Effect of an Interfering Reference Beam on Partially Developed Speckle Contrast
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The present work represents the effect of a coherent reference beam on the contrast of partially developed speckle pattern. An analytical formula for the average speckle contrast in the presence of a coherent reference beam is derived. The theoretical study was showed presence of a reference beam arises mutual interference with the scattered waves contributing in the formation of the speckle pattern. The produced interference pattern modulates the speckle pattern which decreases its contrast. The speckle contrast can be controlled by the ratio between the intensities of the reference and the scattered beams as well as the optical path difference between the reference and the object beam. Experimental work is carried out showing the speckle pattern is modulated by interference pattern due to the presence of a reference beam. It leads to decrease the speckle contrast with increasing the intensity of the reference beam with respect to the intensity of the diffused beam from the rough object. The experimental results are in good agreement with the presented theoretical study.

Keywords: Speckle contrast, Speckle interferometry, Effect of coherent reference beam, Speckle non-contact technique.

Introduction
The statistical characteristics of surface roughness are of great importance in many industrial fields. It is a parameter of the surface quality which is closely associated with the performance of material surfaces. Speckle formation by surface reflection or transmission of light through materials is a noncontact technique used for testing the quality of surfaces with respect to their functional and optical properties for various loading conditions. Speckle contrast, speckle correlation and the speckle power spectrum density are the most optical noncontact techniques used for measuring surface roughness and give information about its statistical characteristics [1-11]. In some industrial applications and medical investigations speckle pattern is considered to be noise [12-115]. The effect of the light spectral broadening on the speckle contrast is presented in [13-15]. The speckle contrast versus the speckle size and aperture is investigated [19]. A coherent reference beam has a significant effect on the contrast of speckle interferometry[20]. It adjusts the speckle contrast and allows the calculation of the surface phase modulation due to the surface deformation caused by an external stress on the surface[21].

The present work deals with the effect of an external coherent beam on the contrast of partially developed speckle pattern formed by transmitted scattered beams through a rough glass material. A new analytical derivative of the speckle contrast in the presence of coherent reference beam is obtained. The statistical average of product of two dependent random variables is obtained without facing the correlation between them. Explanation is given for the reason of reducing the speckle contrast in the presence of interfering reference beam and controlling this effect. Through the variation of the ratio of the intensities between the reference and object beams, the speckle contrast can be controlled. The reference beam and the scattered beams are considered to be monochromatic of the same wavelength.
Theory

Let us consider the interference between a direct reference beam and a resultant field of scattered waves from a diffuser. The reference beam and the resultant scattered field are assumed to be plane polarized and monochromatic of the same wavelength \( \lambda \) originated from a point light source through a beam splitter.

The intensity of the produced speckle interferogram \( I_{\text{int.}} \) will be given by

\[
I_{\text{int.}} = \left[ A_r e^{i \phi_r} + A_s e^{i \phi_a} \right] \left[ A_r e^{-i \phi_r} + A_s e^{-i \phi_a} \right]
\]

Where at the point of interference on the observation plane, \( (\varphi_l = 2\pi \ell_l / \lambda) \) is the phase of the reference beam due to its optical path length from the beam splitter up to the speckle observation plane. \( \varphi_o = (2\pi \ell_o / \lambda) + \varphi_s \) is the phase of the object beam due to its optical path length \( \ell_o \) from the beam splitter up to the speckle observation plane in addition to the phase \( \varphi_s \) which is acquired by the object beam due to the scattering process of the diffuser. \( \varphi_l \) and \( \varphi_o \) are two independent variables. \( A_r \) and \( A_s \) are the real amplitudes of the reference beam and the resultant amplitude of the scattered waves on the observation plane, respectively.

Equation (1) can be rewritten in the form

\[
I_{\text{int.}} = A_r^2 + A_s^2 + 2A_r A_s \cos(\Delta \varphi - \phi_s) \tag{2}
\]

\( \Delta \varphi = (\pi/\lambda)(\ell_r - \ell_o) \) is the phase of the reference beam with respect to the scattered object beam due to their optical path difference \( (\ell_r - \ell_o) \).

