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ASSESSING THE ACCURACY OF THE GEOID HEIGHTS CALCULATED BY INTERPOLATION AND EXTRAPOLATION FROM GLOBAL GEOID MODELS IN NORTHERN EGYPT

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ABSTRACT

Global Geoid Models (GGMs) have an important role in height transformation to convert the ellipsoidal height into orthometric height in several engineering applications. Based on recent research findings, and by pre-assessment of five GGMs; namely EGM2008, EIGEN-6C4, GECO, SGG-UGM-1, and XGM2019e_2159, it was found that the global models XGM2019e_2159 and GECO showed promising results by comparing against 165 GNSS/Level control points in the north of Egypt. In light of the above, the geoid heights calculated from the XGM2019e_2159 and GECO using interpolation and extrapolation methods have been assessed in this region. For each GGM, Geoid heights have been computed and, then, compared with the known control points. In the interpolation part, statistical analysis has been performed with accomplished results indicating that, the errors were about 0.04 m and 0.05 m in the Mediterranean coast and for the Nile River in the delta area were about 0.06 m and 0.05 m, respectively. In the region of Northern Egypt, the leveling of the 3rd class could be replaced by the XGM2019e_2159 or GECO model after aligning it with the control points every 50 km by interpolation approach. On the other approach, the geoid height for a distance of 25 km can be obtained from the global models XGM2019e_2159 and GECO using the extrapolation method with an error of 0.08 m. In order to improve the results obtained by the extrapolation method, Artificial Neural Network (ANN) has been used to create a local geoid in the area of the Mediterranean coast and evaluated over the Delta region. The results showed small error values of about 0.04 m and 0.05 m in the first 50 km using XGM2019e_2159 and GECO.

Keywords: Global Geoid Models, GNSS/Level Elevations, Interpolation, Extrapolation, RMSE, ANN.

تقييم دقة ارتفاعات الجيود المحسوبة عن طريق الاستيفاء والاستقراء من نماذج الجيود العالمية في شمال مصر

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الملخص

تلعب نماذج الجيود العالمية (GGMs) دورًا هاما في تحويل الارتفاعات الجيوديسية الي ارتفاعات ارثومترية في العديد من التطبيقات الهندسية. استنادًا إلى نتائج الأبحاث الحديثة ، ومن خلال التقييم المسبق لخمسة من نماذج الجيويد العالمية ؛ وبالتحديد EGM2008 و EIGEN-6C4 و GECO و SGG-UGM-1 و XGM2019e_2159 ، فقد وجد أن النموذجين العالميين XGM2019e_2159 و GECO أظهروا نتائج جيدة من خلال المقارنة مع ١٦٥ نقطة تحكم GNSS/Level في شمال مصر. و في ضوء ما سبق ، تم تقييم ارتفاعات الجيويد المحسوبة من النماذج العالمية XGM2019e_2159 و GECO باستخدام طرق الاستيفاء والاستقراء في هذه المنطقة. تم حساب ارتفاعات الجيويد لكل نموذج، ثم مقارنتها بنقاط التحكم المعروفة. باستخدام طريقة الاستيفاء و بعد إجراء التحليلات الاحصائية اشارت النتائج إلى أن الأخطاء كانت حوالي ٠,٠٤ م و ٠,٠٥ م في منطقة ساحل البحر الأبيض المتوسط بينما كانت حوالي ٠,٠٦ م و ٠,٠٥ م في منطقة الدلتا. و عليه يمكن استبدال ارتفاعات ميزانية من الدرجة الثالثة بارتفاعات نموذج XGM2019e_2159 أو نموذج GECO بعد محاذاتها مع نقاط التحكم كل ٥٠ كم من خلال طريقة الاستيفاء. و من ناحية أخرى ، يمكن الحصول على ارتفاع الجيود لمسافة ٢٥ كم من النموذجين العالميين XGM2019e_2159 و GECO باستخدام طريقة الاستقراء بخطأ ٠,٠٨ م. و من أجل تحسين النتائج التي تم الحصول عليها من طريقة الاستقراء ، تم استخدام الشبكة العصبية الاصطناعية (ANN) لإنشاء جيويد محلي في منطقة ساحل البحر الأبيض المتوسط وتقييمها على منطقة الدلتا. و قد أظهرت النتائج قيم أخطاء صغيرة حوالي ٠,٠٤ م و ٠,٠٥ م في أول ٥٠ كم باستخدام النماذج العالمية XGM2019e_2159 و GECO.

الكلمات المفتاحية : نماذج الجيود العالمية, الارتفاع الجيوديسي والارثومتري, الاستيفاء, الاستقراء, الخطأ التربيعي المتوسط, الشبكة العصبية الاصطناعية.

