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Design of the Rotor Blade for a Wind Mill

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Abstract: The design of a rotor blade requires comprehensive information of the particular wind-mill location. The design itself consists in determining the aerodynamic and geometric characteristics of the rotor blade (local chords, blade twist, airfoils).The blade design has to satisfy the design conditions, gain high aerodynamic efficiency and simultaneously take into consideration blade structural constraints (diameter, number of blades, revolutions).

The paper deals with the design of aerodynamic characteristics of the rotor blade of a wind mill in a chosen design point. The design is specified for the rotor of the diameter 20.4 [m] and revolutions 0.728 [s^{-1}].

Keywords: Wind mill, Propellers, Aerodynamic design

Nomenclature

C	$[-]$	Correction factor (effect of number of blades)
D	$[m]$	Propeller diameter
Q	$[N]$	Drag
L	$[N]$	Lift
N	$[N]$	Circumferential force (plane of rotation)
P	$[W]$	Power
R	$[m]$	Propeller radius
S	$[m^2]$	Propeller disc area
T	$[N]$	Thrust
W	$[W]$	Power developed by the rotor
β_0	$[^0]$	Flow angle
β_1	$[^0]$	Flow angle (induced velocity included) w_1
Γ	$[m^2.s^{-1}]$	Circulation
b	$[m]$	Local blade chord
c_L	$[-]$	Lift coefficient
c_P	$[-]$	Power coefficient

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c_Q	$[-]$	Drag coefficient
c_T	$[-]$	Thrust coefficient
c_L^α	$[-]$	Lift curve slope
h_1, h_2, σ, k	$[-]$	Iteration constants
n	$[s^{-1}]$	Revolutions
r	$[m]$	Local blade radius
t	$[m]$	Airfoil thickness
u_0	$[m.s^{-1}]$	Circumferential velocity
u_1	$[m.s^{-1}]$	Circumferential velocity (incl. induced quantities) u_i
v_0	$[m.s^{-1}]$	Velocity in front of the propeller (flight speed)
v_1	$[m.s^{-1}]$	Axial velocity component (incl. induced velocity) v_i
v_i a u_i	$[m.s^{-1}]$	Induced velocity
w_0	$[m.s^{-1}]$	Resulting velocity
w_1	$[m.s^{-1}]$	Resulting velocity (induced velocity included)
z	$[-]$	Number of blades
α	$[^0]$	Angle of attack
α_0	$[^0]$	Zero lift angle
η	$[-,\%]$	Efficiency
φ	$[^0]$	Angle of blade twist (setting angle)
λ	$[-]$	Advance ratio
μ	$[-]$	Airfoil drag/lift ratio
ω	$[s^{-1}]$	Angular velocity
ρ	$[kg.m^{-3}]$	Mass density of air

Introduction

The paper describes the design of a rotor blade of a wind-mill which gives high aerodynamic efficiency at the design point. Velocity components and local force components are expressed as functions of circulation the distribution of which giving maximum efficiency is calculated. The link between geometric/aerodynamic characteristics makes possible to determine structural parameters of the rotor blade.

Design Specification.

The selection of the design point for the rotor requires comprehensive information of the conditions at the particular location of the wind-mill. The design parameters include wind velocity (average) and the structural parameters – the diameter, number of blades, and rotor revolutions.

In the design process it is to determine the rotor geometric and aerodynamic characteristics (distribution of blade chords, distribution of blade twist, airfoils). These parameters have to conform to the design specifications and moreover achieve the aerodynamic efficiency as high as possible.

The essential propeller characteristics are introduced as follows:

Advance ratio

$$\lambda = \frac{v_0}{n.D} \tag{1}$$

Thrust and drag

$$T = \rho.n^2.D^4.c_T \tag{2}$$

$$Q = \rho.n^2.D^4.c_Q \tag{3}$$

Power

$$P = \rho.n^3.D^5.c_P \tag{4}$$

The rotor efficiency is determined by the thrust and power coefficients and the advance ratio[6]

$$\eta = \lambda.\frac{c_T}{c_P} \to \max \tag{5}$$

Set of Propeller Equations

In the following relations the indices 0, 1 describe the flow in front of a propeller (index „$_0$“) and in the propeller disc (index „$_1$“), as indicated in Fig.1.

