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Exploring the Effects of Uncertainty in Structural Design

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Abstract: Modern structures require more complex and accurate designs than ever before. In order to achieve an economic and safe design it is necessary to account for the uncertainties in design variables. This paper presents a technique in which the science of design of experiments (DOE) is employed to assess in modern structure design. A case study of a simple structure that is idealized as a simply supported beam is investigated. The design variables are categorized according to their sources as: load, geometry and material types. Each variable value is assumed to vary within a given numerical distribution. The deflection at the mid span of the structure is considered as the performance response of interest. DOE is applied to evaluate the effect of uncertainty within the design variables on the performance response. It is also used to calculate the interaction effect between design variables. This technique yields a better understanding of how the design variables are interacting and how this will affect the overall performance of the structure. It is believed that the intelligent use of DOE techniques will help in the design of safer and more economic structures.

Keywords: Structure design, design of experiments, design uncertainties, variables variations, safe design.

Introduction

Real-world structures do not behave in a deterministic manner. Instead their behavior varies due to the uncertainty in design variables [**1**]. This uncertainty is inherent in material characteristics, loading conditions, simulation model accuracy, geometric properties, manufacturing precision, actual product usage, etc. A traditional design approach first neglects the fore mentioned uncertainties and assumes that design variables are deterministic, and then applies a safety factor to account for the effects of the neglected uncertainties. While this approach can lead to a safe design, however it can also lead to an overdesigned structure that is uneconomic. Moreover, neglecting the uncertainties within the design variables deprives the designer from understanding their effect on the structure behavior.

This paper presents a technique in which the science of design of experiments (DOE) is employed to assess in modern structure design. A case study of a simple structure that is idealized as a simply supported beam is investigated. The design variables are categorized according to their sources as load, geometry and material types. Each variable value is assumed to vary within a given numerical distribution. The deflection at the mid span of the structure is considered as the performance response of interest. DOE is applied to evaluate the effect of uncertainty within the design variables on the performance response. It is also used to calculate the interaction effect between design variables. This technique yields a better understanding of how the design variables are interacting and how this will affect the overall performance of the structure.

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Method

1. Model description

A case study of a structure design is considered here [2]. A structure is idealized as a simply supported beam as shown in Figure 1. The design variables names, types and nominal values are presented in Table 1. It is assumed that all variables are normally distributed around their nominal values with 10% standard deviation.

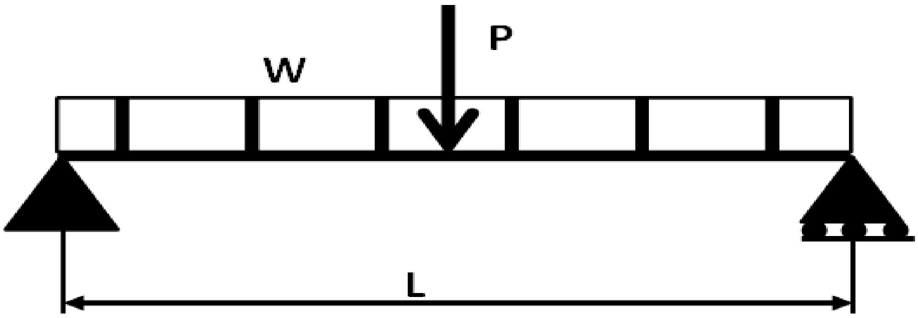

Figure 1 Schematic drawing of a three bar truss.

Table 1 Structure design variables

		Variable Type		Nominal Value
1.	**Load**			
	W	**Distributed load**	**[N/m]**	**35×10^3**
	P	**Concentrated load**	**[N]**	**10^5**
2.	**Geometrical**			
	L	**Structure length**	**[m]**	**10**
	I	**Second moment of Inertia**	**[m^4]**	**9×10^{-4}**
3.	**Material**			
	E	**Young's modulus**	**[Pa]**	**210×10^9**

2. Approach:

The quantitative response that describes the structure design is considered to be the mid-span displacement (*Disp*) that should not exceed 40 mm and its value can be easily calculated using Equation (1) as follows:

$$Disp = \frac{5}{384}\frac{WL^4}{EI} + \frac{1}{48}\frac{PL^3}{EI} \quad (1)$$

At this stage a conventional structure design approach would have plugged in the nominal values of the design variables in Equation (**1**), which gives a safe value of *Disp* = 35.1 mm. This value is 12.5% less than the allowable 40 mm mid-span displacement. However, it will be shown that disregarding the uncertainty within the design variables gives a misleading conclusion of a safe design. In order to unveil the effect of uncertainty within the design variables on the mid-span displacement, design of experiments (DOE) is employed.