The third term \( 2A_r A_s \cos(\Delta \varphi - \phi_s) \) in Eq. (2) represents a mutual interference term between the field of the reference beam \( A_r e^{i \varphi_r} \) and the resultant speckle field \( A_s e^{i \varphi_a} \). Noting that \( A_r^2 = I_r \) is the intensity of the reference beam.

\[
A_s^2 = I_s = a^2 \sum_{m=1}^{N} \sum_{n=1}^{M} \cos(\varphi_m - \varphi_n) \tag{3}
\]

\( I_r \) is the intensity of the speckle pattern in the case of the absence of the reference beam. \( \varphi_o \) and \( \varphi_s \) are two independent random phase values acquired by the scattered waves due to the random elementary \( m \)th and \( n \)th heights of the scatterer respectively. \( a \) is the real wave amplitude scattered due to its transmission through one scatterer of the diffuser and collected by CCD camera.

The scattered waves are considered to be of equal real amplitudes $a$. $N$ is the number of the scatterers contributing in the formation of the speckle pattern.

The following Eqs. give the first and second orders statistics of the average intensity of the speckle interferogram $<I_{int}>$ and $<I_{int}^2>$, respectively.

\[
\langle I_{int} \rangle = I_r + \langle I_s \rangle + 2A_r \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle
\]  
\[
\langle I_{int} \rangle^2 = I_r^2 + \langle I_s \rangle^2 + 4I_r \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle^2 + 2I_r \langle I_s \rangle + 4I_r A_r \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle + 4A_r \langle I_s \rangle \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle
\]  
\[
\langle I_{int}^2 \rangle = I_r^2 + \langle I_s \rangle^2 + 4I_r \langle A_s \cos^2(\Delta \varphi_r - \varphi_s) \rangle + 2I_r \langle I_s \rangle + 4I_r A_r \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle + 4A_r \langle I_s A_s \cos(\Delta \varphi_r - \varphi_s) \rangle
\]

The normalized speckle contrast is given by

\[
C = \left[ \frac{\langle I_{int}^2 \rangle - \langle I_{int} \rangle^2}{\langle I_{int} \rangle} \right]^{1/2}
\]

From the Eqs. (4), (5), (6) and (7) we can get

\[
C = \left\{ \frac{\langle I_s^2 \rangle - \langle I_s \rangle^2 + 4I_r \left[ \frac{\langle A_s^2 \cos^2(\Delta \varphi_r - \varphi_s) \rangle - \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle^2}{\langle I_s \rangle \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle} \right]}{I_r + \langle I_s \rangle + 2A_r \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle} \right\}^{1/2}
\]

By dividing both the nominator and denominator by $N^2$, equation (8-a) can be rewritten as

\[
C = \left\{ \frac{\langle I_s^2 \rangle - \langle I_s \rangle^2 + 4I_r \left[ \frac{\langle A_s^2 \cos^2(\Delta \varphi_r - \varphi_s) \rangle - \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle^2}{\langle I_s \rangle \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle} \right]}{\langle I_s \rangle + \langle I_s \rangle + 2A_r \langle A_s \cos(\Delta \varphi_r - \varphi_s) \rangle} \right\}^{1/2}
\]
Where $I = I_r/Na^2$ and $Na^2$ is the light intensity of the scattered beams contributing to form the speckle pattern. It is proportional to the light intensity incident on the diffuser area. To check the reliability of the proposed model, setting in equation (8-b), it gives the published formula of the speckle contrast [18, 20] which assert the current theoretical view.

To evaluate the mathematical statistical expressions of the speckle contrast given by formula (8-a), we have to take into account that $Ascos\phi$ and $Assin\phi$ are, respectively the real and imaginary components of the complex resultant scattered wave $Ase\phi$. Under this consideration we get

$$\langle A_s \cos \phi_s \rangle = a \int_{-\infty}^{\infty} \sum_{j=1}^{N} \cos \phi_j P(\phi) d\phi$$

Where $P(\phi)$ is the phase probability density distribution of the random independent phases of the scattered waves acquired from the random surface heights.