The relation between the local lift values and the total lift can be expressed as

$$L = \frac{1}{2}\rho.w_1^2.c_L.S = \frac{1}{2}\rho.\int_0^R w_{1\,(r)}^2.c_{L\,(r)}.b_{(r)}.dr \tag{6}$$

$$dL = \frac{1}{2}\rho.w_1^2.c_L.b.dr \tag{7}$$

Introducing the concept of circulation and the Joukovski – Kutta theorem makes possible to write relations

$$\Gamma = \frac{1}{2}\rho.w_1.c_L.b \tag{8}$$

$$dL = \rho.w_1.\Gamma.dr \tag{9}$$

These terms when inserted into (6) give the lift resultant which depends on the distribution of circulation along a propeller blade.

The previous procedure transfers the task of gaining maximum efficiency to the problem of determining the optimum distribution of circulation.

To facilitate the numerical procedures the non-dimensional quantities are introduced

$$\overline{\Gamma} = \frac{\Gamma}{4.\pi.R^2.\omega} \qquad \overline{b} = \frac{z.b}{4.\pi.R} \qquad \overline{r} = \frac{r}{R} \qquad \overline{v}_1 = \frac{v_1}{R.\omega} \qquad \overline{u}_1 = \frac{u_1}{R.\omega} \tag{10}$$

where the velocity components are indicated in Fig.1. [5]

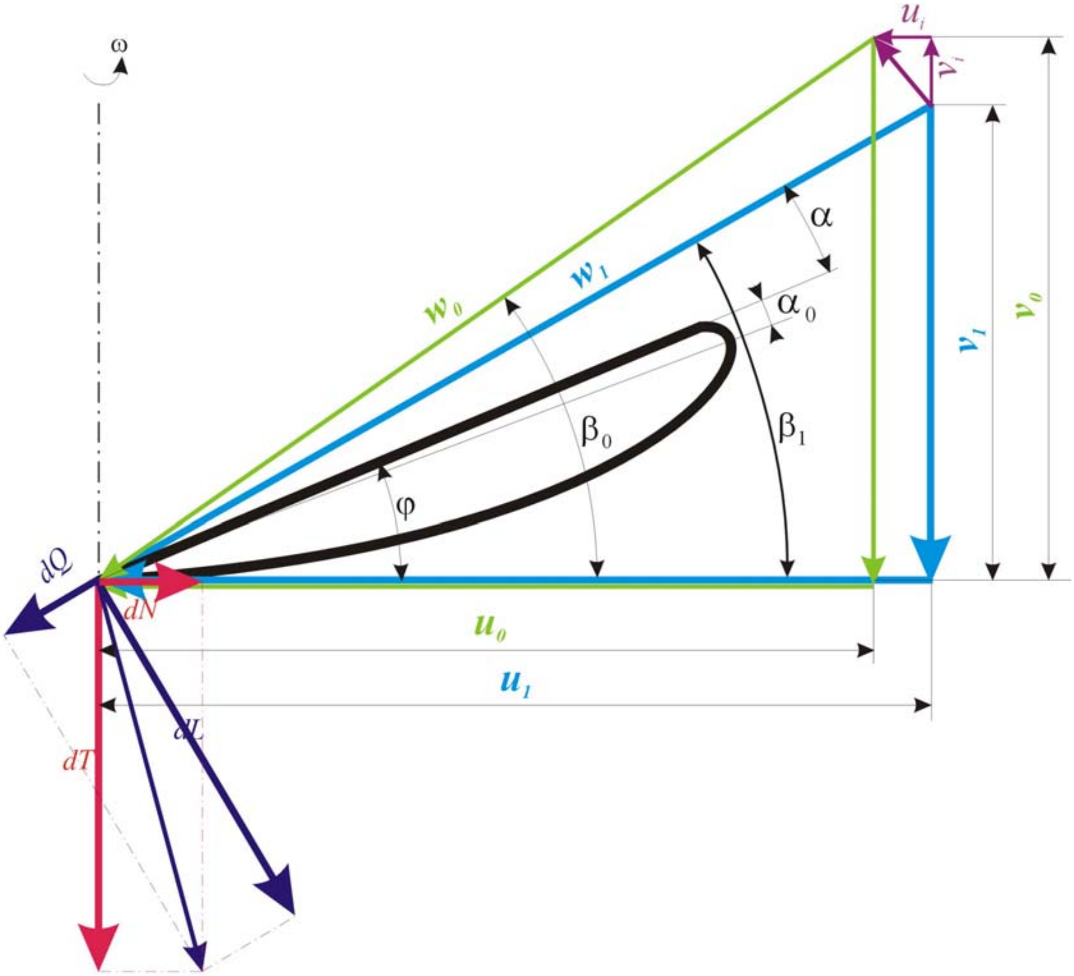

Fig. 1 Velocity and force components

Velocity Relations at a Local Blade Section

The resulting velocity at the local radius including induced effects is composed of axial and tangential components

