Military Technical College
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Cairo, Egypt

12-th International Conference
on
Aerospace Sciences &
Aviation Technology

DYNAMIC TUNING OF PARTICLES NUMBER IN A PARTICLE FILTER USING FUZZY LOGIC SYSTEM TO TRACK HIGH-PERFORMANCE TARGETS

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ABSTRACT

To solve the problem of nonlinear non-Gaussian state space model engendered due to tracking high-performance targets, the particle filters were used. Obviously, a large number of particles provide more accurate calculations for the posterior density, which results in a better tracking performance, especially when tracking a high performance target. Unfortunately, a large number of particles results in more computation load. Therefore, we need a dynamic tuning of the particles number to provide a relative small particles number for a non-maneuverable target, and a relative large particles number to track a maneuverable target. In this paper, a new fuzzy logic system for dynamic tuning of particles number in a particle filter is introduced. The fuzzy logic system is used to choose the suitable number of particles based on the maneuverability of the target of interest. It assigns a large number of particles to track high-performance targets; meanwhile, a smaller number is required to track non-maneuvering targets. The proposed fuzzy logic system showed good performance in tracking both maneuvering and non-maneuvering targets when applied to track-while-scan (TWS) radar.

KEY WORDS

Particle filters, Fuzzy logic systems, Track-while-scan radar, high-performance targets.

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1. INTRODUCTION

A hybrid estimation problem is target tracking. the future trajectory of an object is predicted based on its previous states. A target may thrust, roll, and pitch; which results in a nonlinear model due to aircraft control and turbulence; e.g. military aircraft. To solve such a problem, extended Kalman filter (EKF) and its higher orders are used [1]. An alternative to the EKF is to use the unscented Kalman filter (UKF) which was introduced in to offer superior performance to that of the EKF [2]. The main difficulty in tracking a maneuverable target is the random change in the target motion defined in the model used by the Kalman filter and its extensions. When the target in not maneuvering, including an acceleration state and using colored noise is not the perfect solution to track a maneuverable object because its performance is degraded.

To solve the problem of nonlinear non-Gaussian state space model problem engendered due to tracking a high-performance target, particle filters are used [3]. However, due to the uncertain and incomplete information in case of a maneuverable target, the advantage of the particle filter is degraded. To overcome this weakness, a fuzzy logic particle filter (FLPF) is proposed in [4]. The fuzzy logic has been considered as the key tool to deal with uncertainty problems [5]. However, a large number of particles provide more accurate calculations for the posterior density, which results in a better tracking performance, especially when tracking a high performance target. Unfortunately, a large number of particles results in more computation load. Therefore, we need a dynamic tuning of the particles number to provide a relatively small particles number for a non-maneuverable target, and a larger particles number to track a maneuverable target. We note that when tracking a maneuvering target the smaller is the particles number the larger is the error between the predicted target states and its actual states. On the contrary when tracking a non-maneuvering target, we do not need a large number of particles to get reasonably good results where the error between the predicted states and the actual ones is within a certain threshold.

In this paper, we introduce a new fuzzy logic system for dynamic tuning of particles number in a particle filter. The fuzzy logic system is used to choose the suitable number of particles based on the maneuverability of the target of interest. Therefore, it assigns a large number of particles to track a high-performance target; meanwhile, a smaller number is required to track a non-maneuvering target. This paper is organized as follows. In Section 2, different maneuvering target models are shown. We introduce briefly the fuzzy logic systems (FLS) in Section 3. The proposed FLPF with dynamic tuning of particles number is explained in Section 4. Finally, experimental results are shown in Section 6 followed by a conclusion.