1. INTRODUCTION

In this research, the studied areas are known by their diverse environmental, tourist, and social aspects. As a result, the Egyptian government has recently concentrated on this region, constructing a number of large engineering projects.

Global Navigation Satellite Systems (GNSS) are broadly applied for geodesy and surveying applications. These heights are determined by the ellipsoid's geometry, and so have no physical significance. In engineering applications, the orthometric height i.e. the height above Mean Sea Level (MSL) is used [1]. Determination of the orthometric height from ellipsoidal height for a point, depends on precise estimation of geoid undulation. In practice, estimating the geoid undulations, particularly for local applications, requires control points which both ellipsoidal and orthometric heights are known [2]. Estimation of the geoid heights is a difficult task especially with the lack of data available along the study area. Upon the above, one of the main tasks of geodetic researches is to determine the Earth's global gravity field. Compared to the first-generation global gravity field models derived, it is now possible to represent the Earth’s global gravity field and its variations with better spatial and temporal resolutions using the highly accurate satellite measurements. Many studies in Egypt were conducted to evaluate the GGMs by different approaches, such as: [3–6].

A pre-assessment of the global geoid models in the north of Egypt area was carried out by comparing the geoid heights from the global geoid models, EGM2008, EIGEN-6C4, GECO, SGG-UGM-1, and XGM2019e_2159 with the geoid heights of 165 GNSS/level known points. The results showed that XGM2019e_2159 and GECO are ranked as the best global geoid models in this region, with standard deviation of about 0.13 m and 0.14 m, respectively.

Here's a quick rundown of the geoid that's been used:

GOCE-EGM2008 combined (GECO); a global gravity model which is computed by incorporating the GOCE-only TIM-R5 solution, which is the fifth release (R5) of the time-wise (TIM) model, into the EGM2008 [7]. The EGM2008 geoid undulations are computed on a global spherical grid with a 0.5º resolution by synthesizing the EGM2008 coefficients up to degree 359.
XGM2019e_2159; a combined global gravity field model represented through spheroidal harmonics up to d/o 5399, corresponding to a spatial resolution of 2' (~4 km). the performance of XGM2019e can be considered as globally more homogeneous and independent from existing high-resolution global models [8].
In order to assess the geoid heights calculated by Interpolation and Extrapolation, the results should be compared against independent external sources. It is very common to compare the model-computed geoid heights with GNSS/levelling-derived geoid heights[9–11]. The advantage of this method is that it is suitable for assessing the model outcomes at a regional level or in a particular area but the assessments are only as good as the quality of the external datasets used in the validation. According to the GNSS/levelling method, geoid height value at a point is calculated with Equation. 1 [12].

$$N_{GNSS/LEV} = h - H \qquad (1)$$

With light of the above, the aim of this investigation is to assess the geoid heights calculated from the XGM2019e_2159 and GECO using interpolation and extrapolation methods in Northern Egypt.

2. MATERIALS AND METHODS

2.1. Study Area and Available Data

Fig. 1 depicts the research region in Northern Egypt. There were 87 GNSS/level points along the Mediterranean coast and 78 GNSS/level points along the Nile River in the delta region in the study area. The orthometric height error of the state leveling network is less than 1 cm. In each session, the geodetic height of each point was estimated with an error of no more than 2 cm. The two global geoid models XGM2019e 2159 and GECO were used in this comparison study.

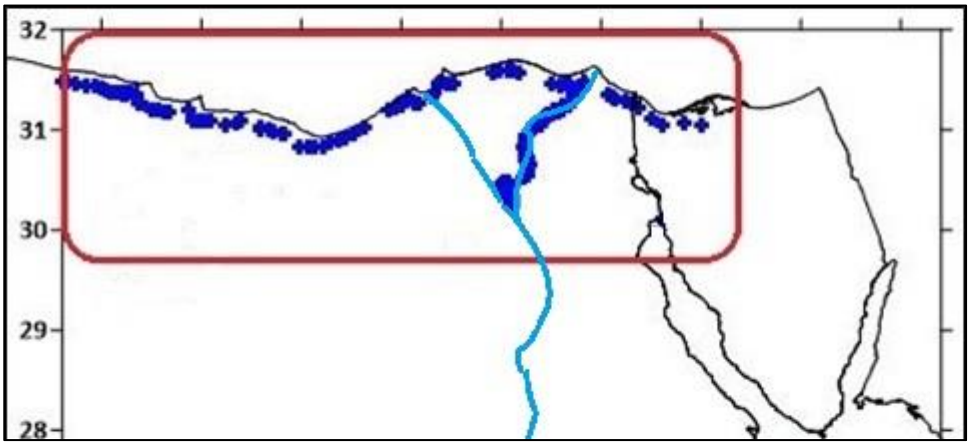

Fig. 1: The study area.