Let $\langle A_s \cos \phi_s \rangle = Nax$ \hspace{1cm} (9)  
with 
$$x = \langle \cos \phi \rangle = \int_{-\infty}^{\infty} \cos \phi P(\phi) d\phi$$ \hspace{1cm} (10)

Similarly, 
$$\langle A_s \sin \phi_s \rangle = Nay$$ \hspace{1cm} (11)

with 
$$y = \langle \sin \phi \rangle = \int_{-\infty}^{\infty} \sin \phi P(\phi) d\phi$$ \hspace{1cm} (12)

For the present work, case of transmission through material of rough surface, is given by

$$\phi = \frac{2\pi}{\lambda} h_r (\mu - 1)$$ \hspace{1cm} (13)

With $\mu$ is the material refractive index and $h_r$ is the random height of the surface roughness.
\[ \langle A_x \cos(\Delta \varphi_r - \varphi_s) \rangle = Na[x \cos \Delta \varphi_r + y \sin \Delta \varphi_r] \quad (14) \]
\[ \langle A_x^2 \cos^2(\Delta \varphi_r - \varphi_s) \rangle = \sin^2 \Delta \varphi_r \langle l_2 \rangle + a^2 \cos 2\Delta \varphi_r \left[ \frac{N}{2} (1 + x_2) + N(N - 1)x^2 \right] + a^2 \sin 2\Delta \varphi_r \left[ \frac{N}{2} y_2 + N(N - 1)xy \right] \quad (15) \]

with
\[ x_2 = \langle \cos 2\varphi \rangle = \int_{-\infty}^{\infty} \cos 2\varphi P(\varphi) d\varphi \quad (16) \]
\[ y_2 = \langle \sin 2\varphi \rangle = \int_{-\infty}^{\infty} \sin 2\varphi P(\varphi) d\varphi \quad (17) \]
\[ \langle l_x A_x \cos(\Delta \varphi_r - \varphi_s) \rangle = a^2 \cos \Delta \varphi_r \langle l_x A_x \cos \varphi_s \rangle + a^2 \sin \Delta \varphi_r \langle l_x A_x \sin \varphi_s \rangle \]
\[ \langle l_x A_x \cos(\Delta \varphi_r - \varphi_s) \rangle = a^2 \cos \Delta \varphi_r \sum_{m=1}^{N} \sum_{j=1}^{N} \cos(\varphi_m - \varphi_j) \cos \varphi_j + a^2 \sin \Delta \varphi_r \sum_{m=1}^{N} \sum_{j=1}^{N} \cos(\varphi_m - \varphi_j) \sin \varphi_j \quad (18) \]

Thus, to evaluate \( \langle l_x A_x \cos(\Delta \varphi_r - \varphi_s) \rangle \), all of the following possible combinations between the random independent values of \( \varphi_m, \varphi_n, \) and \( \varphi_j \) must be taken into consideration: Case I: \( m = n = j \), through Eq. (18) gives

I. \( m = n = j \) \( N \) times

II. \( m \neq n \neq j \) \( N(N - 1)(N - 2) \) times

III. \( \begin{cases} A: \ m = n \neq j \\ B: \ m = j \neq n \\ C: \ n = j \neq m \end{cases} \) \( N(N - 1) \) times

Case I: \( m = n = j \), through Eq. (18) gives
\[ \langle l_x A_x \cos(\Delta \varphi_r - \varphi_s) \rangle = a^2 N(x \cos \Delta \varphi_r + y \sin \Delta \varphi_r) \quad (19) \]

Case II: \( m \neq n \neq j \), through Eq. (18) gives
\[ \langle l_x A_x \cos(\Delta \varphi_r - \varphi_s) \rangle = a^3 N(N - 1)(N - 2) \{ \cos \Delta \varphi_r [(x^3 + y^2x) + \sin \Delta \varphi_r (x^2y + y^3)] \} \quad (20) \]

Case III-A: \( m = n \neq j \), through Eq. (18) gives
\[ \langle l_x A_x \cos(\Delta \varphi_r - \varphi_s) \rangle = a^3 N(N - 1) [x \cos \Delta \varphi_r + y \sin \Delta \varphi_r] \quad (21) \]

Case III-B: \( m = j \neq n \), through Eq. (18) gives

\[ \langle I_s A_s \cos(\Delta \varphi_r - \varphi_s) \rangle = a^3 N(N - 1) \left\{ \cos \Delta \varphi_r \left[ \frac{1}{2} (1 + x_2) x + \frac{1}{2} y_2 y \right] + \sin \Delta \varphi_r \left[ \frac{1}{2} (1 - x_2) y + \frac{1}{2} y_2 x \right] \right\} \]  

(22)

