# Mathematical Encoding in Pre-historic Architecture:Case Study of the Pyramid of Cheops 

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#### Abstract

: Justifying the geometric representation of the Pyramid of Cheops of Giza in Egypt is of great concern to studies on the prehistoric intelligence of a great civilization that used to inhabit the ancient world. This research aimed to define the encoded values of the Pyramid of Cheops in its inter-relation with depicting the Pyramid's volume derived from its building block arrangement without imposing the conventional studies made elsewhere.

We may conclude that construction of the Pyramid of Cheops of Giza in Egypt relates to the graphical representation of any plane of its horizontal building blocks with respect to the distribution of the blocks and the general depiction of the Pyramid's form. (Ghoussayni, Findings of Patterns in Prehistoric Architecture, 2018)

One Sentence Summary: The study contributes a sequence of findings based on a simple justification for the geometry of the Pyramid of Cheops, derived from the patterns of its building blocks and their distribution, from the apex down, as a direct indicator on the graphical representation of the main section of the Pyramid of Cheops of Giza. Main Text: 'The term 'true' pyramid is used to differentiate the true ancient Egyptian pyramids from other more common step pyramids, found all over the world. The fact [is] that the Egyptian pyramids are the only true pyramids in the world culminating in an Apex point 1' (Ole Jørgen Bryn, 2010).


The challenging nature of the geometric representation of the Pyramid of Cheops highlights some founding figures for this depiction, which is explained in the sequence of sketches and diagrams in thispaper.
'Historically, the Egyptian pyramids were regarded as piles of stone that needed to be moved one on top of the other. In his otherwise brilliant book, Lehner states (Ole Jørgen Bryn, 2010) this assumption of only any kind of piling has been shown as untrue as the arrangement of blocks of stone on a horizontal plane according to a geometric organization is the factor acting on the geometric figure of the Cheops Pyramid itself.
'The small piece of desert plateau opposite the village of Gizeh, though less than a mile across, may well claim to be the most remarkable piece of ground in the world. There may be seen the very beginning of architecture, the most enormous piles of building ever raised, the most accurate constructions known, the finest masonry, and the employment of the most ingenious tools' (Petrie, 1990).

The main finding in this paper is that the graphical coordinates of points laid on the surface of the Cheop's Pyramid are in constant relation with the dimension of Pyramid's horizontal section as those points predict the volume of Cheop. This can be obtained by knowing the dimension of each of its block that should be similar or equal to 1 Royal Cubit in each direction.

The horizontal length of each layer is proven an equal to the number of blocks on that horizontal layer multiplied by a constant factor all through the sloping area by which the reported diagram of the section using its area will depict the total number of blocks in the whole Pyramid by which will be able to calculate the volume of the whole Pyramid.

## The principles

Given that the simplest constitution of the pyramid is a block with square geometry, a two-dimensional composition is the simplest recognized figure in the field of graphical understanding, whereas a void is visualized as an empty square. Symmetrical representation should be in at least four directions, namely, the vertical, horizontal, and diagonal directions. Meanwhile, the orthogonal system necessitates fourdirectional symmetry to attain static representation and by this we mean the balance of the building blocks accumulation one over the other.

Any state that does not constitute four- directional symmetry will definitely be considered a non-static state. The question that arises in this case is whether the empty space cubicle is static or dynamic, as represented in movement. The present study expands the figures and their states to calculate the different schemes, with respect to the consideration of single symmetry or sole dimension as movement and of four-way symmetry or fourdimensional states as static.

Figures 1.0 to 8.0 represent the abovementioned expansion. They likewise indicate how the study will attain results to determine the notion of the voidwithin the pyramid and its characteristics.

To start with the graphical presentation of the results the study has obtained from the configuration of equal and stable alignments of the different placements of blocks and their underlying progression the study has set below the findings.

| 0 | 0 | 0 | 0 | 0 | $\pi$ | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | $\pi$ | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | $\pi$ | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Fig. 1.0 Step number one ( $\mathrm{Y}=1$ denoted as a dark pixel) is in static state (four-directional symmetry).

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 2 |  | 2 | 0 | 0 |
| 0 | 0 |  | 2 |  | 0 | 0 |
| 0 | 0 | 2 |  | 2 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Fig. 2.0 Step number two (denoted as $\mathrm{X}=2$ ), which has an addition of four dark pixels added to step numberone ( $\mathrm{Y}=1+4=$ 5), is in static state (four-directional symmetry).

| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $"$ | 3 |  | 4 |  | 3 | 0 |
| 0 |  | 2 |  | 2 |  | 0 |
| 0 | 4 |  | 2 |  | 4 | 0 |
| 0 |  | 2 |  | 2 |  | 0 |
| 0 | 3 |  | 4 |  | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Fig. 3.0 Step number four (denoted as $\mathrm{X}=4$ ) has an addition of eight dark pixels to step number $\mathrm{two}(\mathrm{Y}=5+8=13)$ and is in static state (four-directional symmetry).


