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Part I: Modified Buckling Factor for Columns in Braced and Unbraced Frames with Composite Girders

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KEYWORDS:
Stability functions, unbraced frames, The alignment charts, composite frames, braced frames, composite girders, 15% cracked analysis, stiffness coefficients, and sagging moment.

Abstract— **In the current paper, the influence of composite girders on modified buckling factor values was studied in braced and unbraced frames in which the composite girders' far ends were represented as fixed and rigid. Moreover, the derived formulation is based on the modified stiffness parameter and pursues the same assumptions as in the standard effective length factor (K-factor). Furthermore, the slope-deflection method was used to obtain the modified buckling factor formulae for composite girders in braced and unbraced frames. After that, the relationship between the critical buckling factor and beam length has been demonstrated for practical purposes. In addition, the composite girders' effects on effective length parameters are illuminated using illustrative examples that utilize the various girders' far-end conditions. Eventually, we can draw the conclusion that the modified stiffness factor for composite girders must be considered in the calculations of K-factor for braced and unbraced frames in order to achieve an accurate and economical design.**

I. INTRODUCTION

The determination of effective length factor in structural engineering is one of the most spatial applications in the field of second order analysis and members slenderness [1]. Moreover, the buckling length of steel frame columns has a clear influence on the cost of utilizing cross sections as well as the behavior of structural analysis [2]. To calculate the buckling length factor (K), most codes employ alignment charts for braced and sway frames (AISC, 2016), [Egyptian Design Code of Steel Constructions (LRFD), 2008] and [Egyptian Design Code of Steel Constructions (ASD), 2009] [3][4][5]. In addition, K-parameter is used to simplify frame member design by converting an end-restrained compression member to an equivalent pinned-ended member. Furthermore, the effective length factor is determined by numerically calculating the exact equations or utilizing alignment charts (Fig. 1) [6].

In most design rules and specifications, simpler formulas and charts are provided in practical applications to determine the effective lengths of frame columns [7]. Since 1966, simple formulae have been included in the French Design Rules for Steel Structure, and they have since been integrated into the European Recommendation for Steel Construction [8]. Then, the French rule equations are modified in order to get more accurate closed form formulas for calculating the effective length factors in relation to rotational resistance at the column ends [6]. Moreover, a new buckling length factor (K) formula was developed to accurately estimate the stiffness of column ends [2].

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On the basis of the modified approaches, approximate formulae for calculating the effective length factor are derived by changing some of the old approaches' inappropriate assumptions [9]. Furthermore, the modified G factor used in US code alignment charts was determined using the stiffness parameter calculated for girders on elastic foundations with different far end conditions in braced and unbraced frames [1]. In addition, an entirely new approach was proposed by Chen et al.(1993a) for estimating the K-factor for both braced and unbraced columns within a tapered girder frame with diverse far end conditions [10]. Moreover, there is a straightforward method for figuring up the effective column lengths in multi-story steel frames in the NCCI paper SN008a to BS EN1993-1 [11][12]. A simple approach to calculate approximate values for braced frame buckling loads is established [7]. In addition, five alternative boundary conditions for top and bottom columns are considered to obtain the effective length factor formulas therefore columns in braced frames can be designed with more accuracy [13]. A new alignment chart method for calculating an approximate coefficient for unbraced frame column design is suggested [14][15].Furthermore, there were 2,960 braced simple frames subjected to short-term loads that were simulated to see how various ways of calculating the effective length factor (K) affected the calculations of column strength [16].

Nowadays, steel-concrete composite structures are commonly employed in the construction of buildings [17][18][19] and have a large market share in numerous countries [20] [21].Actually, the reasons for this are based on a variety of advantages that may be achieved by combining two distinct materials into a single component: Significantly increased span width-to-height ratio, high load capacity, high rigidity, lower dead weight, fast construction process, superior structural safety at a reasonable cost, and acceptable fire performance [17] [18] [19].

Buildings and bridges often use continuous composite beams as an economical structural solution owing to extra benefits connected with an optimistic redistribution of internal forces across the component and the ease with which serviceability requirements may be satisfied. On the other hand, the design and analysis of continuous composite beams is rather difficult because of the differences in their behavior in the positive (or sagging) and negative (or hogging) moment zones [18][22]. The negative bending moment in the interior support areas of continuous composite beams causes tension in the concrete slab and compression in the steel, which is undesirable in the design. There have been a number of studies dedicated to developing models for evaluating composite beam behavior, the majority of which concentrated on beams with positive bending moments. A few studies focused on the ultimate bearing capacity of composite beams under hogging moment and the crack growth in concrete slabs [17][23]. It's crucial to understand how composite beams behave structurally under negative moments. Hardly have we found experimental studies in this field. There is little information on the shear connection's effectiveness while the slab is under tension. However, the finite element software ANSYS is used to conceptually evaluate available experimental data on composite steel-concrete beams under negative bending [22]. A method for analyzing the service load of continuous composite beams is indicated. In this method, we must do short-term as well as time-dependent assessments, taking into account the concrete slab cracking, creeping, and shrinking [24].

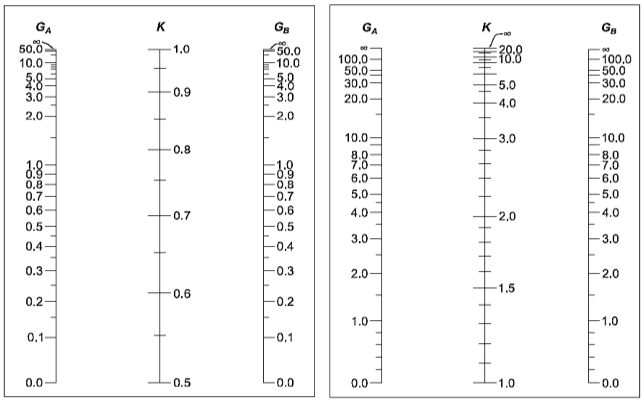
In general, composite girder behavior exhibits significant nonlinearity due to the nonlinearities in each structural component: steel sections, reinforced concrete slabs, and shear connections. Furthermore, nonlinear analysis as well as computer software are essential for the study of composite structures since they are so complicated. Even so, modern design standards like Eurocode 4 (EC4) use linear-elastic analysis with some changes to account for current nonlinearities and make design simpler [17]. When the extreme-fiber tensile stress in concrete reaches twice the mean value of the axial tensile strength specified by EN 1992-1-1 [25], concrete cracking decreases flexural stiffness in hogging moment zones but not in sagging regions. In addition, the variation in relative stiffness must be taken into consideration in elastic analysis. In braced frames, the cracked regions in beams are of fixed extent (Eurocode 4 recommends that the cracked region be 0.15 of the beam length). The extent of cracking in unbraced frames can only be calculated by analysis under design loads by using software [26]. Actually, Eurocode 4 [27] specifies a few straightforward methods for calculating creep, shrinkage, concrete cracking, and shear lag effects. Four girders were numerically evaluated and the results compared using the computer software "Kontinualac". Composite girders are regarded fully connected [28]. As a result, the zone of cracked concrete estimated by "cracked" analysis was significantly less than the length expected in 15% cracked analysis [17].

In this paper, modified column buckling factor formulae for braced and unbraced columns in frames with composite girders are proposed. Moreover, girders' far-end conditions are represented as rigid and fixed. Furthermore, the derivation is performed on continuous composite beams with various flexural stiffness in braced and unbraced frames because the phenomena of concrete cracking decrease flexural stiffness in hogging moment zones but not in sagging regions. In addition, this phenomenon has been taken into account in this derivation. The uncracked and cracked flexural stiffnesses are EI_1 and EI_2, respectively (Fig. 2). I_1: the moment of inertia for steel section (reinforcement is ignored while calculating I_1 in most cases), I_2: the moment of inertia for composite girder, E: Modulus of elasticity as well as L, a and b are lengths indicated in (Fig. 2).