2.2. Research Methodology

To evaluate the accuracy of the geoid heights calculated by interpolation from global models, the following steps were done:

- $N_{GNSS/level}$ and N_{GGM} for 165 control points are calculated.
- $N_{GNSS/level}$ for two control points with a given distance are aligned with N_{GGM} at these two points.
- In the area between the two control points, differences (δ_i) between the geoid heights from the global geoid model after alignment (N_{GGM-P}) and the geoid heights from the GNSS/level points ($N_{GNSS/level}$) are calculated and the results are evaluated using standard deviation (S).

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$$\delta_i = N_{GNSS/level_i} - N_{GGM-P_i} \qquad (2)$$

$$S = \sqrt{\frac{\sum \delta i^2}{n-1}} \quad (3)$$

To evaluate the accuracy of the geoid height calculated from global models by extrapolation, the following were done:

- The difference between the geoid heights for the two-control point is calculated and compared with the difference between the geoid heights from the global model for the same two control points, see Equations 4 and 5 [13].

$$\delta_{AB_i} = \Delta N_{GNSS/level_{AB_i}} - \Delta N_{GGM_{AB_i}} \quad (4)$$

$$S = \sqrt{\frac{\sum \delta_{AB_i}{}^2}{n-1}} \quad (5)$$

3. RESULTS AND DISCUSSION

3.1. Assessing the Accuracy of the Geoid Heights Calculated by Interpolation

In this section, the accuracy of the geoid heights that can be calculated from global models, XGM2019e_2159 and GECO, was evaluated by the interpolation method in Northern Egypt. The study area include 87 GNSS/level points distributed along the coastal Mediterranean about 610 km long (west-east direction), 78 GNSS/level points distributed along the Nile River in the delta area about 150 km long (north-south direction), see **Fig. 2** and **Fig. 3** show the geoid heights from the control points ($N_{GNSS/level}$) and from the global models XGM2019e_2159 (N_{XGM}) and, GECO (N_{GECO}) along the Mediterranean coast and the Nile River in the delta area.

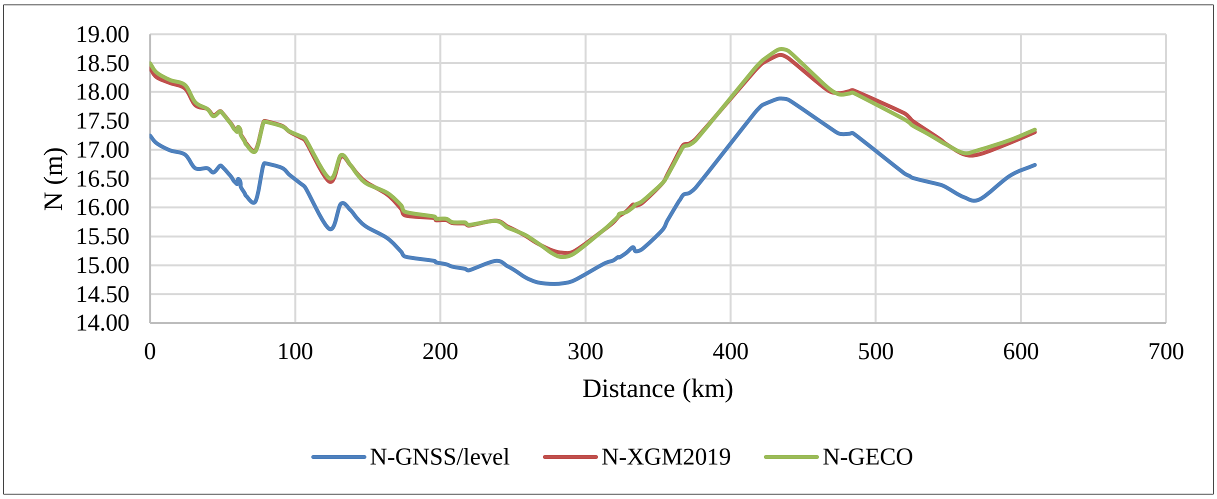

Fig. 2: Geoid heights from GNSS/ level points, XGM2019e_2159, and GECO in the Mediterranean coastal area.