3. Design of experiments

DOE is the science of designing experiments with different influencing variables to understand the behavior of one or more responses. DOE was founded by Sir Ronald A. Fisher, who in the early 1920’s published his landmark book (The Design of Experiments) [**3**]. He formulated the basics of the science in his book where he studied the effect of different fertilizers and other variables (soil condition and moisture content… etc.) in different land plots on the final condition of the crop [**3**]. Now, DOE is widely used in agriculture, industrial design, pharmaceutics, management, marketing, chemical engineering, life science ...etc. [**4**].

DOE is basically the statistical tool for minimizing the number of expensive time consuming experiments while maximizing the information gain from these experiments. With the advancement of computer hardware and software, researchers realized the usefulness of DOE and started using it as means to minimize the burden of computations without sacrificing accuracy as much as possible.

There are many different DOE designs for selecting data points from a multidimensional design space [**4**]. The full factorial design is selected here for the application of DOE. In full factorial design, the design is composed of all the combinations of design variables at all levels. For example, for a system with two design variables in which each variable can take a value from one of two values (levels), a full factorial design then has $2^2 = 4$ points. Thus for a design with k variables and l levels, there are l^k data points. In the present study, there are five variables (*W*, *P*, *L*, *E*, and *I*). It is assumed that each variable can vary within one standard deviation (*SD*) around its nominal value, for example, *W* can assume values as *W-SD*, *W* and *W+SD*. This means that each design variable has three levels, which yields $3^5=243$ design points.

It should be noted here that 243 is a relatively large number that could have demanded a lot of computational resources e.g. when using finite element analysis of a large structure. In the current study the *Disp* function is a simple function that can be easily calculated at virtually no computational cost. However, there are cases where the design response is complicated that demands large computations; such as when evaluating vehicle structure response under crash [**5**]. In such cases, approximate model building techniques are used to create simple surrogate models that are easy to calculate instead of the initial complex models. For more information on these subjects, the reader is referred to reference [**6**].

Results and Discussion

The mid-span displacement is calculated using Equation (1) at each design point of the 243 points. For the comparison purpose, each design variable (x) is normalized and centered with respect to its nominal value (x_{norm}) according to Equation (2). Since the values of the design variables are assumed to vary up and down with one standard deviation, hence Equation (2) will result in -1, zero and +1 as shown in Table 2 which lists the first ten design variables and their associated responses (Disp).

$$x_{norm} = \frac{x - x_{norm}}{x_{norm}} \tag{2}$$

Table 2 Sample of the first 10 design variables and their associated responses

ID	W	P	L	E	I	Disp [mm]
1	-1	-1	-1	-1	-1	26.5
2	-1	-1	-1	-1	0	28.5
3	-1	-1	-1	-1	1	30.4
4	0	-1	-1	-1	-1	27.5
5	0	-1	-1	-1	0	29.5
6	0	-1	-1	-1	1	31.4
7	1	-1	-1	-1	-1	28.5
8	1	-1	-1	-1	0	30.4
9	1	-1	-1	-1	1	32.4
10	-1	-1	0	-1	-1	39

The effect of uncertainty within the design variables on the mid-span displacement (*Disp*) can be understood from two important graphs [**7**]. The graphs are produced using the whole 243 design points resulting from Equation (2). The two graphs are presented in the following sections along with detailed discussion.

