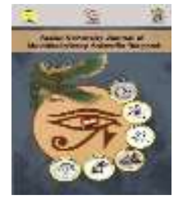
Assiut University Journal of Multidisciplinary Scientific Research (AUNJMSR)
Faculty of Science, Assiut University, Assiut, Egypt.
Printed ISSN: 2812-5029
Online ISSN: 2812-5037
Vol. 51(2): 115 – 134 (2022)
https://aunj.journals.ekb.eg/



Exact Solutions to a Class of Schamel Nonlinear Equations Modeling Dust Ion-acoustic Waves in Plasma

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Article Info

Article History:
Received: 30/12/2021
Accepted: 30/3/2022
Online: 1/5/2022

Keywords:
The extended Kudryashov method; Nonlinear evolution equation ; Exact solution; Schamel equations; Ion-acoustic waves.

Abstract

In this paper, we apply the extended Kudryashov method to construct some new exact solitary wave solutions of three important physical models, Schamel-nonlinear Schrödinger (S-NLS) equation, Schamel Korteweg-de Vries (S-KdV) equation, Schamel Korteweg-de Vries Burgers (S-KdVB) equation. In addition, this method is applied to seek solutions of Schamel equation modeling ion-acoustic waves in plasma and dust plasma. With the aid of symbolic computation, explicit exact solutions of these equations are expressed in terms of the hyperbolic functions, the trigonometric functions, and the exponential function. The stability of some exact solutions is computationally studied.

INTRODUCTION

Nonlinear partial differential equations (NLPDEs) arise in number of scientific models to study many phenomena in physics and other fields. The salient feature of nonlinear evolution equations (NLEEs) appearing in mathematical physics is the study of traveling wave solution, solitary wave solution, periodic wave and kink-antikink wave solutions. The investigation of these solutions helps one to well understand the nonlinear physical phenomena. Over the last year, many methods have developed for finding exact solutions to NLEEs such as, the tanh-function method and its various extension [1]-[3], the homogeneous balance method [4, 5], the Jacobi elliptic function method [6]-[8], the F-expansion method and its extension [9]-[12], the sub-ODE method [13, 14] and other methods [15]-[17]. The Kudryashov method was presented by Kudryashov [18]-[20]

and it has been applied by many authors [21]-[23]. The extended Kudryashov method [24, 25] is direct effective and provides more new exact solutions by taking the combination of all solutions of the Bernoulli and Riccati equations.

In this paper, we apply the extended Kudryashov method to construct some new explicit exact solutions of S-NLS equation, S-KdV equation, S-KdVB equation and Schamel equation. The physical signification of exact solutions of these equations are that they describe various natural phenomena such as solitons, solitary wave and kink wave solutions.

The rest of this work is organized as follows: In section 2, we give simple descriptions of the extended Kudryashov method. In section 3, we use this method to obtain the abundant exact solutions of the S-NLS equation, S-KdV equation, S-KdVB equation and Schamel equation. Finally, conclusion of the paper is given in the last section.

SUMMARY OF THE EXTENDED KUDRYASHOV METHOD

In this section, we summarize the essential steps of the extended Kudryashov method [24, 25] in the following steps:

Consider a NLPDE with the independent variables x and t

$$G(u, u_t, u_x, u_{tt}, u_{xx}, \ldots.) = 0, \tag{2.1}$$

where the left hand side of Eq. (2.1) is a polynomial in $u(x,t)$ and its various partial derivatives and the subscripts are used for partial derivatives.

Step 1: Assume that the transformation $u(x,t) = u(\xi)$, $\xi = k(x - c\,t)$, where c is the speed of traveling wave and k is the wave number to be determined later. Consequently, the NLPD Eq. (2.1) is transformed into an ordinary differential equation (ODE)

$$H(u, u', u'', \ldots) = 0, \tag{2.2}$$

where $u' = \frac{du}{d\xi}$ and H is a polynomial of u and its various derivatives.

Step 2: We propose the solutions of Eq. (2.2) to be a polynomial in the form

$$u(x,t) = u(\xi) = A_0 + \sum_{k=1}^{N} \sum_{i+j=k} A_{ij}\phi^{i}(\xi)\,\psi^{j}(\xi) + \sum_{k=1}^{N} \sum_{i+j=k} B_{ij}\phi^{-i}(\xi)\,\psi^{-j}(\xi), \tag{2.3}$$

where A_0, A_{ij}, B_{ij} $(i, j = 0, 1, 2, \ldots, N)$ are constants to be determined such that $A_N \neq 0$ and N is a positive integer that can be determined by balancing the nonlinear term (s) with the highest derivative term appearing in Eq. (2.2). The functions $\phi(\xi)$ and $\psi(\xi)$ satisfy the following Bernoulli and Riccati equations, respectively:

$$\frac{d\phi}{d\xi} = R_2\,\phi^2(\xi) - R_1\,\phi(\xi), \quad R_2 \neq 0, \tag{2.4}$$

$$\frac{d\psi}{d\xi} = S_2\,\psi^2(\xi) - S_1\,\psi(\xi) + S_0, \quad S_2 \neq 0, \tag{2.5}$$

where R_2, R_1, S_2, S_1 and S_0 are constants. The solutions of the Bernoulli equation are denoted by

$$\phi = \begin{cases} \frac{R_1}{R_2 + R_1 \exp(R1\,\xi + \xi_0)}, & R_1 \neq 0. \\ \frac{-1}{R_2\,\xi + \xi_0}, & R_1 \doteq 0. \end{cases} \tag{2.6}$$

It is known that the Riccati equation (2.5) admits the following solutions:

$$\psi = \begin{cases} \frac{-S_1}{2\,S_2} - \frac{\sqrt{\mu}}{2\,S_2}\; tanh\,(\frac{\sqrt{\mu}}{2}\,\xi + \xi_0), & \mu > 0, \\ \frac{-S_1}{2\,S_2} - \frac{\sqrt{\mu}}{2\,S_2}\; coth\,(\frac{\sqrt{\mu}}{2}\,\xi + \xi_0), & \mu > 0, \\ \frac{-S_1}{2\,S_2} + \frac{\sqrt{-\mu}}{2\,S_2}\; tan\,(\frac{\sqrt{-\mu}}{2}\,\xi + \xi_0), & \mu < 0, \\ \frac{-S_1}{2\,S_2} - \frac{\sqrt{-\mu}}{2\,S_2}\; cot\,(\frac{\sqrt{-\mu}}{2}\,\xi + \xi_0), & \mu < 0, \\ \frac{-1}{S_2\,\xi + \xi_0}, & \mu \doteq 0, \end{cases} \tag{2.7}$$

where $\mu = S_1^2 - 4\,S_0\,S_2$ and ξ_0 is an arbitrary real constant.
Step 3: Substituting Eq. (2.3) with Eq. (2.4) and Eq. (2.5) into the ODE Eq. (2.2) and equating each coefficients of $\phi^i(\xi)\,\psi^j(\xi)$ where $(i,j = 0,1,2.\dots)$ to zero yields a system of algebraic equations for $c, k, A_0, A_{ij}, B_{ij}(i,j = 0,1,2,\dots, N)$.
Step 4: Using the Maple or Mathematica to solve the system for $R_1, R_2, S_0, S_1, S_2, k, A_0,\ A_{ij}, B_{ij}$ and c. Substituting the solutions for coefficients into Eq. (2.3), then concentration formulas of traveling wave solutions of Eq. (2.1) can be obtained.

APPLICATIONS OF THE EXTENDED KUDRYASHOV METHOD

Through this section, we use the extended Kudryashov method to find exact solutions of S-NLS equation, S-KdV equation, S-KdVB equation and Schamel equation.