TABLE I

Comparison Between General Equations for The Stability Analysis of Braced and Unbraced Frames with Constant Cross-Section Steel Girders and Proposed Equations for The Stability Analysis of Braced and Unbraced Frames with Composite Girders

Types of frame	General equations	Proposed equations
Braced frame	$\frac{G_A\, G_B}{4}\left(\frac{\pi}{K}\right)^2 + \frac{(G_A+G_B)}{2}\left(1 - \frac{\pi/K}{\tan(\pi/K)}\right) + \frac{2\tan(\pi/K)}{\pi/K} = 1$	$\frac{G_A\, G_B}{4\alpha}\left(\frac{\pi}{K}\right)^2 + \frac{(\alpha G_A+G_B)}{2\alpha}\left(1 - \frac{\pi/K}{\tan(\pi/K)}\right) + \frac{2\tan(\pi/K)}{\pi/K} = 1$
Unbraced frame	$\frac{G_A\, G_B\,(\pi/K)^2-36}{6(G_A+G_B)} = \frac{\pi/K}{\tan(\pi/K)}$	$\frac{G_A\, G_B\,(\pi/K)^2-36\alpha}{6(\alpha G_A+G_B)} = \frac{\pi/K}{\tan(\pi/K)}$



(a) braced frames (b) unbraced frames

Fig. 1. Alignment charts for effective length factor

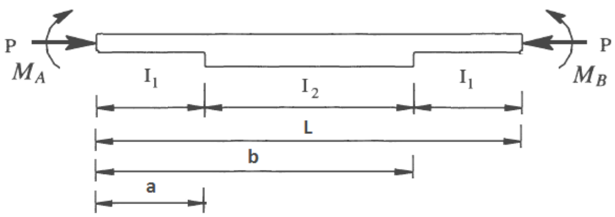


Fig. 2. Composite girder with rigid or fixed far end

In Table I: The two subscripts A and B refer to the points at two ends of the beam-column while G is defined by:

$$G = \frac{\sum (EI/L)_{columns}}{\sum (EI/L)_{beams}}$$

Where: L is the unsupported length of the column.

I is the moment of inertia perpendicular to the plane of buckling of the columns and the beams [4].

The difference between fixed and rigid joint:

The both fixed and rigid joints transfer induced moments, shearing forces, and normal forces from one structural member to another [29]. The fixed joints prevent rotation but the rigid joints permit rotation.

The modified k-factor formulae:

The differential equation for a composite girder is as follows:

$$EI\, y'' = -M_a + \frac{M_a - M_b}{L}\, x - p\, y \qquad (1)$$

Where:

M_a , M_b are bending stiffness.

Stiffness coefficients for fixed and rigid far end:

We obtain the girder stiffness coefficients by solving the previous formula: Using the slope-deflection equations for composite girders with varied far-end conditions in a braced column frame. The notation (parameters N, O, X, and U) will be used in order to simplify the expressions. Parameters N, O, X, and U are demonstrated in detail in the appendix.

$$S_{NN} = \frac{-U}{XO-UN} \qquad (2)$$

$$S_{NF} = \frac{O}{XO-UN} \qquad (3)$$

Braced frame (sway prevented):

The general formula for the stability analysis of a sway-prevented frame with composite girder and variable girder far ends conditions, as determined by (4), is:

$$\frac{G_A\, G_B}{4\alpha}\left(\frac{\pi}{K}\right)^2 + \frac{(\alpha G_A + G_B)}{2\alpha}\left(1 - \frac{\pi/K}{\tan(\pi/K)}\right) + \frac{2\tan(\pi/K)}{\pi/K} = 1 \qquad (4)$$

$$G = \frac{\sum (EI/L)_{columns}}{\sum (EI/L)_{beams}} \qquad (5)$$

The actual value of the stiffness modification factor:

The modification factor for composite girder stiffness in braced frames is determined by dividing the bending stiffness of composite girders by the bending stiffness of conventional girders. The parameter is derived:

When the far end of the girder is fixed:

$$\alpha = \frac{-U}{2(UN-XO)} \qquad (6)$$

When the far end of the girder is rigid:

$$\alpha = \frac{U+O}{2(UN-XO)} \qquad (7)$$

Unbraced frame (sway permitted):

The general formula for the stability analysis of a sway-permitted frame with composite girder and variable girder far ends conditions, as determined by (8), is:

$$\frac{G_A\, G_B\, (\pi/K)^2 - 36\alpha}{6(\alpha G_A + G_B)} = \frac{\pi/K}{\tan(\pi/K)} \qquad (8)$$

$$G = \frac{\sum (EI/L)_{columns}}{\sum (EI/L)_{beams}} \qquad (9)$$

The actual value of the stiffness modification factor:

The modification factor for composite girder stiffness in unbraced frames can be given as:

When the far end of the girder is fixed:

$$\alpha = \frac{U}{6(UN-XO)} \qquad (10)$$

When the far end of the girder is rigid:

$$\alpha = \frac{U-O}{6(UN-XO)} \qquad (11)$$

II. FORMULA VERIFICATION

When the moment of inertia ratio (I_1 / I_2) equals one as well as values of a and b equal zero, the composite girder under sagging moment approaches a constant cross-sectional girder.

Therefore, the modification factors of columns in frames for a restraining composite girder become:

The previous equations are exactly the same as the values proposed in ECP to consider the effects of far end conditions of restraining girders in using the alignment charts as illustrated in table II.

TABLE II
The modification factors of columns in frames for a restraining composite girder with various far end conditions at constant cross-sectional girder approach

Far end condition for composite girder	**Sidesway prevented**	**Sidesway permitted**
Fixed far end	$\alpha =$ 2.0	$\alpha =$ 2/3
Rigid far end	$\alpha =$ 1.0	$\alpha =$ 1.0

III. Modified Buckling Factor (K_{CR}) For Composite Girder With Rigid Far End In Braced And Unbraced Frames

In this work, the effect of the studied parameters on the modified buckling factor (K_{cr}) is summarized as follows:

1) The moment of inertia ratio (I_1 / I_2) for the composite steel girder varied from 0.025 to 0.975 with two cases of braced and unbraced frames.
2) The far end of the composite steel girder is rigid in braced and unbraced frames.
3) The composite steel girder's far end is fixed into braced and unbraced frames.
4) In the steel frames, the column base is fixed, and another one is hinged.

Figs. from 3 to 10 Show that the relation between the modified buckling factor (K_{cr}) and the moment of inertia ratio (I_1 / I_2) is nonlinear for all parameters studied. The modified buckling factor "K_{cr}" grows as the moment of inertia ratio climbs from 0.025 to 0.975. In addition, as the parameter (G_B) is increased, the modified buckling factor (K_{cr}) raises.

(1) The effect of changing the column base from fixed to hinged at the far end of the composite beam is rigid in braced frames:

From Figs. 3 and 4, the parameters such as the far end of the composite beam being rigid, the frame being braced, and the ratio a/L = 0.15 are constant while the column base is changed from fixed to hinged. From the relation between the moment of inertia ratio (I_1 / I_2) and the modified buckling factor (K_{cr}), it is noticed that the modified buckling factor increased from a range of 2.83% to 6.05%. Also, due to the rise in the parameter (G_B) from 0.1 to 5, the (K_{cr}) increases from 26.14% to 31.39%.

(2) The influence of converting the far end of the composite beam from rigid to fixed (Figs. 3 and 5):

The relationship between the modified buckling factor (K_{cr}) and the moment of inertia ratio (I_1 / I_2) for composite beams when the column base is fixed and a/L = 0.15 in braced frames. Regarding Figs. 3 and 5, it can be seen from this relationship that the (K_{cr}) grew from 2.53% to 6.32% with the moment of inertia ratio rose from 0.025 to 0.975. In addition, the modified buckling factor (K_{cr}) raised from 24.96 % to 29.58 % when the parameter (G_B) is increased from 0.1 to 5.