1. Main Effect Graph

It presents the change in mean response produced by the change in the level of a design variable. It is called the main effect because it relates to the primary effect of the design variable. The data is analyzed using MATLAB and plotted in Figure 2, where it shows that both the distributed load (*W*) and the concentrated load (*P*) have little positive effect on the mean value of the response (*Disp*). This is indicated by the small slope of their main effects graph as shown in Figure 2. It can also be concluded that the structure's length (*L*) has the largest positive effect on (*Disp*). Finally, both modulus of elasticity (*E*) and moment of inertia (*I*) have identical negative effect on (*Disp*). It should be noted that negative effects are desirable since this means that the mid-span displacement will decrease, whereas positive effects imply that the mid-span displacement will increase which is undesired. It is also helpful for the designer to visualize the main effect of the design variables in a quantitative manner in a bar plot as shown in Figure 3, where it shows the percentage change in the mean value of (*Disp*) when the level of each design variable changes from low (-1) to high (+1). It can be concluded from both Figure 2 and Figure 3 that the structure's length (*L*) can have a profound effect on its behavior, therefore, special attention must be given to its inspection before installation.

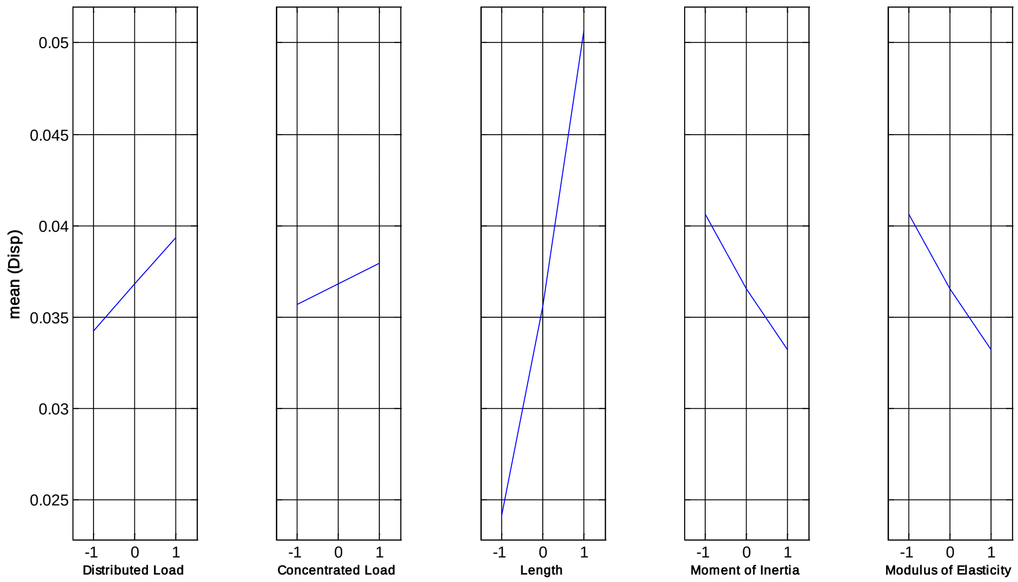

Figure 2 Main effects graph of the design variables on the mean (*Disp*)

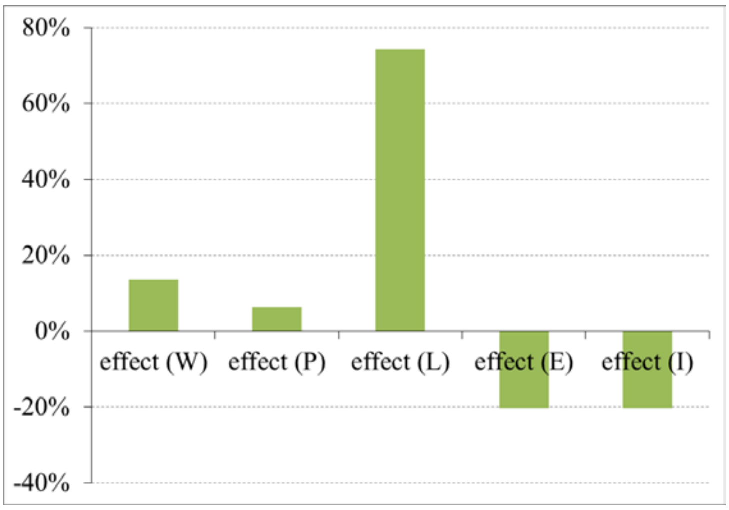

Figure 3 Bar graph of the percentage main effect on the mean (*Disp*)

