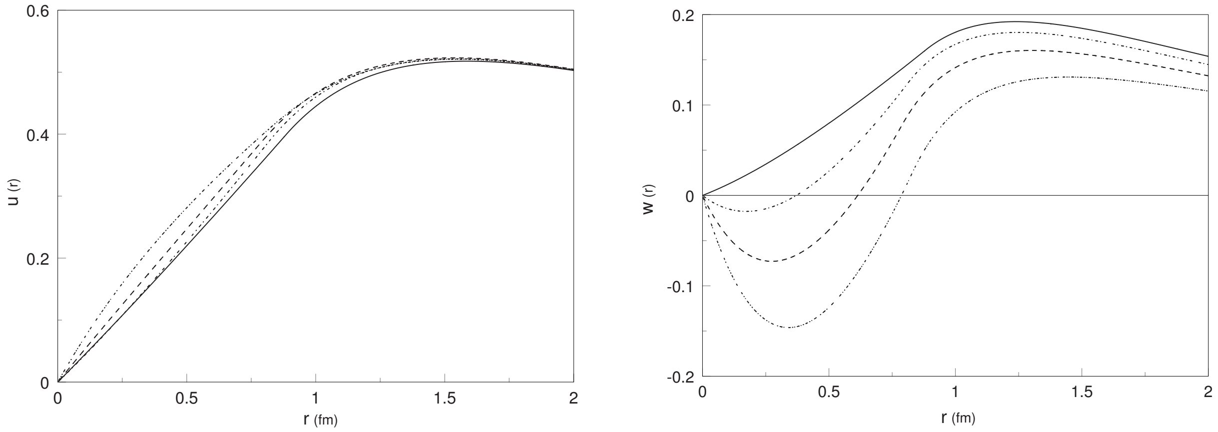

Fig. 1: The radial deuteron wave functions $u(r) and w(r)$ when the interaction with no hard-core radius, i.e., $r_C = 0\ fm$, at different pairs of λ and R_{cut}, where, the solid curve is for ($\lambda = 1.25718\ fm^{-3}$, $R_{cut} = 0.8825\ fm$), the dashed-dotted curve is for ($\lambda = 6.0981\ fm^{-3}$, $R_{cut} = 0.81\ fm$), the dashed curve is for ($\lambda = 10.016\ fm^{-3}$, $R_{cut} = 0.79\ fm$) and the dashed-double dotted curve is for ($\lambda = 11.99358\ fm^{-3}$, $R_{cut} = 0.8275\ fm$).

$$k^2 = \frac{M}{\hbar^2} E, \quad (12)$$

$$P = V_C, \quad (13)$$

$$S = 2\sqrt{2} V_T, \quad (14)$$

$$Q = \frac{6}{r^2} + V_C - 2V_T \quad (15)$$

3 Results and discussion

3.1 Sharp Separation (Cut-off)

We made a sharp separation between the short-range (SR) , i.e., the core region, and the long- and medium-range LR+MR parts of the nucleon-nucleon interaction. The nonlocal potential with nonlocality strength parameter λ is repulsive and short ranged to simulate the quark model QM in the short-range region $r_C < r \leq R_{cut}$. The one-pion exchange potential was used in the long- and medium-range LR+MR region $r > R_{cut}$, where r_C and R_{cut} are the hard-core, if it is assumed, and the cut-off radii, respectively.

3.2 The core region described by the quark model alone

We chose $r_C = 0\ fm$, i.e., the interaction without assuming a hard-core. In this case, the short-range SR part of the interaction, or core region, is completely described by the quark model. Both the nonlocality strength parameter λ and the cut-off radius R_{cut} were adjusted to reproduce the deuteron binding energy $E_b = -2.224644 MeV$ at different pairs of λ and R_{cut}.