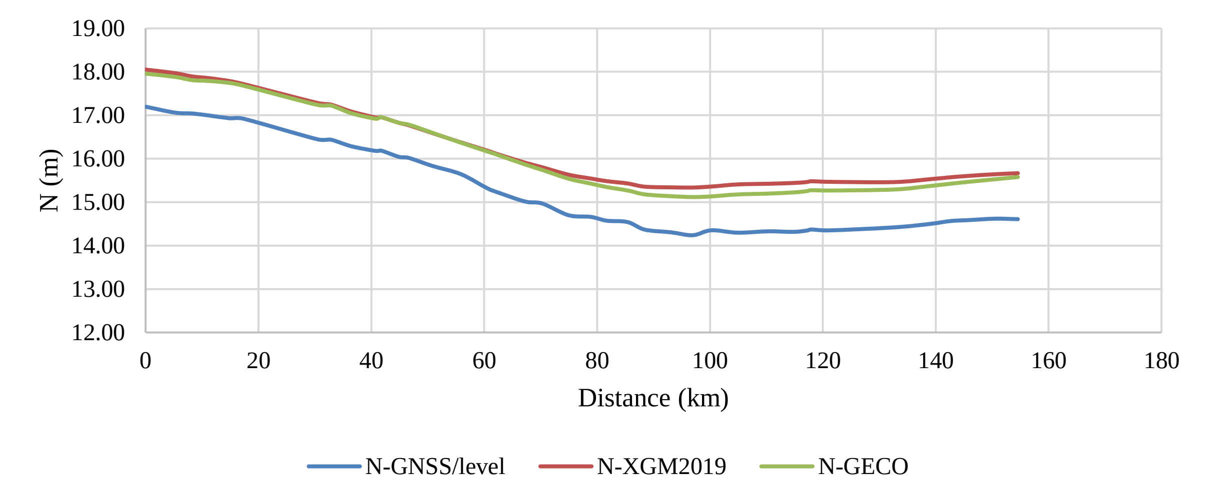

Fig. 3: Geoid heights from GNSS/level points, XGM2019e_2159, and GECO along the Nile in the Delta area.

To assess the accuracy of the geoid heights calculated by the global models XGM2019e_2159 and GECO by interpolation method, the geoid heights from XGM2019e_2159 and GECO were aligned with the geoid heights of the two control points. Then the values of the new heights of the geoid XGM2019e_2159 (N_{XGM-P}) and GECO (N_{GECO-P}) were calculated in the area between two control points. To assess the new values of the geoid heights for XGM2019e_2159 and GECO, the differences between (N_{XGM-P}) or (N_{GECO-P}) against the geoid heights of control points ($N_{GNSS/level}$) in this region were calculated. For the Mediterranean coast, the geoid heights from XGM2019e_2159 and GECO were aligned every 600 km, 300 km, 100 km, 50 km and 25 km. While for the Nile River, the geoid heights from XGM2019e_2159 and GECO were aligned every 150 km, 50 km and 25 km. **Fig. 4** and **Fig. 6** show the results obtained, and **Fig. 5** and **Fig. 7** show the geoid heights of the models XGM2019e_2159 and GECO after alignment every 50 km.

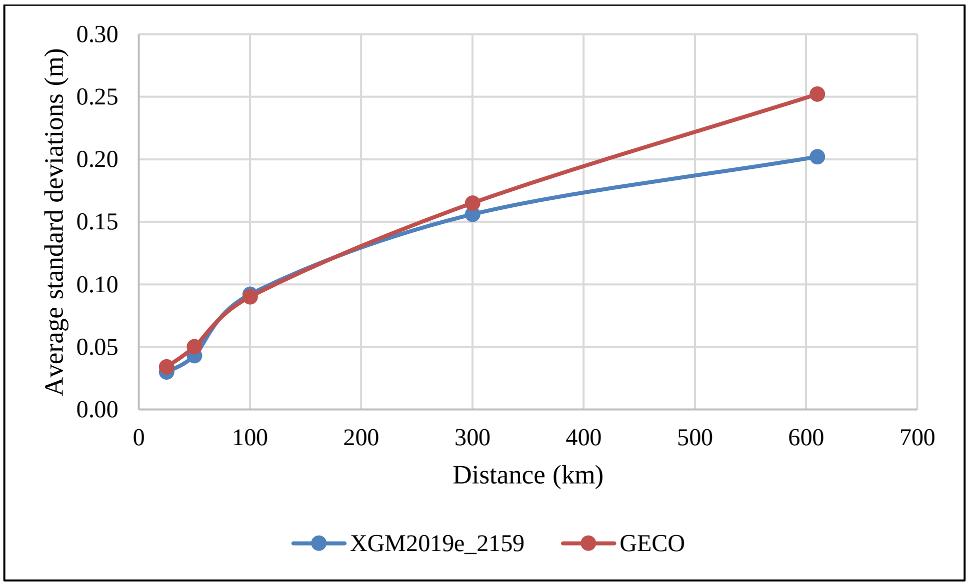

Fig. 4: Average standard deviation values for the Mediterranean coast area.