2. MANEUVERING TARGET MODELS

The most commonly used such models are those known as state-space models, in form stated in (1) and (2) with additive noise,

$$x_{k+1} = F_k\left(x_k, u_k\right) + \nu_k \tag{1}$$

$$z_k = H_k x_k + \omega_k \tag{2}$$

where u_k is a control input vector. In target tracking, the control input u is usually not known and assumed (approximately) to be constant. The choice of the kinematic model is not a trivial problem, where target dynamics, accuracy of approximations, sensor coordinate system, among others, must be taken into account. Various kinematic models proposed for tracking of a target moving in the horizontal plane can be comprised from the following standard motion model from kinematics shown in Fig.1:

$$\dot{x} = v\cos\varphi \tag{3}$$

$$\dot{y} = v\sin\varphi \tag{4}$$

$$\dot{v} = a_t \tag{5}$$

$$\dot{\varphi} = \frac{a_n}{v} \tag{6}$$

where (x,y) are the target Cartesian coordinates, v is the ground speed (air speed added to wind speed), φ is the velocity heading angle, and a_n and a_t are the normal and tangential acceleration components in the horizontal plane, respectively.
The c*oordinated turn* (CT) motion is characterized by $a_n \neq 0$, $a_t = 0$; i.e., the target is moving in circular, constant-speed trajectory. Such motion is preferably specified in terms of the turn rate $\dot{\varphi}$. In the CT model with unknown turn rate, the turn rate is included as a state component, to be estimated. Two models to estimate $\dot{\varphi}$ are:
(a) Wiener process model:

$$\dot{\varphi}_{k+1} = \dot{\varphi}_k + w_k \tag{7}$$

(b) first-order Markov process model:

$$\dot{\varphi}_{k+1} = e^{-\frac{T}{\tau_{\dot{\varphi}}}} \dot{\varphi}_k + w_k \tag{8}$$

where $\tau_{\dot{\varphi}}$ is the correlation time constant for the turn rate, and w is a zero-mean white noise of a suitable level, which can be determined exactly the same way as for the corresponding models for acceleration. Consequently, the value of $\dot{\varphi}_k$ replaces $\dot{\varphi}$ in the transition matrix F_k. where:

$$x_{k+1} = F_k(x_k) + \nu_k = \begin{bmatrix} 1 & 0 & \dfrac{sin\,\dot{\varphi}T}{\dot{\varphi}} & -\dfrac{1-cos\,\dot{\varphi}T}{\dot{\varphi}} \\ 0 & 1 & \dfrac{1-cos\,\dot{\varphi}T}{\dot{\varphi}} & \dfrac{sin\,\dot{\varphi}T}{\dot{\varphi}} \\ 0 & 0 & cos\,\dot{\varphi}T & -sin\,\dot{\varphi}T \\ 0 & 0 & sin\,\dot{\varphi}T & cos\,\dot{\varphi}T \end{bmatrix} (x_k) + \nu_k \tag{9}$$

where ν_k is a non-Gaussian distributed noise process with covariance matrix Q_k given by: [6]

$$Q_k = cov(\nu_k) = S_w \times \begin{bmatrix} \dfrac{2(\dot{\varphi}_k T - sin(\dot{\theta}_k T))}{\dot{\varphi}^3} & 0 & \dfrac{1-cos(\dot{\varphi}_k T)}{\dot{\varphi}^2} & \dfrac{\dot{\varphi}_k T - sin(\dot{\varphi}_k T)}{\dot{\varphi}^2} \\ 0 & \dfrac{2(\dot{\varphi}_k T - sin(\dot{\varphi}_k T))}{\dot{\varphi}^3} & -\dfrac{\dot{\varphi}_k T - sin(\dot{\varphi}_k T)}{\dot{\varphi}^2} & \dfrac{1-cos(\dot{\varphi}_k T)}{\dot{\varphi}^2} \\ \dfrac{1-cos(\dot{\varphi}_k T)}{\dot{\varphi}^2} & -\dfrac{2(\dot{\varphi}_k T - sin(\dot{\varphi}_k T))}{\dot{\varphi}^2} & T & 0 \\ \dfrac{\dot{\varphi}_k T - sin(\dot{\varphi}_k T)}{\dot{\varphi}_k^{\ 2}} & \dfrac{1-cos(\dot{\varphi}_k T)}{\dot{\varphi}^2} & 0 & T \end{bmatrix} \tag{10}$$

3. INTRODUCTION TO FUZZY LOGIC SYSTEMS

A fuzzy logic system (FLS) is a nonlinear mapping from the input to the output space. As shown in Fig.2, there are three modules that characterize a FLS: fuzzifier, inference engine with a rule base, and defuzzifier [7].