2. Interaction-Effects Graph

When the levels of the design variables vary simultaneously, this can introduce a change in the response value. In other words, the difference in the response change due to the main effect of one design variable is not the same at all levels of the other design variables. This is defined as the interaction-effect. Figure 4 shows the interaction-effect between the design variables. It can be seen that there is a small interaction-effect between *W* and *P*, which is illustrated by the small slope in the upper left corner plot in Figure 4. On the other hand, there is a strong interaction effect between (*W* and *L*) and (*P* and *L*) as represented in the second left plot in the upper row and in the first left plot in the second row in Figure 4. It can also be concluded that although the interaction-effect between (*E* and *L*) and (*I* and *L*) is small, nevertheless there is a significant change in the response value when the structure length value changes from level to level. Finally, it can be concluded that the structure length (*L*) has an influencing effect on the mid-span displacement (Disp) when its value changes between levels as represented in the two plots on its right in Figure 4.

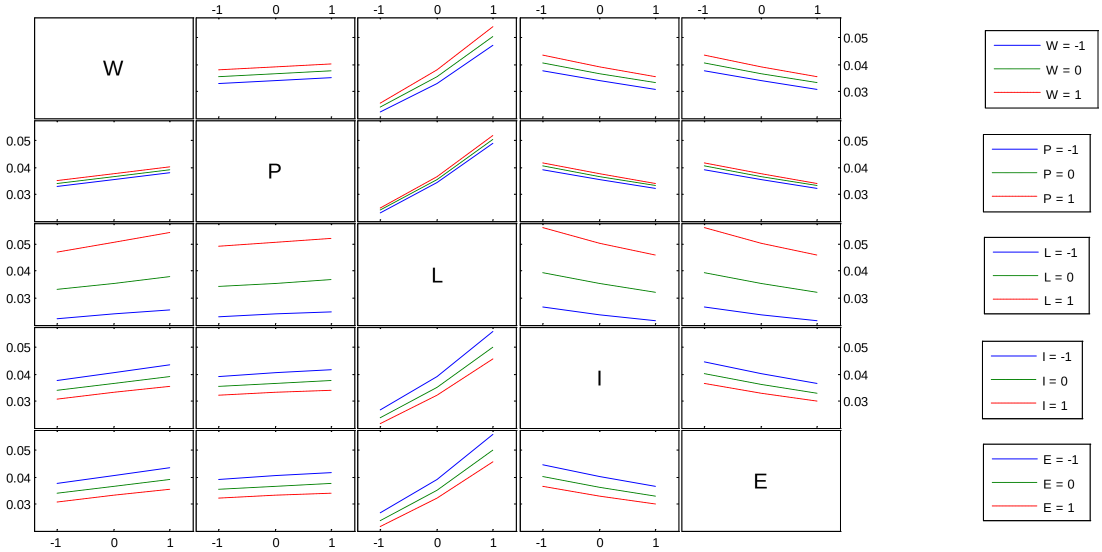

Figure 4 Interactions effect graph between the design variables on the mean (*Disp*)

Effect of Uncertainty on Structure Safety

Finally, to emphasize the importance of the consideration of uncertainty in structure design, It is interesting to note that out of the 243 different structure designs, 84 have filed, i.e., their mid-span displacement (*Disp*) is larger than the allowable 40 mm. This means that there is a 35% probabilty of structure failure. This is illustrated in Figure 5 which shows the histogram of Disp. The data shows a normal distribution as shown by the fitted line. This behavior is in accordance with our assumptions that the input design variables are normally distributed, hence the output function will also be normally distributed. In other words, it should be expected that when input design variables values are uncertain, the output response value will also be uncertain. Hence, the safety of the design will also be uncertain.