(3) The effect of the base column changed from fixed to hinged while the far end of the composite beam is fixed and the frame is braced.

Concerning Figs. 5 and 6, the parameters such as far end of composite beam is fixed, braced frame and a/L = 0.15 are constant while the base of column is changed from fixed to hinged. A raise from 0.025 to 0.975 in the moment of inertia ratio (I_1 / I_2) for composite beams reveals that (K_{cr}) grows from 2.44% to 6.83%. As demonstrated in Figs. 5 and 6, the (K_{cr}) climbs from 24.96 % to 31.33 % as the parameter (G_B) increases from 0.1 to 5.0.

(4) The influence of the composite beam's far end altered from rigid to fixed as given in Figs. 4 and 6:

The modified buckling factor (K_{cr}) raises in the range of 2.44% to 6.83% owing to a gradual growth in the moment of inertia (I_1 / I_2) for the composite beam, as seen in Figs. 4 and 6. In addition, resulting in an increase of parameter (G_B) from 0.1 to 5, the modified buckling factor goes up from 25.93% to 31.39%.

(5) The effect of the base column converted from fixed to hinged at the far end of the composite beam is rigid in unbraced frames:

As illustrated in Figs. 7 and 8, when the column base is changed from fixed to hinged, the parameters, which include the far end of the composite beam being rigid in unbraced frames and the ratio a/L = 0.06, remain unchanged. Furthermore, the modified buckling factor (K_{cr}) rises from a range of 5.89 % to 25.5 % based on the relationship between the moment of inertia ratio (I_1 / I_2) and the modified buckling factor (K_{cr}). Additionally, the modified buckling factor increases from 52.35 % to 116.87 % as a result of a progressive rise in parameter (G_B) from 1 to 100, as shown in Figs. 7 and 8.

(6) The influence of changing the composite beam's far end from rigid to fixed (Figs. 7 and 9):

In Figs. 7 and 9, the (K_{cr}) goes up from 5.59 % to 24.94 % when the moment of inertia ratio rises from 0.025 to 0.975, the column base is fixed, and a/L = 0.06 in unbraced frames. Furthermore, when the parameter (G_B) was increases from 1 to 100, the modified buckling factor (K_{cr}) raises from 50.79%. to 78.41 %.

(7) The effect of using a hinged base column instead of a fixed one when the composite beam's far end is fixed and the frame is unbraced:

The parameters as with the far end of the composite beam is fixed, the braced frame, and a/L = 0.06 are constant in Figs. 9 and 10, but the base of the column is altered from fixed to hinged. In addition, the moment of inertia ratio (I_1 / I_2) for composite beams increases from 0.025 to 0.975, indicating that (K_{cr}) grows from 5.59 % to 26.73 %. Also, due to the rise in the parameter (G_B) from 1 to 100, the (K_{cr}) increases from 50.79% to 117.89%, as shown in Figs. 9 and 10.

(8) The effect of the composite beam's far end converted from rigid to fixed as illustrated in Figs. 8 and 10:

Because of a significant growth in the moment of inertia (I_1 / I_2) for the composite beam, the modified buckling factor (K_{cr})

rises from 12.92 % to 26.73 %, as shown in Figs. 8 and 10. Furthermore, the modified buckling factor increases from 94.15 % to 117.89 % as a result of steadily rising parameter (G_B) from 1 to 100.

Eventually, it is immediately apparent that the stiffness modification factor (α) (which is indicated in equations (6,7,10 and 11)) depends on the beam length. However, it is found that we can neglect the beam length in the calculation of the stiffness modification when the moment of inertia ratio (I_1 / I_2) becomes constant.

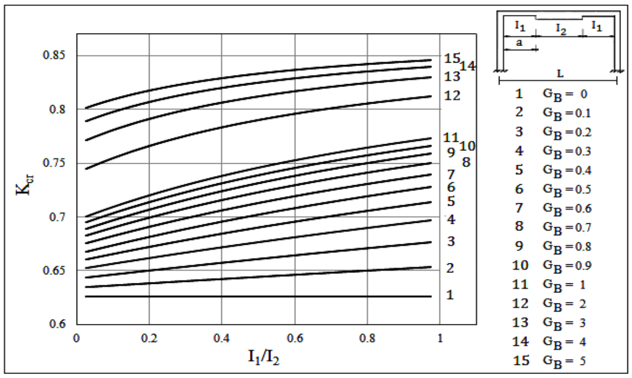


Fig. 3. Effective length factor for composite girder with rigid far end in braced frames (Fixed base) (a/L = 0.15)

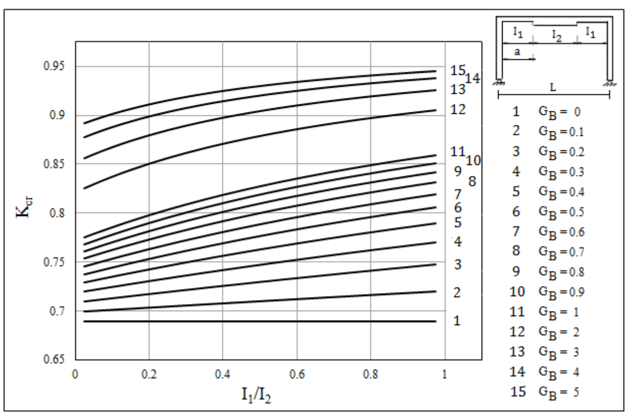


Fig. 4. Effective length factor for composite girder with rigid far end in braced frames (Hinged base) (a/L = 0.15)

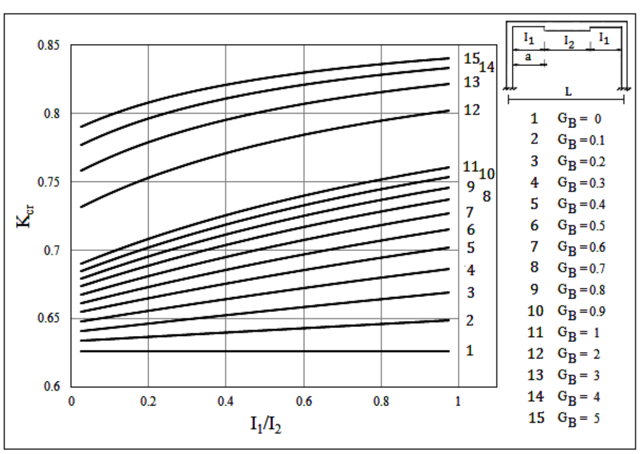


Fig. 5. Effective length factor for composite girder with fixed far end in braced frames (Fixed base) (a/L = 0.15)

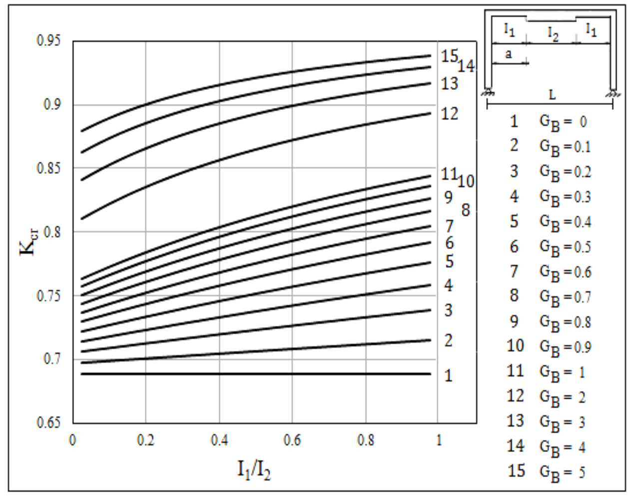


Fig. 6. Effective length factor for composite girder with fixed far end in braced frames (Hinged base) (a/L = 0.15)