3.1 The S-NLS equation

In this section, we apply the extended Kudryashov method to construct exact traveling wave solutions of the S-NLS equation [26] which is given by

$$i\,u_t + u_{xx} + |u|^{\frac{1}{2}}\,u = 0, \tag{3.1}$$

where the real part of u denotes the electrostatic potential. In order to derive these solutions, we look for the solution of this equation in the form

$$u(x,t) = U(\xi)\ e^{i\,\Theta}, \quad \xi = k\,(x - c\,t), \quad \Theta = k_1\,x + k_2\,t, \tag{3.2}$$

where k, c, k_1 and k_2 are real constants to be determined and $U(\xi)$ is a real function of ξ. Setting Eq. (3.2) into Eq. (3.1) and separating the imaginary and the real parts of the resulting complex ODE, we have

$$U'\,(2\,k_1 - c) = 0, \tag{3.3}$$

$$k^2\,U'' - ({k_1}^2 + k_2)U + U^{\frac{3}{2}} = 0. \tag{3.4}$$

From Eq. (3.3) we obtain $k_1 = \frac{c}{2}$ and it is convenient to use the transformation $U(\xi) = v^2(\xi)$. Thus, Eq. (3.4) has the form

$$2\,k^2((v')^2 + v\,v'') - ({k_1}^2 + k_2)v^2 + v^3 = 0. \tag{3.5}$$

Balancing v^3 with $v\,v''$ in Eq. (3.5), we have $N = 2$. Thus, the extended Kudryashov method gives the solution in the form

$$v(\xi) = A_0 + A_{10}\,\phi(\xi) + A_{01}\,\psi(\xi) + A_{20}\,\phi^2(\xi) + A_{02}\,\psi^2(\xi) + A_{11}\,\phi(\xi)\,\psi(\xi)$$

$$+ \frac{B_{10}}{\phi(\xi)} + \frac{B_{01}}{\psi(\xi)} + \frac{B_{20}}{\phi^2(\xi)} + \frac{B_{02}}{\psi^2(\xi)} + \frac{B_{11}}{\phi(\xi)\,\psi(\xi)}, \tag{3.6}$$

where $A_0, A_{10}, A_{01}, A_{20}, A_{02}, A_{11}, B_{10}, B_{01}, B_{20}, B_{02}$ and B_{11} are constants. Setting Eq. (3.6) into Eq. (3.5) and using Eq. (2.4) and Eq. (2.5), thus Eq. (3.5) becomes a polynomial in $\phi(\xi)\,\psi(\xi)$. Putting each coefficients of $\phi^i(\xi)\,\psi^j(\xi)$ to zero, yields a system of algebraic equations for $A_0, A_{10}, A_{01}, A_{20}, A_{02}, A_{11}, B_{10}, B_{01}, B_{20}, B_{02}, B_{11}, R_1, R_2, S_0, S_1, S_2, k, k_1, k_2$ and c. Solving this system of algebraic equations, we get the following results:

Case 1:
$c = c,\ A_{10} = 20\,k^2 R_1 R_2,\ A_{20} = -20\,k^2 {R_2}^2,\ k_2 = 4\,k^2 {R_1}^2 - {k_1}^2,\ A_0 = A_{01} = A_{02} = A_{11} = B_{10} = B_{01} = B_{20} = B_{02} = B_{11} = 0,\ R_1 = R_1,\ R_2 = R_2,\ k = k,\ S_0 = S_0,\ S_1 = S_1,\ S_2 = S_2.$ (3.7)

Substituting Eq. (3.7) into Eq. (3.6) with the transformation $U(\xi) = v^2(\xi)$ and using Eq. (3.2), we get the soliton solution of Eq. (3.1) as the following:

$$u(x,t) = \left[\frac{20\, R_2 k^2 R_1{}^3 \exp(\, R_1 \xi + \xi_0\,)}{(\, R_2 + R_1 \exp(\, R_1 \xi + \xi_0\,)\,)^2} \right]^2 \quad e^{i\Theta}, \tag{3.8}$$

where $\xi = k\,(x - c\,t)$, $\Theta = k_1\,x + k_2\,t$ with c, k and k_2 are given in Eq. (3.7), R_1 and R_2 are arbitrary constants.

Case 2:

$c = c,\ A_0 = -20\,k^2 S_0 S_2,\ B_{01} = -20\,k^2 S_0 S_1,\ B_{02} = -20\,k^2 {S_0}^2,\ R_1 = R_1,\ R_2 = R_2,\ A_{10} = A_{01} = A_{20} = A_{02} = A_{11} = B_{10} = B_{20} = B_{11} = 0,\ S_0 = S_0,\ S_1 = S_1,\ S_2 = S_2,\ k = k,\ k_2 = 4\,k^2\left({S_1}^2 - 4\,S_0 S_2\right) - {k_1}^2\ .$ (3.9)

From Eq. (3.9), Eq. (3.6) and using $U(\xi) = v^2(\xi)$ with Eq. (3.2), we obtain the hyperbolic type solutions of Eq. (3.1) as

$$u(x,t) = \left[\frac{5\,({k_1}^2 + k_2)\,S_0 S_2\ \ sech^{\,2}(\frac{\sqrt{{k_1}^2 + k_2}}{4\,k}\,\xi + \xi_0\,)}{[\frac{\sqrt{{k_1}^2 + k_2}}{2\,k}\tanh(\frac{\sqrt{{k_1}^2 + k_2}}{4\,k}\,\xi + \xi_0\,) + S_1]^2} \right]^2 e^{i\Theta}, \tag{3.10}$$

$$u(x,t) = \left[\frac{-5\,({k_1}^2 + k_2)\,S_0 S_2\ \ csch^{\,2}(\frac{\sqrt{{k_1}^2 + k_2}}{4\,k}\,\xi + \xi_0\,)}{[\frac{\sqrt{{k_1}^2 + k_2}}{2\,k}\coth(\frac{\sqrt{{k_1}^2 + k_2}}{4\,k}\,\xi + \xi_0\,) + S_1]^2} \right]^2 e^{i\Theta}, \tag{3.11}$$

and the trigonometric type solutions

$$u(x,t) = \left[\frac{5\,({k_1}^2 + k_2)\,S_0 S_2\ \ sec^{\,2}(\frac{\sqrt{-({k_1}^2 + k_2)}}{4\,k}\,\xi + \xi_0\,)}{[\frac{\sqrt{-({k_1}^2 + k_2)}}{2\,k}\tan(\frac{\sqrt{-({k_1}^2 + k_2)}}{4\,k}\,\xi + \xi_0\,) - S_1]^2} \right]^2 e^{i\Theta}, \tag{3.12}$$

$$u(x,t) = \left[\frac{5\,({k_1}^2 + k_2)\,S_0 S_2\ \ csc^{\,2}(\frac{\sqrt{-({k_1}^2 + k_2)}}{4\,k}\,\xi + \xi_0\,)}{[\frac{\sqrt{-({k_1}^2 + k_2)}}{2\,k}\cot(\frac{\sqrt{-({k_1}^2 + k_2)}}{4\,k}\,\xi + \xi_0\,) - S_1]^2} \right]^2 e^{i\Theta}. \tag{3.13}$$

Case 3:

$c = c,\ A_0 = -40\,k^2 S_0 S_2,\ A_{02} = -20\,k^2 {S_2}^2,\ B_{02} = -20\,k^2 {S_0}^2,\ R_1 = R_1,\ R_2 =$

$$R_2\,,\ k=k,\ k_2=-64\,k^2S_0S_2-\ {k_1}^2,\ A_{01}=A_{10}=A_{20}=A_{11}=B_{10}=B_{01}=B_{20}=B_{11}=0,\ S_0=\ S_0,\ S_2=S_2,\ \ S_1=0. \tag{3.14}$$

