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AMPLITUDE AND PHASE MODULATED PULSE TRAINS FOR CLUTTER REJECTION IN RADAR

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ABSTRACT

In radar systems the very low side-lobes that can be achieved with amplitude and phase modulated pulse trains make their use particularly attractive in systems requiring a large dynamic range. Moreover, the excellent self-clutter rejection performance is obtained without sacrificing the ability for the separation of close targets (no main lobe widening). The additional expense of encoding and decoding in amplitude and phase may be justified for radars that must cope with land clutter or operate in a dense-target environment. This paper presents a method for generating finite length sequences that have high energy ratio and low sidelobe energy in their autocorrelation function. The method is computer based and generates the desired sequence of any given length in an iterative manner starting with a sequence that has relatively low sidelobes in its autocorrelation function. It is shown that energy ratio of polyphase sequences that have low sidelobe energy , is improved through the process of clipping.

KEY WORDS

Radar ,Clutter, Range, Resolution, Sidelobe,

INTRODUCTION

The estimation of target resolution performance is probably the most difficult problem to solve in modern high performance radar systems. In some cases the interfering objects may themselves be targets of interest, whereas in others they may be undesirable scatterers The optimum receiver for maximum resolution is not necessarily a matched filter receiver (MFR). In practice, however, the typical target situation is too complex and not enough prior information is available to implement anything but a MFR or an approximation. For good target resolution, it has thus been necessary to retain a MFR and optimize the signal waveform so as to

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reduce the mutual interference (self-clutter) between targets[1-3]. Moreover, the advent of solid state antenna arrays had its impact on system design in two ways. First peak power limitations have necessitated the use of waveforms with long durations in order to achieve the required signal energy over a desired range. Secondly, the ability to switch the beam at high speeds gives the radar a multi-function capability, thus requiring the flexibility to enable a variety of waveforms to be employed [4,5]. Theoretical studies that provide the basis for technical advances have not, so far, solved the general signal design problem. The knowledge of the properties of pulse trains, a class of signals particularly well suited to digital processing is, therefore, of increasing practical importance. Two properties of such sequences are of interest. One is the energy ratio E, defined as the ratio of the total energy of the sequence to the energy of the largest pulse. The other property is the total sidelobe energy SLE i.e. the energy in the sidelobes of the autocorrelation function, since the self-clutter power at the output of the matched filter is directly proportional to this quantity [1-2]. For a given sequence $\{a_i\}$ of length (n+1) these quantities are given by

$$E = \frac{\text{Total Energy of the sequence}}{\text{Energy of the largest pulse}} = \frac{\sum_{i=0}^{n} a_i^2}{\max\left|a_n\right|^2} \qquad (1)$$

$$SLE = 2\sum_{i=1}^{n} r_i^2 \qquad (2)$$

$r_i = \sum_{j=0}^{n} a_i a_{i+j}$, i= 0,1,2 . . .n is the autocorrelation function of the sequence $\{a_i\}$.

This paper presents a method for generating finite length sequences that have high energy ratio and low sidelobe energy in their autocorrelation function. It has been shown that a sequence formed from the weighting sequence of an optimum inverse filter that is designed for a sequence of the same length, and having low sidelobe energy, also has a good auto-correlation function, better than the sequence itself. This result is here utilized to generate a sequence of low side-lobe energy by repeating the process of inversion a number of times. The method is computer based and generates the desired sequence of any given length in an iterative manner starting with a sequence that has relatively low sidelobes in its autocorrelation function. It is also suggested that limiting or “clipping” to some predetermined level, the value of the largest pulse of a polyphase low sidelobe energy sequence generates clipped sequences of high energy ratio while still keeping low sidelobe in its autocorrelation function.

THE PROPOSED METHOD

Inverse Filter

The optimum inverse (or least error energy) filter [6,7] problem can be formulated with reference to Fig. 1. The error sequence $\{e_i\}$ represents the difference between the desired filter output sequence $\{d_i\}$ and the actual filter output sequence $\{c_i\}$. $\{a_i\}$

is the input sequence of length (n+1). The desired filter output sequence in the present case, consists of a unit 'spike' at some time index ‘k’. The optimization problem is to select the filter weighting sequence $\{f_i\}$ of length (m+1) such that the energy of the error sequence $\{e_i\}$ is minimized. The error energy I is given by

$$I = \sum_{t=0}^{m+n} e_t^2 = \sum_{t=0}^{m+n} [d_t - \sum_{s=0}^{m} f_s a_{t-s}]^2 \tag{3}$$

