13th International Conference on
AEROSPACE SCIENCES & AVIATION TECHNOLOGY,
ASAT- 13, May 26 – 28, 2009, E-Mail: asat@mtc.edu.eg
Military Technical College, Kobry Elkobbah, Cairo, Egypt
Tel : +(202) 24025292 – 24036138, Fax: +(202) 22621908

Reduced-Order Local Optimal Controller for a Higher Order System

M. Ashry[*], Zukhra Kamalova[**], and Tim Breikin[***]

Abstract: In this paper, a reduced order local optimal controller is designed for a higher order system. A reduced order model is obtained for the higher order system and its parameters are used for the reduced order local optimal controller. Also, genetic algorithm is used with the reduced order local optimal controller structure to design the controller parameters instead of obtaining them from the reduced order model. These results obtained are compared with the results obtained from full order local optimal controller. Finally, analogy between reduced order local optimal controller and PI controller parameters is represented. As such, this reduced order local optimal controller can be used for tuning PI controller parameters. Experimental results on a lab-based test rig confirm the effectiveness of the reduced order local optimal controller.

Keywords: Local optimal controller, genetic algorithm, and PI controller.

1 Introduction

Local optimal control is a control approach described by Lyantsev et al. (2004). It is used to control multivariable systems as well as Single Input-Single Output (SISO) systems. Local Optimal Controller (LOC) design is based on the model parameters of the system to be controlled. Usually full order models are used to design the LOC (Ashry et al., 2008a,b). In this paper, reduced order model parameters are used to design the LOC. In addition, Genetic Algorithm (GA) is used with the reduced order LOC structure to design the controller parameters instead of obtaining them from the reduced order model. The obtained results are compared with the results of LOC designed using full order model parameters. Finally, an analogy is established between the conventional digital PI controller and the reduced order LOC. As such, new approach fortuning PI controller parameters can be proposed. Experimental results on a lab-based test rig confirm the effectiveness of the proposed techniques.

The paper is organized as follows: Section 2 contains a brief description of the system to be controlled and the test rig. Section3 presents system identification results to provide the full order and reduced order models. In Section 4 full order and reduced order local optimal

[*] Egyptian Armed Forces, mmaashry@yahoo.com
[**] PhD student, Control Systems Centre, The University of Manchester, Sackville Street Building, Manchester, UK, PO BOX 88, M60 1QD., zukhra.kamalova@postgrad.manchester.ac.uk
[***] Lecturer, Control Systems Centre, The University of Manchester, Sackville Street Building, Manchester, UK, PO BOX 88, M60 1QD, t.breikin@manchester.ac.uk

controllers are designed and a comparison between them is made. In Section 5 the GA is used with the reduced order LOC structure to design the controller parameters. Furthermore, the GA-tuned LOC is compared with the GA-tuned PI controller where the objective function to be minimized is the same for both controllers. In Section 6 a relation between the reduced orders LOC parameters and the digital PI controller parameters is introduced. As such a digital PI controller is designed based on the reduced order LOC and the results compared with the genetically tuned digital PI controller. Concluding remarks are provided in Section 7.

2 The Test Rig Process Description

The system rig is the Bytronic Process Control Unit (PCU), which is based around a fluid flow process, where flow and temperature can be controlled. This system is a multivariable system fully described by Bytronic international ltd (1998). In the case studied SISO system is considered. In this system a fluid is pumped from a sump in a closed path and then drained back to the sump while the flow rate of the fluid is monitored. As such, the system input is the DC voltage to the pump, and the system output is the fluid flow rate. The system is connected to a power supply unit that provides the input power to all elements of the system. Also the process inputs and outputs are connected to a computer control module that works as an interface between the PCU and PC-based controller. The system has been modified by replacing the existing Input-Output (I/O) interface module with a *NI PCI−DIO−96* digital I/O card to enable working in Matlab environment using real-time windows target toolbox (Matlab, 2007b). The overall block diagram of the system with I/O interface is illustrated in Figure 1.

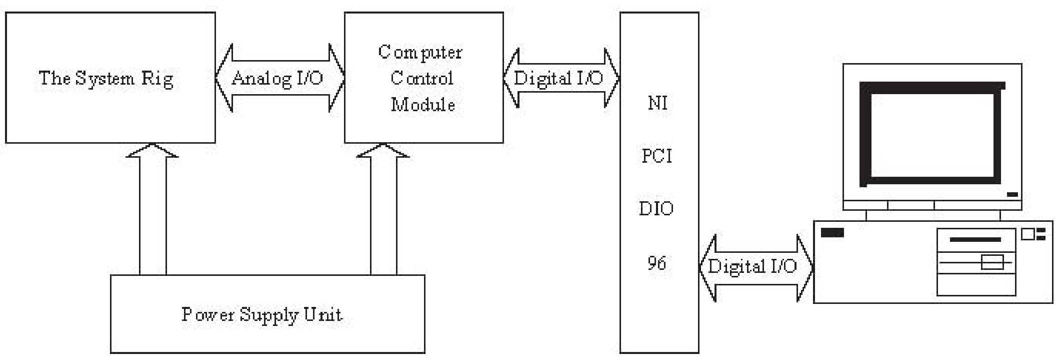

Figure 1: The block diagram of the I/O interface system.

3 Open Loop System Identification

As mentioned before for the system under consideration, the input is the DC voltage to the pump and the output is the fluid flow rate. Prior to the system identification several initial open loop tests have been performed to determine the characteristics of the system and design the excitation signal (Ljung, 1999), based on which the system's time constant is determined to be 700 ms and its cut-off frequency is 6 rad/s. As such, the sampling time is chosen as 125 ms and the frequency band for the excitation signal is chosen accordingly. Figure 2 shows the output step response of the system.

The input voltage to the pump can vary between 0-12V. The I/O curve for this system is shown in Figure 3. From this curve, the system can be considered as linear when working between 3-9 V.

According to Figure 3, the excitation signal should vary between 3-9 V in amplitude and its power spectrum should be flat for frequencies from 0-6 rad/s. Chirp and multi-sine signals can be considered as the excitation signals satisfying the mentioned requirements (Ljung, 1999). Two sets of I/O data are obtained for system identification using these two inputs as different excitation signals. Figure 4 and Figure 5 show the I/O data using chirp and multi-sine as excitation signals respectively.

