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VARIATION OF SOIL SUBGRADE MODULUS UNDER CIRCULAR FOUNDATION

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ABSTRACT

In this paper, the distribution of soil subgrade modulus underneath circular plate was investigated using PLAXIS 2D program. The modeling procedure included the soil with its special characteristics underneath a thin circular plate subjected to different loading cases. Six soil types varying from very loose to very dense sandy soils were tested to consider the general behavior of soil. In addition, nine load intensities were applied to the plate to investigate their effects on the variation of the soil subgrade modulus. Then, multiple regression analysis was carried out to examine a proposed parabolic formula that describes the variation profile of the soil subgrade modulus. The results of multiple regression show an excellent agreement with the data obtained from PLAXIS numerical analysis. The proposed formula can be very handy for soil-structure interaction programs that use only Winkler's model to represent the soil.

KEYWORDS: Subgrade Modulus, Soil Structure Interaction, Circular Foundation, PLAXIS, Regression Analysis.

تغير معامل التربة تحت الاساسات الدائرية

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الملخص

تهدف هذه الدراسة إلى إيجاد علاقة تصف تغير معامل رد فعل التربة تحت البلاطات الدائرية. تضمنت الدراسة استخدام برنامج اسمه (PLAXIS 2D) يعتمد على طريقة العناصر المحددة لتمثيل الاساسات والتربة بخصائصها المختلفة. شملت عملية التحليل استخدام ستة أنواع مختلفة من التربة الرملية تتراوح من تربة ضعيفة مفككة جدا إلى تربة كثيفة جدا بالإضافة الي تحميل الاساسات بمجموعة مختلفة من الأحمال. بعد ذلك تم تحليل نتائج معامل رد فعل التربة باستخدام التحليل المتعدد لمنحي الانحدار وهو تحليل احصائي يهدف لإيجاد علاقة تصف نقاط البيانات على المنحنى. نتائج التحليل المتعدد لمنحني الانحدار أظهرت توافقاً ممتازاً مع منحني معامل رد فعل التربة باستخدام علاقة تربيعية بالإضافة الي تقارب النتائج مع بيانات مستخدمة في أبحاث سابقة مما يشير الي قابلية اعتماد نتائج هذا البحث في الحياة العملية استخدام نتائجه في برامج التحليل الانشائي التي تعتمد على طريقة العناصر المحددة في عملية التحليل بدلا من الاعتماد على نموذج (Winkler) فقط كأداة لتمثيل التربة مما يجعل تمثيل التربة أكثر واقعية.

الكلمات المفتاحية: معامل التربة، تفاعل بنية التربة، الاساسات الدائرية، PLAXIS، تحليل منحني الانحدار.

1. INTRODUCTION

A wide range of engineering applications are concerned with the soil-structure interaction such as wind turbine foundation, mat foundation of building and foundation of liquid tanks. The simplest model to idealize the behavior of the soil is Winkler's model, in which the soil is represented by a set of linear separated springs that has a constant stiffness value. The contribution of Winkler's foundation depends on the transverse displacement of the plate at the spring location.

However, considering the soil interacting as separated units while neglecting the shear rigidity of the foundation and the collaboration between the individual springs leads to unrealistic results in many cases. For instance, Smith et al. [1] conducted experimental studies to measure the contact pressure underneath circular plates and they found a remarkable variation of the soil subgrade modulus in the radial direction. Gómez et al. [2] explained that the subgrade modulus cannot be constant but variable depending on many factors. Furthermore, for a flexible foundation subjected to uniform loading, the settlement of the foundation at the center is higher than at the edges. As a result, the contact pressure is maximum at the edges. Dey et al. [3] provided a differential equation of the flexural analysis of beam resting on elastic medium. they showed how the type of loading can influence the contact pressure and the distribution of the subgrade modulus beneath the beam.

Larkela et al. [4] utilized finite element software called PLAXIS 3D to investigate the soil-structure interaction and proved that the subgrade modulus is variable especially when the foundation is loaded with uniform pressure besides taking into account the soil plasticity role in the estimation of the contact pressure. Mohamed Saad Eldin [5] investigated the behavior of thin and thick plates resting on different sand soils using PLAXIS 3D. The study included different load cases and different aspect ratios of the plate in addition to modeling the soil as a continuous continuum representing the soil characteristics.