The solution to this optimization problem is given by Proakis [6]. According to him, if there is no constraint on the admissible values of $\{f_j\}$ then the optimum filter coefficient must satisfy the relation $\partial I/\partial f_j = 0$, for j = 0, 1,.......,m. The result of performing the indicated operation is

$$\sum_{s=0}^{m} f_s r_{j-s} = g_j \qquad j=0, 1, 2,\dots,m \tag{4}$$

where $r_{j-s} = \sum_{t=0}^{2n} a_{t-s} a_{t-j}$ is the auto-correlation coefficient (for index, j-s) of the input sequence $\{a_i\}$, and $g_j = \sum_{t=0}^{m+n} d_t a_{t-j}$; j = 0, 1, … ,m are the cross-correlation coefficients of the sequence $\{d_i\}$ with sequence $\{a_i\}$. Eqn. (4) provides a set of (m+1) linearly independent expressions in (m+1) unknowns.

The finite length inverse filter for a given input sequence obtained by solving the set of Eqn. (4) is not an exact inverse. The filter, however, is optimum for the chosen length m+1. For a given filter length m+1, the inverse filter weighting sequence is computed for each value of ‘k’. The value of ‘k’ which minimizes I is then selected [7].

ACF of the Filter Weighting Sequence

If the length of the inverse filter is made equal to the length of input sequence i.e. when m=n then the (n+1) equations (Eqn. 4) can be written in matrix form as

$$\mathbf{RA} = \mathbf{G} \tag{5}$$

A is a column vector containing (n+1) tap gains of the filter, **G** is a vector containing (n+1) discrete cross correlation coefficients of the input sequence $\{a_i\}$ and the desired output $\{d_i\}$. **R** is the (n+1) x (n+1) symmetric and positive definite auto-correlation matrix [6,7].

If the input sequence $\{a_i\}$ has near ideal auto-correlation function such as the Huffman or Barker sequences[1,8,9], then $r_j = 0$ for $j \neq 0$ or n, and also $r_o >> r_n$, therefore, neglecting r_n, the auto-correlation matrix **R** becomes a diagonal matrix and the filter coefficients f_j from Eqn. (5) are given by

$$f_j = g_j/r_0 \qquad j = 0, 1, \dots, n \tag{6}$$

If the input sequence is such that the optimum value of k is (n+1), the cross-correlation coefficients are

$$g_j = a_{n-j} \qquad j = 0, 1, 2, ..., n \qquad (7)$$

The inverse filter coefficients under these conditions are from Eqn. (6)

$$f_j = a_{n-j}/r_0; \qquad j = 0, 1, ..., n \qquad (8)$$

The side lobe energy of the sequence $\{a_i\}$is

$$(SLE)_A = 2.\sum_{i=1}^{n} r_i^2 \qquad (9)$$

If the inverse filter tap gains $\{f_j\}$ given by Eqn. (8) are taken as a sequence then the side lobe energy of this sequence $\{f_j\}$ is

$$(SLE)_F = 2.\sum_{i=1}^{n} r_i'^2 = 2.\sum_{i=1}^{n} \frac{r^2}{r_0^2} \qquad (10)$$

where $\{ r'_i\}$ are the side lobes of the auto-correlation function of the sequence $\{f_i\}$.

Now $$(SLE)_F = \frac{1}{r_0^2} 2.\sum_{i=1}^{n} r_i^2 = \frac{1}{r_0^2}(SLE)_A$$

Therefore, since for $n > 1$, $r_o^2 > 1$,

$$(SLE)_F < (SLE)_A \qquad (11)$$

Eqn. (11) is derived when $r_j = 0$, for $j \neq 0$ or n. In practice $r_j \neq 0$ for $j \neq 0$ or n, i.e. the auto-correlation function always has some side-lobes; however, if the main lobe is of considerably greater magnitude than the side-lobes, the condition derived in Eqn. (11) will still hold good, though the filter coefficients f_j will not be exactly equal to the value given by Eqn. (8).