Substituting Eq. (3.14) into Eq. (3.6) with $U(\xi)=v^2(\xi)$ and using Eq. (3.2), we get the exact solutions of Eq. (3.1) as follow:

$$u(x,t)=[\frac{-5\,({k_1}^2+k_2)}{16}]^2\ sech^{\,4}(\frac{\sqrt{{k_1}^2+k_2}}{8\,k}\ \xi+\xi_0)\ \ csch^{\,4}(\frac{\sqrt{{k_1}^2+k_2}}{8\,k}\ \xi+\xi_0)\ \ e^{i\Theta}, \tag{3.15}$$

$$u(x,t)=[\frac{-5\,({k_1}^2+k_2)}{16}]^2\ sec^{\,4}(\frac{\sqrt{-({k_1}^2+k_2)}}{8\,k}\ \xi+\xi_0)\ \ csc^{\,4}(\frac{\sqrt{-({k_1}^2+k_2)}}{8\,k}\ \xi+\xi_0)\ \ e^{i\Theta}. \tag{3.16}$$

Case 4:

$$c=c,\ A_0=-20\,k^2S_0S_2,\ A_{01}=-20\,k^2S_1S_2,\ A_{02}=-20\,k^2{S_2}^2,\ A_{10}=A_{20}=A_{11}=B_{10}=B_{01}=B_{20}=B_{02}=B_{11}=0,\ k_2=4\,k^2\left({S_1}^2-4\,S_0S_2\right)-{k_1}^2,\ S_0=S_0,\ S_1=S_1,\ S_2=S_2,\ k=k,\ R_1=R_1,\ R_2=R_2. \tag{3.17}$$

Setting these values into Eq. (3.6) with $U(\xi)=v^2(\xi)$ and Eq. (3.2), we obtain the solitary wave solutions of Eq. (3.1) in the form

$$u(x,t)=[\frac{5\,({k_1}^2+k_2)}{4}]^2\ sech^{\,4}(\frac{\sqrt{{k_1}^2+k_2}}{4\,k}\ \xi+\xi_0)\ \ e^{i\Theta}, \tag{3.18}$$

$$u(x,t)=[\frac{-5\,({k_1}^2+k_2)}{4}]^2\ csch^{\,4}(\frac{\sqrt{{k_1}^2+k_2}}{4\,k}\ \xi+\xi_0)\ \ e^{i\Theta}, \tag{3.19}$$

$$u(x,t)=[\frac{5\,({k_1}^2+k_2)}{4}]^2\ sec^{\,4}(\frac{\sqrt{-({k_1}^2+k_2)}}{4\,k}\ \xi+\xi_0)\ \ e^{i\Theta}, \tag{3.20}$$

$$u(x,t)=[\frac{5\,({k_1}^2+k_2)}{4}]^2\ csc^{\,4}(\frac{\sqrt{-({k_1}^2+k_2)}}{4\,k}\ \xi+\xi_0)\ \ e^{i\Theta}, \tag{3.21}$$

and the intensity of solution (3.18) is given by

$$|u(x,t)|^2=[\frac{5\,({k_1}^2+k_2)}{4}]^4\ sech^{\,8}(\frac{\sqrt{{k_1}^2+k_2}}{4\,k}\ \xi+\xi_0)$$

Equation (3.4) can be written in the form of an energy-like equation

$$\frac{1}{2}\,(U')^2 + V(U) = 0, -V(U) = \frac{({k_1}^2+k_2)}{2\,k^2}\;U^2 - \frac{2}{5\,k^2}\;U^{\frac{5}{2}}.$$

It is clear that $V(U) = 0$ and $\frac{dV}{dU} = 0$ at $U = 0$. So, there exist a point U_c (the amplitude of solitary wave) such that $V(U_c) = 0$. The existence condition for solitary wave solution (3.18) is $\frac{d^2V}{dU^2}|_{U=0} < 0$, which implies that ${k_1}^2 + k_2 > 0$.

We draw plots the intensity of solution (3.18) and its position at $t = 1$ in Fig.(1.a) and Fig.(1.b) with the choice of parameters as $k_1 = 0.3, k_2 = 1.55, k = 1, c = 0.6$ and $\xi_0 = 1$.

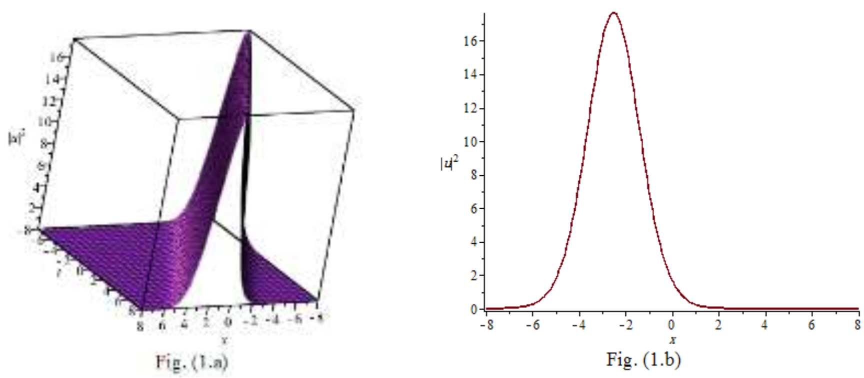

Figure 1: Fig.(1.a) The intensity of solution (3.18) with the parameters $k_1 = 0.3, k_2 = 1.55, k = 1, c = 0.6$ and $\xi_0 = 1$. Fig.(1.b) Its position with the same parameters.

3.2 The S-KdV equation

We consider the S-KdV [27] which is established in plasma physics in the study of ion acoustic solitons that reads

$$u_t + \alpha\,u^{\frac{1}{2}}\,u_x + \beta\,u\,u_x + \delta\,u_{xxx} = 0, \tag{3.22}$$

where β, α and δ are constants. This equation becomes the KdV equation (when $\alpha = 0$) and the Schamel equation (when $\beta = 0$). Due to various applications of Schamel equation and S-KdV equation, exact traveling wave solutions of these equations have given in [28, 29]. To derive some exact solutions of S-KdV equation, we use the transformation

$$u(x,t) = v^2(x,t), \;\; v(x,t) = v(\xi), \;\;\; \xi = k(x - c\,t). \tag{3.23}$$

Then Eq. (3.22) becomes

$$-c\,v\,v' + \alpha\,v^2\,v' + \beta\,v^3\,v' + \delta\,k^2(v\,v''' + 3\,v'\,v'') = 0. \tag{3.24}$$

Integrating Eq. (3.24) and setting the constant of integration equal to zero, we have

$$-6\,c\,v^2 + 4\,\alpha\,v^3 + 3\,\beta\,v^4 + 12\,\delta\,k^2((v')^2 + v\,v'') = 0. \tag{3.25}$$

Balancing procedure gives $N = 1$. Therefore, we may choose the ansatz

$$v(\xi) = A_0 + A_{10}\phi(\xi) + A_{01}\psi(\xi) + \frac{B_{10}}{\phi(\xi)} + \frac{B_{01}}{\psi(\xi)}, \tag{3.26}$$

where $A_0, A_{10}, A_{01}, B_{10}, B_{01}$ are constants. The left hand side of Eq. (3.25) becomes a polynomial in $\phi(\xi)\,\psi(\xi)$ after substituting Eq. (3.26) into Eq. (3.25) and using Eq. (2.4) and Eq. (2.5). Setting each coefficients of $\phi^i(\xi)\,\psi^j(\xi)$ to zero, we obtain the algebraic equations for $A_0, A_{10}, A_{01}, B_{10}, B_{01}, R_1, R_2, S_0, S_1, S_2, k$ and c. Solving these equations, we get the following results:

Case 1:

$$c = -\frac{16}{75}\frac{\alpha^2}{\beta}, B_{01} = \pm 2\sqrt{\frac{-3\,\delta}{\beta}}\,S_0\,k, S_1 = \pm\frac{(5\,\beta\,A_0 + 2\,\alpha)}{5\,k\sqrt{-3\,\beta\,\delta}}, S_2 = -\frac{1}{60}\frac{(5\,\beta\,A_0 + 4\,\alpha)A_0}{\delta\,k^2 S_0},$$

$$A_0 = A_0, A_{10} = A_{01} = B_{10} = 0, R_1 = R_1, R_2 = R_2, S_0 = S_0, k = k. \tag{3.27}$$

In this case, we obtain the solutions of Eq. (3.22) as follow:

$$u(x,t) = \left[\frac{2\,A_0\,\alpha(\,\tanh(\frac{1}{15}\sqrt{-\frac{3\,\alpha^2}{\beta\,\delta\,k^2}}\,\xi + \xi_0\,) \mp 1\,)}{2\,\alpha\tanh(\frac{1}{15}\sqrt{-\frac{3\,\alpha^2}{\beta\,\delta\,k^2}}\,\xi + \xi_0\,) \pm 5\,\beta\,A_0 \pm 2\,\alpha}\right]^2, \tag{3.28}$$

$$u(x,t) = \left[\frac{2\,A_0\,\alpha(\,\coth(\frac{1}{15}\sqrt{-\frac{3\,\alpha^2}{\beta\,\delta\,k^2}}\,\xi + \xi_0\,) \mp 1\,)}{2\,\alpha\coth(\frac{1}{15}\sqrt{-\frac{3\,\alpha^2}{\beta\,\delta\,k^2}}\,\xi + \xi_0\,) \pm 5\,\beta\,A_0 \pm 2\,\alpha}\right]^2. \tag{3.29}$$

Case 2:

$$c = -\frac{16}{75}\frac{\alpha^2}{\beta}, A_{01} = \pm 2\sqrt{\frac{-3\,\delta}{\beta}}\,S_2\,k, S_1 = \mp\frac{2}{15}\frac{\alpha\sqrt{\frac{-3\,\delta}{\beta}}}{k\,\delta}, S_2 = S_2,$$

$$A_0 = A_{10} = B_{10} = B_{01} = 0, R_1 = R_1, R_2 = R_2, S_0 = 0, k = k. \tag{3.30}$$

Using Eq. (3.30) and Eq. (3.26) with Eq. (3.23), we construct the solutions of Eq. (3.22) as

$$u(x,t) = \frac{4\,\alpha^2}{25\,\beta^2}\,[1 \pm \tanh\,(\frac{\alpha}{5\sqrt{-3\,\beta\,\delta}}\,(\,x + \frac{16\,\alpha^2}{75\,\beta}\,t\,) + \xi_0)\,]^2, \tag{3.31}$$

$$u(x,t) = \frac{4\,\alpha^2}{25\,\beta^2}\left[1 \pm \coth\left(\frac{\alpha}{5\sqrt{-3\,\beta\,\delta}}\left(x + \frac{16\,\alpha^2}{75\,\beta}\,t\right) + \xi_0\right)\right]^2. \tag{3.32}$$

The solutions (3.31) represent kink-type solitary wave and antikink-type solitary wave solutions (depending upon the choice of sign). Moreover, the solutions (3.32) are singular traveling wave solutions. In figure 2 we plot the solution (3.31) in Fig.(2.a) and its position with different values of t in Fig.(2.b), Fig.(2.c) and Fig.(2.d). The evolution graphs indicate that the solutions (3.31) can propagate stable with the parameters $\alpha = \beta = 0.5, k = \xi_0 = 1, \delta = -1$.

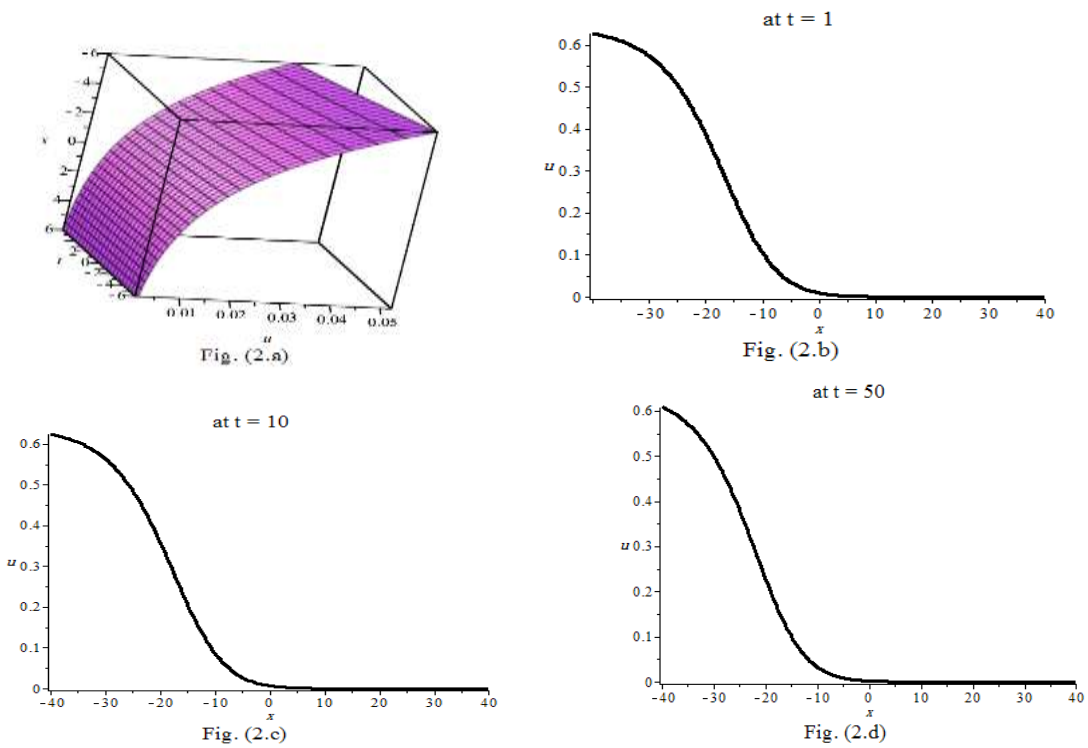

Figure 2: Fig.(2.a) The solution (3.31) with the parameters $\alpha = \beta = 0.5, k = \xi_0 = 1, \delta = -1$ when (+) sign is taken. Fig.(2.b), Fig.(2.c) and Fig.(2.d) The solution (3.31) with these parameters and with $t = 1, t = 10$ and $t = 50$, respectively.

Case 3:

$$c = -\frac{16}{75}\frac{\alpha^2}{\beta}, A_{01} = \pm 2\sqrt{\frac{-3\,\delta}{\beta}}\,S_2\,k, S_1 = \pm\frac{(5\,\beta\,A_0 + 2\,\alpha)}{5\,k\sqrt{-3\,\beta\,\delta}}, S_0 = -\frac{1}{60}\frac{(5\,\beta\,A_0 + 4\,\alpha)A_0}{\delta\,k^2 S_2},$$

$$A_0 = A_0, A_{10} = B_{10} = B_{01} = 0, R_1 = R_1, R_2 = R_2, S_2 = S_2, k = k. \tag{3.33}$$

In this case we get the solutions (3.31) and (3.32).