$$\overline{w}_1 = \sqrt{\left(\overline{u}_1^{\,2} + \overline{v}_1^{\,2}\right)} \tag{11}$$

$$\overline{u}_1 = \overline{u}_0 + \overline{u}_i \tag{12}$$

$$\overline{v}_1 = \overline{v}_0 + \overline{v}_i \tag{13}$$

or by means of (12) and (13)

$$\bar{u}_1 = \bar{r} + \bar{u}_i \qquad (14) \qquad\qquad \bar{v}_1 = \frac{\lambda}{.\pi} + \bar{v}_i \qquad (15)$$

The simplest vortex system of a propeller is represented by a system of concentric vortex cylinders with a continuous distribution of circulation (by Joukovski) which gives the following relations for velocity components as functions of circulation

$$\bar{u}_1 = \bar{r} + \frac{\overline{\Gamma}}{\bar{r}} \qquad (16)$$

$$\bar{v}_1 = \frac{\lambda}{2.\pi} + \sqrt{\left(\left(\frac{\lambda}{2.\pi}\right)^2 - \overline{\Gamma}.(1+\overline{\Gamma})\right)} \qquad (17)$$

The vortex system of question does not fully conform to experimental results because it strictly corresponds to the infinite number of blades. As a consequence the error increases with the advance ratio, the propeller power loading and the decrease in number of blades.

It is recommended to improve the error by a correction factor depending on the advance ratio and number of blades. [2]

$$C = 1 + 1{,}803.(\frac{1}{z})^{1{,}16}.\lambda^2 - 0{,}459.(\frac{1}{z})^{1{,}062}.\lambda^3 + 0{,}0243.(\frac{1}{z})^{0{,}835}.\lambda^4 \qquad (18)$$

Substituting into (16), (17) the velocity components become

$$\bar{u}_1 = \bar{r} + C.\frac{\overline{\Gamma}}{\bar{r}} \qquad (19)$$

$$\bar{v}_1 = \frac{\lambda}{2.\pi}.(2-C) + C.\sqrt{\left(\left(\frac{\lambda}{2.\pi}\right)^2 - \overline{\Gamma}.(1+\overline{\Gamma})\right)} \qquad (20)$$

The following step is to specify links between kinematic and structural quantities by the geometric parameters (angle of blade twist, local sectional chord, airfoil thickness) to conform the flow relations (Fig. 1)

$$\beta_1 = arctg\,\frac{\bar{v}_1}{\bar{u}_1} \qquad (21)$$

$$\varphi = \beta_1 - \alpha \qquad (22)$$

$$c_L = c_L^{\alpha}\left[arctg\,\frac{\bar{v}_1}{\bar{u}_1} - (\varphi + \alpha_0)\right] \qquad (23)$$

By analyzing the previous relations it is possible to get to the conclusion that the structural parameters φ, b a t of the propeller at the particular blade section are determined by the local value of circulation $\overline{\Gamma}$. [3]

Distribution of Optimum Circulation

Based on the previous considerations on velocity and geometric relations the task to calculate and design the propeller blade is simplified to the determination of optimum distribution of circulation.

The specific method to solve this problem and utilize its result for a real propeller design follows the procedures described in Ref. [3]

An iteration method was used where the successive steps lead to the final value of local circulation. The numerical process is terminated when the accuracy criterion for two consecutive steps is satisfied $\left|\ \overline{\Gamma} - \overline{\Gamma}_{(-1)}\ \right| \leq 1.10^{-4}$.