In this study, a two-dimensional finite element soil-structure interaction software was used to model the behavior of a thin circular plate where the foundation experiences uniform loading. A set of 31 models was created by PLAXIS 2D for different soil types and the results of subgrade modulus were computed. A nonlinear regression analysis was performed on the results obtained from PLAXIS 2D and a parabolic model for the variation of the subgrade modulus was developed to show the accuracy of the fitted curve to the data points. The adopted model shows an excellent agreement with the data points. In addition, results of the multiple regression were provided and can be very useful to describe the distribution of subgrade modulus underneath elastic foundation.

2. FOUNDATION AND SOIL MODELING

2.1. Modeling of Foundation

The adopted study case for the foundation is axisymmetric circular plate subjected to different values of uniform loadings. The plate has a radius of 1.0m with thickness of 0.01m to ensure that the plate behaves as a thin plate (see **Fig. 1**). Moreover, the plate is made of isotropic material with constant flexural rigidity in the radial direction and modeled as an elastic material. A coarseness factor of 0.1 has been used for the automatic mesh to discretize the plate.

PLAXIS 2D provides an option to model axisymmetric structural plates as a simplification from the two-dimensional analysis to one-dimensional analysis only in the radial direction considering the tangential deformation is identical zero.

2.2. Modeling of Soil

Modified Mohr-Coulomb model was utilized in the present work to model the soil as an elasto-plastic material where five parameters are required to represent the soil; theses parameters are modulus of elasticity (E), Poisson's ratio (v), friction angle (φ), cohesion (c) and dilatancy angle (Ψ).

The dimensions of study field contours were 10.0m in x-direction and 8.0m depth in y-direction and the dimensions of the soil model are taken to be 5.0m with 8.0m depth. These dimensions were found to be adequate to represent the soil as no deformations were observed at the boundaries (see **Fig. 2**). The soil has been meshed with a coarseness factor of 0.1 to fine the mesh as possible.

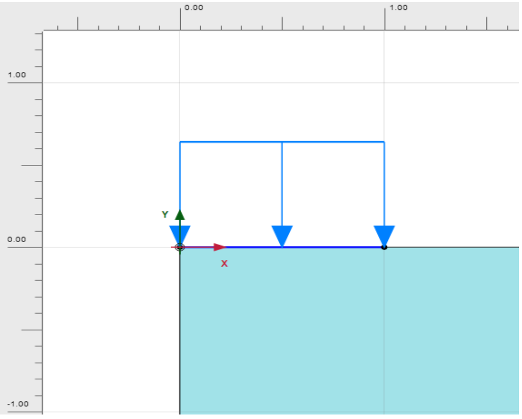

Fig. 1: Plate dimensions.