Deuteron properties and low energy parameters have been listed in Table 1. It can be seen from Table 1 that the binding energy can be reproduced but the values of deuteron properties such as the quadrupole moment Q_d, the *rms* radius r_d, the asymptotic S-state amplitude A_S and the asymptotic ratio η and the low energy parameters such as the triplet scattering length a_t and the effective range parameter r_t are less than the corresponding experimental data. As λ increases, the cut-off radius R_{cut} decreases. This means that the truncated OPEP becomes more attractive to compensate for the increasing nonlocal repulsion, in order to maintain the value of the binding energy at $E_b = -2.224644\ MeV$. In this case, some properties of deuteron such as Q_d, P_D, r_d and η decrease with increasing λ values. But the other properties such as A_S, a_t, r_t, and the shape parameter P first decrease and then increase again with increasing λ values. Figure (1) shows the radial deuteron wave functions at different pairs of λ and R_{cut}. We noted a node appears in the D-wave function $w(r)$ as λ increases with positive values.

3.3 The effect of assuming a hard-core together with the quark model

The strong short-range repulsion between nucleons is sometimes called the hard-core interaction. The presence of a hard-core in the interaction means that $V(r) = \infty$ and $\Psi(r) = 0$ for $r \leq r_C$.

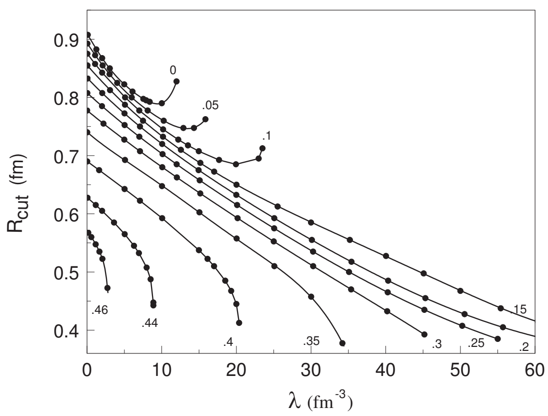

Fig. 2: The relation between the cut-off radius R_{cut} and the nonlocality strength parameter λ, when the hard-core radius r_C is constant at different values. The graphs are labeled by their values of r_C.

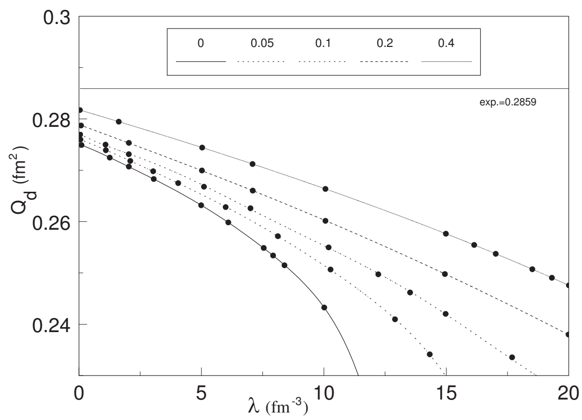

Fig. 3: The variations of the deuteron properties and low energy parameters when the hard-core radius r_C is constant at different values; i.e., Deuteron quadruple moment Q_d as function of the nonlocality strength parameter λ. Some curves have not been drawn to make the figures clear.

In this part, we introduced a hard-core with a radius r_C in the interaction where this produces an infinite short-range local repulsion in the inner part. The values of the hard-core radii were chosen to be 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.44, and 0.46 fm to study the effect of increasing the short-range repulsion on deuteron properties and low energy parameters, in the presence of the nonlocal potential representing the quark model.

At each value of r_C, both λ and R_{cut} were adjusted to reproduce the deuteron binding energy at $E_b = -2.224644 MeV$. It was found that by introducing a hard-core in the interaction, deuteron properties such as Q_d, r_d, A_S and η and low energy parameters such as a_t and r_t are relatively increasing to be closer to the experimental data. At each value of r_C, as λ increases with positive values, this means that the nonlocal potential becomes more repulsive. Then R_{cut} decreases and the truncated OPEP becomes more attractive to maintain the same value of the binding energy . In this case, all the properties of deuteron and low energy parameters decrease as λ increases. This means that these properties decrease as the LR+MR attraction increases. Figure (2) illustrates the relation between R_{cut} and λ for different constant values of r_C. From Figure (3) to Figure (10) show the dependence of deuteron properties and low energy parameters on λ.