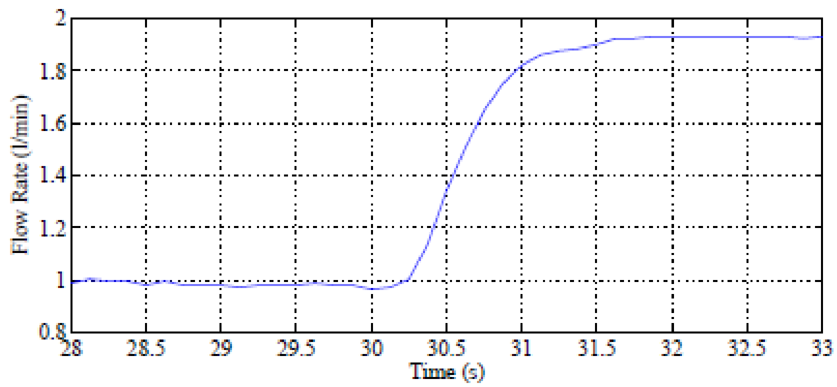

Figure 2: The output step response of the system.

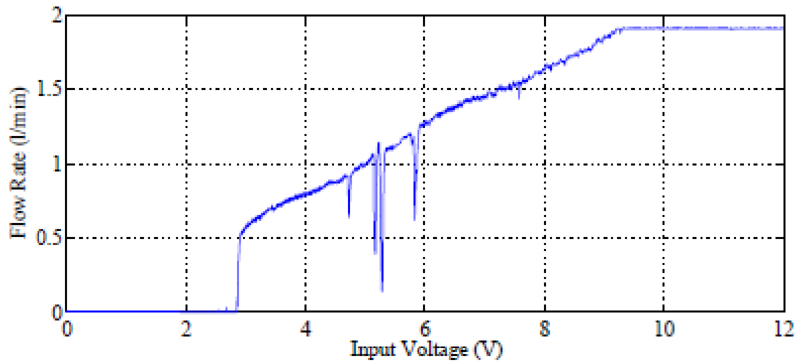

Figure 3: The I/O curve for the pump.

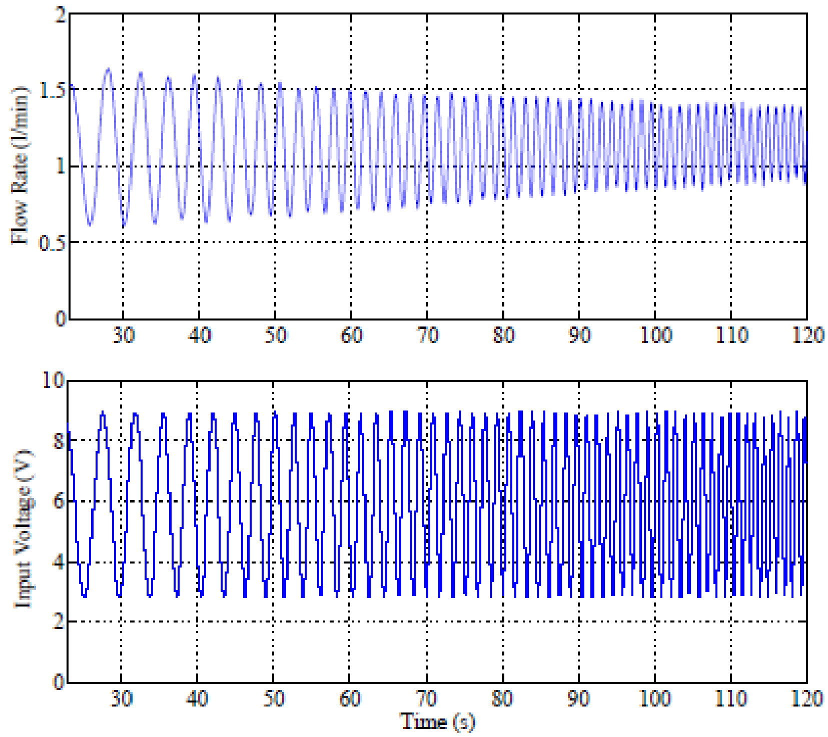

Figure 4: The input voltage to the pump (lower) and the output flow rate.

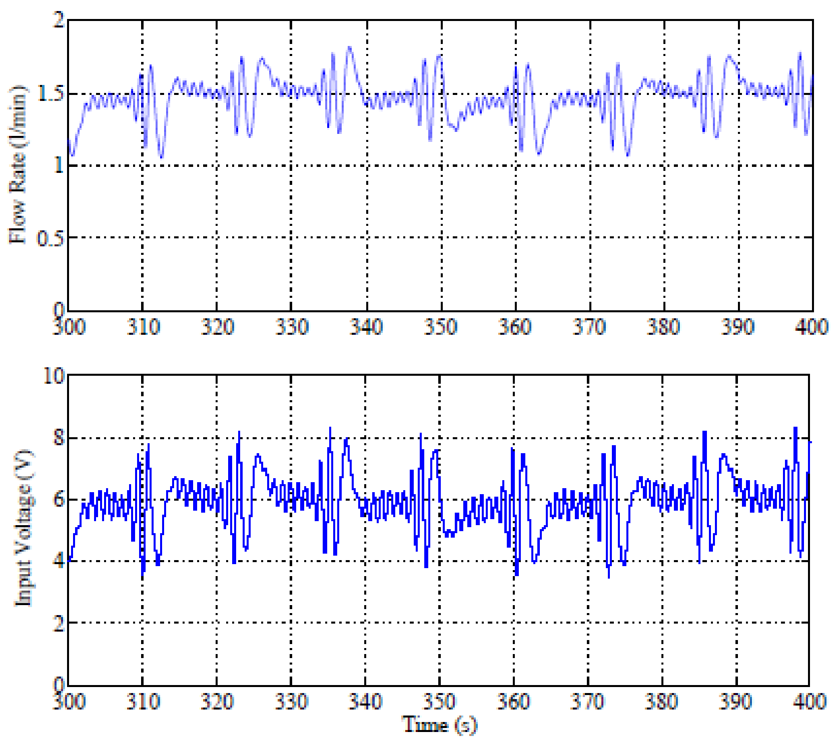

Figure 5: The input voltage to the pump (lower) and the output flow rate.