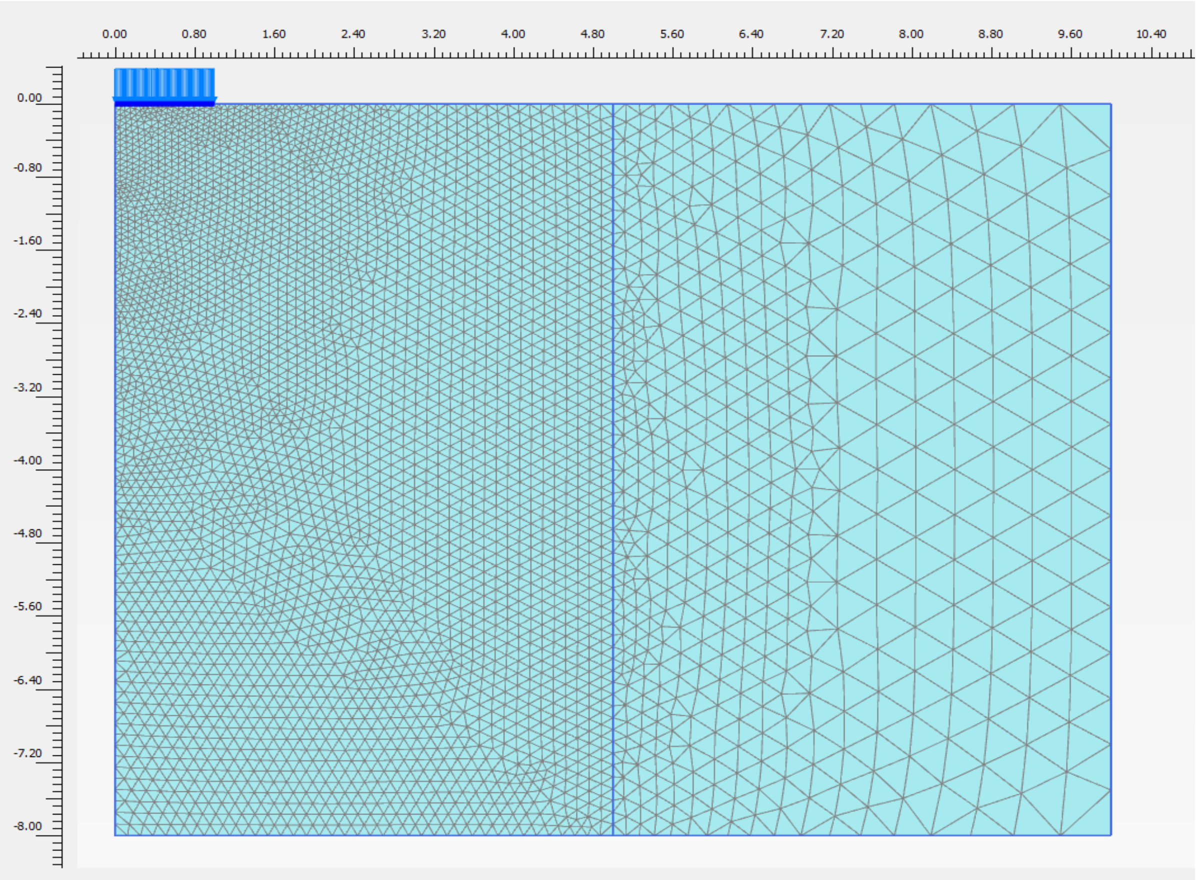

Fig. 2: Soil dimensions and meshing.

2.3. Study Cases

Six different sandy soils, that have various densifications, were considered in the present work to investigate the behavior of the soil subgrade modulus. The mechanical properties of the plate and different soils are presented in

Table. 1. For each soil type, nine different uniform loading intensities were applied on the plate as shown in **Table. 2** to form a total number of 31 models.

Table. 1: Mechanical properties for plate and different soils.

Material	Mechanical Properties				
	Dry unit weight $[\gamma_d]$ kN/m^3	Relative density $[D_r]$	Modulus of elasticity $[E]$ kN/m^2	Poisson's ratio $[\nu]$	Angle of internal friction $[\varphi]$
Plate	2.7	-	70000	0.30	-
Soil 1 (very loose)	13.5	15%	7000	0.25	17
Soil 2 (loose to very loose)	14.0	20%	10000	0.30	20
Soil 3 (medium)	14.5	25%	15000	0.25	32
Soil 4 (medium to loose)	16.0	30%	10000	0.30	30
Soil 5 (dense)	17.0	60%	30000	0.35	35
Soil 6 (very dense)	18.0	80%	50000	0.40	40

Table. 2: Applied loads to each soil type.

Soil Type	Applied Load (kN/m2)
Soil 1 and Soil 2	10,15,20,25,30 and 35
Soil 3	25,30,35,50,100 and 200
Soil 4	20,25,30,35,50 and 100
Soil 5	35,50,100 and 200
Soil 6	50,100 and 200

3. REGRESSION ANALYSIS

Regression analysis is considered as the "best guess" when using a set of data to predict certain information by fitting a set of points to a graph. There are different types of regression analysis, for instance, linear, multiple and nonlinear regression. One can judge the accuracy of the fitted curve using different parameters, for example, R-squared (the coefficient of determination) ranging from 0 to 1, P-value used in hypothesis testing and S-value (the standard error) representing how close is the curved fitted line to the data points.

In the present work, multiple regression was employed to refine a parabolic model that was utilized by Rad and Shariyat [6] to describe the variation of soil subgrade modulus. The only difference between multiple regression and simple linear regression is the number of independent variables. In our case, two independent variables are considered to construct the model. The first one is the grid point location (r/R) and the second one is the loading value (q). Different models were tested to find the most descriptive one that fits all the studied soils with high accuracy level taking into account the simplicity of the model. This multiple regression analysis was carried out using Microsoft Excel and the results are presented in the next section.

4. RESULTS AND DISCUSSION

4.1. Results of PLAXIS 2D

The results of subgrade modulus were obtained from PLAXIS models by taking cross section underneath the foundation and computing the values of subgrade modulus along the radial direction of the plate. The results of subgrade modulus for the studied soils are illustrated in **Figs. 3** to **8**.The plots reveal that the subgrade modulus tends to be parabolically distributed and correlates well with a second-degree parabolic variation. This outcome agreed with Larkela et al. [4]. The results show that the curvature goes upward with a higher slope when the foundation experience low a level of loading or when the soil responses in linear elastic range. On the other hand, the slope is decaying and the curvature is going downward in case of intense loading when the soil is in plastic range until yielding of the soil.