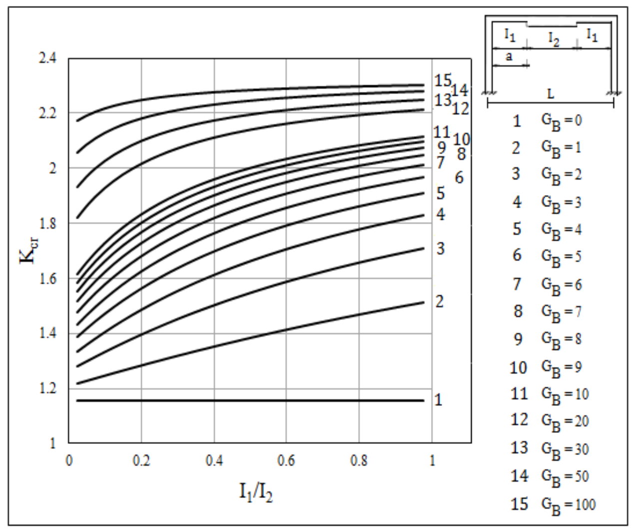


Fig. 7. Effective length factor for composite girder with rigid far end in unbraced frames (Fixed base) (a/L = 0.06)

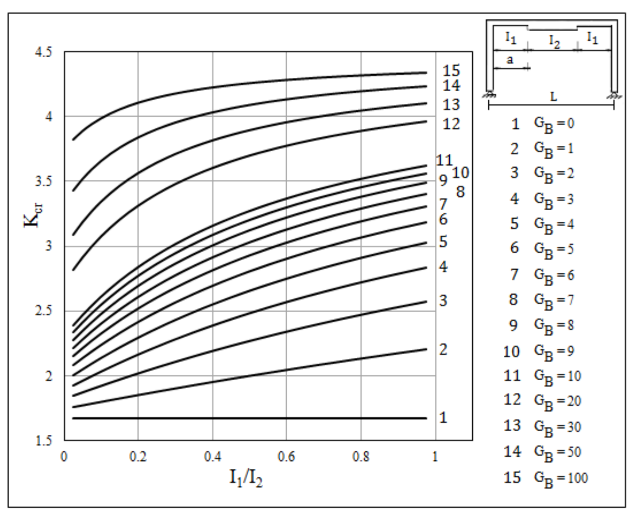


Fig. 8. Effective length factor for composite girder with rigid far end in unbraced frames (Hinged base) (a/L = 0.06)

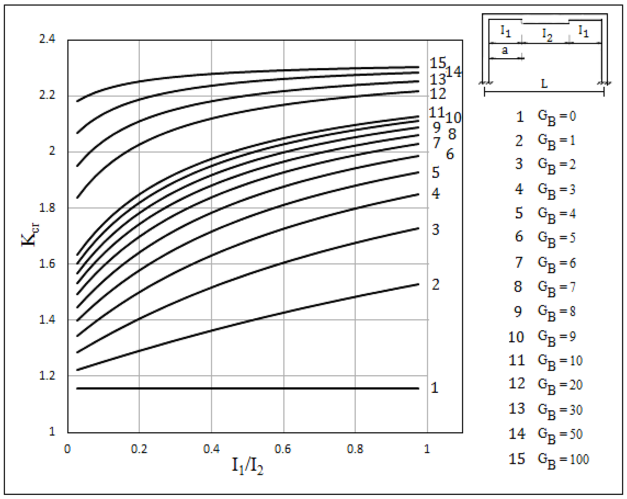


Fig. 9. Effective length factor for composite girder with fixed far end condition in unbraced frames (Fixed base) (a/L = 0.06)

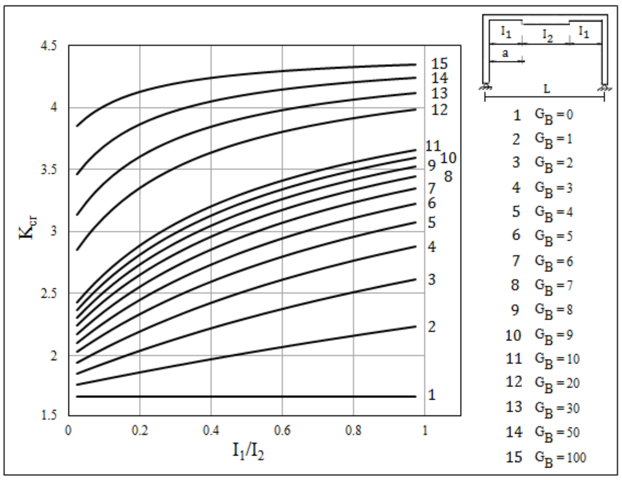


Fig. 10. Effective length factor for composite girder with fixed far end condition in unbraced frames (Hinged base) (a/L = 0.06)

IV. The Relationship Between The Critical Buckling Factor And The Beam Length

As is seen, the following is a brief description of the influence of the various parameters studied on the modified buckling factor (K_{cr}):

1) The composite-steel girder (L_b) lengths ranged from 4 to 15 meters, with two types of braced and unbraced frames.
2) In both braced and unbraced frames, the far end of the composite steel girder is rigid.
3) The column base is fixed in the steel frames, while another is hinged.

Overall, the most striking feature of the following graphs is that they illuminate the substantial rise in the modified buckling factor (K_{cr}) of the composite girders by gradually increasing the length of the beam (L_b) at the different IPE cross-sections. Moreover, it is clear that the more IPE section numbers are used in the composite girder, the more the modified buckling factor significantly grows. Concerning the IPE300 curve, there is a slight decline in the modified buckling factor when the beam length (L_b) is about 9 m, then it goes up considerably until the end of the curve. Actually, the main reason that the indicated dip happened is that the neutral axis of the composite beam has shifted from the concrete slab to the steel cross section.

(1) The effect of using a hinged base column instead of a fixed one when the composite beam's far end is rigid and the frame is braced:

Firstly, the far end of the composite beam is rigid, and the frame is braced. In addition, the ratio a/L equals 0.15. These are unchanged in Figs. 11 and 12, while the column base is changed from fixed to hinged. It can be seen from the relationship between the length of the composite-steel girder (L_b) and the modified buckling factor (K_{cr}) that the modified buckling factor rises from 8.62 % to 12.1 %. Figs. 11 and 12 demonstrate the percentage between the IPE section number and the modified buckling factor (K_{cr}). The modified buckling factor goes up from 3.54 % to 6.5 % when the IPE section number increases from IPE100 to IPE600, according to this percentage.

(2) The base column's effect changed from fixed to hinged, when the composite beam's far end is rigid and the frame is unbraced:

Figs. 13 and 14 show that the far end of the composite beam remains rigid, the frame is unbraced, and a/L = 0.03 remain constant, whereas the base of the column is converted from fixed to hinged. The length of composite-steel beam (L_b) is increased from 4m to 15m, while (K_{cr}) increases from 11.56 % to 23.29 %. Furthermore, the modified buckling factor raises from 16.21% to 28.76% if the IPE section number grows from IPE100 to IPE600.

(3) The influence of converting the column base from fixed to hinged, while the composite beam's far end is rigid (Figs. 15 and 16):

The parameters, which include the far end of the composite beam being rigid in unbraced frames and the ratio a/L = 0.06, are constant whenever the column base is converted from fixed to hinged, as seen from Figs. 15 and 16.

Moreover, depending on the proportion between the composite beam's length (L_b) and the modified buckling factor (K_{cr}), the modified buckling factor (K_{cr}) rises from 14.6 %.to 24.03 %. In addition, as illustrated in Figs. 15 and 16, the modified buckling factor increases from 13.87 % to 23.19 %. when the IPE section number grows from IPE100 to IPE600.

(4) The consequence of utilizing a hinged base column rather than a fixed one when the frame is unbraced at the rigid composite beam’s far end as well as a/L equals 0.09:

When the column base is altered from fixed to hinged in Figs. 17 and 18, the relationship between the composite-steel girder's length (L_b) and the modified buckling factor (K_{cr}) indicates that the modified buckling factor goes up from 16.72 % to 24.62 %. As shown in Figs. 17 and 18, there is a relationship between the proportion of IPE sections and the modified buckling factor (K_{cr}). As the IPE section number grows, the modified buckling factor rises from 11.89 % to 18.87 %, based on this relationship.