The maximum value of r_C that could be used here must correspond to the removal to the nonlocal repulsion, i.e., $\lambda = 0$. This can be determined as the following. The relation between the nonlocality strength parameter λ and the cut-off radius R_{cut} can be given by a polynomial with an order n where

$$R_{cut} = a_o + a_1\lambda + a_2\lambda^2 + ... + a_n\lambda^n \quad (16)$$

a_o is equal to R^o_{cut} when $\lambda = 0\ fm^{-3}$.

We took all points determined for λ and R_{cut} when r_C is constant which are illustrated in figure (2) and then fitted by polynomials of (16) with order five in λ to find R^o_{cut} for each fixed value of r_C. The relation between R^o_{cut} and r_C is drawn in figure 11 . This polynomial is used to extrapolate the relation between R^o_{cut} and r_C until a value of r_C is reach at which $r_C = R^o_{cut}$. This is illustrated in Figure 11.

From Figure 11, the intersection point of the extrapolation of the curve and the straight line $R_{cut} = r_C$ is equal to 0.47784 fm which satisfies the polynomial and the straight line in the same time. This point is approximately equal to that of Glendenning and Kramer [8] $r_C = 0.46863\ fm$. We can use only a hard-core up to this radius and OPEP outside. The small difference $0.47784 - 0.46863 = 0.0092\ fm$, may be attributed to the more recent values of the masses of the pion and the nucleon we used.

3.4 The effect of the variation of the repulsion in the core region with outer constant attraction

In this case, the cut-off radius is assumed to be constant at $R_{cut} = 0.6\ fm$ and also at $0.8\ fm$. This means that the effect of the LR+MR attraction of the OPEP is fixed. The value of $R_{cut} = 0.8\ fm$ is equal to that used in the Paris potential which delimits the phenomenological and the theoretical parts [6,5].

When $r_C = 0\ fm$, i.e. the interaction without a hard-core, and $R_{cut} = 0.8\ fm$, then the short-range SR region is completely described by the quark model. The binding energy of deuteron can only be fitted at

Table 1: Deuteron properties and the low energy parameters of the truncated potentials with different cut-off radius R_{cut} and constant hard-core radius $r_C = 0\,fm$ where, the potential is nonlocal in the region $r_C < r < R_{cut}$ with strength parameter λ and it is $OPEP$ in the region $r > R_{cut}$. In each case, the binding energy of the deuteron is $E_b = -2.224644 MeV$. The number of nodes in the radial deuteron wave functions is N.

r_C fm	λ fm^{-3}	R_{cut} fm	Q_d fm^2	P_D	A_S $fm^{-\frac{1}{2}}$	η	r_d fm	a_t fm	r_t fm	P	N u	N w
0	0.10471	0.9085	0.2749	7.0862	0.8642	0.0266	1.9262	5.3146	1.6066	0.0168	0	0
	1.25718	0.8825	0.2725	6.8298	0.8639	0.0264	1.9254	5.3134	1.6045	0.0169	0	0
	2.03722	0.8675	0.2707	6.6620	0.8636	0.0262	1.9247	5.3124	1.6028	0.0170	0	0
	3.05422	0.8500	0.2683	6.4463	0.8633	0.0260	1.9239	5.3111	1.6007	0.0172	0	1
	4.98887	0.8225	0.2632	6.0321	0.8628	0.0255	1.9223	5.3090	1.5971	0.0176	0	1
	6.09810	0.8100	0.2598	5.7846	0.8626	0.0253	1.9215	5.3083	1.5959	0.0177	0	1
	7.54438	0.7975	0.2549	5.4410	0.8626	0.0248	1.9208	5.3082	1.5955	0.0176	0	1
	7.92820	0.7950	0.2534	5.3445	0.8626	0.0247	1.9206	5.3083	1.5957	0.0176	0	1
	8.39500	0.7925	0.2515	5.2232	0.8627	0.0245	1.9204	5.3087	1.5961	0.0175	0	1
	10.0160	0.7900	0.2433	4.7603	0.8631	0.0238	1.9200	5.3106	1.5990	0.0171	0	1
	11.99358	0.8275	0.2208	4.0812	0.8628	0.0221	1.9155	5.3064	1.5944	0.0200	0	1
Exp.			0.2859^a		0.8846^b	0.0268^c	1.9547^d	5.419^c	1.764^e			
			± 0.0003		± 0.0014	± 0.0007	± 0.0019	± 0.007	± 0.006			

(a) [11]; (b) [12]; (d)[13]; (c) [14,15]; (e) [16].