Iteration constants

$$h_1 = \frac{\lambda}{2.\pi}.(2 - C) + C.\frac{\left(\frac{\lambda}{2.\pi}\right)^2 + \frac{3.\overline{\Gamma}_{(-1)}}{2}}{\sqrt{\left[\left(\frac{\lambda}{2.\pi}\right)^2 + \overline{\Gamma}_{(-1)}\right]}} \qquad h_2 = C.\frac{\left(\frac{\lambda}{2.\pi}\right)^2 + \frac{3.\overline{\Gamma}_{(-1)}}{4}}{\sqrt{\left[\left(\frac{\lambda}{2.\pi}\right)^2 + \overline{\Gamma}_{(-1)}\right]}} \qquad \sigma = \frac{k.\bar{r} - \mu}{1 + k.\mu.\bar{r}}$$

form auxiliary quantities for the final relation

$$\overline{\Gamma} = \bar{r}.\left(\frac{\bar{r} + \sigma.\left(h_1 - h_2.\overline{\Gamma}_{(-1)}\right)}{2.C - \bar{r}.\sigma.h_2}\right) \tag{24}$$

The calculated values represent the optimum distribution of circulation directly giving the distribution of local chords and setting angles. Moreover it enables to calculate even the thrust and power coefficients. [2]

Design of the Rotor Blade

The method described above was applied to the blade design conforming to the demand of the VEZ Kosice comp.(East Slovakia Energy Ent.). The blade design and its realization were intended as a replacement for an older type of a wind- mill.

The given input data

D = 20,40 [m], v_0 = 8,00 [m.s^{-1}], n = 0,728 [s^{-1}] = 43,70 [min^{-1}]

determine $\lambda = 0{,}5384$ and C = 1.088.

Subsequently the optimum distribution of circulation was calculated giving the process in Fig. 2.

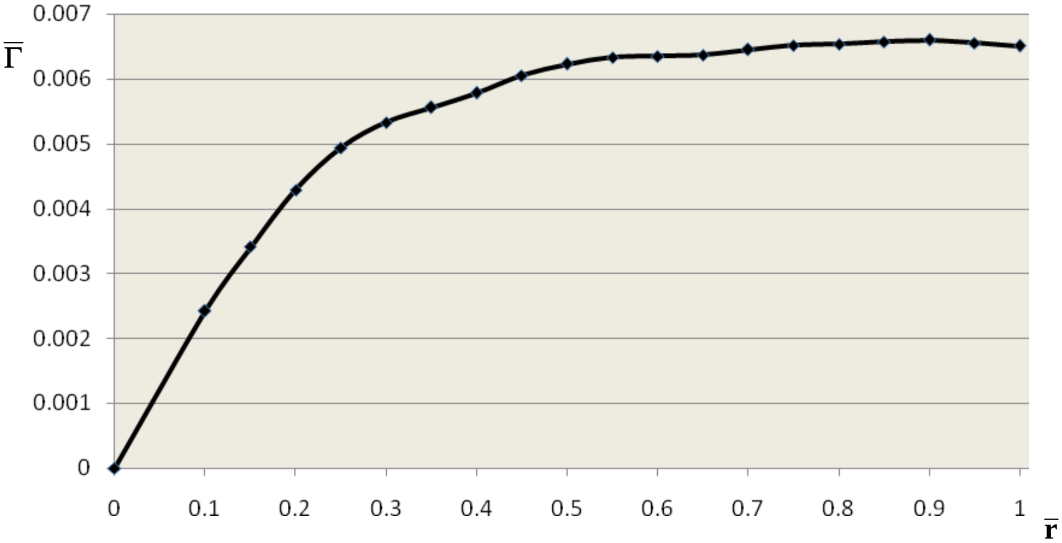

Fig.2. Distribution of circulation $\overline{\Gamma}$ along the propeller blade

Table 1. contains the values of circulation and velocity components (19), (20), (11). The next step consists in calculating the lift distribution $c_L.\overline{b}$ from (8) which in connection with the characteristics of the RAF 6 airfoil leads to resulting geometric parameters of the designed blade (Table 2.), Figs.3 & 4..

Table 1. Distribution of circulation and velocity components on the propeller blade

$\overline{r}$	$\overline{\Gamma}$	$\overline{u}_1$	$\overline{v}_1$	$\overline{w}_1$
0	0	0	0	0
0.1	0.00242	0,12420	0,155816	0,19926
0.15	0.00341	0,17273	0,148313	0,22767
0.2	0.00429	0,22148	0,140725	0,26240
0.25	0.00494	0,26977	0,134445	0,30141
0.3	0.00533	0,31777	0,130210	0,34342
0.35	0.00556	0,36578	0,127982	0,38752
0.4	0.00578	0,41429	0,125606	0,43291
0.45	0.00605	0,46329	0,122111	0,47911
0.5	0.00623	0,51247	0,118330	0,52596
0.55	0.00633	0,56152	0,116763	0,57353
0.6	0.00635	0,61059	0,116411	0,62159
0.65	0.00637	0,65981	0,116116	0,66995
0.7	0.00645	0,70921	0,114880	0,71845
0.75	0.00652	0,75869	0,113592	0,76715
0.8	0.00654	0,80818	0,113156	0,81606
0.85	0.00657	0,85773	0,112650	0,86510
0.9	0.00660	0,90734	0,112015	0,91423
0.95	0.00655	0,95690	0,112929	0,96354
1	0.00651	1,00652	0,113705	1,01292