(5) The result of using a hinged base column instead of a fixed one while the far end of the composite girder is rigid in unbraced frame:

Furthermore, since the far end of the composite beam is rigid, and a/L = 0.12, which are constants in Figs. 19 and 20, but the base of the column is now hinged rather than fixed. The (K_{cr}) goes up from 18.29 % to 25.09 % when the length of composite-steel beam (L_b) is raised from 4 meters to 15 meters. Also, the modified buckling factor grows from 10.13 % to 15.49 % whenever the IPE section number is increased from IPE100 to IPE600.

(6) The base column's influence converted from fixed to hinged when a/L equalized 0.15 and the far end of the composite beam is rigid (Figs. 21 and 22):

While the base of the column is altered from fixed to hinged in Figs. 21 and 22, the parameters such as the rigid composite beam's far end, unbraced frame, as well as a/L = 0.15 remain constant. Moreover, increasing the length of composite-steel beam (L_b) from 4 m to 15 m results in (K_{cr}) rising from 19.34 % to 25.33 %. Additionally, Figs. 21 and 22 illustrate that if the IPE section number grows from IPE100 to IPE600, the (K_{cr}) grows from 8.58 % to 12.76 %.

(7) The result of utilizing an unbraced frame rather than a braced one as demonstrated in Figs. 11 and 21:

Firstly, the far end of the composite beam is rigid, with a/L = 0.15 in addition, the column base is fixed. When the length of composite-steel beam (L_b) is increased from 4 meters to 15 meters, the (K_{cr}) rises from 11.57 % to 21.05 %. Secondly, IPE section number rises from IPE100 to IPE600, and so the modified buckling factor increases from 6.35 % to 10.13 %.

(8) The consequence of using an unbraced frame instead of a braced one as seen in Figs. 12 and 22:

To begin, the composite beam's far end is rigid, with a/L = 0.15, and the column base is hinged. The (K_{cr}) goes up from 12.1 % to 25.33 % when the length of composite-steel beam (L_b) is grown from 4 meters to 15 meters. Second, the modified buckling factor increases from 6.5 % to 12.76 % when the IPE section number grows from IPE100 to IPE600.

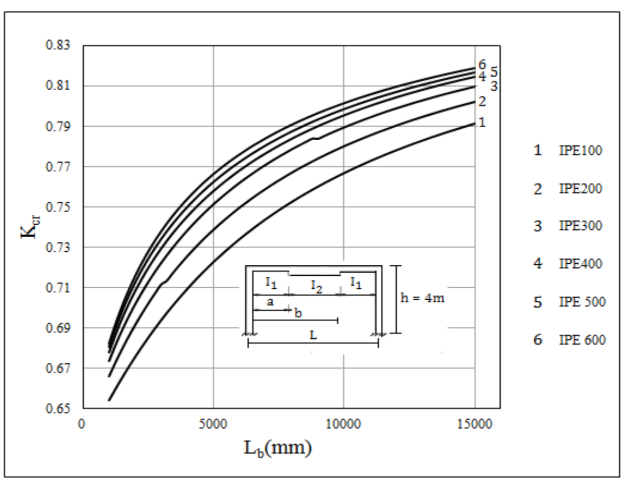


Fig. 11. The relationship between the critical buckling factor and the beam length in braced frame with rigid far end girder (a/L = 0.15, fixed base, center spacing = 2000 mm, t_c = 120 mm, E_c = 220 t/cm^2, $I_{col} = I_{steel\ beam}$)

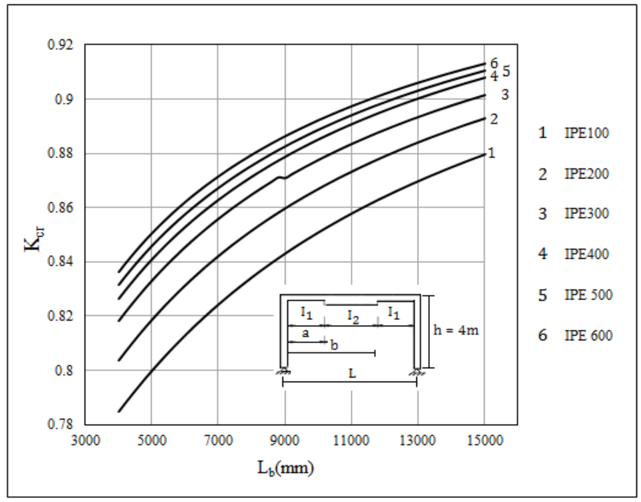


Fig. 12. The relationship between the critical buckling factor and the beam length in braced frame with rigid far end girder (a/L = 0.15, hinged base, center spacing = 2000 mm, t_c = 120 mm, E_c = 220 t/cm^2, $I_{col} = I_{steel\ beam}$)

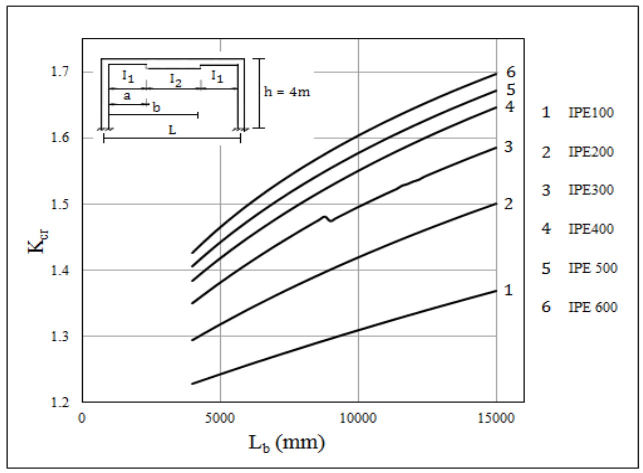


Fig. 13. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.03, fixed base, center spacing = 2000 mm, t_c = 120 mm, E_c= 220 t/cm^2, $I_{col} = I_{steel\ beam}$)

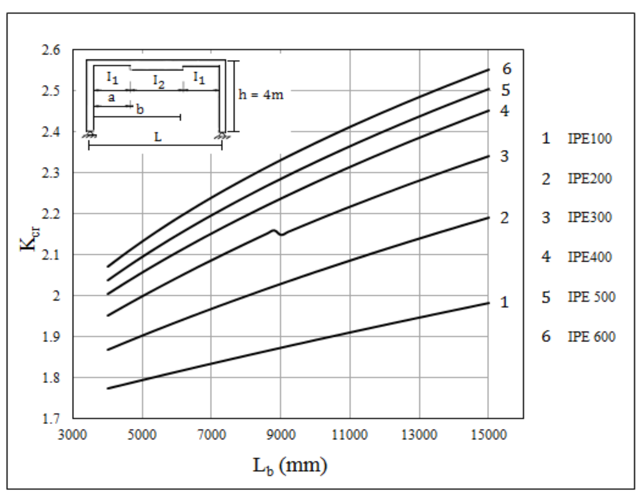


Fig. 14. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.03, hinged base, center spacing = 2000 mm, t_c = 120 mm, E_c = 220 t/cm^2, $I_{col} = I_{steel\ beam}$)

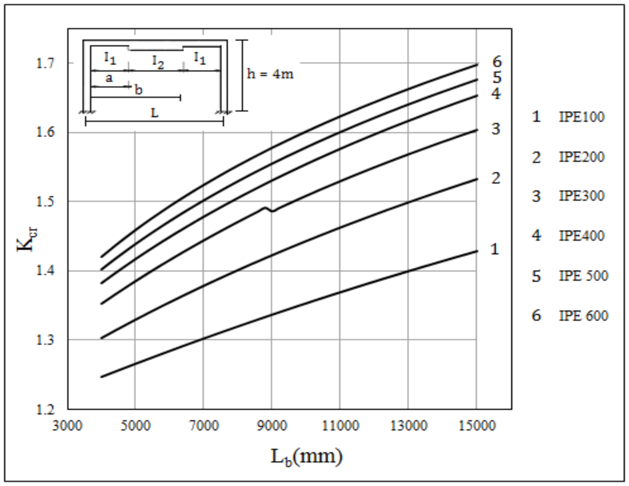


Fig. 15. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.06, fixed base, center spacing = 2000 mm, t_c = 120 mm, E_c= 220 t/cm^2, I_{col} = $I_{steel\ beam}$)