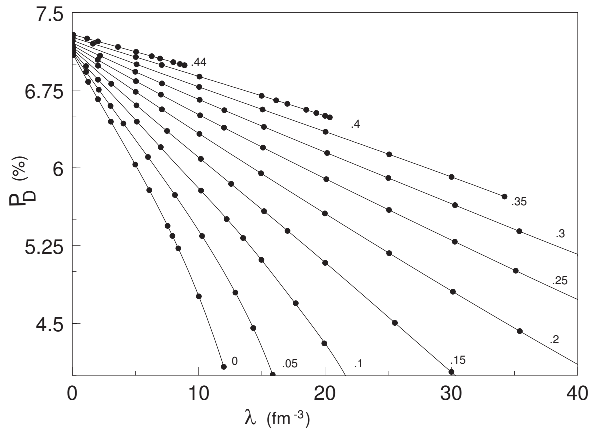

Fig. 4: D-state propability P_D as function λ.

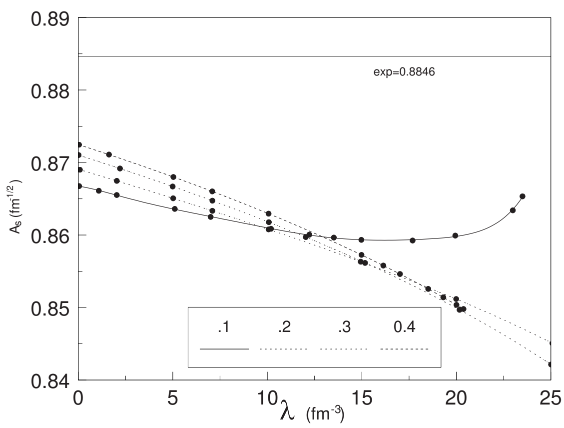

Fig. 5: Asymptotic S-state amplitude A_S as function of λ,.

$\lambda = 7.20687\ fm^{-3}$. As λ increases, the binding energy E_b decreases and increases again and is equal to the deuteron binding energy when $\lambda = 11.21\ fm^{-3}$. This corresponds to the first excited state of the deuteron.

When introducing a hard-core radius r_C in the interaction to produces a local short-range repulsion in the inner part, the values of the nonlocality strength parameter λ decreases as r_C increases. Then, deuteron properties and low energy parameters increase and become more consistent with the experimental data when the nonlocality becomes attractive and the local short-range repulsion caused by the hard-core is relatively large. The results are listed in Table 2. In this particular case when λ is negative, the nonlocal potential is no longer simulates the quark model, because we have attraction instead of repulsion in this case.

When the LR+MR attraction is constant at $R_{cut} = 0.6\ fm$, it was found that the repulsion of the nonlocality alone, i.e. the repulsion of the quark model is not sufficient to compensate the attraction of the OPEP. So, there must be an infinite repulsion to compensate this attraction with the QM in the SR region. The radius of

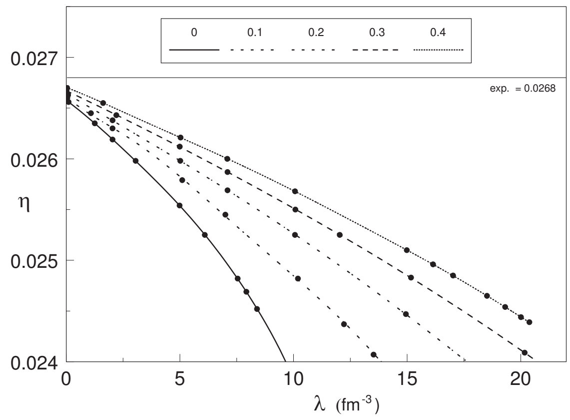

Fig. 6: Asymptotic ratio η as function of λ,.