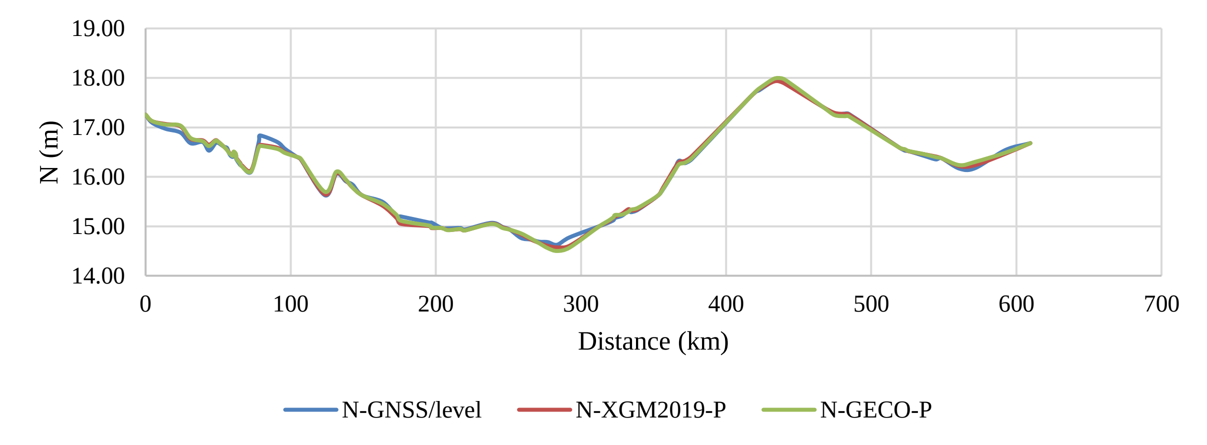

Fig. 5: Geoid heights from XGM2019e_2159 and GECO after alignment with control points every 50 km on the Mediterranean coast area.

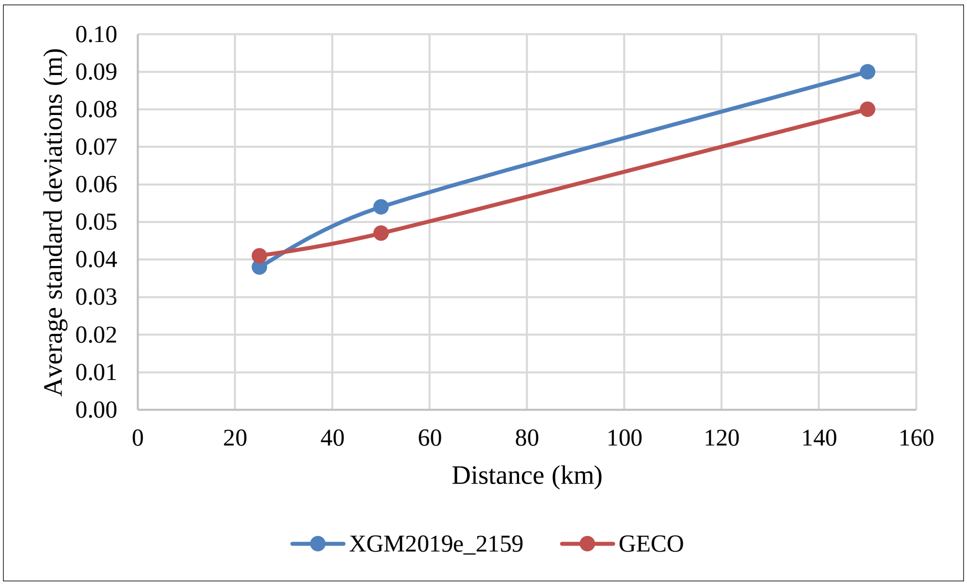

Fig. 6: Average standard deviation values along the Nile River in the delta area.