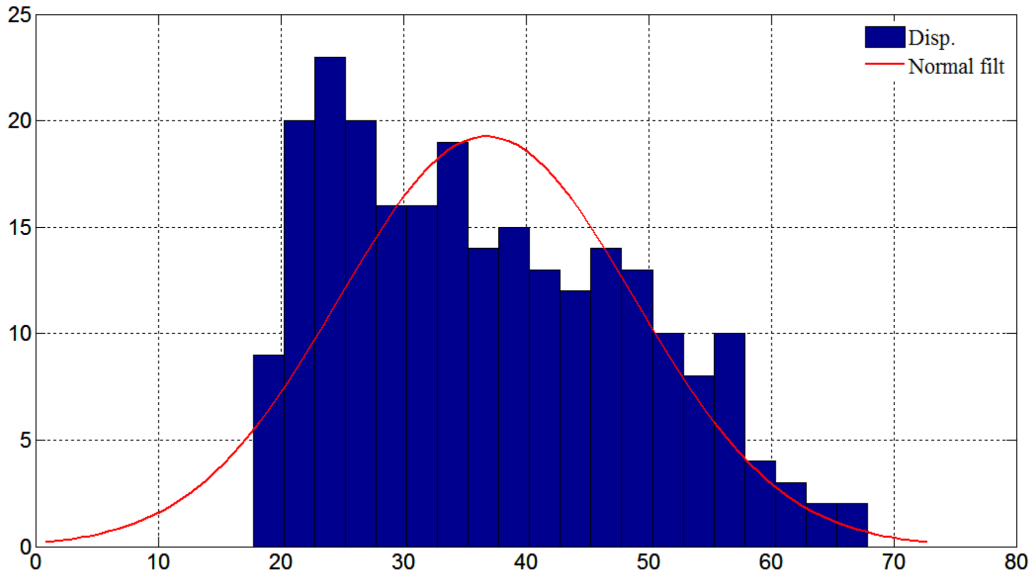

Figure 5 Histogram of mid-span displacement *(Disp)*

Effect of Changing the Limit of Uncertainty

In this section, the effect of changing the limit of uncertainty is studied. The standard deviation (SD) is changed from 10% to 5% and all calculations are repeated again. The main effects plot is presented in Figure 6 where the interaction effects are presented in Figure 7. The two figures show that changing the limit of uncertainty (SD) from 10% to 5% has no effect on the trend of change in the value of the response (Disp). However the upper and lower limits on the response (Disp) have been lowered, which resulted in halving the values of main effects as shown in Figure 8. Finally, Figure 9 shows over-plotting the histograms of the response (Disp) resulting from the two cases, i.e., when SD = 10% and SD = 5%. The figure shows that an uncertainty range of 5% resulted in a narrower distribution of the response (Disp). It can be concluded from Figure 9 that the design will fail only 66 times out of the total 243 times, i.e., when the response (Disp) is larger than the limit (40 mm). Compared with 84 failure times occurrences when SD = 10%. This means that failure expectation probability has decreased from 35% to 27%.

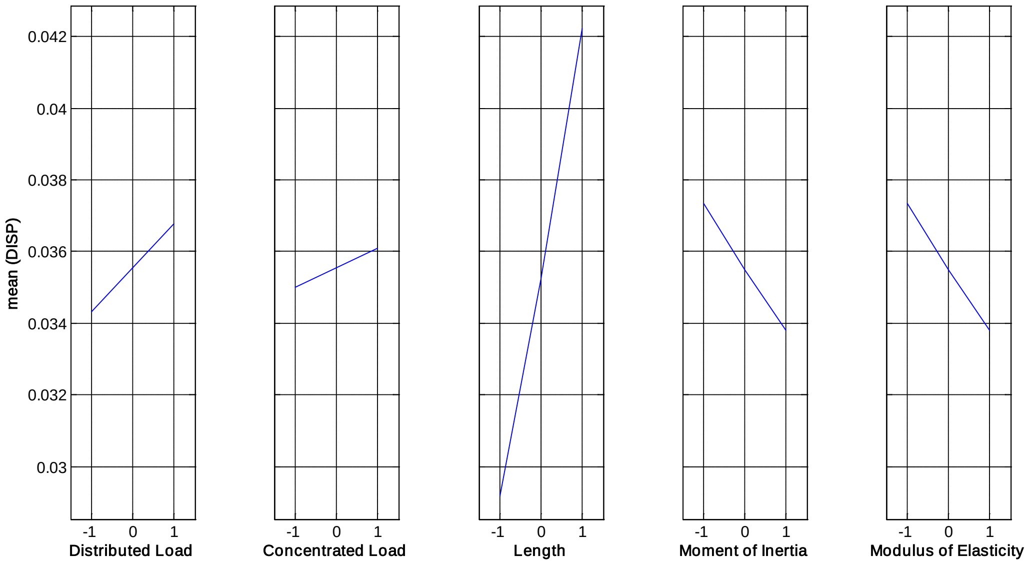

Figure 6 Main effects graph of the design variables on the mean (Disp)