4.2. Developed Variation Model

In the present work, a parabolic function was developed to describe the variation in the subgrade modulus as follows:

$$k(r) = k_0\left(1+\left(\alpha-\frac{q}{\beta}\right)\left(\frac{r}{R}\right)^2\right) \tag{1}$$

where k_0 denotes for the reference value of subgrade modulus at the center of the plate, α and β are constant values depend on the soil characteristics, r is the location of grid point and R is the outer radius of the circular plate.

The results of the fitted curves show agreement with the data points from PLAXIS as they have a very high R-squared value that reflects the confidence level of the proposed model.

After performing the multiple regression, the results of the parameter α and β were computed for the six soils as shown in **Table. 3**. Overall, the values of α and β have less values for loose soils and increasing for medium and dense soils. Furthermore, the range of value of the term including α and β together (coefficient of $(r/R)^2$) is close to the values used in [6].

Table. 3: Values of α and β for different soil types

Soil No	α	β	R-squared
1	0.30	45	98%
2	0.52	40	98%
3	0.30	315	99%
4	0.39	200	99%

5	0.39	690	98%
6	0.41	1160	97%

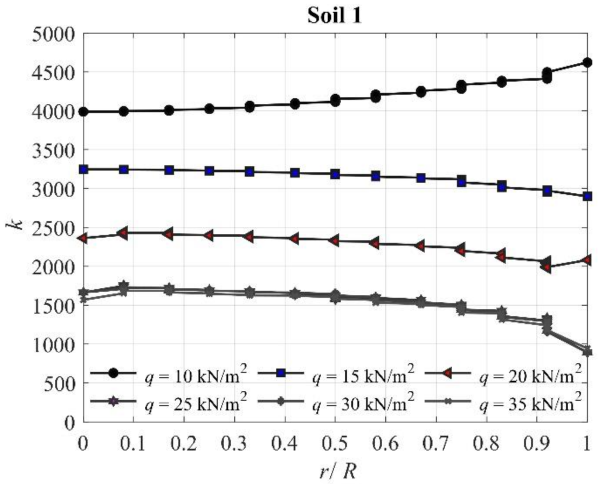

Fig. 3. Subgrade modulus for soil 1 (very loose)

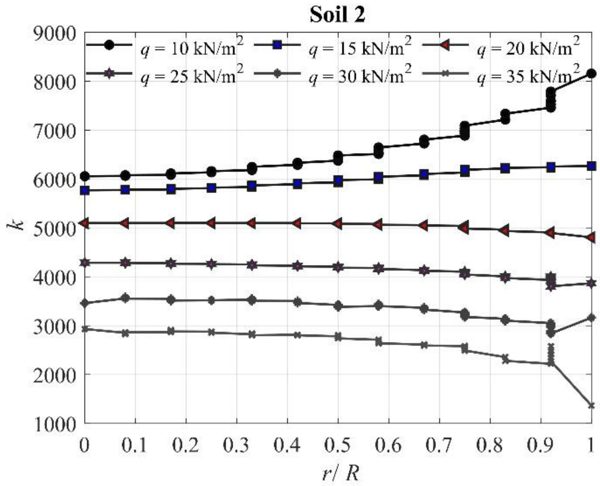

Fig. 4. Subgrade modulus for soil 2 (loose to very loose)

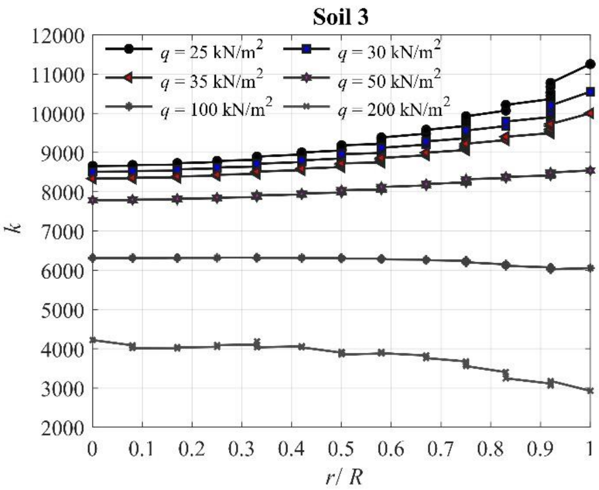

Fig. 5. Subgrade modulus for soil 3 (medium)