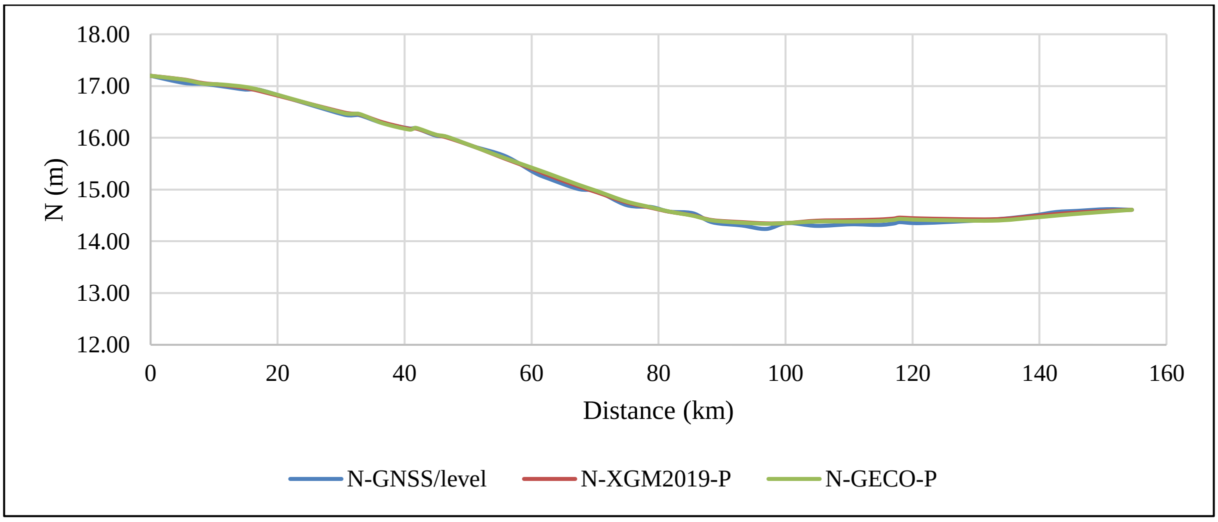

Fig. 7: Geoid heights from XGM2019e_2159 and GECO after alignment with control points every 50 km along the Nile River in the delta area.

Fig. 4 and **Fig. 6** show the average standard deviation values of the differences between the geoid heights from the XGM2019e_2159 and GECO models after alignment against the geoid heights from the control points ($N_{GNSS/level}$). With the control points every 50 km alignment for the Mediterranean coast region, the standard deviation was ± 0.04 m and ± 0.05 m. While for delta area was ± 0.06 m and ± 0.05, respectively.

As indicated in the technical specifications of leveling networks in Egypt, the Allowable closing error in the leveling line should be no more than 12 $\sqrt{d}$, where d is the shortest distance between the two ends in km. So, on the territory of Northern Egypt, the leveling of the 3rd class could be replaced by the XGM2019e_2159 or GECO model after aligning it with the control points every 50 km.

To determine the corrected geoid height at any point (N_{P_i}), the following equation can be used:

$$N_{P_i} = N_{GGM_i} - \Delta N_A + \sin\alpha * d_{A_i} \qquad (6)$$

Where N_{GGM_i}: geoid height from XGM2019e_2159 or GECO;

$\Delta N_A = N_{GGM_A} - N_{GNSS\backslash level_A}$;

$\alpha = \tan^{-1}\frac{\Delta N_B}{d_{AB}}$;

d_{AB}: distance from A to B and d_{A_i}: distance from A to new point.

3.2. Assessing the Accuracy of the Geoid Heights Calculated by Extrapolation

In this section, the accuracy of the geoid heights that can be calculated from global models XGM2019e_2159 and GECO was evaluated by the extrapolation method in Northern Egypt. The difference between the geoid heights of two control points was calculated and compared with the difference from global models for these two points. These differences were calculated between points along the Mediterranean coast and along the Nile in the delta region, see Equations 7 and 8 [13]. Three different cases with a distance between each point of 10 km, 25, and 50 km were studied. **Fig. 8** and **Fig. 9** show the results obtained using the XGM2019e_2159 and GECO models.

$$\delta_{AB} = \Delta N_{GNSS/level(AB)} - \Delta N_{GGM(AB)} \qquad (7)$$

$$S = \sqrt{\frac{\sum \delta i^2}{n-1}} \qquad (8)$$

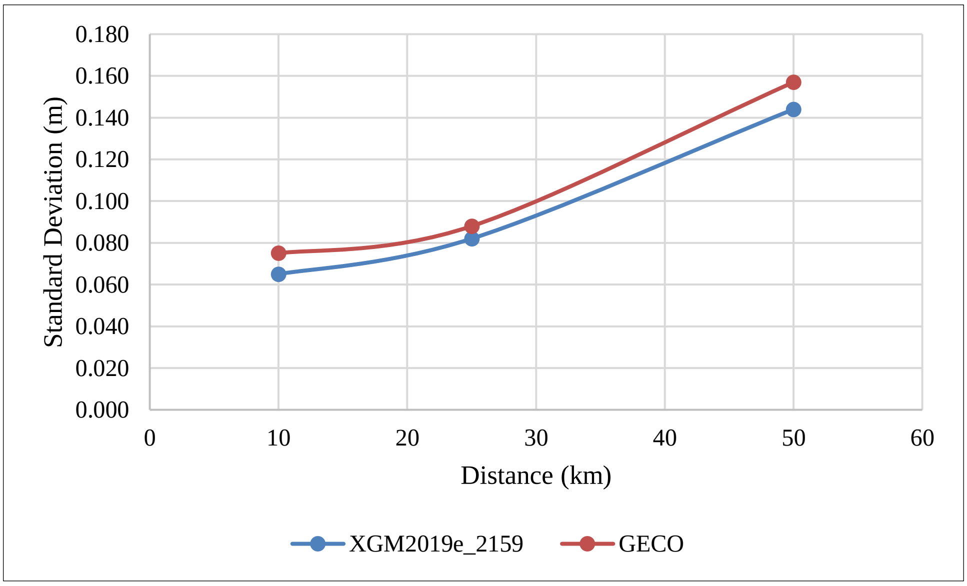

Fig. 8: The values of standard deviation on the Mediterranean coast by extrapolation.