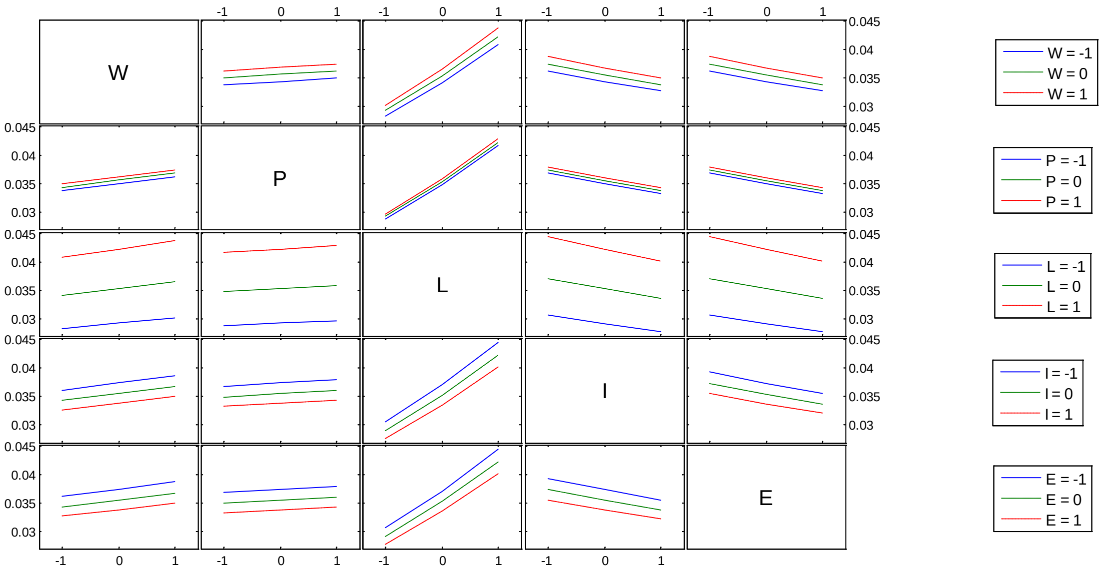

Figure 7 Interactions effect graph between the design variables on the mean (Disp)