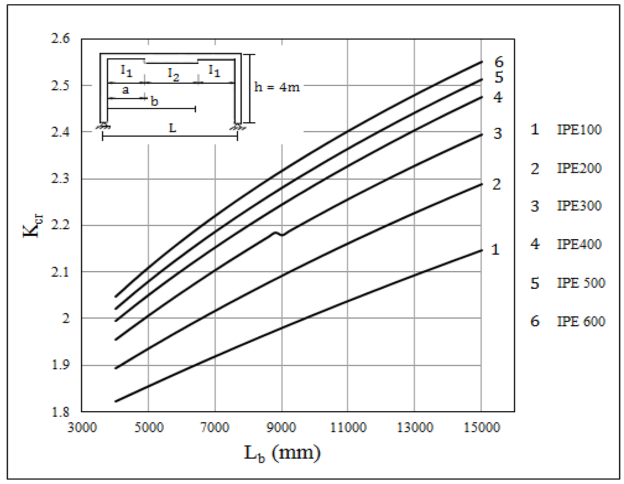


Fig. 18. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.09, hinged base, center spacing = 2000 mm, t_c = 120 mm, E_c= 220 t/cm^2, I_{col} = $I_{steel\ beam}$)

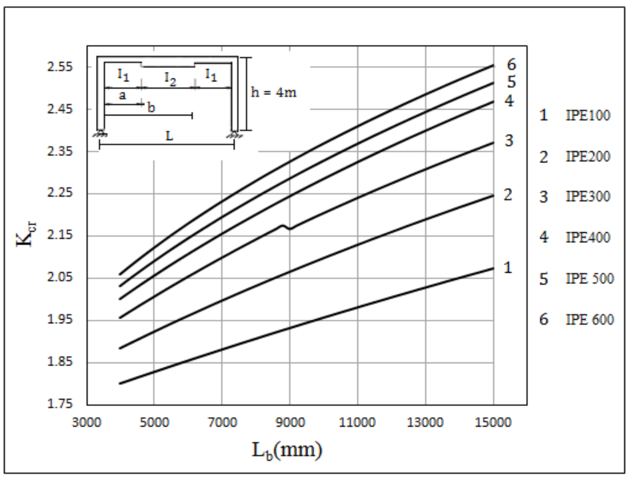


Fig. 16. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.06, hinged base, center spacing = 2000 mm, t_c = 120 mm, E_c= 220 t/cm^2, I_{col} = $I_{steel\ beam}$)

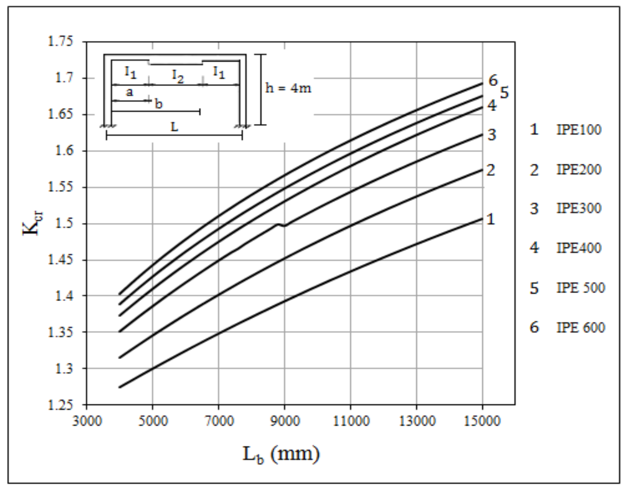


Fig. 19. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.12, fixed base, center spacing = 2000 mm, t_c = 120 mm, E_c= 220 t/cm^2, I_{col} = $I_{steel\ beam}$)

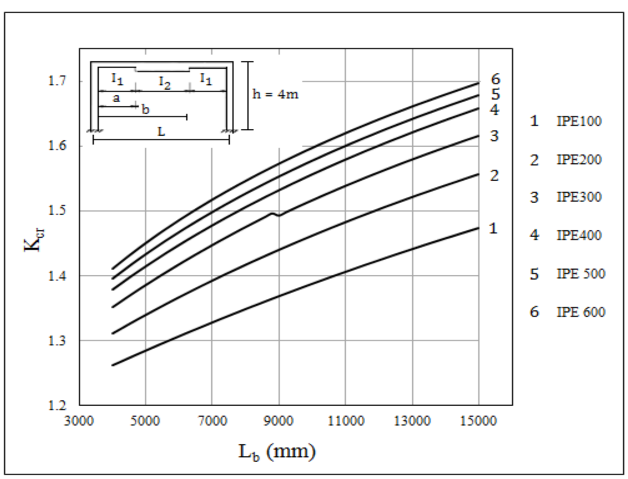


Fig. 17. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.09, fixed base, center spacing = 2000 mm, t_c = 120 mm, E_c= 220 t/cm^2, I_{col} = $I_{steel\ beam}$)

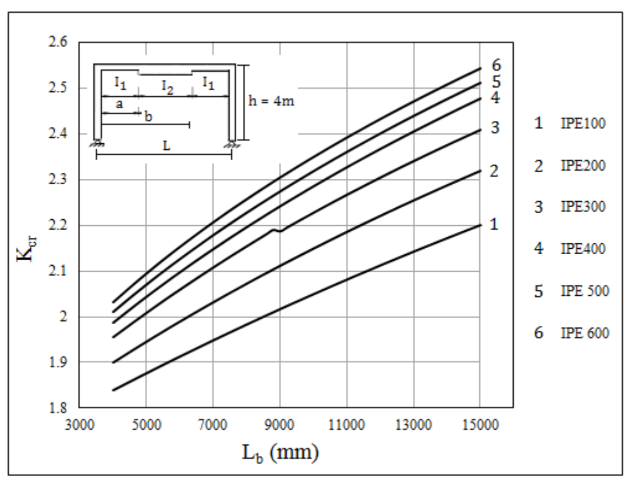


Fig. 20. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.12, hinged base, center spacing = 2000 mm, t_c = 120 mm, E_c= 220 t/cm^2, I_{col} = $I_{steel\ beam}$)

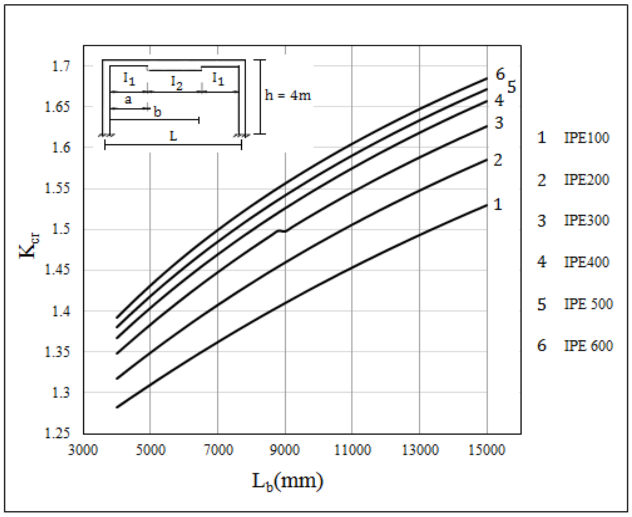


Fig. 21. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.15, fixed base, center spacing = 2000 mm, t_c = 120 mm, E_c= 220 t/cm^2, $I_{col} = I_{steel\ beam}$)