3.1 Full Order (Second Order) System Model

Using these two I/O data sets the system model is obtained. System identification toolbox of Matlab and Output Error (OE) method (Ljung, 2007, 1999) are used with data set 1 (chirp input) to give the model m1 and with dataset 2 (multi-sine input) to give the model m2. The model structure is given in (1) and the estimated parameters are given in Table 1.

$$y(i) = -a_1 y(i-1) - a_2 y(i-2) + b_1 u(i-1) + b_2 u(i-2) \tag{1}$$

In addition, GA is used to estimate the model parameters of the same model structure given in (1). This GA is used with data set 1 and data set 2 to give models m3 and m4respectively. The models estimated parameters are also given in Table 1.

Table 1: Second order (full order) model parameters.

Model	a_1	a_2	b_1	b_2
m_1	-0.9078	0.1557	0.0136	0.0336
m_2	-0.8323	0.1378	0.0167	0.0472
m_3	-0.8348	0.1318	0.0042	0.0462
m_4	-0.8358	0.1360	0.0187	0.0448

The two sets of output data generated by each experiment mentioned above are used to validate the different models obtained. The models fitness values with each set of output data are listed in Table 2. For each model two fitness values are presented, one is using the same data used to produce the model and the other is the cross validation. The four models are subjected to a cross validation, i.e., to validate the model generated using certain set of I/O data with the output data of the other set. The cross validation is represented as bold in Table 2. From this table, the model m1 is chosen to be the full order (second order) model of the system.

Table 2: Fitness of each model with the output data sets from each experiment.

Model	Fitness	
	Experiment1	Experiment2
m_1	91.89%	80.18%
m_2	72.92%	90.95%
m_3	92.39%	78.04%
m_4	72.84%	90.9%

3.2 Reduced Order (First Order) System Model

The same I/O data sets are used to obtain a reduced order (first order) model. Models M_1 and M_2 are obtained using OE method of system identification toolbox for I/O dataset 1 and I/O data set 2 respectively. Models M_3 and M_4 are obtained using GA for I/O data set 1 and I/O data set 2 respectively. The model structure for all reduced order models is given in (2). The models parameters are given in Table 3.

$$y(i) = -ay(i-1) + bu(i-1) \tag{2}$$

Table 3: First order (reduced order) model parameters.

Model	a	b
M_1	-0.8065	0.0461
M_2	-0.7484	0.0581
M_3	-0.8019	0.0455
M_4	-0.7563	0.0585

As in Section 3.1 the two sets of output data generated by each experiment are used to validate the different models obtained. The models fitness values for each set of output data are listed in Table 4. The cross validation for each model is represented as bold in Table 4. From this table, the model M1 is chosen to be the reduced order (first order) model of the system.

Table 4: Fitness of each first order model with the output data sets from each experiment.

Model	Fitness	
	Experiment1	Experiment2
M_1	75.07%	72.46%
M_2	66.33%	77.37%
M_3	74.79%	72.5%
M_4	66.43%	77.47%

4 System Control

The main purpose of this section is to design a reduced order LOC for the pump system under consideration and to compare its response with the full order LOC response. Figure 6 shows the closed loop step response of the pump system without any controller when it is subjected to the reference input shown in the figure.

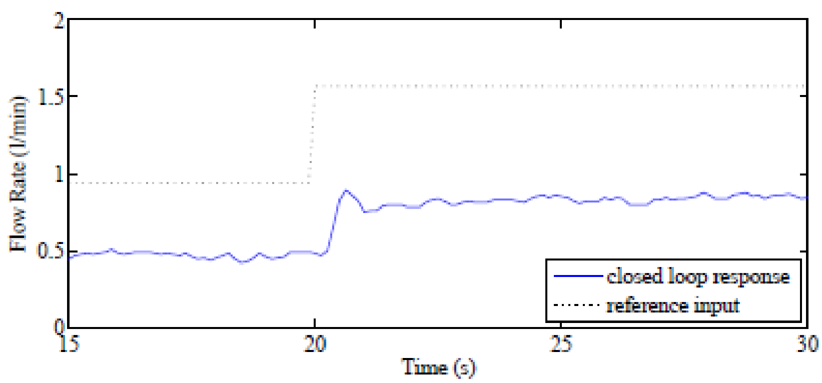

Figure 6: The reference input and the closed loop response of the pump.

4.1 The Full Order LOC Design

The LOC is designed and implemented as proposed by Lyantsev et al. (2004). For the pump model, the following equation is driven from the model equation given in (1).

$$\delta u(i) = \frac{1}{b_1}\left(\frac{1}{h}(r(i) - y(i)) + a_1 y(i) + (a_2 - a_1)y(i-1) - a_2 y(i-2) - b_2 u(i-1) + b_2 u(i-2)\right) \tag{3}$$

where:

- a_1, a_2, b_1, b_2 are the model parameters in (1),
- h is the LOC tunable parameter,
- $r(i)$ is the reference input for the closed loop system.

From (3), the full order LOC for the pump system which is a non-minimum phase system is designed as in the block diagram shown in Figure 7 (Ashry et al., 2008b).

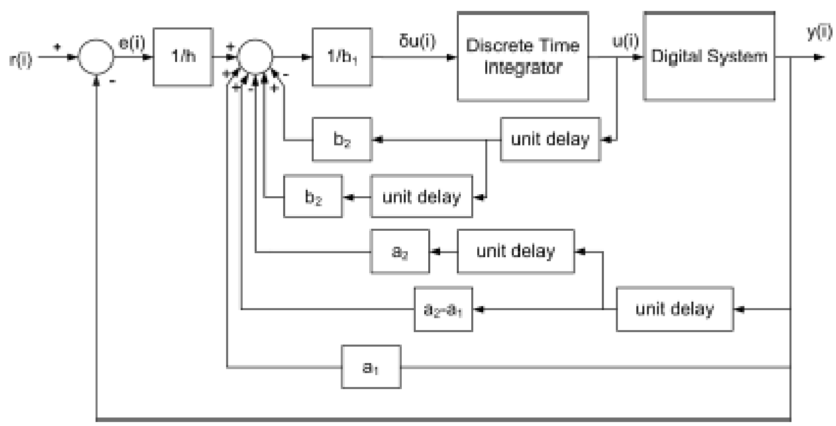

Figure 7: Block diagram of second order LOC.