Case III-C: \( n = j \neq m \), through Eq. (18) gives

\[ \langle I_s A_s \cos(\Delta \varphi_r - \varphi_s) \rangle = a^3 N(N - 1) \left\{ \cos \Delta \varphi_r \left[ \frac{1}{2} (1 + x_2) x + \frac{1}{2} y_2 y \right] + \sin \Delta \varphi_r \left[ \frac{1}{2} (1 - x_2) y + \frac{1}{2} y_2 x \right] \right\} \]  

(23)

Equations (19), (20), (21), (22), and (23) give

\[ \langle I_s A_s \cos(\Delta \varphi_r - \varphi_s) \rangle = a^3 N^2 [x \cos \Delta \varphi_r + y \sin \Delta \varphi_r] + a^3 N(N - 1) \left[ (1 + x_2) x + y_2 y \right] + \sin \Delta \varphi_r \left[ y_2 x + (1 - x_2) y \right] + a^3 N(N - 1) (N - 2) \left[ \cos \Delta \varphi_r (x^3 + y^2 x) + \sin \Delta \varphi_r (x^2 y + y^3) \right] \]  

(24)

The speckle contrast \( C \), given by Eq. (8-a), is a general formula of the speckle contrast. It can be calculated through Eq. (14), (15) and (24) for a given random phase probability density distribution \( P(\varphi) \).

The statistical density distribution of the random phases \( \varphi \) is mostly considered to be a Gaussian probability of zero mean value given by

\[ P(\varphi) = \frac{1}{\sqrt{2\pi}\sigma_\varphi} \exp \left( -\frac{\varphi^2}{2\sigma_\varphi^2} \right) \]  

(25)

With \( \sigma_\varphi \) is the standard deviation of the random phases.

\[ \sigma_\varphi = 2\pi/\lambda (\mu - 1) ((h_r e^2)^{(1/2)}) = 2\pi/\lambda (\mu - 1) \sigma_R \]

with \( \sigma_R \) is the roughness standard deviation.

It gives

\[ x = \exp \left( -\frac{\sigma_\varphi^2}{2} \right) \]

\[ x_2 = \exp \left( -2\sigma_\varphi^2 \right) \]

and \( y_2 \) are equal zero.

Note that, \( <I_s> \) and \( <I_2> \) are given previously by [18, 20]

Results and Discussions

Theoretical results

The dependence of the speckle contrast $C$ on the ratio is demonstrated through the computation of formula (8-b) considering $\rho = 100$, $\rho = 1000$, $N = 2000$ and $N = 10000$ as shown in Figs. 1(a-d) for different $\sigma$ values. From the figures, it is evident that the speckle contrast increases with increasing $\sigma$ and the density $N$ of the scattered waves contributing in the formation of the speckle pattern. The increase in $\sigma$ is attributed to the requirement of the formation of the speckle pattern; this means that as $\sigma$ increases, the variation of the heights of the surface increases therefore, the contrast increases. In this context as $\sigma$ decreases, all the roughness heights have the same value and the surface become not rough and the contrast in this case be zero. As the number of the scattered waves, within an illuminated scattering area, contribute to form the speckle pattern are increased, the amplitude of each scattered wave decreases. This act as the ratio of the reference beam intensity to that of one scattered wave increases. Therefore, the reference beam will be more effective in decreasing the speckle contrast.

The decrease in the speckle contrast as increases can be attributed to the mutual interference between the reference beam and the scattered beams. As a result of the presence of the coherent reference beam, the speckle pattern is modulated by the interference pattern. As increases, the intensity of the reference beam compared with the intensity of the scattered beam is increased therefore the visibility of the interference pattern modulating the speckle pattern decreases and act as a background, therefor the speckle contrast decreases.