Case 4:

$$c=\frac{4\,(25\,\beta^2 A_{01}B_{01}-4\,\alpha^2)}{75\,\beta},A_0=\frac{-4\,\alpha}{5\,\beta},A_{01}=A_{01},B_{01}=B_{01},S_0=\pm\,\frac{\sqrt{\frac{-3\,\beta}{\delta}}\,B_{01}}{6\,k},$$

$$S_1=\pm\,\frac{2\,\alpha}{5\,k\,\sqrt{-3\,\beta\,\delta}},S_2=\pm\,\frac{\sqrt{\frac{-3\,\beta}{\delta}}\,A_{01}}{6\,k},A_{10}=B_{10}=0,R_1=R_1,R_2=R_2,k=k. \quad (3.34)$$

From Eq. (3.34) and Eq. (3.26) with Eq. (3.23), we construct the new traveling wave solutions of Eq. (3.22)

$$u(x,t)=\left[\frac{(4\alpha^2-25\beta^2A_{01}B_{01})\;sech^{\,2}\left(\frac{\sqrt{\frac{75\,\beta^2A_{01}B_{01}-12\,\alpha^2}{\beta\,\delta\,k^2}}}{30}\,\xi+\xi_0\right)}{5\delta\sqrt{\frac{-\beta}{3\,\delta}}\,\left(\mp\,\beta\sqrt{\frac{75\,\beta^2A_{01}B_{01}-12\,\alpha^2}{\beta\,\delta}}\tanh\left(\frac{\sqrt{\frac{75\,\beta^2A_{01}B_{01}-12\,\alpha^2}{\beta\,\delta\,k^2}}}{30}\,\xi+\xi_0\right)+2\,\alpha\,\sqrt{\frac{-3\,\beta}{\delta}}\right)}\right]^2, \quad (3.35)$$

$$u(x,t)=\left[\frac{(25\beta^2A_{01}B_{01}-4\alpha^2)\;csch^{\,2}\left(\frac{\sqrt{\frac{75\,\beta^2A_{01}B_{01}-12\,\alpha^2}{\beta\,\delta\,k^2}}}{30}\,\xi+\xi_0\right)}{5\delta\sqrt{\frac{-\beta}{3\,\delta}}\,\left(\mp\,\beta\sqrt{\frac{75\,\beta^2A_{01}B_{01}-12\,\alpha^2}{\beta\,\delta}}\coth\left(\frac{\sqrt{\frac{75\,\beta^2A_{01}B_{01}-12\,\alpha^2}{\beta\,\delta\,k^2}}}{30}\,\xi+\xi_0\right)+2\,\alpha\,\sqrt{\frac{-3\,\beta}{\delta}}\right)}\right]^2. \quad (3.36)$$

Case 5:

$$c=-\frac{16}{75}\,\frac{\alpha^2}{\beta},A_0=-\frac{2}{5}\,\frac{\alpha}{\beta},\;B_{01}=B_{01},\;A_{01}=\frac{\alpha^2}{25\,\beta^2\,B_{01}},\;S_0=\pm\,\frac{B_{01}}{2\,k\,\sqrt{\frac{-3\,\delta}{\beta}}},\;S_1=0,$$

$$S_2=\pm\,\frac{\alpha^2\,\sqrt{\frac{-3}{\beta\,\delta}}}{150\,B_{01}\,k\,\beta},\;A_{10}=B_{10}=0,\;R_1=R_1,\;R_2=R_2,\;k=k. \quad (3.37)$$

Substituting Eq. (3.37) into Eq. (3.26) with Eq. (3.23), we have the soliton -like solutions of Eq. (3.22)

$$u(x,t)=\frac{\alpha^2}{25\beta^2}\left[2\mp\left(\tanh\left(\frac{\alpha}{10\sqrt{-3\beta\delta}}\left(x+\frac{16\alpha^2}{75\beta}t\right)+\xi_0\right)+\coth\left(\frac{\alpha}{10\sqrt{-3\beta\delta}}\left(x+\frac{16\alpha^2}{75\beta}t\right)+\xi_0\right)\right)\right]^2. \quad (3.38)$$

Case 6:

$$c=-\frac{16}{75}\,\frac{\alpha^2}{\beta},\;A_{10}=\pm\,2\,\delta\,k\,R_2\,\sqrt{\frac{-3}{\beta\,\delta}},\;R_1=\pm\,\frac{2\,\alpha\,\sqrt{\frac{-3}{\beta\,\delta}}}{15\,k},\;S_0=S_0,\;S_1=S_1,\;S_2=S_2,$$

$$A_0=A_{01}=B_{10}=B_{01}=0,\;R_2=R_2,\;k=k. \quad (3.39)$$

Using Eq. (3.39) and Eq. (3.26) with Eq. (3.23), we obtain the following traveling wave solutions of Eq. (3.22):

$$u(x,t) = \left[\frac{\mp 12\, R_2 \alpha\, k}{\beta\left(2\,\alpha \sqrt{\frac{-3}{\beta\,\delta}}\, \exp\left(\pm\left(\frac{2\,\alpha\sqrt{\frac{-3}{\beta\,\delta}}\,\xi \pm 15\,\xi_0 k}{15\,k} \right)\right) \pm 15\, R_2 k \right)} \right]^2, \tag{3.40}$$

where R_2 is an arbitrary constant.

Case 7:

$$c = -\frac{16}{75}\,\frac{\alpha^2}{\beta},\; A_0 = \frac{-4\,\alpha}{5\,\beta},\; A_{10} = \pm\, 2\,k\,R_2\sqrt{\frac{-3\,\delta}{\beta}},\; R_1 = \pm\,\frac{2\,\alpha}{5\,\beta\,k\sqrt{\frac{-3\,\delta}{\beta}}},\; S_0 = S_0,\; S_1 = S_1,$$

$$S_2 = S_2,\; A_{01} = B_{10} = B_{01} = 0,\; R_2 = R_2,\; k = k. \tag{3.41}$$

Using Eq. (3.41) into Eq. (3.26) with Eq. (3.23), we get new soliton solutions of Eq. (3.22) as,

$$u(x,t) = \left[\frac{\mp 8\,\alpha^2\sqrt{3}\, \exp\left(\frac{15\,k\,\xi_0\sqrt{-\delta\,\beta} \pm 2\,\alpha\,\sqrt{3}\,\xi}{15\,k\sqrt{-\delta\,\beta}} \right)}{5\,\beta\left(15\,k\,R_2\sqrt{-\delta\,\beta} \pm 2\,\alpha\,\sqrt{3}\, \exp\left(\frac{15\,k\,\xi_0\sqrt{-\delta\,\beta} \pm 2\,\alpha\,\sqrt{3}\,\xi}{15\,k\sqrt{-\delta\,\beta}} \right)\right)} \right]^2, \tag{3.42}$$

where R_2 is an arbitrary constant.