Figure 8 shows the response of the model with the proposed full order LOC using Simulink for different values of the controller parameter h (h=2, 3, 4, and 5)

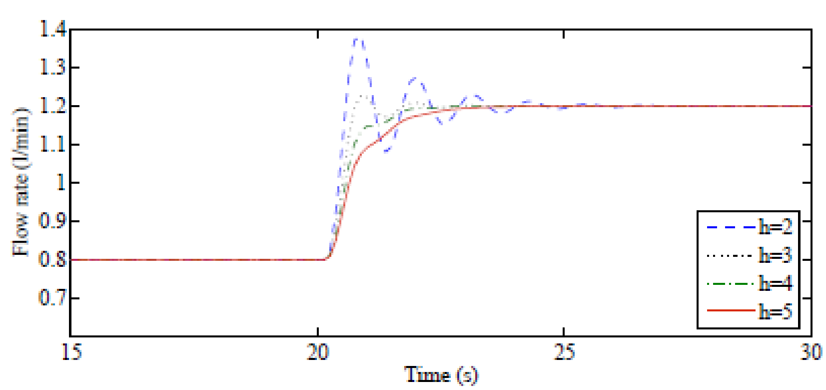

Figure 8: Simulated output response of full order LOC for different h.

For $h = 5$ the controller is designed for the pump system and Figure 9 shows the output response for both real and simulated system. From this figure it is clear that the real and simulated outputs are approximately identical.

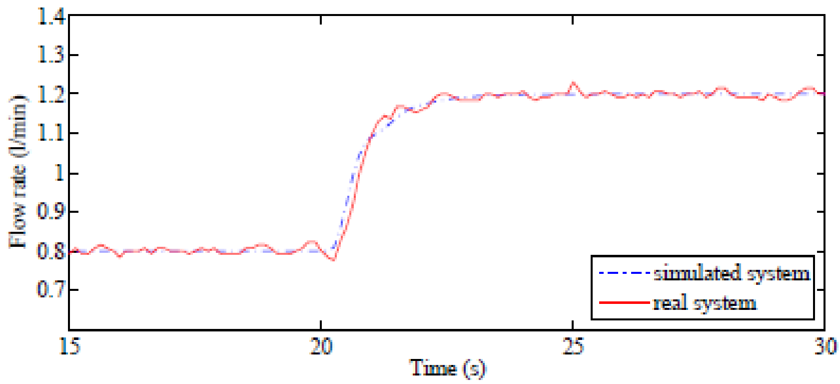

Figure 9: The output step response for both real and simulated system, *h* = 5.

4.2 The Reduced Order LOC Design

The reduced order LOC is also designed and implemented as proposed by Lyantsev et al. (2004), but the model used is the reduced order model represented in (2). As such the following equation is obtained.

$$\delta u(i) = \frac{1}{b}(\frac{1}{h}(r(i) - y(i)) + ay(i) - ay(i-1)) \quad (4)$$

where:

a, b are the model parameters in (2).

From (4), the reduced order LOC of the pump system is designed as described by Ashry et al. (2008b). The detailed block diagram of the system controlled by the first order LOC is illustrated in Figure 10.

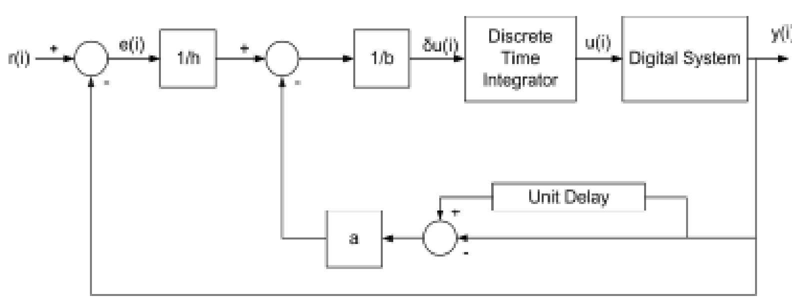

Figure 10: Block diagram of the LOC for first order model.

Figure 11 shows the simulated response of the full order system model with the proposed reduced order LOC using Simulink for different values of the controller parameter h (h=1, 2, and 3).

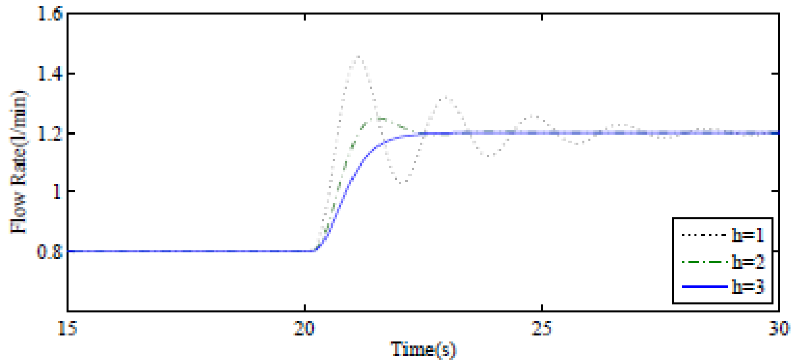

Figure 11: Simulated output response of reduced order LOC for different h.

It is clear from this figure that the reduced order LOC gives acceptable response at *h*=3, which is different from that of the full order LOC (*h*=5). For *h*=3 the controller is designed for the pump system and Figure 12 shows the output response for both real and simulated systems. From this figure it is clear that the real and simulated outputs are very similar. This indicates good robust performance of the reduced order LOC.

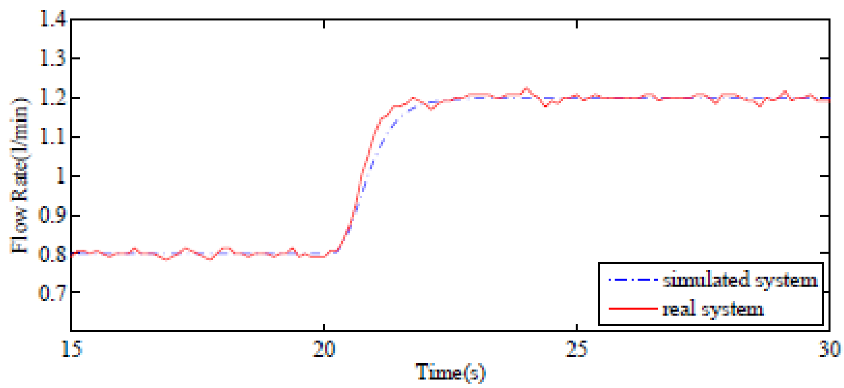

Figure 12: The output response for both real and simulated reduced order LOC, *h* = 3.