A fuzzifier maps a crisp object to a membership function. Generally, fuzzifiers are divided into singletons and non-singletons. Even though singleton fuzzifiers are easier to use and commonly used, non-singleton fuzzifiers are used in case of presence of uncertainties (e.g., high-noise measurements).

In fuzzy logic, there is an important inference rule called *generalized modus ponens*, defined as:

- Premise 1: "*a* is *A**";
- Premise 2: "IF *a* is *A* THEN *b* is *B*";
- Consequence: "*b* is *B**."

where fuzzy set *A** is not the necessarily the same – but similar – as rule antecedent fuzzy set *A*, and fuzzy set *B** is not necessarily the same – but similar – as rule consequent.

In order to be used in the real world, the fuzzy output needs to be interfaced to the crisp domain by the defuzzifier. This fuzzy output will be a membership function that provides the degree of membership of several possible crisp outputs. Hence, the point corresponding to the highest degree of membership in the fuzzy output has to be chosen, which is called *max* defuzzification. Unfortunately, in most practical cases the situation is not that simple since there might be many points having the same maximum degree of membership in the fuzzy output. Moreover, choosing the maximum point of the membership function is an operation that discards most of the information contained in the membership function itself. Consequently, we need a technique that takes into account all the points in the support of this fuzzy output, weighing the points with high membership degree more than the ones with small or no membership degree. This corresponds to a *center of gravity* (COG) operation, as shown in Fi.3.

4. FLPF WITH DYNAMIC TUNING OF PARTICLES NUMBER

In the FLPF algorithm, we treat the maneuver as an abrupt change in the angular turn rate $\dot{\varphi}_k$ at a time k affecting the transition matrix F_k as well as the covariance matrix Q_k given in (9) and (10), respectively. Since F_k and Q_k are considered to be time varying due to target's maneuver and probably nonlinear, a fuzzy logic system is used to estimate the angular turn rate and, thus, predict the target status x_{k+1} for tracking purposes. The key idea is to find the value of $\dot{\varphi}_k$, and hence F_k, that minimizes the particle filter residual at every instant k. The residual error can be seen graphically in Fig. 4.
Assume the target state vector is given by:

$$X_k = \begin{bmatrix} x_k & y_k & \dot{x}_k & \dot{y}_k \end{bmatrix} \tag{11}$$

where x_k and y_k represent the target's position at time k; as well $\dot{x}_k$ and $\dot{y}_k$ represent the target's velocity such that the velocity is constrained to some set V. The radar measurements are modeled as:

$$z_k = H_k X_k + \omega_k = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ tan^{-1}\left(\dfrac{y_k}{x_k}\right) \end{bmatrix} + \omega_k \tag{12}$$

where ω_k is a zero mean non-Gaussian noise with covariance R_k given by:

$$P_k = \begin{bmatrix} \sigma_{x,k}^2 & \sigma_{xy,k}^2 \\ \sigma_{yx,k}^2 & \sigma_{y,k}^2 \end{bmatrix} \tag{13}$$

which is given by:

$$P_k = A_2 R_0 A_2^T \tag{14}$$

knowing that:

$$A_2 = \begin{bmatrix} cos(\dot{\varphi}T) & -sin(\dot{\varphi}T) \\ sin(\dot{\varphi}T) & cos(\dot{\varphi}T) \end{bmatrix} \tag{15}$$

$$P_0 = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & r^2\sigma_\theta^2 \end{bmatrix} = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \tag{16}$$

where A_2 is the rotational matrix, σ_r^2 and σ_θ^2 are the range and azimuth measurement noise respectively.