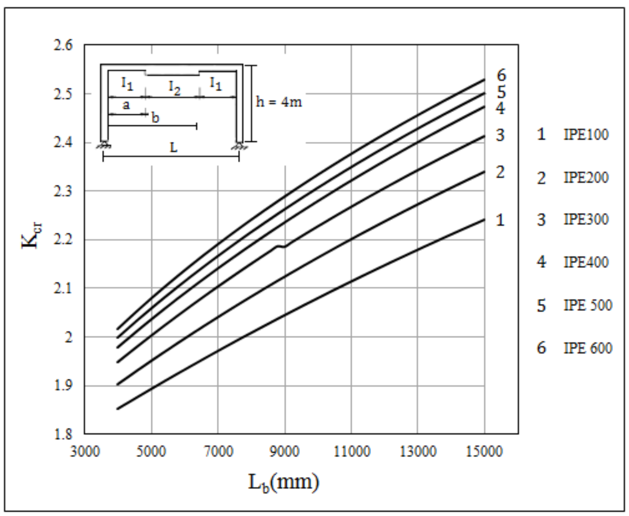


Fig. 22. The relationship between the critical buckling factor and the beam length in unbraced frame with rigid far end girder (a/L =0.15, hinged base, center spacing = 2000 mm, t_c = 120 mm, E_c= 220 t/cm^2, $I_{col} = I_{steel\ beam}$)

V. Illustrative Examples

Example 1

A simple unbraced frame with composite girder is shown in Fig. 23. Assume ŕ = 0.06. Composite beam with a span length L equals 7000 mm; the steel profile (IPE360) is characterized by A_s = 7270 mm2 and Is = 162700000 mm4. The solid slab is 120 mm thick and modulus of elasticity = 220 t/cm2; the adjacent beams spacing is 3000 mm. Steel column height equals 4000 mm; the steel profile (IPE500) is characterized by A_s = 11600mm2 and I_s = 482000000mm4. Determine the effective length and the effective length factor K for column AB. Recalculate the answers in the case of altering the steel profile of the beam from (IPE360) to (IPE450).

Note: the steel beam profile (IPE450) is characterized by A_s = 9880mm2 and I_s = 337400000mm4.

Case 1: the steel beam profile = (IPE360):

1) Suggested solution:

G_A= 1(Fixed base)

$I_{comp\ beam.}$ = 411393652.2mm^4

$I_{steelbeam.}/I_{comp\ beam.} = I_1 / I_2 = 0.395$

$\alpha = 0.809$

$G_B = \dfrac{\sum E_c I_c/L_c}{\sum E_b I_b/L_b} = 5.184$

K_1= 1.774 is the result obtained by referring to the alignment chart.

Effective length = K *L_{col}= 7.096m

2) ECP(LRFD) alignment chart solution:

$I_{beam} = I_{avg} = \dfrac{I_{steel\ beam.} + I_{comp\ beam.}}{2} = 287046826.1\text{mm}^4$

$G_B = \dfrac{\sum E_c I_c/L_c}{\sum E_b I_b/L_b} = 2.939$

K_1 = 1.56 is the result obtained by referring to the alignment chart.

Effective length = K * L_{col} = 6.240m

The ratio between ECP(LRFD) alignment chart solution and suggested solution = (1.56 - 1.774) / 1.774 = -12.06 %.

Case 2: the steel beam profile = (IPE450):

1) Suggested solution:

G_A= 1(Fixed base)

$I_{comp\ beam.}$ = 752118366.045mm^4

$I_{steelbeam.}/I_{comp\ beam.} = I_1 / I_2 = 0.449$

$\alpha = 0.736$

$G_B = \dfrac{\sum E_c I_c/L_c}{\sum E_b I_b/L_b} = 2.5$

K_2 = 1.59 is the result obtained by referring to the alignment chart.

Effective length = K *L_{col}= 6.345m

2) ECP(LRFD) alignment chart solution:

$I_{beam} = I_{avg} = \dfrac{I_{steel\ beam.} + I_{comp\ beam.}}{2} = 544759183\text{mm}^4$

$G_B = \dfrac{\sum E_c I_c/L_c}{\sum E_b I_b/L_b} = 1.548$

K_2 = 1.39 is the result obtained by referring to the alignment chart.

Effective length = K * L_{col} = 5.56m

The ratio between ECP(LRFD) alignment chart solution and suggested solution = (1.39 - 1.59) / 1.59 = -12.58%.

Fig. 23. Simple unbraced frame

Example 2

A simple braced frame with composite girder which has fixed far end as is shown in Fig. 24. Composite beam with a span length L equals 7500 mm; the steel profile (HEA280) is characterized by A_s = 9730 mm2 and Is = 136700000 mm4. The solid slab is 100 mm thick as well as modulus of elasticity = 220 t/cm2; the adjacent beams spacing is 2500 mm. Steel column height equals 4200mm; the steel profile HEB340) is characterized by A_s = 17100mm2 and I_s = 366600000mm4. Calculate the effective length and the effective length factor K for column AB.

Suggested solution:

G_A= 1 (Fixed base)
$I_{comp\ beam.}$ = 300891965.45mm^4
$I_{steelbeam.}$ /$I_{comp\ beam.}$ = I_1 / I_2 = 0.4543
α = 1.965
$G_B = \frac{\sum E_c I_c / L_c}{\sum E_b I_b / L_b} = 4.79$
K = 0.828 is the result obtained by referring to the alignment chart.
Effective length = K * L_{col} = 3.478m

ECP(LRFD) alignment chart solution:

$I_{beam} = I_{avg} = \frac{I_{steel\ beam.} + I_{comp\ beam.}}{2}$ = 218795982.7mm^4
$G_B = \frac{\sum E_c I_c / L_c}{2 * \sum E_b I_b / L_b} = 1.496$
K = 0.8 is the result obtained by referring to the alignment chart.
Effective length = K * L_{col} = 3.36m
The ratio between ECP(LRFD) alignment chart solution and suggested solution = (0.8 - 0.828) / 0.828 =
-3.38%

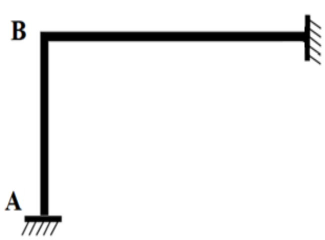


Fig. 24. Braced frame with fixed far end girder

VI. Conclusions And Summary

This paper illustrates the calculation method of the effective length factor for columns in braced and unbraced frames with composite girders. The far ends of the girders are depicted as fixed and rigid. Furthermore, the equations for the modified buckling factor have been proven using methodology based on parameter (α). Moreover, the parameter (α) has been deduced in its closed form. In addition, the relationship between the modified buckling factor and beam length has been elucidated for practical purposes.

Finally, A number of conclusions may be made from the results of the present research. The following is a summary of the research results:

1) In both cases of Example 1, the results of the suggested solution indicate a moderate fall in the modified K-parameter (K_{cr}) with a tedious rise in the moment of inertia ratio (I_1 /I_2). On the other hand, it's noticeable from the results of the ECP(LRFD) alignment chart solution in the two cases of Example 1 that the K-parameter decreases slowly when the average moment of inertia (I_{avg}) goes up slightly.
2) A summary of the results from examples 1 and 2 revealed that:

- The ratio of K-factor values between the ECP (LRFD) alignment chart solution and the suggested solution for a braced frame is -3.38 %, while for the first case in an unbraced frame it is -12.06 % and for the second case in an unbraced frame it is -12.58 %.
- Regarding the unbraced frame, the modified stiffness factor for composite girders enhances the effective length calculation and achieves an accurate and economical design.
- Moreover, results illuminate a slight improvement in the modified buckling factor for braced frames with composite girders, but still more accuracy than conventional solution.