4.3 Comparison Between Full and Reduced Order LOC

Figure 13 shows the simulated output response of the system with full and reduced order LOC, and Figure 14 shows the real output response as obtained from the test rig for both full and reduced order LOC.

From these two figures it is clear that the results obtained using reduced order LOC approximately the same as that of the full order LOC.

5 Parameters Estimation of LOC Using GA

In this section GA is used to estimate the controller parameters ($a, b,$ and h) for the reduced order LOC structure shown in Figure 10. Minimization of the Mean Squared Error (MSE) between the reference input and the system's output and avoidance of overshoot are used as a multi-objective function to estimate these three controller parameters genetically. GA toolbox of Matlab (Matlab, 2007a) is used to estimate the three controller parameters.

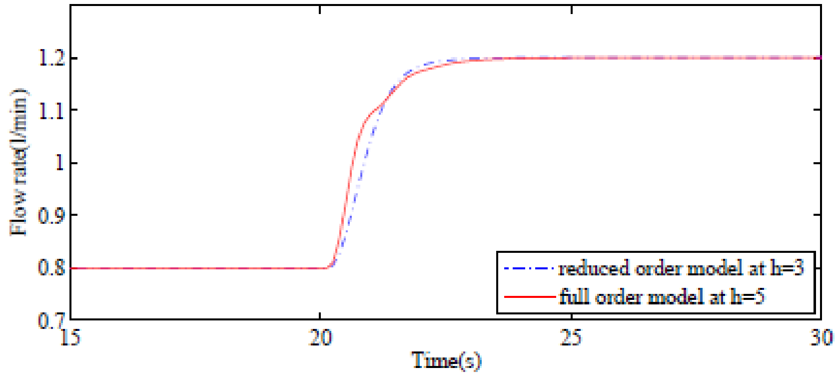

Figure 13: The simulated output response of full and reduced order LOC.

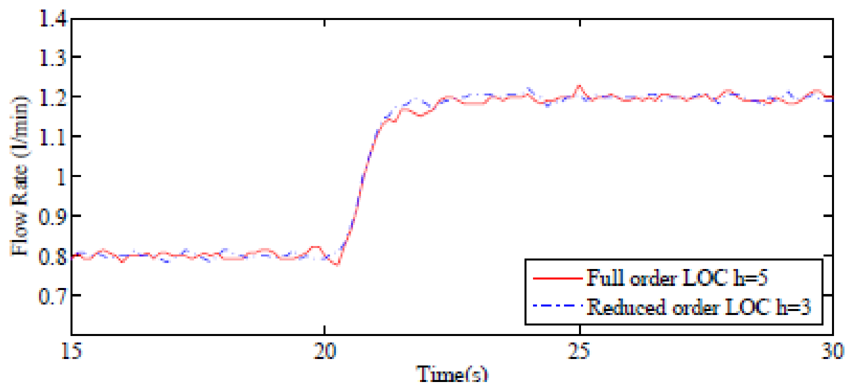

Figure 14: The measured output response of full and reduced order LOC.

It is found that for each value of h there are unique values of a and b that satisfy the objective function.

Table 5 represents the values of a and b for different values of h (h=1, 2, 3) and the value of the objective function (Zmin) obtained by the two GA approaches stated.

Table 5: LOC parameters calculated using GA toolbox of Matlab for the first order model.

	a	b	Z_{min}
h=1	-2.9623	0.1178	0.0142
h=2	-1.5562	0.0446	0.0126
h=3	-0.8303	0.0347	0.0132

Figure 15 shows the simulated output response for the controller parameters calculated genetically.

From Table 5 and Figure 15, it is clear that there are values of a and b satisfying the objective function for each value of h. As such, infinite number of solutions satisfying the objective function may be found. Whereas, for the normal reduced order LOC design, fixed values of a and b are the reduced order model parameters and the LOC parameter h is the only tuneable parameter to get the required response. Always, the value of h is increased gradually until obtaining overshoot free output response.

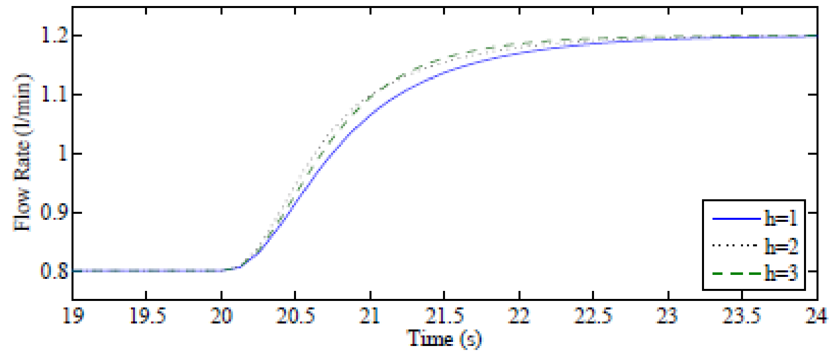

Figure 15: Simulated output response for *h* = 1, 2, 3

Figure 16 shows the output response of the system controlled by reduced order (first order) LOC, where the controller parameters are tuned using GA toolbox and h is chosen to be 3.