Table 2. Final aerodynamic and geometric characteristics

$\bar{r}$	$c_L.\bar{b}$	c_L	$b[m]$	$t[\%]$	$\alpha[^o]$	$\varphi[^o]$
0	0.0	0.0000				
0.1	0.02429	0.6278	1.240	23.00		
0.15	0.02996	0.8020	1.197	22.16		
0.2	0.03274	0.9085	1.155	21.33	4.53	27.89
0.25	0.03279	0.9441	1.113	20.50	4.98	21.50
0.3	0.03105	0.9301	1.070	19.66	4.56	17.72
0.35	0.02851	0.8887	1.028	18.83	3.63	15.65
0.4	0.02641	0.8584	0.986	18.00	3.04	13.82
0.45	0.02496	0.8484	0.943	17.16	2.82	11.93
0.5	0.02372	0.8438	0.901	16.33	2.66	10.33
0.55	0.02210	0.8244	0.859	15.50	2.44	9.29
0.6	0.02046	0.8035	0.816	14.66	2.27	8.51
0.65	0.01903	0.7881	0.774	13.83	2.24	7.74
0.7	0.01795	0.7860	0.732	13.00	2.55	6.64
0.75	0.01700	0.7908	0.689	12.16	2.93	5.57
0.8	0.01604	0.7946	0.647	11.33	3.23	4.73
0.85	0.01519	0.8049	0.605	10.50	3.59	3.89
0.9	0.01445	0.8241	0.562	9.66	4.11	2.92
0.95	0.01361	0.8389	0.520	8.83	4.68	2.04
1	0.01286	0.8625	0.478	8.00	5.32	1.11

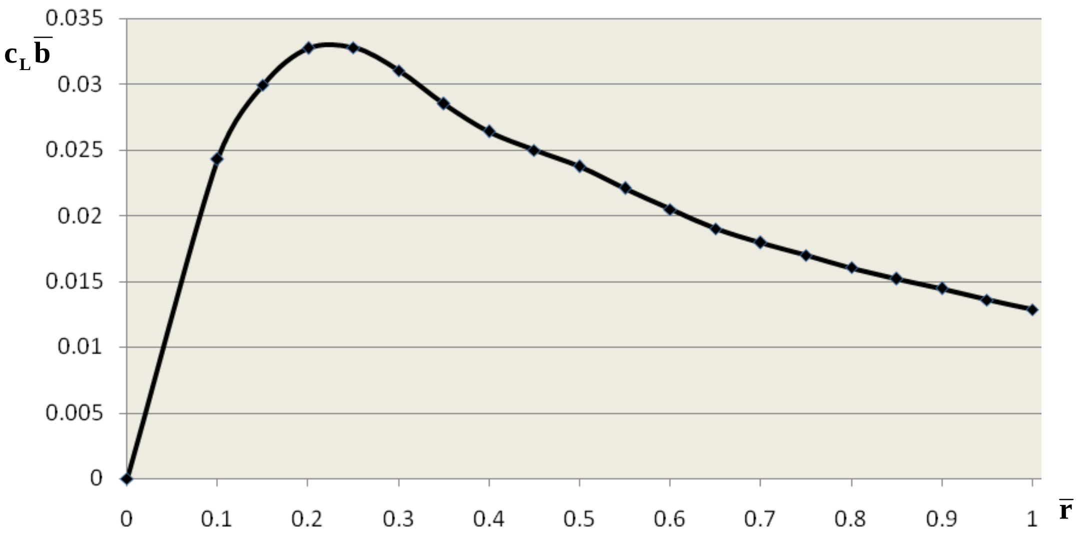

Fig. 3. Lift distribution $c_L.\bar{b}$

For the design parameters given as wind velocity v_0 = 8.00 [m.s^{-1}] and rotor revolutions n = 0.73 [s^{-1}] resulting parameters were calculated