3.3 The S-KdVB equation

The S-KdVB equation is given as

$$u_t + \alpha\, u^{\frac{1}{2}}\, u_x + \beta\, u_{xx} + \delta\, u_{xxx} + u\, u_x = 0, \tag{3.43}$$

where α, β and δ are constants. The S-KdVB equation containing a square root nonlinearity describes the nonlinear propagation of ion-acoustic shocks in a dusty plasma with dust charge fluctuations and small deviation from isothermally of electrons [30]. Using the transformation (3.23) into S-KdVB equation, we get

$$-c\, v\, v' + \alpha\, v^2\, v' + \beta\, k\, ((v')^2 + v\, v'') + \delta\, k^2 (v\, v''' + 3\, v'\, v'') + v^3\, v' = 0. \tag{3.44}$$

Integrating this equation with respect to ξ and setting the integration constant equal to zero, we have

$$-6\, c\, v^2 + 4\,\alpha\, v^3 + 12\,\beta\, k\, v\, v' + 12\,\delta\, k^2 ((v')^2 + v\, v'') + 3\, v^4 = 0. \tag{3.45}$$

Balancing v^4 and $v\,v''$ in Eq. (3.45), we obtain $N = 1$. Thus, we get the general formal solution as in Eq. (3.26). Substituting Eq. (3.26) into Eq. (3.45) with Eq. (2.4) and Eq. (2.5), the left hand side of Eq. (3.45) becomes a polynomial in $\phi(\xi)\,\psi(\xi)$. Equating the coefficients of $\phi^i(\xi)\,\psi^j(\xi)$ to zero, yields a system of algebraic equations for $A_0, A_{10}, A_{01}, B_{10}, B_{01}, R_1, R_2, S_0, S_1, S_2, k$ and c that can be solved to get the following results:

Case 1:

$$c = \frac{2\,(\mp\beta\sqrt{-3\,\delta}\,(14\,\alpha^2\,\delta+9\,\beta^2)+24\,\alpha^3\delta^2+21\,\alpha\,\beta^2\delta\,)}{75\,\delta\,(\pm\sqrt{-3\,\delta}\beta-3\,\alpha\,\delta)}, B_{01} = \pm 2\sqrt{-3\,\delta}S_0 k, A_0 = A_0, S_0 = S_0,$$

$$S_1 = \frac{\sqrt{-3\,\delta}\beta\pm\delta\,(2\,\alpha+5\,A_0)}{5\,k\delta\sqrt{-3\,\delta}}, S_2 = \frac{A_0(\pm 2\sqrt{-3\,\delta}\alpha\,\beta\mp 5\sqrt{-3\,\delta}\beta\,A_0+12\,\delta\,\alpha^2+15\,\delta\,A_0\alpha+6\,\beta^2)}{60\,k^2 S_0\delta\,(\pm\sqrt{-3\,\delta}\beta-3\,\alpha\,\delta)},$$

$$A_{10} = A_{01} = B_{10} = 0, R_1 = R_1, R_2 = R_2, k = k. \tag{3.46}$$

Substituting Eq. (3.46) into Eq. (3.26) with Eq. (3.23), we get the exact solutions of Eq. (3.43) as,

$$u(x,t) = \left[-\frac{(\,[3\alpha\,(-\delta)^{\frac{3}{2}}\mp\beta\delta\sqrt{3}\,]\,k\sqrt{\lambda}\tanh(\frac{\sqrt{3\lambda}}{30}\xi+\xi_0)+\sqrt{-3\delta}\alpha\,\beta\pm 6\delta\alpha^2\pm 3\beta^2\,)A_0\delta}{(\sqrt{-3\,\delta}\beta\mp 3\alpha\,\delta\,)(\pm k(-\delta)^{\frac{3}{2}}\sqrt{\lambda}\tanh(\frac{\sqrt{3\,\lambda}}{30}\xi+\xi_0)\mp\sqrt{-3\,\delta}\beta-2\,\alpha\,\delta-5\,\delta\,A_0\,)}\right]^2, \tag{3.47}$$

$$u(x,t) = \left[-\frac{(\,[3\alpha\,(-\delta)^{\frac{3}{2}}\mp\beta\delta\sqrt{3}\,]\,k\sqrt{\lambda}\coth(\frac{\sqrt{3\lambda}}{30}\xi+\xi_0)+\sqrt{-3\delta}\alpha\,\beta\pm 6\delta\alpha^2\pm 3\beta^2\,)A_0\delta}{(\sqrt{-3\,\delta}\beta\mp 3\alpha\,\delta\,)(\pm k(-\delta)^{\frac{3}{2}}\sqrt{\lambda}\coth(\frac{\sqrt{3\,\lambda}}{30}\xi+\xi_0)\mp\sqrt{-3\,\delta}\beta-2\,\alpha\,\delta-5\,\delta\,A_0\,)}\right]^2, \tag{3.48}$$

where

$$\lambda = \frac{8\,\alpha^2\beta\,\delta\sqrt{-3\,\delta}+3\,\beta^3\sqrt{-3\,\delta}\pm 12\,\alpha^3\delta^2\pm 3\,\alpha\,\beta^2\delta}{\delta^2 k^2(\sqrt{-3\,\delta}\beta\mp 3\,\alpha\,\delta)}. \tag{3.49}$$

We plot the position of the solution (3.47) with the parameters $\alpha = 0.5, \beta = 0.3, k = \xi_0 = 1, \delta = -1, A_0 = 2$ and different values of t in figure 3. We show that this solution can propagate stable.

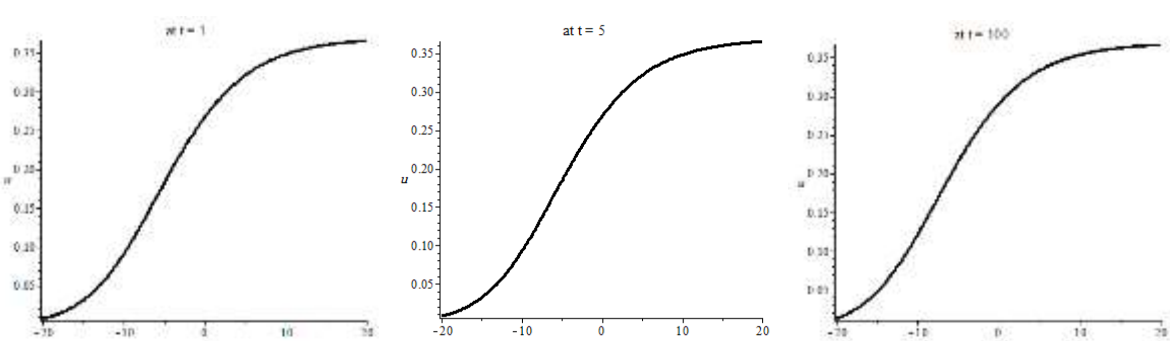

Figure 3: The solution (3.47) studied when (+) sign is taken with the parameters $\alpha = 0.5, \beta = 0.3, k =$

$\xi_0 = 1, \delta = -1, A_0 = 2$ and with $t = 1, t = 5$ and $t = 100$, respectively.

Case 2:

$$c = \frac{2\,(\mp 2\sqrt{-3\,\delta}\alpha\,\beta - 8\,\delta\,\alpha^2 - 9\,\beta^2)}{75\,\delta},\ A_{01} = \pm\, 2\sqrt{-3\,\delta}\,S_2\,k,\ S_1 = \frac{\mp 2\sqrt{-3\,\delta}\alpha - 3\,\beta}{15\,k\,\delta},$$

$$A_0 = A_{10} = B_{10} = B_{01} = 0,\ R_1 = R_1,\ R_2 = R_2,\ S_0 = 0,\ S_2 = S_2,\ k = k. \qquad (3.50)$$

From Eq. (3.50) and Eq. (3.26) with Eq. (3.23), we get the solutions of Eq. (3.43) as,

$$u(x,t) = \frac{-1}{75\,\delta}\,(2\sqrt{-3\,\delta}\,\alpha\ \pm\ 3\,\beta\,)^2\,[\,\tanh(\frac{(2\sqrt{-3\,\delta}\,\alpha \pm 3\,\beta)\xi}{30\,k\delta} + \xi_0\,) \mp\ 1\,]^2, \qquad (3.51)$$

$$u(x,t) = \frac{-1}{75\,\delta}\,(2\sqrt{-3\,\delta}\,\alpha\ \pm\ 3\,\beta\,)^2\,[\,\coth(\frac{(2\sqrt{-3\,\delta}\,\alpha \pm 3\,\beta)\xi}{30\,k\delta} + \xi_0\,) \mp\ 1\,]^2. \qquad (3.52)$$

These solutions are kink-type solitary wave solutions and singular traveling wave solutions, respectively. It is obvious that these types of solutions arise due to the combined effect of the nonlinear term containing α and the dissipative term containing β. In figure 4, we plot the position of the solution (3.51) with the parameters $\alpha = \beta = 0.5, k = \xi_0 = 1, \delta = -1$ with different values of t. We show that the solitary wave solution (3.51) propagates stable.