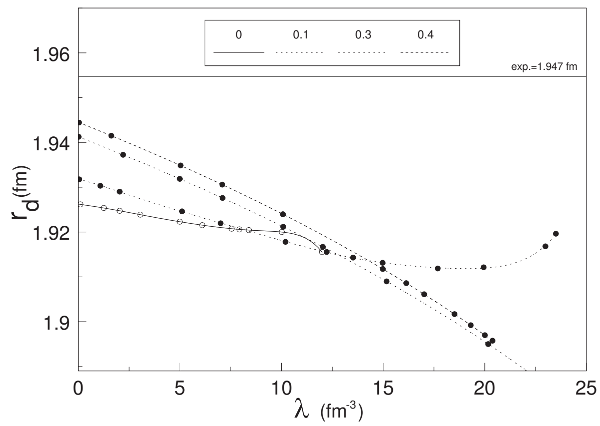

Fig. 7: Deuteron *rms* radius r_d as function of λ.

this hard-core begins from $r_C = 0.14 fm$. This can be seen from Table 2.

Table 3 gives the pairs of λ and r_C which can reproduce the experimental data of Q_d, r_d, A_S, η, a_t and r_t.

Figure 11 shows the radial dependence of the OPEP components on r which are truncated at a cut-off radius $R_{cut} = 0.6\ fm$ and $R_{cut} = 0.8\ fm$. The potential is equal to zero in the region $r_C - R_{cut}$ (dotted curves) and the OPEP starts from the cut-off radius R_{cut} and outworks (solid curves).

Recently, Entem *et al.*[17] present NN potentials through five orders of chiral effective field theory ranging from leading order (LO) to next to next to next to leading order (N4LO). The NN potentials are fit to the world NN data below the pion-production threshold of the year 2016. The potential of the highest order (N4LO) reproduces the world NN data, which is the highest precision ever accomplished for any chiral NN potential to date. Their family of potentials is nonlocal and, generally, of soft character. This feature is reflected in the fact that the predictions for the triton binding energy (from two-body forces only) converges to about 8.1 MeV at the highest orders. This leaves room for moderate three-nucleon forces.

4 Conclusion

In this work, we studied deuteron properties and low energy parameters of a potential model. In this model, we used a nonlocal potential to simulate the quark model to describe the short-range SR interaction and OPEP for the long- and medium-range LR+MR interaction.

When $r_C = 0\ fm$, i.e., the interaction without a hard-core, the short-range SR part, core region, is described by the quark model. The binding energy E_b can be fitted but other properties of the nucleon-nucleon NN interaction do not agree with the experimental values.

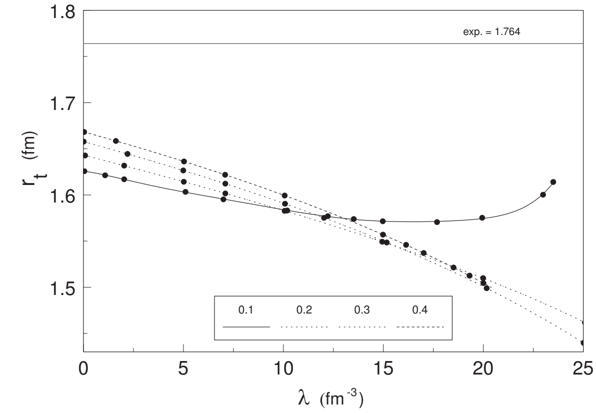

Fig. 8: Effective range parameter r_t as function of λ.

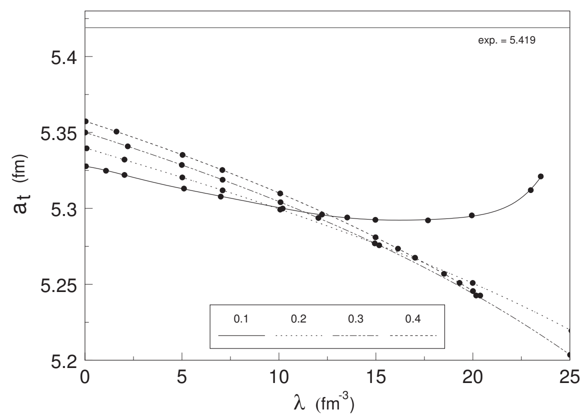

Fig. 9: The triplet scattering length as function of λ.