Eqn.(11) indicates that a sequence formed from the weighting sequence of an inverse filter that is designed for a sequence of the same length, and having low sidelobe energy, also has a good auto-correlation function, better than the sequence itself. This fact is here utilized to generate a sequence of low side-lobe energy by repeating the process of inversion a number of times. Thus new sequences having lower side-lobe energy are generated iteratively by successively taking the coefficients of the inverse filter as a sequence and computing the corresponding inverse filter coefficients. The initial or the starting sequence for this iterative process can be any sequence of desired length, having low side-lobe energy. The method can, therefore, be used to generate both amplitude and phase modulated pulse trains for any desired length. As an example, Fig.2 shows the variation in the sidelobe energy and the energy ratio of sequences obtained at different stages of inverse filtering (computer runs) starting from a Barker code of length 13. It can be seen that the sidelobe energy as well as the energy ratio reduces with the computer runs. The decrease in energy ratio, however, is compensated by choosing a waveform of larger length.

CHOICE OF STARTING SEQUENCE

The choice of the starting sequence is important in order that the iterative process converges to some final value. It can be seen from earlier discussion that in order that the sequence obtained from the inverse filter coefficients has lower sidelobe energy, the starting sequence must fulfill the following three conditions

1. The filter should be of the same length as the sequence for which it is designed.
2. The optimum pulse position 'k' should be equal to n where n is the length of the starting sequence.
3. It should have low side-lobe level in its autocorrelation function.

There are many methods for generating sequences of reasonably good autocorrelation sequences that can be used for selecting the starting sequence and these are available in the published literature[1,8-10].

CLIPPING OF POLYPHASE SEQUENCES

A figure of merit for the energy distribution of a sequence is the energy ratio defined by Eqn(1). It is evident that the maximum value of the energy ratio will be attained for purely phase modulated pulse trains such as binary sequences, having all element amplitudes equal to unity. In this case the energy ratio will be equal to n (the length of the sequence). The energy ratio depends on two independent variables, namely, the total energy E of the pulse train and the magnitude of the largest pulse denoted by max $|a_n|$. The energy ratio of a polyphase sequence of given length can, therefore, be increased by decreasing the energy contained in the largest pulse max $|a_n|$. This is done by process of 'clipping' where the largest pulse in the sequence is clipped to some predetermined level. The clipping level is defined as

$$\mathrm{CLIP} = \frac{\max(\mathrm{Pulse.Height}) - \min(\mathrm{Pulse.Height})}{100}.x \qquad (12)$$

x is the percentage at which the clipping is required. The side lobe energy in the ACF of the clipped sequence will be greater than that of the unclipped sequence and this is achieved at the cost of an increase in the sidelobe energy in its autocorrelation function. As the clipping level is increased, the energy ratio of the resulting sequence increases, while the side lobe energy also increases. In the limit when the energy ratio is maximum, the resulting sequence will be a binary sequence, but still having an ACF with small side lobe levels. The process of clipping thus provides a means to generate sequences where energy ratio and side lobe energy is a compromise between those of polyphase and binary sequences. What level of clipping should be chosen would however, depend upon the actual requirements and test conditions.

Figure 3 shows the variation of energy ratio and side lobe energy for various levels of clipping starting from the Huffman sequence of length 13. It can be seen from the Figure that as the clipping level is increased the energy ratio of the resulting clipped sequence increases while the side lobe energy also increases. For full clipping such that all the sequence elements have same value, the resulting sequence is the binary sequences of corresponding length. It is interesting to mention here that full clipping of the Huffman sequence of length 13 results in the Barker sequence of same length.

Thus the method of clipping can be used to advantage for generating either binary sequences or non-binary sequences having low side lobe energy and high energy ratio. The main requirement of this method is that the starting sequence which is a non-binary sequence, must have very low side lobe energy. This method can, therefore, be used only in those cases where non-binary sequences having good auto-correlation properties are available or a method is available for the generation of such sequences as has been proposed in this paper. In some applications a complete suppression of the side-lobes may not be required, instead they can be kept to a specified low level. Consequently clipping may be used to advantage for improving the energy ratio.

CONCLUSIONS

In modern complex target environment and in the absence of enough prior available information, it has become necessary to retain a matched filter processor and to optimize the signal waveform in order to reduce mutual interference (self-clutter) between targets. The knowledge of the properties of the pulse trains, a class of signals particularly well suited to digital processing is therefore, of increasing practical importance. the additional expense of encoding and decoding in amplitude and phase may be justified for radars that must cope with land clutter or operate in a dense-target environment.