3) Concerning braced and unbraced frame Figs. 3 to 10, it's clear that the relationship between the moment of inertia ratio (I_1/I_2) and the modified buckling factor (K_{cr}) grows progressively by using a hinged column base instead of a fixed one when all other studied parameters remain constant.
4) While all other examined parameters stay constant, the modified buckling factor (K_{cr}) moderately rises when the parameter (G_B) grows as a consequence of altering the composite beam's far end from rigid to fixed as illustrated in Figs. 3 to 10.
5) Regarding the unbraced frame Figs. 11 to 22, it's remarkable that the relationship between the length of the composite beam and the modified buckling factor (K_{cr}) increases gradually when changing the column base from hinged to fixed.
6) Furthermore, whenever the IPE section number grows, the modified buckling factor rises as well, as seen in Figs. 11 to 22.
7) Last but not least, the result of using an unbraced frame instead of a braced one demonstrate that there is a significantly growing in the modified K-parameter (K_{cr}) with the gradual lengthening of the composite beam as shown in Figs. 11 to 22.
8) Graphed charts make design procedures easier to follow.

APPENDIX

Parameters N, O, X, and U are given by:

$$N = (\frac{1}{{K_1}^2L^2} + n\,(\frac{K_2\cos(K_2a)}{\cos(K_1a)} - (\frac{\frac{K_2\sin(K_2a)}{\cos(K_1a)}}{\cos(K_2a) + \frac{K_2}{K_1}\tan(K_1a)\sin(K_2a)}\,[\frac{K_2}{K_1}\tan(K_1a)\cos(K_2a) - \sin(K_2a)])) + \frac{1}{\cos(K_1a)}(\frac{\sin(K_1a)}{K_1L} + \frac{1}{r{K_2}^2L^2} - \frac{1}{{K_1}^2L^2} - \frac{K_2\sin(K_2a)}{\cos(K_2a) + \frac{K_2}{K_1}\tan(K_1a)\sin(K_2a)}\,(\frac{1}{{K_1}^2L}\,[\cos(K_1a) + \tan(K_1a)\sin(K_1a) - 1] + \frac{1}{r{K_2}^2L^2}[\frac{\tan(K_1a)}{K_1} - a] + \frac{1}{{K_1}^2L^2}[\frac{-\tan(K_1a)}{K_1} + a] + \frac{1}{r{K_2}^2L})))$$

$$O = (\frac{-1}{{K_1}^2L^2} + m\,(\frac{K_2\cos(K_2a)}{\cos(K_1a)} - (\frac{\frac{K_2\sin(K_2a)}{\cos(K_1a)}}{\cos(K_2a) + \frac{K_2}{K_1}\tan(K_1a)\sin(K_2a)}\,[\frac{K_2}{K_1}\tan(K_1a)\cos(K_2a) - \sin(K_2a)])) + \frac{1}{\cos(K_1a)}(\frac{-1}{r{K_2}^2L^2} + \frac{1}{{K_1}^2L^2} - \frac{K_2\sin(K_2a)}{\cos(K_2a) + \frac{K_2}{K_1}\tan(K_1a)\sin(K_2a)}\,(\frac{-1}{r{K_2}^2L^2}\,[\frac{\tan(K_1a)}{K_1} - a] - \frac{1}{{K_1}^2L^2}\,[\frac{-\tan(K_1a)}{K_1} + a])))$$

$$X = \Bigg(\frac{-K_1(\sin(K_1L) + \frac{\cos(K_1L)}{\tan(K_1L)})}{(\cos(K_1b) - \frac{\sin(K_1b)}{\sin(K_1L)})}\,\Big[\frac{\cos(K_2b)}{\cos(K_2a) + \frac{K_2}{K_1}\tan(K_1a)\sin(K_2a)}\Big(\frac{1}{{K_1}^2L}[\cos(K_1a) + \tan(K_1a)\sin(K_1a) - 1] + \frac{1}{r{K_2}^2L^2}[\frac{\tan(K_1a)}{K_1} - a] + \frac{1}{{K_1}^2L^2}[\frac{-\tan(K_1a)}{K_1} + a] + \frac{1}{r{K_2}^2L}\Big) + n\,[\sin(K_2b) + \frac{\cos(K_2b)}{\cos(K_2a) + \frac{K_2}{K_1}\tan(K_1a)\sin(K_2a)}(\frac{K_2}{K_1}\tan(K_1a)\cos(K_2a\sin(K_2a))] + \frac{b}{r{K_2}^2L^2} - \frac{1}{r{K_2}^2L} - \frac{b}{{K_1}^2L^2} + \frac{1}{{K_1}^2L}\Big] + \frac{1}{{K_1}^2L^2}\Bigg)$$

$$U = \Bigg(\frac{-K_1(\sin(K_1L) + \frac{\cos(K_1L)}{\tan(K_1L)})}{(\cos(K_1b) - \frac{\sin(K_1b)}{\sin(K_1L)})}\,\Big[\frac{\cos(K_2b)}{\cos(K_2a) + \frac{K_2}{K_1}\tan(K_1a)\sin(K_2a)}\Big(\frac{-1}{r{K_2}^2L^2}[\frac{\tan(K_1a)}{K_1} - a] - \frac{1}{{K_1}^2L^2}[\frac{-\tan(K_1a)}{K_1} + a]\Big) + m\,[\sin(K_2b) + \frac{\cos(K_2b)}{\cos(K_2a) + \frac{K_2}{K_1}\tan(K_1a)\sin(K_2a)} * (\frac{K_2}{K_1} * \tan(K_1a)\cos(K_2a) - \sin(K_2a))] - \frac{b}{r{K_2}^2L^2} + \frac{b}{{K_1}^2L^2} - \frac{\sin(K_1b)}{{K_1}^2L\sin(K_1L)}\Big] + \frac{1}{K_1L\tan(K_1L)} - \frac{1}{{K_1}^2L^2}\Bigg)$$

Where:

$$K = \sqrt{\frac{P}{EI}}$$

$$r = \frac{I_2}{I_1}$$

AUTHORS CONTRIBUTION

The research strategies were devised by:

N.S. Mahmoud and S.M. Abdrabou. S.A.Eltawil compiled the research data, modified the buckling factor formula for columns in braced and unbraced frames with composite girders, and drew the charts.

N.S. Mahmoud and S.M. Abdrabou supervised the derivation and conducted out the observations.

At all stages, the authors discussed and checked the chart's results, as well as commented on those results.

The solved example consequences were analyzed by *N.S. Mahmoud, S.M. Abdrabou, and S.A. Eltawil*.

The paper was written and edited *by S.A. Eltawil* under the supervision of *N.S. Mahmoud and S.M. Abdrabou*.

The work reported in this publication was equally contributed by all authors.

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TITLE ARABIC:

الجزء الأول: عامل الانبعاج المعدل للأعمدة في الإطارات المثبتة والغير مثبتة ذات الكمرات المركبة.

ARABIC ABSTRACT:

في البحث الحالي، تم دراسة تأثير الكمرات المركبة على القيم المعدلة لعامل الانبعاج الخاص بالأعمدة في الإطارات المثبتة والغير مثبتة حيث تم تمثيل الركائز البعيدة للكمرات المركبة على أنها ثابتة وجاسئة. علاوة على ذلك، تعتمد الصيغ المشتقة على معاملات الصلابة المعدلة وتتبع نفس الافتراضات المتبعة في حساب عامل الطول الفعال القياسي. بالإضافة إلى ذلك، تم استخدام طريقة الانحراف الانحدارى للحصول على صيغ معدلة لعامل الانبعاج الخاص بالأعمدة في الإطارات ذات الكمرات المركبة سواء كانت إطارات مثبتة أو غير مثبتة. بعد ذلك، تم توضيح العلاقة بين عامل الانبعاج المعدل وطول الكمرة وذلك لأغراض عملية. علاوة على ذلك، تم عمل أمثلة توضيحية لإبراز تأثيرإستخدام الكمرات المركبة ذات الركائز المختلفة على معاملات الطول الفعالة للأعمدة.