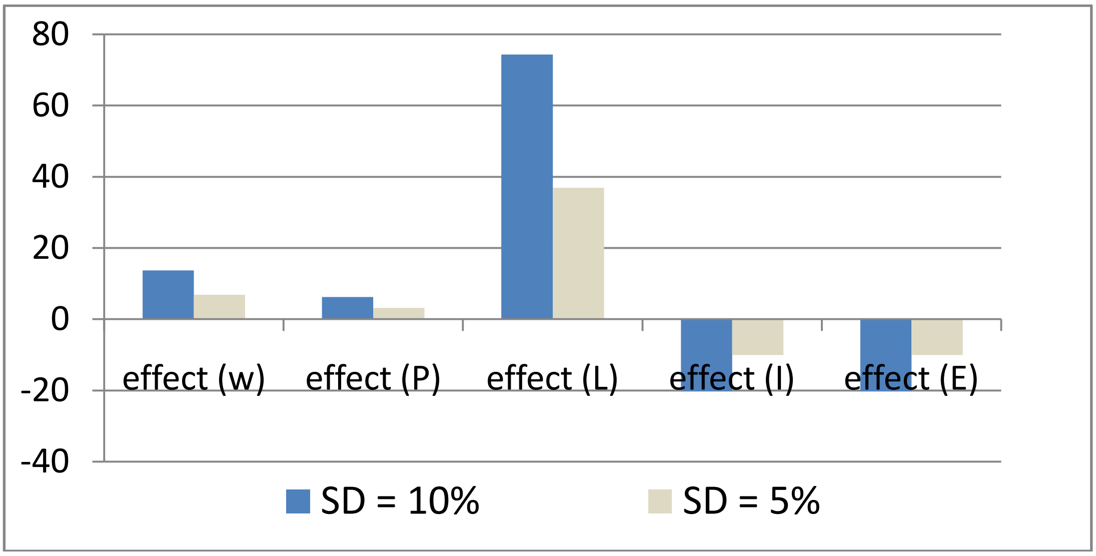

Figure 8 Bar graph illustrating a comparison between the effects of SD change

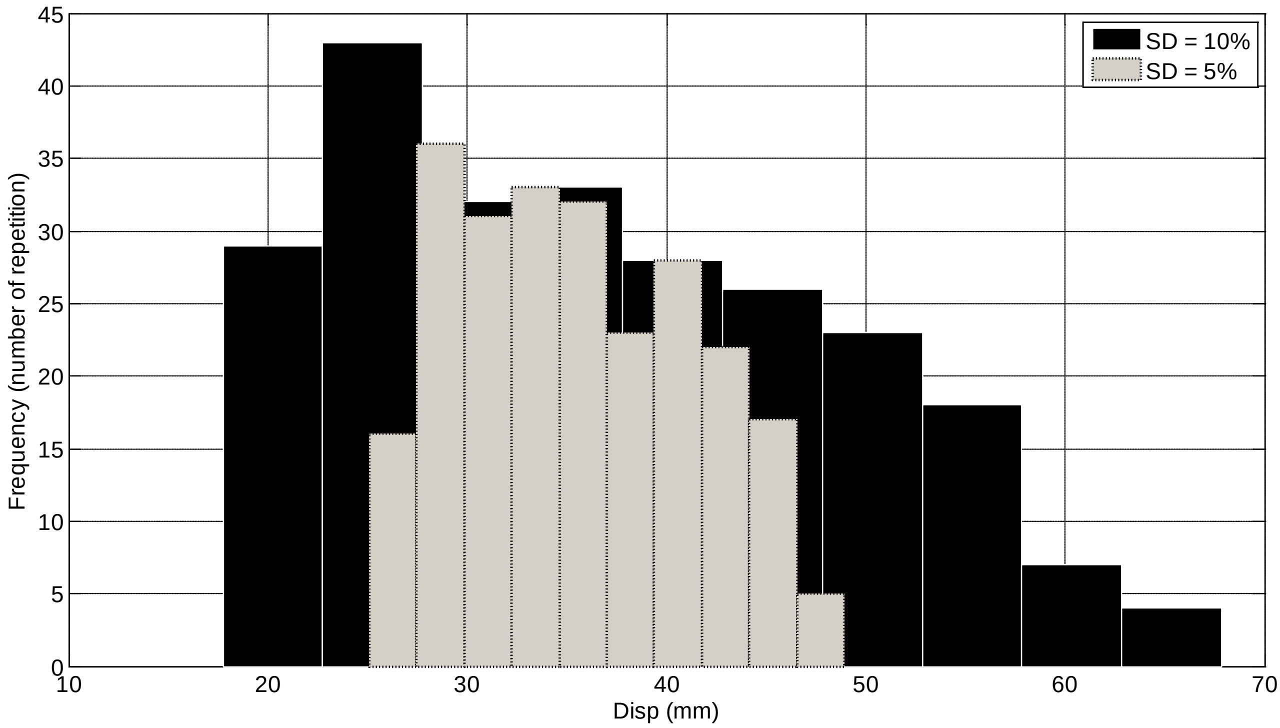

Figure 9 Histogram comparison showing the effect of SD = 10% and SD = 5%

Concluding Remarks

It has been demonstrated that the uncertainty within the design variables in structure design cannot be disregarded as their effect on its behavior can lead to unpredicted unsafe designs. It has been shown that the application of DOE has helped in attaining better understanding of both main and interaction-effects of the design variables on structure behavior. It has been demonstrated that the range of uncertainty has no effect on the *trend* of response value change. However, it has an effect of response values. Using meaningful graphs helps the designer in realizing the importance and the level of effect of uncertainty in the design variables values. This in effect will lead to safer and more economic structure designs.

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