Experimental results

In order to validate the proposed theoretical model, Mach Zehnder interferometer was employed to determine the dependence of the speckle contrast on the ratio of the intensities between the references and the scattered beams. A He-Ne laser light sources is considered. Figure 2 shows a schematic diagram of the Mach Zehnder interferometer. A polarizer (P) is employed for obtaining a polarized light, a beam expander (BE) is considered to obtain parallel beam. A He-Ne laser beam (l= 632.8 nm, 12mW) is split into two beams using non-polarizing beam splitter BS1. The beams propagate toward two 45° tilted mirrors M1 and M2. The reflected beams from M1 and M2 are recombined to form the modulated speckle pattern with the interference fringes on the array of a CMOS camera. These patterns are stored on a computer storage media and automatically processed using MATLAB code to compute the speckle contrast for different rough surfaces and different ratios. The camera is 8 bit and 1920 x 2560 resolution with square pixels of 2.2 µm. The experiment is carried out for five rough surfaces of different roughness standard deviation as follow $\sigma_R = 0.58$, $\sigma_R = 0.89$, $\sigma_R = 1.5$, $\sigma_R = 3.6$, $\sigma_R = 3.69$ and different ratios $\rho = 0.1$, $\rho = 0.2$, $\rho = 0.3$, $\rho = 0.4$, $\rho = 0.5$, $\rho = 0.65$, $\rho = 0.8$ for each rough surface.

Figures 3 and 4 represent for demonstration a set of the recorded speckle patterns and their scanning intensities along a horizontal line across the speckle pattern in the observation plane. The chosen set are for $\sigma_R = 1.5$, $\sigma_R = 3.6$. The effect of the reference beam is presented for $\epsilon = 0$, $\epsilon = 0.2$, $\epsilon = 0.3$, $\epsilon = 0.4$, $\epsilon = 0.5$, $\epsilon = 0.65$, $\epsilon = 0.8$ for each rough surface.

Figure 5 shows the experimental behavior of the measured speckle contrast $C$ with the ratio of the reference beam intensity to that of the diffused beam incident. The figure shows that the speckle contrast decreases with increasing the ratio and increases with increasing the roughness standard deviation. These results are in consistency with the theoretical computation of speckle pattern as produced from eq.(8-b).

\[
\langle I_s \rangle = a^2[N + N(N - 1)x^2]
\]
\[
\langle I_s^2 \rangle = a^4\{N^2 + 2N^2(N - 1)x^2 + N(N - 1)[(1 + x_2^2) + 2(N - 2)(x^2 + x_2x^2) + (N - 2)(N - 3)x^4]\}
\]
\[
\langle I_s \rangle^2 = a^4\{N(N - 1)[1 + x_2^2 + 2(N - 2)(x^2 + x_2x^2) - (4N - 6)x^4]\}\]
Fig. 1. The dependence of the average speckle contrast $C$ on the ratio $\epsilon$ computed theoretically for $D\phi_r = 0$, $\sigma_j = 2.5$, 3 and 5 for (a) $N = 100$, (b) $N = 1000$, (c) $N = 2000$ and (d) $N = 10000$.

Fig. 2. Schematic diagram of the experimental setup.
Fig. 3. The speckle pattern (left) and its intensity distribution (right) for (a), (b) (c) considering a rough surface of $\sigma_r = 1.5$. 

Fig. 4. The speckle pattern (left) and its intensity distribution (right) for (a), (b) (c) considering a rough surface of $\sigma_s = 3.6$.

Fig. 5. The dependence of the average speckle contrast $C$ on the ratio $\varepsilon$ for different He-Ne laser 632.8 nm.
Conclusion

The speckle contrast can be controlled by the variation of the ratio of the reference and the object beam intensities. As the ratio increases, the intensity of the reference beam compared with the intensity of the scattered beam is increased therefore the visibility of the interference pattern modulating the speckle pattern decrease and act as a background, therefore the speckle contrast decreases.

The speckle contrast increases with increasing the roughness standard deviation. It is attributed to the principal requirement of formation speckle pattern. The roughness standard deviation is a measure of the mean square random differences between the surface roughness heights. Decreasing the roughness standard deviation means that the roughness heights tend to be of a single value. This means the surface is not more rough and the contrast of the speckle pattern tends to be of zero value. Therefore, an increase in the roughness standard deviation leads to increase the speckle contrast.

For rough surface of higher scatterers density, the number of the scattered waves within an illuminated scattering area contribute to form the speckle pattern, will increase. In this case the ratio $\epsilon$ of the reference beam intensity to that of one scattered wave increases. Therefore, the reference beam will be more effective in decreasing the speckle contrast. The case of $\epsilon = 0$ means that the reference beam is not present, the speckle contrast given by the present derived formula (26) tends to the published formula for the speckle contrast [16, 17]. It emphasis the present theoretical view.

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