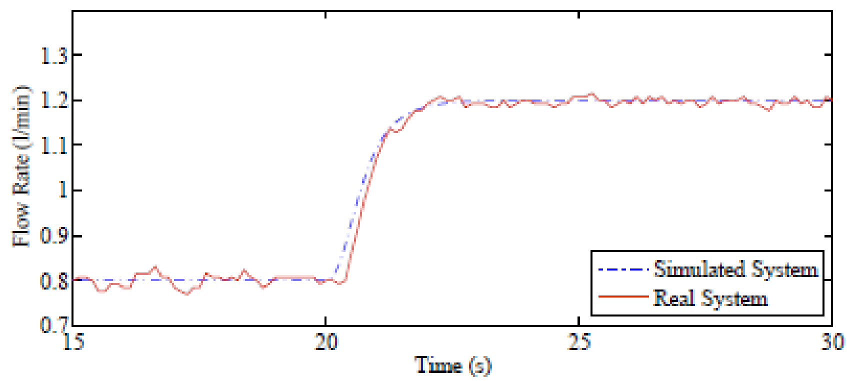

Figure 16: The real and simulated output responses of the system controlled by GA-based LOC of first order model structure when *h* = 3.

5.1 Comparison Between GA-Tuned LOC and GA-Tuned PI

In this subsection, a comparison between GA-tuned LOC (GA-LOC) and GA-tuned PI (GA-PI) is presented. Figure 17 shows the simulated results of GA-LOC for the reduced order model, and the GA-PI controller. It should be noted that, the objective function for both controllers is the same.

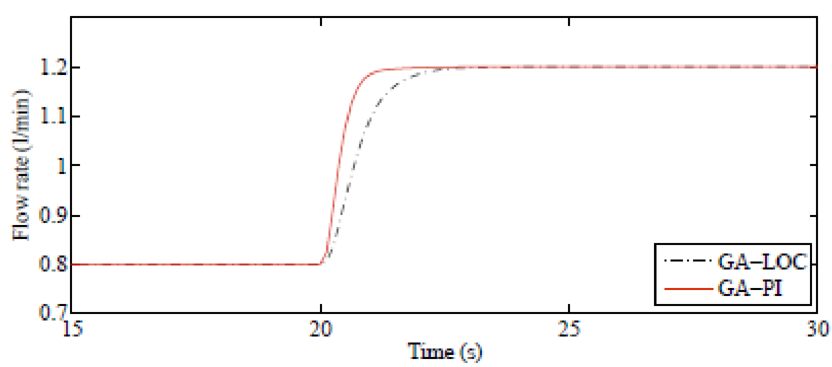

Figure 17: The simulated output response of the system controlled by GA-PI and GA-LOC.

The figure shows faster output responses of the GA-PI than that of GA-LOC.

For more investigation, consider Figure 16 for the real and simulated output responses of the GA-LOC. Also, consider Figure 18 for the real and simulated output response of GA-PI.

From these figures, the real and the simulated output responses of the GA-LOC are identical for the reduced order model structure. The output responses of the real system for GA-PI have overshoot, which is not the same for the simulated output response. From these results, it can be deduced that the GA-LOC is better than GA-PI from robust performance point of view, although the same objective function is used with these controller structures to obtain the controllers' parameters.

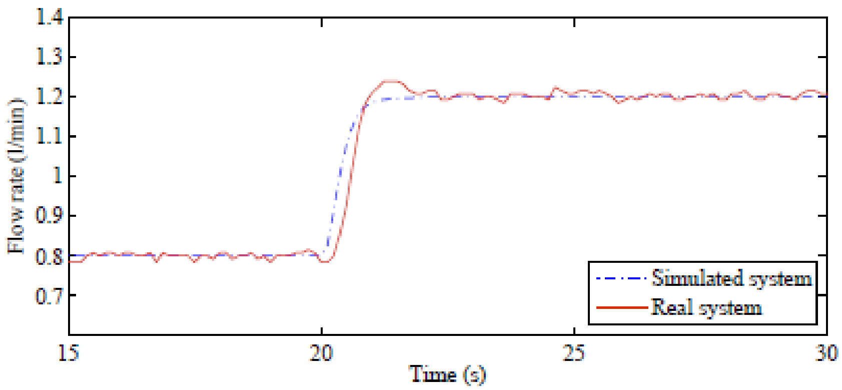

Figure 18: The output step response of the system controlled by GA-PI controller (real and simulated).

6 Analogy between LOC and Digital PI Controller

In this section a relation between the reduced order (first order) LOC parameters and the digital PI controller parameters is deduced. As such, the LOC parameters can be used for tuning the digital PI controller parameters. Figure 19 shows the block diagram of the system controlled by digital PI controller.

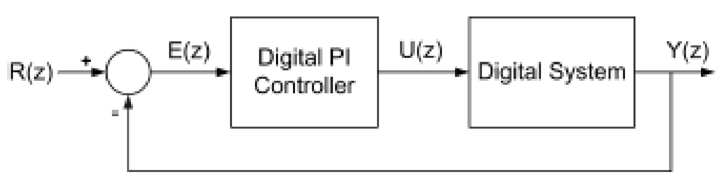

Figure 19: The block diagram of digital PI controlled system.

According to discrete time control theory (Ogata, 1995), the digital PI controller in Figure 19 can be represented in Z-domain as in (5).

$$\frac{U(z)}{E(z)} = Kp + \frac{K_I}{1 - z^{-1}} \tag{5}$$

And so, the following relation is obtained

$$u(i) = u(i-1) + (K_P + K_I)e(i) - K_P e(i-1) \tag{6}$$

From the block diagram of Figure 10 and by denoting $\delta u(i) = x(i)$, the following equation can be obtained for the first order LOC.

$$\begin{aligned} x(i) &= \frac{1}{b}[\frac{1}{h}e(i) + a(y(i) - y(i-1))] \\ &= \frac{1}{b}[\frac{1}{h}e(i) + a(y(i) - y(i-1) - r(i) + r(i-1))] \end{aligned} \tag{7}$$

where $r(i) = r(i-1)$ represents constant reference input to closed loop system. So, the following equation is deduced.

$$x(i) = \frac{1}{b}[(\frac{1}{h} - a)e(i) + ae(i-1)] \tag{8}$$

For the discrete time integrator shown in Figure 10, the following equations are deduced, where T_s is the sampling time.

$$U(z) = \frac{T_s}{1 - z^{-1}} X(z) \tag{9}$$

$$u(i) - u(i-1) = T_s x(i) \tag{10}$$

$$u(i) = u(i-1) + \frac{T_s}{b}[(\frac{1}{h} - a)e(i) + ae(i-1)] \tag{11}$$

From (6) and (11), the following relations can be deduced by equating the coefficients of $e(i)$ and $e(i-1)$. These relations define the digital PI controller parameters in terms of the LOC parameters.

$$\begin{aligned} K_P &= -\frac{aT_s}{b}, \\ K_I &= \frac{T_s}{bh} \end{aligned} \tag{12}$$

From the relations deduced in (12), the digital PI controller parameters can be calculated from the reduced order LOC parameters (*a*, *b*, and *h*).