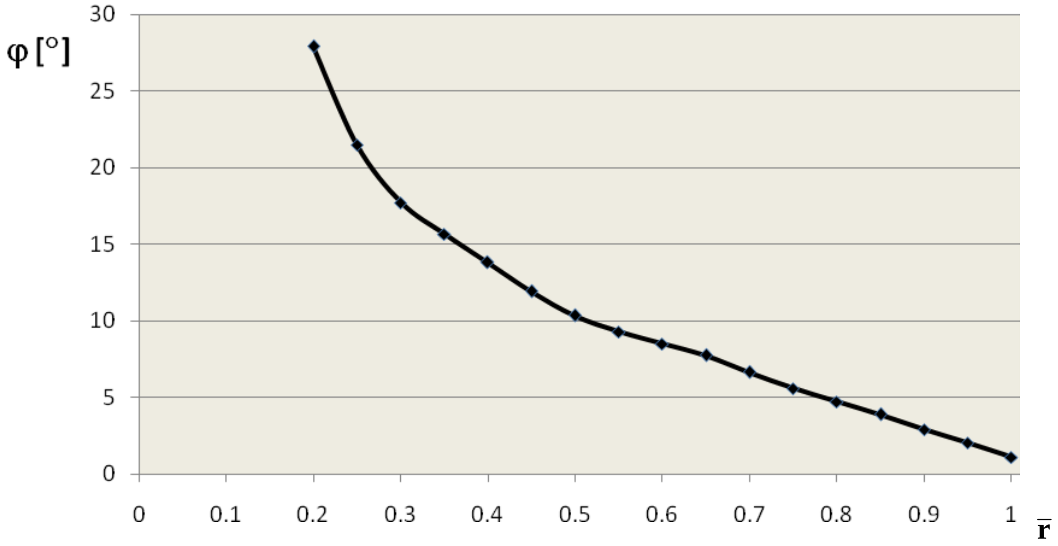

Fig.4. Distribution of blade twist $\varphi[^{O}]$

$c_T = 0{,}025569 \quad \eta = 44{,}919[\%] \; T = 11563{,}3[N] \; Q = 1882{,}58[N]$

Then the total wind mill power W is calculated from

$$W = z.n.\int_0^R N_{(r)}.dr \tag{25}$$

Calculating the integral (25) by means of values in Table 3 gives the total power $W = 47{,}484[kW]$

Conclusion

The given method to design the rotor blade results from the distribution of optimum circulation which conforms to the selected design conditions and achieves high aerodynamic efficiency. Circulation relates to the lift distribution which includes geometric and aerodynamic characteristics and makes possible to determine structural parameters of the blade. As an example a particular rotor design is presented giving the rotor geometry, its aerodynamic characteristics and power developed by the wind mill.

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Table 3. Rotor force components and power

R $[m]$	L/z $[N]$	Q/z $[N]$	T/z $[N]$	N/z $[N]$	W/z $[W]$	W $[W]$
10.2	0	0.0	0.0	0	534.2	2136
9.69	286.6	7.0	286.3	13.5	1233.7	4934
9.18	559.3	13.5	558.6	30.8	2041.5	8166
8.67	817.7	19.6	816.2	51.4	2923.7	11694
8.16	1061.5	25.4	1059.0	74.4	3828.0	15312
7.65	1291.1	30.8	1287.3	99.1	4732.7	18930
7.14	1505.7	36.0	1500.3	125.6	5641.5	22566
6.63	1704.8	40.9	1697.4	153.7	6529.2	26116
6.12	1888.5	45.6	1878.9	182.8	7364.7	29458
5.61	2057.3	50.1	2045.2	212.2	8136.3	32545
5.1	2211.3	54.5	2196.4	241.4	8833.8	35335
4.59	2350.4	58.6	2332.6	270.4	9456.6	37826
4.08	2474.6	62.6	2453.6	298.8	10002.7	40010
3.57	2584.1	66.6	2559.5	326.6	10471.2	41884
3.06	2679.4	70.6	2651.0	353.4	10864.8	43459
2.55	2761.0	74.9	2728.4	379.7	11197.3	44789
2.04	2829.4	79.5	2791.7	405.9	11477.1	45908
1.53	2885.2	83.9	2841.0	432.2	11702.1	46808
1.02	2929.0	87.9	2876.4	458.0	11871.1	47484
0.51	2948.0	89.7	2890.8	470.6	11871.1	47484