Table 2: Deuteron properties and the low energy parameters of the truncated potentials with different hard-core radius r_C and constant cut-off radius R_{cut} (share cut-off) where, the potential is nonlocal in the region $r_C < r < R_{cut}$ with strength parameter λ and it is *OPEP* in the region $r > R_{cut}$. In each case, the binding energy of the deuteron is $E_b = -2.22464 MeV$. The number of nodes in the radial deuteron wave functions is N.

R_{cut} fm	λ fm^{-3}	r_C fm	Q_d fm^2	P_D	A_S $fm^{-\frac{1}{2}}$	η	r_d fm	a_t fm	r_t fm	P	N u	N w
0.6	-12.62468	0.50	0.2977	7.6889	0.8825	0.0277	1.9661	5.4071	1.7388	-0.0047	0	0
	0.35352	0.45	0.2817	7.2835	0.8726	0.0267	1.9447	5.3580	1.6691	0.0040	0	0
	9.33714	0.40	0.2676	6.9080	0.8637	0.0258	1.9256	5.3137	1.6051	0.0141	0	0
	15.34015	0.35	0.2555	6.5511	0.8564	0.02050	1.9098	5.2768	1.5505	0.0248	0	0
	19.27743	0.30	0.2453	6.1884	0.8509	0.0242	1.8977	5.2490	1.5086	0.0348	0	0
	21.93826	0.25	0.2364	5.7807	0.8475	0.0235	1.8897	5.2315	1.4814	0.0425	0	1
	24.10796	0.20	0.2275	5.2501	0.8463	0.0228	1.8863	5.2260	1.4719	0.0461	0	1
	25.71434	0.17	0.2203	4.7836	0.8472	0.0221	1.8870	5.2306	1.4781	0.045	0	1
	26.47370	0.16	0.2169	4.5702	0.8479	0.0218	1.8880	5.2344	1.4835	0.0439	0	1
	27.52292	0.15	0.2124	4.2944	0.8491	0.0213	1.8897	5.2403	1.4921	0.0421	0	1
	29.37940	0.14	0.2045	3.8638	0.8512	0.0205	1.8930	5.2512	1.5082	0.0384	0	1
0.8	-29.12650	0.50	0.3142	8.1628	0.8925	0.0288	1.9875	5.4555	1.8066	-0.0113	0	0
	-17.15710	0.44	0.3030	7.8861	0.8853	0.0281	1.9721	5.4206	1.7582	-0.00652	0	0
	-11.36965	0.40	0.2965	7.7181	0.8812	0.0277	1.9632	5.4001	1.7293	-0.0033	0	0
	-5.96237	0.35	0.2896	7.5202	0.8766	0.0273	1.9533	5.3775	1.6974	0.0007	0	0
	-2.08168	0.30	0.2836	7.3286	0.8727	0.0269	1.9450	5.3584	1.6699	0.0045	0	0
	-0.86052	0.28	0.2815	7.2521	0.8714	0.0268	1.9420	5.3516	1.6602	0.0060	0	0
	0.68592	0.25	0.2786	7.1357	0.8695	0.0266	1.9380	5.3425	1.6469	0.0081	0	0
	2.65315	0.20	0.2743	6.9327	0.8670	0.0263	1.9324	5.3298	1.6282	0.0113	0	0
	4.06076	0.15	0.2704	6.7087	0.8650	0.0261	1.9279	5.3200	1.6137	0.0139	0	0
	5.10635	0.10	0.2668	6.4458	0.8636	0.0258	1.9246	5.3130	1.6033	0.0159	0	0
	5.99304	0.05	0.2628	6.1047	0.8628	0.0255	1.9223	5.3089	1.5969	0.0173	0	1
	7.20687	0.00	0.2561	5.5239	0.8626	0.0250	1.9209	5.3081	1.5954	0.0177	0	1
Exp.			0.2859 $\pm$0.0003		0.8846 $\pm$0.0014	0.0268 $\pm$0.0007	1.9547 $\pm$0.0019	5.419 $\pm$0.007	1.764 $\pm$0.006			

Table 3: The values of the hard-core radius r_C and the nonlocality strength parameterλ of the truncated potentials which fit the experimental values of the deuteron properties and the low-energy parameters when the cut-off radius R_{cut} is constant at $R_{cut} = 0.6 fm$ and $R_{cut} = 0.8 fm$.