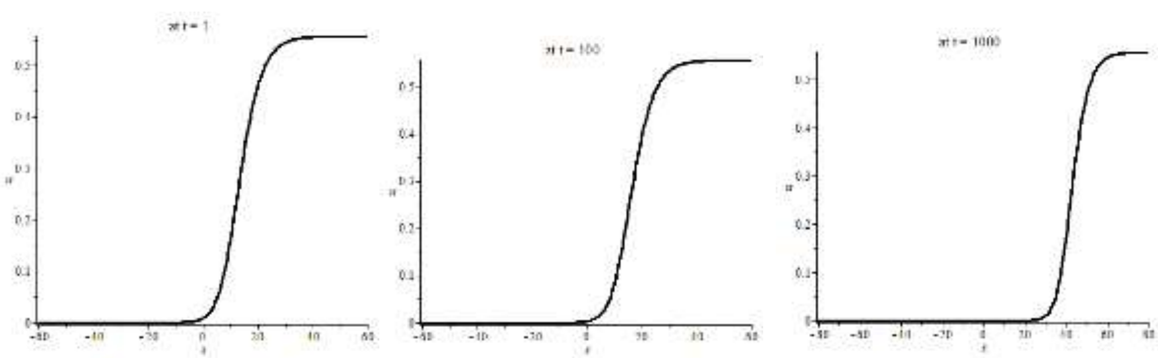

Figure 4: The position of the solution (3.51) when (+) sign is taken with the parameters $\alpha = \beta = 0.5, k = \xi_0 = 1, \delta = -1$ with $t = 1, t = 100$ and $t = 1000$, respectively.

Case 3:

$$c = \frac{2\,(\mp\,\beta\sqrt{-3\,\delta}\,(14\,\alpha^2\delta + 9\,\beta^2) - 24\,\alpha^3\delta^2 - 21\,\alpha\,\beta^2\delta\,)}{75\,\delta\,(\pm\sqrt{-3\,\delta}\beta + 3\,\alpha\,\delta)},\ A_0 = A_0,\ A_{01} = \pm\, 2\sqrt{-3\,\delta}\,S_2\,k,$$

$$S_0=\frac{A_0(\pm 2\sqrt{-3\,\delta}\alpha\,\beta\mp 5\sqrt{-3\,\delta}\beta\,A_0-12\,\delta\,\alpha^2-15\,\delta\,A_0\alpha-6\,\beta^2)}{60\,k^2S_2\delta\,(\pm\sqrt{-3\,\delta}\beta+3\,\alpha\,\delta)},\ A_{10}=B_{10}=B_{01}=0,$$

$$S_1=\frac{-\beta\sqrt{-3\,\delta}\pm\delta\,(2\,\alpha+5\,A_0)}{5\,k\,\delta\sqrt{-3\,\delta}},\ R_1=R_1,\ R_2=R_2,\ S_2=S_2,\ k=k. \tag{3.53}$$

By setting these values in Eq. (3.26) with Eq. (3.23), we obtain the kink-type solutions and singular solutions of Eq. (3.43)

$$u(x,t)=\left[\frac{\mp(\ k(-\delta)^{\frac{3}{2}}\sqrt{\lambda}\tanh(\frac{1}{30}\sqrt{3\,\lambda}\,\xi+\xi_0)+\sqrt{-3\,\delta}\,\beta\pm 2\,\alpha\,\delta\ )}{5\,\delta}\right]^2, \tag{3.54}$$

$$u(x,t)=\left[\frac{\mp(\ k(-\delta)^{\frac{3}{2}}\sqrt{\lambda}\coth(\frac{1}{30}\sqrt{3\,\lambda}\,\xi+\xi_0)+\sqrt{-3\,\delta}\,\beta\pm 2\,\alpha\,\delta\ )}{5\,\delta}\right]^2, \tag{3.55}$$

where λ given by Eq. (3.49).

Case 4:

$$c=\frac{-2\,(3\,\beta\,(\beta\pm\frac{2}{3}\sqrt{-3\,\delta}\,\alpha)+8\,\delta\,\alpha^2+6\,\beta^2)}{75\,\delta},\ A_{10}=\pm 2\sqrt{-3\,\delta}\,R_2\,k,\ R_1=\frac{\beta\pm\frac{2}{3}\sqrt{-3\,\delta}\,\alpha}{5\,k\,\delta},$$

$$A_0=A_{01}=B_{10}=B_{01}=0,\ R_2=R_2,\ k=k,\ S_0=S_0,\ S_1=S_1,\ S_2=S_2. \tag{3.56}$$

From Eq. (3.56) and Eq. (3.26) with Eq. (3.23) the solutions of Eq. (3.43) become,

$$u(x,t)=\left[\frac{\pm 2\,R_2\sqrt{-3\,\delta}\ \ (\beta\pm\frac{2}{3}\sqrt{-3\,\delta}\alpha)}{5\,\delta\left(R_2+\frac{\left(\beta\pm\frac{2}{3}\sqrt{-3\,\delta}\alpha\right)\ exp\left(\frac{(\beta\pm\frac{2}{3}\sqrt{-3\,\delta}\alpha)\xi}{5\,k\delta}+\xi_0\right)}{5\,k\delta}\right)}\right]^2, \tag{3.57}$$

where R_2 is an arbitrary constant. This family of solutions describes a new exact solutions.

Case 5:

$$c=\frac{2\,\alpha^6\beta^6[\alpha\sqrt{3}\,(\frac{72\,(-\delta)^{\frac{7}{2}}}{\beta^6}+\frac{69\,(-\delta)^{\frac{5}{2}}}{\alpha^2\beta^4}-\frac{290\,(-\delta)^{\frac{3}{2}}}{\alpha^4\beta^2}-\frac{47\,(-\delta)^{\frac{1}{2}}}{\alpha^6})\mp\beta\,\delta^3(\frac{306}{\beta^6}+\frac{345}{\alpha^2\beta^4\delta}-\frac{292}{\alpha^4\beta^2\delta^2}+\frac{9}{\alpha^6\delta^3})]}{75\,\delta\,[9\sqrt{3}(-\delta)^{\frac{5}{2}}\alpha^5+30\sqrt{3}(-\delta)^{\frac{3}{2}}\alpha^3\beta^2+5\sqrt{-3\,\delta}\alpha\,\beta^4\pm(45\alpha^4\beta\,\delta^2-30\alpha^2\beta^3\delta+\beta^5)]}$$

$$A_0 = \frac{\pm 2\,(\sqrt{-3\,\delta}\,\beta \mp 2\,\alpha\,\delta)}{5\,\delta},\ A_{10} = -\frac{6\,k\,R_2(\sqrt{-3\,\delta}\alpha \pm \beta)\delta}{\sqrt{-3\,\delta}\beta \mp 3\,\alpha\,\delta},\ A_{01} = B_{10} = B_{01} = 0,\ R_2 = R_2,$$

$$R_1 = \frac{-(7\,\alpha^2\beta\,\sqrt{3}(-\delta)^{\frac{3}{2}} \pm 6\,\alpha^3\delta^2 + \beta^3\sqrt{3}\sqrt{-\delta} \mp 8\,\alpha\,\beta^2\delta)}{5\,\delta\,k(3\,\sqrt{3}(-\delta)^{\frac{3}{2}}\alpha^2 + \sqrt{3}\sqrt{-\delta}\beta^2 \mp 6\,\alpha\,\beta\,\delta\,)},\ S_0 = S_0,\ S_1 = S_1,\ S_2 = S_2,\ k = k. \quad (3.58)$$