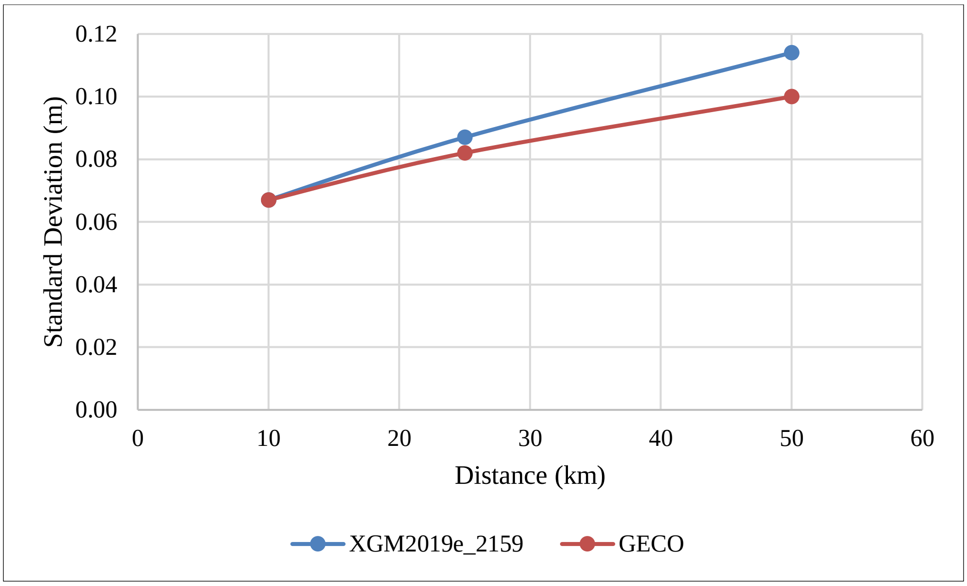

Fig. 9: The values of standard deviation along the Nile River in the delta area by extrapolation.

It can be seen from **Fig. 8** and **Fig. 9** that in the north of Egypt region, the geoid height differences for a distance of 25 km can be obtained from the global models XGM2019e_2159 and GECO using the extrapolation method with an error of 0.08 m. Therefore, in the next step, this method will be used to calculate the values of the geoid heights at a distance of 25 km south of the control points located along the Mediterranean coast, see **Fig. 10**. The new geoid heights were calculated using the following equation:

$$N_{new_B} = N_{GNSS\backslash level_A} + \Delta N_{GGM_{AB}} \qquad (9)$$

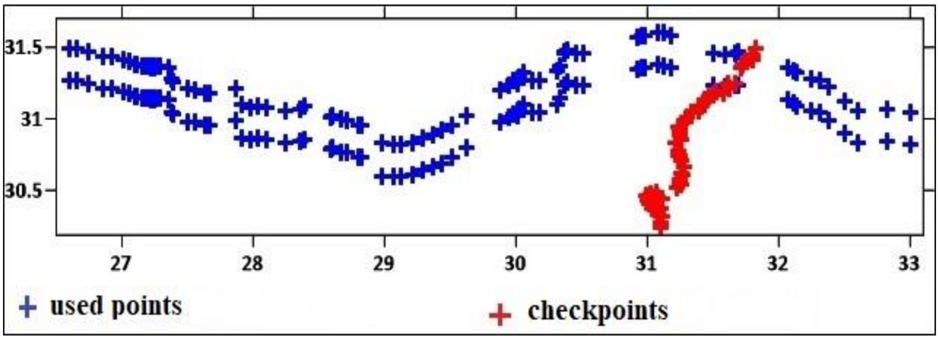

Fig. 10: Location of 87 GNSS/leveling points, 87 new points, and 78 checkpoints.

After the geoid heights were calculated for the new 87 points, based on the geoid height difference from the models XGM2019e_2159 and GECO, they were used with the 87 control points to create a geoid model for this region. ANN was used to interpolate the differences (ΔN_{ANN_i}) between N_{XGM_i} or N_{GECO_i} and $N_{GNSS/level_i}$ in this area. The final geoid height (N_{F_i}) at any point in this area is calculated using the following equations:

$$N_{F_{XGM_i}} = N_{XGM_i} + \Delta N_{ANN_i} \qquad (10)$$

$$N_{F_{GECO_i}} = N_{GECO_i} + \Delta N_{ANN_i} \qquad (11)$$

Table 1 shows a comparison between $N_{GNSS/level_i}$ for 165 points and N_{F_i} derived from the created models.