R_{cut}	pair	$Q(fm^2)$	$r_d(fm)$	$A_S(fm^{-1/2})$	η	$a_t(fm)$	$r_t(fm)$
0.8	$\lambda(fm^{-3})$	-3.48544232	-4.34634009	-16.07544124	-1.21547517	-16.67380426	-18.4479701
	$r_C(fm)$	0.32017515	0.33125569	0.43389129	0.28606986	0.43768123	0.44823782
0.6	$\lambda(fm^{-3})$	-2.72006029	-2.53651085	-15.86354457	-0.92751210	-16.37144744	-18.32391158
	$r_C(fm)$	0.46361249	0.46283617	0.51227148	0.45581599	0.51445402	0.52394315
	Exp. value	0.2859	1.950	0.8846	0.0268	5.419	1.764

By introducing a hard-core in the interaction which causes a short-range local repulsion in the inner region in the presence of the quark model, deuteron properties such as the quadruple moment Q_d, the *rms* radius r_d, the asymptotic S-state amplitude A_S and the asymptotic ratio η and the low energy parameters such as the triplet scattering length a_t and the effective range parameter r_t can have improved values when the local short-range repulsion is increased. Complete agreement with the experiment has not been reach, which may be due to the simplicity being used.

Then, we concluded that the quark model QM can only reproduce deuteron binding energy but other properties of the nucleon-nucleon NN interaction needs a high local short-range repulsion in the inner region if it is to be fitted with the quark model QM.

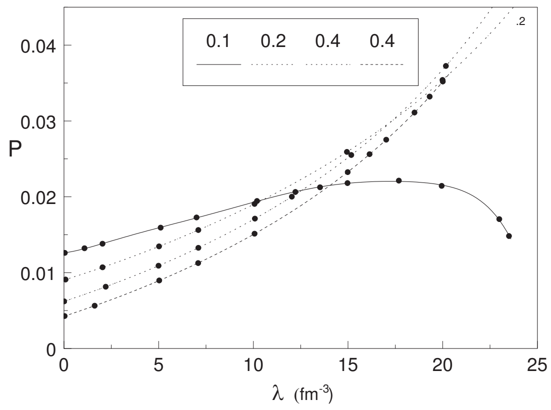

Fig. 10: The shape parameter as function of λ.

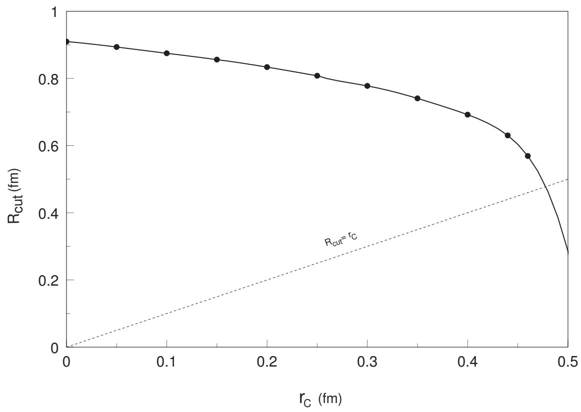

Fig. 11: The relation between R^{o}_{cut} and r_C when $\lambda = 0\ fm^{-3}$. The intersection point of the extension of the curve and the straight line is equal to 0.47784 fm.

A main conclusion that one can deduce from the comparison done here is that a definite and fixed form for the NN interaction is still a critical challenge . In fact, Naghdi [18,19] has and studies different models and forms for the strong nuclear force, which almost all give similar results while have different structures, then the nuclear force will obviously become meaningless. A certain statement is that although some quantitative correspondences are present among the potentials, there are some other quantitative differences.

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