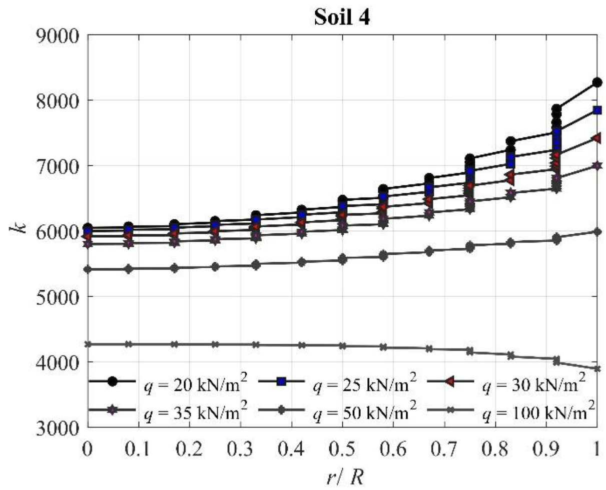

Fig. 6. Subgrade modulus for soil 4 (medium to loose)

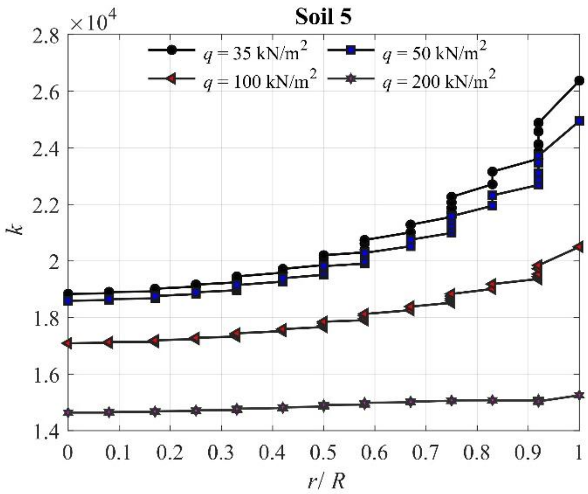

Fig. 7. Subgrade modulus for soil 5 (dense)

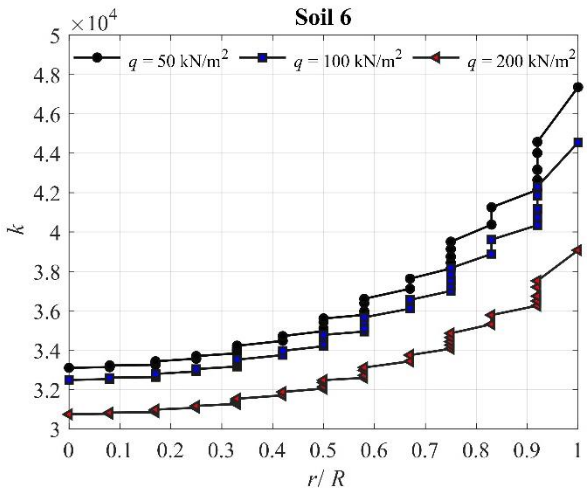

Fig. 8. Subgrade modulus for soil 6 (very dense)

5. SUMMARY AND CONCLUSIONS

The present study provides a formula to describe the variation in the soil subgrade modulus underneath a thin circular plate subjected to uniform loading. The study includes six different soil types ranging from very loose to very dense soils to investigate the behavior of subgrade modulus of them. A set of 31 PLAXIS models was developed for the six soils with different loading cases to compute the values of the subgrade modulus. Furthermore, nonlinear regression analysis was utilized to judge the accuracy of the adopted formula and the results show a good agreement with the data points. Based on the reported results in the present work, conclusions can be summarized as follows:

1) The distribution of soil subgrade modulus is nonuniform along the radial direction and considering it constant is far from the real-world system.
2) It is recommended for structural design programs based on finite element method to provide tools to represent the soil-structure interaction instead of depending on Winkler's model.
3) The profile of subgrade modulus can be expressed as a parabolic function in the radial direction as this formula coincides well with the soil response.
4) The reported results in the present work can be used as a guide to model the real behavior of the soil underneath elastic foundation.
5) Further studies can be conducted to investigate the behavior of additional soil types to develop a more-generalized overview about the distribution of subgrade modulus.

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