Table 1: Comparison between N_{F_i} and $N_{GNSS/level_i}$ for the165 points.

	GECO	XGM2019e_2159
Average (м)	0.003	0.002
Standard deviation (м)	0.039	0.028
Range (м)	0.220	0.179
Minimum (м)	-0.108	-0.086
Maximum (м)	0.113	0.093

Seventy-eight checkpoints along 150 km of the Nile River in the delta area were used to assess the external accuracy of these models in that area.

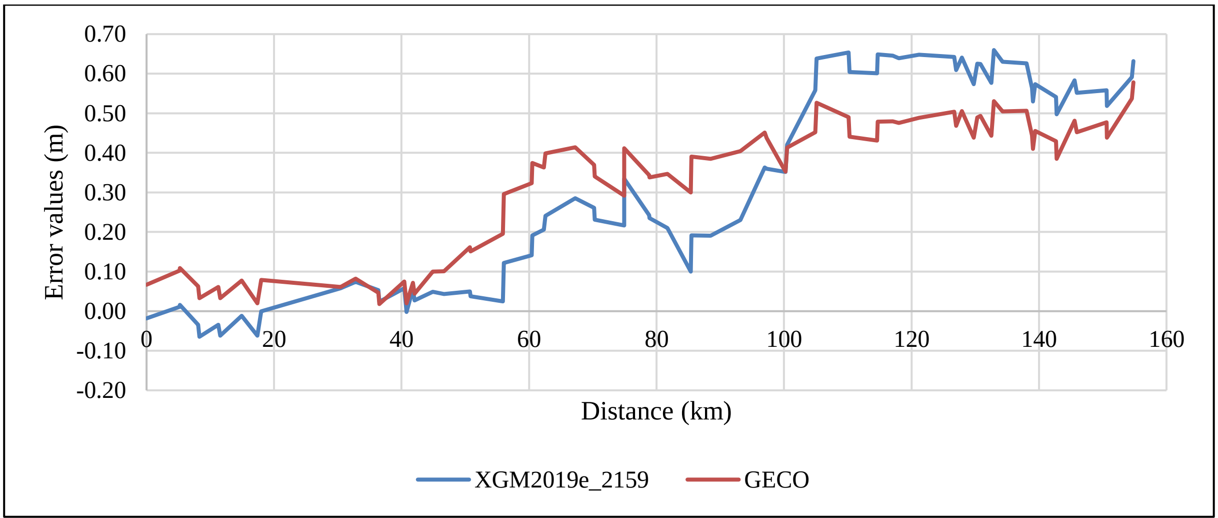

Fig. 11: Values of model errors at checkpoints over a length of 150 km.

Fig. 11 shows the error values of the model at checkpoints of more than 150 km. From **Fig. 11** it can be seen that the error value in the first 50 km is small, then gradually increases with increasing the direction to the south. The difference between the $N_{GNSS/level}$ of the 74 checkpoints and the $N_{F_{XGM_i}}$ or $N_{F_{GECO_i}}$ of the first 50 km with standard deviation of about 0.04 m and 0.05 m, respectively.

SUMMARY AND CONCLUSIONS

In various engineering applications, the need to convert GNSS-based ellipsoidal heights into MSL-based orthometric heights has lately increased. This study assessed the geoid heights of global geoid models XGM2019e_2159 and GECO calculated by interpolation and extrapolation methods. In the part of interpolation method, the average standard deviation values of the differences between the geoid heights from the XGM2019e_2159 and GECO models and the geoid heights from the GNSS/level points after alignment with control points every 50 km for the Mediterranean coast area were about 0.04 m and 0.05 m and for the Nile River in the delta area were about 0.06 m and 0.05 m, respectively. Therefore, on the territory of Northern Egypt, the leveling of the 3rd class could be replaced by the XGM2019e_2159 or GECO model after aligning it with the control points every 50 km. Using the extrapolation method, the geoid height for a distance of 25 km can be obtained from the global models XGM2019e_2159 and GECO with an error of about 0.08 m. After that, new local geoid has been created in the Mediterranean coast area by ANN using the 87 GNSS/LEVEL control points and the new extracted 87 points towards the south. The created geoid was evaluated over the delta region. The results showed small error values in the first 50 km, then gradually increased with direction to the south. The differences between the $N_{GNSS/level}$ of the 78 checkpoints and the $N_{F_{XGM_i}}$ or $N_{F_{GECO_i}}$ in the first 50 km have standard deviation of about 0.04 m and 0.05 m, respectively.

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