6.1 Design of Digital PI Controller Based on the Reduced Order LOC

In this subsection (12) is used to calculate the digital PI controller parameters (KP and KI) based on the reduced order LOC parameters (*a, b,* and *h*). Figure 20 shows the output response for the real and simulated systems controlled by the digital PI controller tuned using the relations given in (12). It is clear from this figure that the output response of the real system is slightly different from that of the simulated one.

Figure 21 shows the simulated output responses of the reduced order LOC and that of the digital PI controller designed using (12).From this figure, it is clear that the responses are approximately the same but not identical. That is because of the assumption of $r(i) = r(i-1)$ in (7) is not valid at the step time. This can be confirmed by considering the disturbance rejection behavior of both controllers at fixed reference input *r*. Figure 22 shows the block diagram for the controlled system when it is subjected to disturbance. This can be considered as leakage in the pipes of the real system. Disturbance step input is applied at $t = 15$ s. The output responses of both controllers are shown in Figure 23, when they are subjected to step disturbance. From this figure, the two output responses are identical.

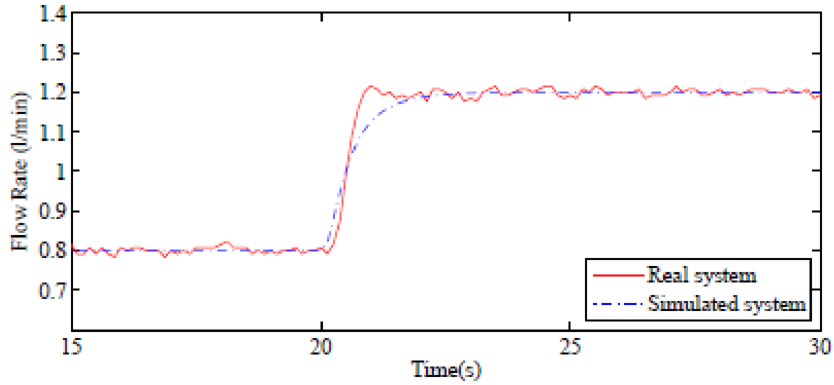

Figure 20: The output response for the real and simulated systems controlled by digital PI controller tuned using the first order LOC.

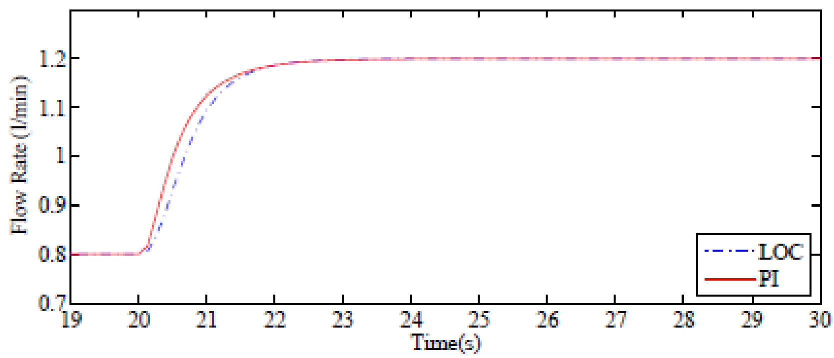

Figure 21: The simulated output response of the reduced order LOC and the digital PI controller deduced.

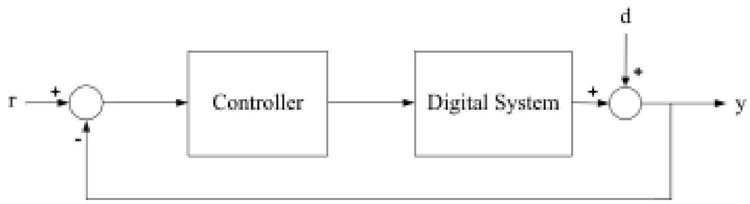

Figure 22: The block diagram for the controlled system when it is subjected to disturbance.

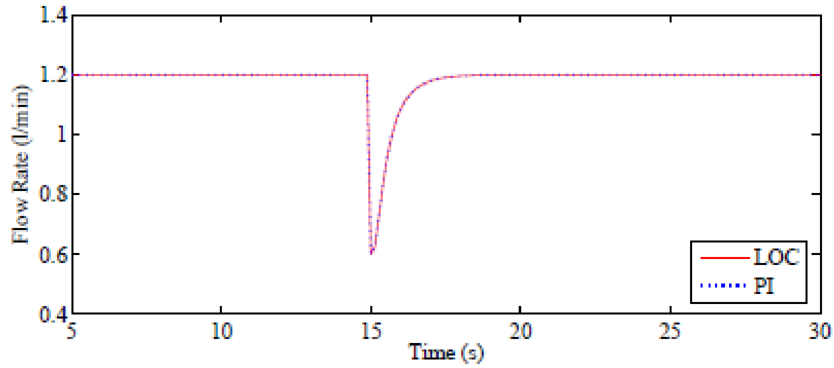

Figure 23: The simulated output responses of both controllers, when they are subjected to step disturbance.

Figure 24 shows the output responses obtained from the test rig for both reduced order LOC and digital PI controller tuned using the relations given in (12). In this case, the responses are not exactly the same and a faster response of PI controller can be noticed (similar to results in Figure 20).

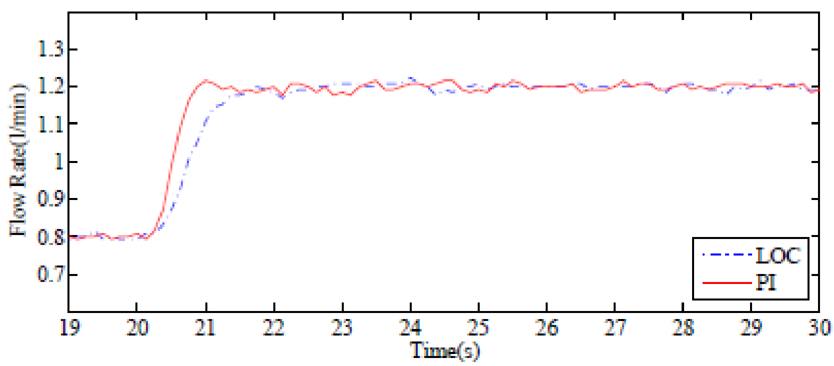

Figure 24: The real output responses of the reduced order LOC and the digital PI controller deduced.