It is shown that the inverse of a sequence that has low sidelobe levels in its autocorrelation function, has sidelobe energy that is lower than the original sequence. This fact is here utilized to generate poly-phase sequences of lower sidelobe levels and higher energy ratios. The inverse of the sequence is here determined in the form of the coefficients of the optimum inverse filter that is designed to have the same length as the sequence itself. Thus new sequences having lower side-lobe energy are generated iteratively by successively taking the coefficients of the inverse filter as a sequence and computing the corresponding inverse filter coefficients. The starting sequence can be of any length. However, it should have low sidelobe levels in its autocorrelation function and the optimum pulse position in the design of the inverse filter should be at a time index equal to the length of the sequence. Energy ratio of polyphase sequences which have low sidelobe energy is improved by clipping the largest pulse. Although, clipping improves the energy ratio, the sidelobe energy also increases with the level of clipping.

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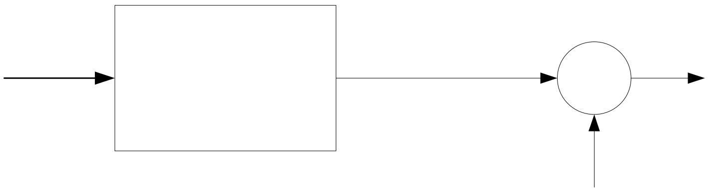

Fig.1 The optimum inverse filter problem

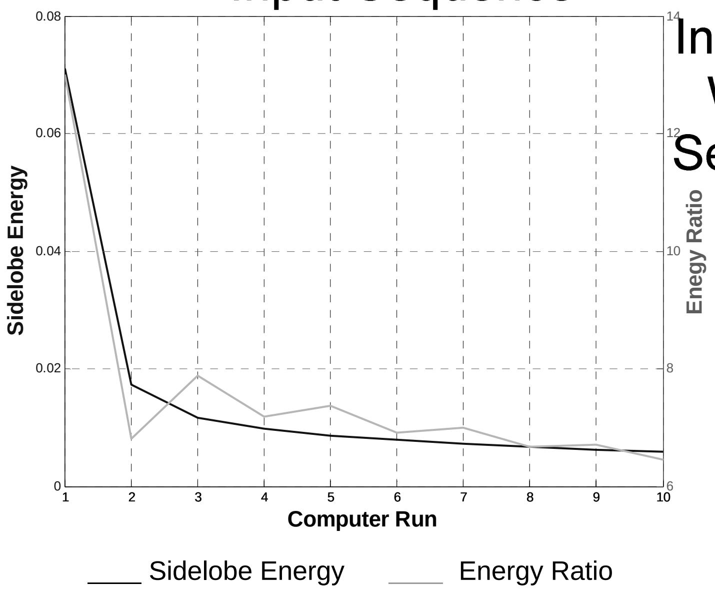

Fig.2 Variation in sidelobe energy and energy ratio with computer runs for the Barker sequence of length 13

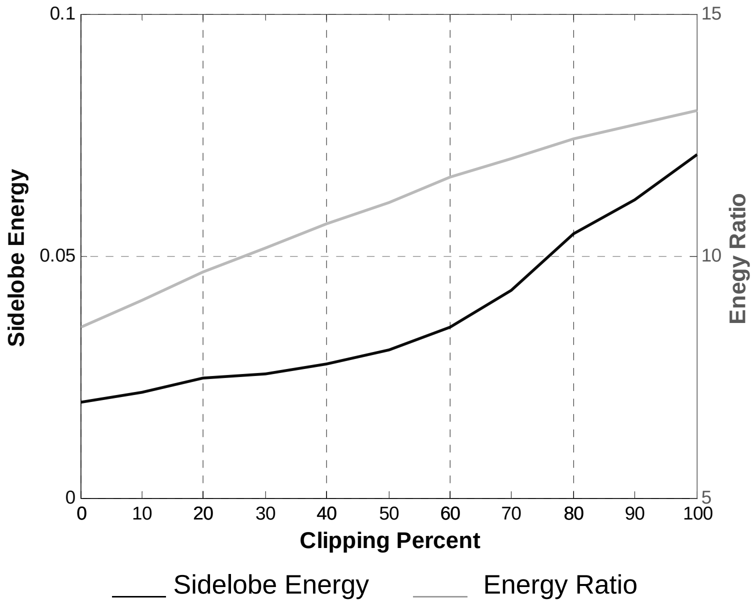

Fig. 3 Sidelobe energy and energy ratio for various levels of clipping a 13-element Huffman sequence