Fig. 4.0 Step number seven (denoted as $\mathrm{X}=7$ ) has an addition of 12 dark pixels to step number four $(\mathrm{Y}=13+12=25)$ and is in static state (four-directional symmetry).

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 3 |  | 0 |  | 3 | 0 |
| 0 |  | 2 |  | 2 |  | 0 |
| 0 | 0 |  | 2 |  | 0 | 0 |
| 0 |  | 2 |  | 2 |  | 0 |
| 0 | 3 |  | 0 |  | 3 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Fig. 5.0 Step number three (denoted as $\mathrm{X}=3$ ) has an addition of four dark pixels to step number two $(\mathrm{Y}=5+4=9)$ and is in static state (four-directional symmetry).

| $\because$ | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\because$ | 3 |  | 4 |  | 3 | 0 |
| $\circ$ |  | 2 |  | 2 |  | 0 |
| $\circ$ | 4 |  | 2 |  | 4 | 0 |
| $\because$ |  | 2 |  | 2 |  | 0 |
| $\because$ | 3 |  | 4 |  | 3 | 0 |
| $\square$ | 0 | 0 | 0 | 0 | 0 | 0 |

Fig. 6.0 Step number four (denoted as $\mathrm{X}=4$ ) has an addition of eight dark pixels to step number two $(\mathrm{Y}=5+8=13)$ and is in static state (four-directional symmetry).


Fig. 7.0 Step number five (denoted as $\mathrm{X}=5$ ) has an addition of 12 dark pixels to step number two $(\mathrm{Y}=5+12=17)$ and is in static state (four-directional symmetry).

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Fig. 8.0 Step number six (denoted as $\mathrm{X}=6$ ) has an addition of 16 dark pixels to step number two ( $\mathrm{Y}=5+16=21$ ) and is in static state (four-directional symmetry).
In any form, the symmetrical distribution of infinite negative pixels ( n ) and infinite positive pixels ( $\mathrm{n}+1$ ) is always $2 \mathrm{n}+1$, which denotes four- directional symmetry as true static state (where $\mathrm{Y}=4 \mathrm{X}-3$; ' X ' is the number of steps starting from X $=2$ and ' Y ' is the number of blocks arranged at a single level of a given step). Giventhat ( n ) is always a multiple of four and applicable with respect to the singular positive pixel (Fig. 1.0), any kind of addition of positive pixels ( $\mathrm{n}+1$ ) has to be associated with the number of equal negative pixels ( $n$ ), to maintain symmetry with respect to the original pixel mentioned in Figure 1.0. Thesame logic is applicable in Figures 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, and 8.0. Whenever the singular positive pixel at the center ceases to be exhibited, then the state of infinite pixels will become asymmetrical, as the original positive pixel has to be replaced (as ( $\mathrm{n}+1$ ) will become n ). Consequently, the whole state ( $\mathrm{n}+\mathrm{n}=2 \mathrm{n}$ ) will cease to become symmetrical. In any symmetrical figure, the minimal number of positive pixels has to be zero or four. In all cases, the symmetrical line has to pass through the centre (singular pixel); otherwise, a symmetrical axis cannot be achieved.
Superimposition of above configuration with the Pyramid of Cheops in Giza, Egypt.

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When expanding the figures obtained in the above section and as proof to the importance of the above findings seen as ( $\mathrm{X}, \mathrm{Y}$ ), there comes the statement that so many superimpositions are occurring when overlaying the section of the Pyramid of Giza (Cheops) with the graphical representation resulting from the hypothetical shapes of different configurations and states of figure with 'true pyramid' marks. (Ghoussayni, Findings of Patterns in Pre-historic Architecture, 2018), see Fig. 9.0


Fig. 9.0 Demonstration of the superimpositions(Ghoussayni, Findings of Patterns in Pre-historic Architecture, 2018)

The demonstration in Figure 9.0 shows the first superimposition as the alignment between the graph of Cheops's section (having an angle of $76^{\circ}$ at the apex) when cast upon the graph obtained from the different coordinates, which are in turn obtained from the state figures. The figure shows matched superimpositions. The second superimposition, indicated in X , is the match between the location of the opening/shaft at the face of the Pyramid in the section and state, shown in Figure 8.0, for coordinate (6, 21).

The third superimposition, indicated in X , is the length of the polyline (from different coordinates) when comparing the length using the graph with the length of the polyline created from drawing the length of the tunnel inside the Pyramid. The fourth superimposition obtained, indicated in X, represents the length of distance between coordinates calculated as equal to the length of the polyline taken from the triangular corner of the Pyramid, and then added to the perpendicular line on the second side of the triangle, which in turn coincides with the horizontal alignment taken from the upper level of the upper King's chamber (fifth and sixthsuperimposition).