From Eq. (3.58) and Eq. (3.26) with Eq. (3.23), we obtain the traveling wave solutions of Eq. (3.43) as

$$u(x,t) = \left[A_0 + \frac{A_1 R_1}{R_2 + R_1\,\exp(\,\xi\,R_1 + \xi_0\,)}\right]^2. \quad (3.59)$$

Case 6:

$$c=\frac{-2\alpha^7\beta^7\delta^4[\alpha\beta\sqrt{-3\,\delta}(\pm(\frac{192}{\beta^8}+\frac{528}{\beta^6\delta\,\alpha^2}+\frac{460}{\alpha^4\beta^4\delta^2}+\frac{177}{\alpha^6\beta^2\delta^3}+\frac{27}{\alpha^8\delta^4}))-(\frac{144}{\beta^6}+\frac{424}{\alpha^2\beta^4\delta}+\frac{297}{\alpha^4\beta^2\delta^2}+\frac{63}{\alpha^6\delta^3})]}{75\,\delta\,[\pm\sqrt{-3\,\delta}(24\,\alpha^6\delta^3+30\,\alpha^4\beta^2\delta^2+15\,\alpha^2\beta^4\delta+3\,\beta^6)-36\,\alpha^5\beta\,\delta^3-35\,\alpha^3\beta^3\delta^2-9\,\alpha\,\beta^5\delta]}$$

$$A_0=\frac{\mp\sqrt{-3\,\delta}\beta-2\,\alpha\,\delta}{5\,\delta},\ A_{01}=\frac{\pm 4\sqrt{-3\,\delta}\alpha^3\delta\pm\sqrt{-3\,\delta}\alpha\,\beta^2-8\,\alpha^2\beta\,\delta-3\,\beta^3}{200\,k\,S_0\delta\,(\pm\sqrt{-3\,\delta}\beta-3\,\alpha\,\delta)},\ B_{01}=\pm 2\sqrt{-3\,\delta}S_0k,$$

$$S_2=\frac{\pm 28\sqrt{-3\,\delta}\alpha^3\beta\,\delta+24\,\alpha^4\delta^2\pm 9\sqrt{-3\,\delta}\alpha\,\beta^3-18\,\alpha^2\beta^2\delta-9\,\beta^4}{1200\,k^2S_0\delta^2(\pm\sqrt{-3\,\delta}\alpha\,\beta+6\,\delta\,\alpha^2+3\,\beta^2)},\ A_{10}=B_{10}=0,\ R_1=R_1,$$

$$R_2=R_2,\ S_0=S_0,\ S_1=0,\ k=k. \tag{3.60}$$

In this case, we get the soliton-like solutions of Eq. (3.43) as

$$u(x,t)=\left[A_0+\frac{A_{01}\,S_0S_2\tanh(\sqrt{-S_0S_2}\,\xi+\xi_0)-B_{01}\,S_2^2\coth(\sqrt{-S_0S_2}\,\xi+\xi_0)}{S_2\sqrt{-S_0S_2}}\right]^2, \tag{3.61}$$

$$u(x,t)=\left[A_0+\frac{A_{01}\,S_0S_2\coth(\sqrt{-S_0S_2}\,\xi+\xi_0)-B_{01}\,S_2^2\tanh(\sqrt{-S_0S_2}\,\xi+\xi_0)}{S_2\sqrt{-S_0S_2}}\right]^2, \tag{3.62}$$

$$u(x,t)=\left[A_0+\frac{A_{01}\,S_0S_2\tan(\sqrt{S_0S_2}\,\xi+\xi_0)+B_{01}\,S_2^2\cot(\sqrt{S_0S_2}\,\xi+\xi_0)}{S_2\sqrt{S_0S_2}}\right]^2, \tag{3.63}$$

$$u(x,t)=\left[A_0+\frac{A_{01}\,S_0S_2\cot(\sqrt{S_0S_2}\,\xi+\xi_0)+B_{01}\,S_2^2\tan(\sqrt{S_0S_2}\,\xi+\xi_0)}{S_2\sqrt{S_0S_2}}\right]^2. \tag{3.64}$$

3.4 The Schamel equation

Let us consider the Schamel equation [31] modeling dust ion-acoustic waves in plasmas

$$u_t+u^{\frac{1}{2}}\,u_x+\delta\,u_{xxx}=0, \tag{3.65}$$

where δ is a constant. Schamel [31] derived this equation and a simple solitary wave solution of having $sech^4$ profile has obtained. We use the transformation (3.23) to derive some exact solutions of Schamel equation. Thus, Eq. (3.65) reduces to

$$-c\,v\,v'+v^2\,v'+\delta\,k^2(v\,v'''+3\,v'\,v'')=0. \tag{3.66}$$

Integrating Eq. (3.66) with respect to ξ and putting the integration constant equal to zero, we get

$$-3\,c\,v^2+2\,v^3+6\,\delta\,k^2((v')^2+v\,v'')=0. \tag{3.67}$$

We note that this equation is similar to Eq. (3.5). Similarly, using the extended Kudryashov method we can get many types of solutions for the Schamel equation.

The solitary wave solution of Schamel equation is obtained by introducing the conditions $u \to 0,\ u' \to 0$ and $u'' \to 0$ as $|\xi| \to \infty$. Therefore, we can obtain from Eq. (3.67) the following ODE:

$$\frac{1}{2}\,(u')^2 = -F(u) = \frac{c\,u^2}{2\,\delta\,k^2} - \frac{4}{15\,\delta\,k^2}\,u^{\frac{5}{2}}.$$

This equation is an energy equation of classical particle which is known as Sagdeev potential equation. The solitary wave solution of the energy equation reads

$$u = u_0\ sech^{\,4}(k\,(x - c\,t)),$$

where the width of the wave $\frac{1}{k}$ and the amplitude u_0 are $4\sqrt{\frac{\delta}{c}}$ and $\frac{225\,c^2}{64}$, respectively. It is seen that as the speed c increase, the amplitude increase while the width decrease. This solution derives the shape of a compressive nature and observations made in space [32].

Remark 1: The solution (3.18) coincides with the solution given in [26]. Note that the solution (3.31) is exactly the same solutions obtained by Khater and Hassan [28] and Hassan [33], [34]. The solutions (3.40) have similar structures to the solutions given in [35]. Also, we note that the solutions (3.31), (3.38) and (3.40) are coincide with the solutions given in [34], [35] and other obtained solutions of S-KdV equation are new. In [36], nonplanar Schamel Burgers equation is derived.

Remark 2: The solutions (3.51) are the same as the results obtained in [30], [33] and other traveling wave solutions of the S-KdVB equation, to the best of our knowledge, are new. If the value of β in (3.50) is very small, the solutions (3.51) will be close to exact solutions defined by (3.31). Solutions obtained in this paper have checked by Maple software.

CONCLUSION

In this paper, the extended Kudryashov method is used to investigate exact solutions of S-NLS equation, S-KdV equation, S-KdVB equation and the Schamel equation. The exact solutions in solitary wave, soliton and soliton-like are obtained for these equations, which have several applications in plasma physics and may be useful for studying the physical interpolation of each equation. Moreover, we compared our results with some of existing results in the literature. Although, some of exact solutions of S-NLS equation, S-KdV equation and S-KdVB equation reported in literature, we observe that some of our results are newly constructed.

Our new solutions insure that the extended Kudryashov method can be used to construct many new exact solutions of NLEEs. Also, we note that some solutions may be develop singularity at a finite point. Often bell shaped sech solution and kink shaped tanh solution model wave phenomena in plasma. The results show that the exact solutions (3.31), (3.47) and (3.51) in terms of tanh function can propagate stable.

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