6.2 Comparison between Digital PI Tuned Using Reduced Order LOC and Digital PI Tuned Genetically

GA is used for tuning the controller parameters (KP and KI). The multi-objective function is used to minimize the MSE of the tracking error and eliminate the overshoot. Figure 18 shows the response of the pump controlled by genetically tuned digital PI controller. Two curves illustrated are obtained from simulated model, and the real PCU system.

The difference observed between the simulated model and the real system is because of model uncertainty is involved in the controller parameters tuning made by GA. Table 6 represents the digital PI controller parameters obtained by both tuning techniques discussed.

Table 6: PI Controller parameters tuned by GA and reduced order LOC.

	KP	KI
Tuning using LOC	2.185	0.903
Tuning using GA	3.15	1.33

Figure 25 shows the simulated output responses of the digital PI controller tuned using the two techniques discussed.

From this figure, it is obvious that the response of the genetically tuned PI is slightly faster than the response of the PI tuned using the reduced order LOC. The measured output responses obtained from the test rig for the PI controllers tuned by the two techniques discussed are shown in Figure 26.

From this figure it has been noticed that both controllers approximately have the same speed. However, it has been found that the PI controller tuned using LOC has better response from the overshoot point of view. This means that PI tuned using LOC has a good robust performance compared with the PI tuned genetically. But in general, the digital PI controller does not show robust performance like that of the LOC. Figure 9 and Figure 12 confirm the excellent robust performance of the full and reduced order LOC.

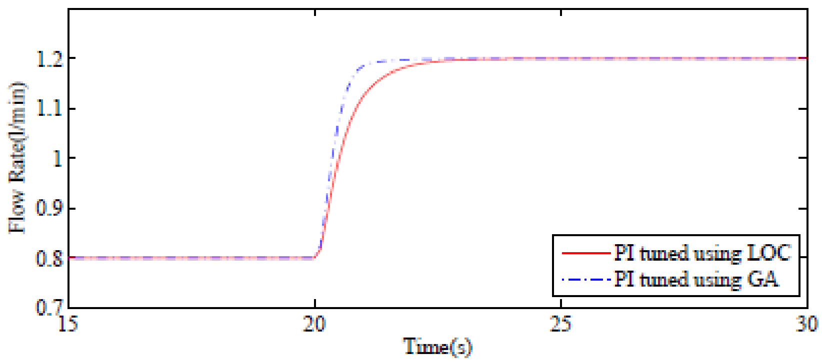

Figure 25: Simulated output response of PI controller tuned genetically and using reduced order LOC.

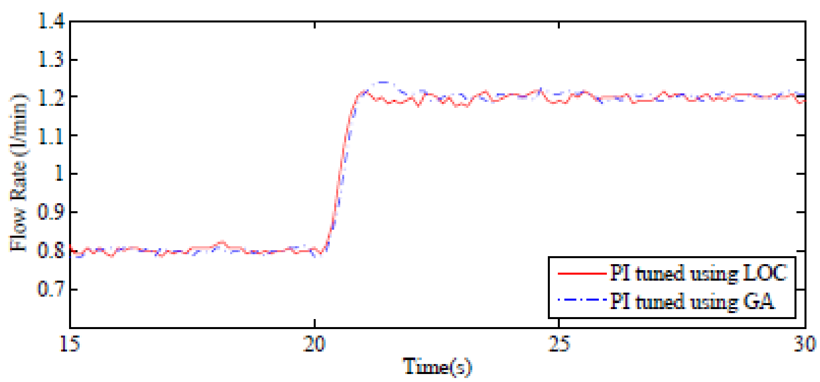

Figure 26: Real output response of PI controllers tuned genetically and using reduced order LOC.

7 Conclusion

Application of reduced order LOC for a higher order system has been studied in this paper. A reduced order (first order) LOC is designed for a higher order (second order) system. This reduced order LOC gives similar results to that obtained using full order LOC, when tested by simulation and on the real system test rig.

Application of GA for tuning controller parameters for both first order LOC and PI structures has been investigated. Better results are obtained using LOC tuned genetically than those obtained using PI controller tuned genetically, where the same objective function is used for both controllers.

Finally, a relationship between the first order LOC and the digital PI controller is deduced. As such, the first order LOC parameters can be easily used for tuning the digital PI controller parameters. This method gives similar results to that obtained by digital PI controller parameters tuned genetically. The experimental results confirm the effectiveness of the reduced order LOC and also the effectiveness of the digital PI controller tuned using the reduced order LOC.

References

[1] Ashry, M., Abou-Zayed, U., and Breikin, T. Comparative robustness study of multivariable controllers for a gas turbine engine. In *The 27th IASTED international conference for modelling, identification, and control,* Innsbruck, Austria, 2008a, pp. 27–32,

[2] Ashry, M., Abou-Zayed, U., and Breikin, T. Control of multivariable systems using modified local optimal controller. In *The 17th IFAC world congress*, Seoul, Korea, 2008b, pp. 8767–8772.

[3] Bytronic international ltd. *Documentation for Bytronic process control unit.* Bytronic international Ltd., 1998.

[4] Ljung, L. *System identification theory for the user.* Prentice-Hall, 1999.

[5] Ljung, L. *System identification toolbox 7.* Matlab, The MathWorks, 2007.

[6] Lyantsev, O., Breikin, T., Kulikov, G., and Arkov, V. Optimal multi-variable control of gas turbine engines. *International journal of systems science,* 2004, 35:79–86.

[7] Matlab, *Genetic algorithm and direct search tool-box 2.* Matlab, The MathWorks, 2007a.

[8] Matlab, *Real-time windows target 3.* Matlab, The MathWorks, 2007b.

[9] Ogata, K. *Discrete-time control systems.* Prentice-Hall, 1995.