Fig. 10.0
The different coordinates of points depicted in Figures 1.0 to 8.0 (in blue) and the depiction for the Pyramid of Cheops at 1:5 scale according to royal cubits ( $\mathrm{Y}=4 \mathrm{X}-3$ ) (in magenta)

The square is the basic unit for all formations, and notably summed to five units per general state of any configuration, five is the first primary unit that exhibits four symmetrical alignments. This type of alignment is known as the four- dimensional state, in which propellant units usually for the case of the square unit will have to connect through the corners.

## Expanding the theory

To deal with this issue using the findings in calculating the horizontal base of any triangle derived from the Pyramid's section has been found to match with the number of blocks using that layering by which the study could constitute the number of blocks of the whole pyramid from only obtaining the accumulated number of blocks in every layer of the Pyramid and eventually multiply these figures with the blockswidth and height as an equity to the Pyramid's volume without going through the conventional formula of multiplication of Pyramids base with its height and divided by three (by simple mathematics).
This new technique stands as an alternative method to indicate that any horizontal layer is only in its area as the number of blocks multiplied by the blockarea.
The triangle's area was supposedly unknown but the area of the trapezoid resulted in aligning three identical triangular shapes will result in an area of equal to the area of each triangle multiplied by three (by convention); in other words, if you want to find an area ofthe triangular section you have to divide the area of the trapezoid by three (byconvntion) as shown on fig. 11.0.


Fig. 11.0
Method used to obtain Triangle's Area (Source: simple mathematics)

How to portray our above result with the subject of this writing? Well, you haveto state that $(1.5 \mathrm{~B} / 3)=0.5 \mathrm{~B}$ is exactlythe width of the base of a small triangle having its apex coinciding with the apex of the original triangle but having its base pass through the midpoint of each of the original triangle's sides, which is known to us that this small base is half the distance of the original triangle's base (byconvention).

In our search for the relation between the base of each triangle resulted in drawing same apex triangle with different base width the study found that the width of the base from the midpoints of the original sides down is having the same value as the number of positive blocks at the small base's level (new finding).


Fig. 12.0 Relation between the width of the base with the number of positive blocks at that base (Source: F.Ghoussayni)

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Fig. 13.0 Method used to obtain total of positive and negative blocks (Source: by convention) Which could only mean that if $0.5 \mathrm{~B}=\mathrm{Y}$; i.e:
$\mathrm{Y}=23$, No. of Blocks in each layer is $=\mathrm{Y}^{\wedge} 2+1=529 \mathrm{Y}=34$, No. of Blocks= $\mathrm{Y}^{\wedge} 2+1=1157$
$\mathrm{Y}=111$, No. of Blocks= $\mathrm{Y}^{\wedge} 2+1=12359 \mathrm{Y}=221$, No. of Blocks=$Y^{\wedge} 2+1=48841$

Total No. of Blocks $=\sum\left(\mathrm{Y}^{\wedge} 2\right)($ as $\mathrm{Y}=25$ till $200(=\mathrm{n}(\mathrm{n}+1)(2 \mathrm{n}+1) / 6$ (by convention) where $\mathrm{n}=\mathrm{Y}$ (proven with the case of Pyramid of Cheops)

How to find the sum of $\mathrm{Y}^{\wedge} 2$ ? The study can simply consider the pyramid whose base is equal to Y (proven hereafter) with an Apex of $(10,25)$ hence the sum of $Y^{\wedge} 2=$ sum of $n^{\wedge} 2=2,686,700$ Blocks (where each block is 1x1x1 Royal Cubit) 'How many Quadratic blocks in apyramid of Cheops are supposed to be" by now you could have the answer found.
How to find the volume? You can simply know the No. of Blocks and then multiply by the volume of each block which is equivalent to 1 Royal Cubit by 1 Royal Cubit by 1 Royal Cubit.

Key in explaining existential acuteness and the extent it inhabits the transition. Transitional occupation from the apex down of the Pyramid of Cheops shows matter in its conditional state of balance. This highlights the importance of implementing the patterns of void andmatter depicted by the Pyramid of Cheops as a factor within any equation.

If you are looking for a conclusion on the blank space in the universe, humans may state that darkness is yet built from matter as much as light is but the difference between darkness and light is that darkness doesn't appear withiin the domain of humans.

## Conclusions

The study presented derivative values on balanced states into the depiction of the Pyramid of Cheops. New contributions to the subject confirm the remarkable engineering details that reveal in novel light the true nature of the pyramids.

There can only be honest approaches if they were in the first place made with scientific integrity and mathematical goodness, since such an amazing structure or even a beacon of Architectural truthfulness will have to be studied with upright methodologies and simple practices.

In concluding on the prospects of human nature, try to identify the occupational behavior that replenishes with holding moral significance and deeds that reside in deep.
Space is still considered as the nurturing factor of human observations on distance, time, and velocity. In any experiment, a working platform is needed, one with clear, fixed, and reliable context. To advance from one level to another, a little bit of void in between is needed.

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## References and Notes:

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