For the FLS, we use singleton fuzzification and the k^{th} *multiple-input single-output* (MISO) rule. The inputs are the k^{th} position residual r_k and angular turn rate $\dot{\varphi}_k$ where:

$$r_k = \sqrt{(x_k - \tilde{x}_k)^2 + (y_k - \tilde{y}_k)^2} \tag{16}$$

where x_k and $\tilde{x}_k$ represent the actual and estimated target's *x*-coordinate, respectively. The same notation is used for the *y*-coordinates. It is easy to prove that:

$$\dot{\varphi}_k = \cos^{-1} \frac{[(x_k - x_{k-1})(\tilde{x}_k - x_{k-1})][(y_k - y_{k-1})(\tilde{y}_k - x_{k-1})]}{\sqrt{(x_k - x_{k-1})^2 + (y_k - y_{k-1})^2} \times \sqrt{(\tilde{x}_k - x_{k-1})^2 + (\tilde{y}_k - y_{k-1})^2}} \tag{17}$$

In a typical computation, the angular turn rate $\dot{\varphi}_k$ can be approximated by Taylor's expansion to be:

$$\begin{aligned} \dot{\varphi}_k &= \tan^{-1}\frac{e_k}{R_k} = \frac{e_k}{R_k} - \frac{1}{3}\left(\frac{e_k}{R_k}\right)^3 + \frac{1}{5}\left(\frac{e_k}{R_k}\right)^5 - \frac{1}{7}\left(\frac{e_k}{R_k}\right)^7 + \cdots \\ &= \tan^{-1} u_k = u_k - \frac{1}{3}(u_k)^3 + \frac{1}{5}(u_k)^5 - \frac{1}{7}(u_k)^7 + \cdots \end{aligned} \tag{18}$$

Now, we will partition the input space u_k into seven simple partitions on the same interval mentioned above, and we will partition the output space $\dot{\varphi}_k$ into five membership functions as shown in Fig.5 and Fig.6, respectively. In these figures, we use the same the abbreviations refer to the linguistic variables mentioned in Table 1. Then, we develop six simple rules, listed in Table 2, that follow the system dynamics. The fuzzy associative memory (FAM) table for these rules is given in Table 3.

Obviously, a large number of particles provide more accurate calculations for the posterior density, which results in a better tracking performance. Unfortunately, a large number of particles results in more computation load. Therefore, we need a dynamic tuning of the particles number to provide a relative small particles number for a non-maneuverable target, and a relative large particles number to track a maneuverable target.

We note that when tracking a maneuvering target the smaller is the particles number the larger is the error between the predicted target states ($\hat{X}_k$) and its actual states (X_k). On the contrary when tracking a non-maneuvering target, we do not need a large number of particles to get reasonably good results where the difference between $\hat{X}_k$ and X_k is within a certain threshold.
We define the universe of discourse of the input variable, $|u_k|$, by the interval [0,u_{max}], and the universe of discourse for the output N_k^* is the interval [500,5000] particles.
First, we will partition the input space $|u_k|$ into four partitions on its interval, and we will partition the output space N_k^* into three membership functions as shown in Fig.7 and Fig.8, respectively.
In these figures, the abbreviations Z, S, M, and L refer to the linguistic variables “Zero”, “Small”, “Medium”, and “Large”. Then, we develop four rules, listed in Table 4, that emulate the system dynamics. The fuzzy associative memory (FAM) table for these rules is given in Table 5.
For example, if the input $|u_k|$ = 0.09625, we can notice that it fires the first rule only. Meanwhile, if the input u_k = 0.28875, the first and second rules are fired. For the case of the input u_k = 0.14435, the first, second, and third rules are fired; meanwhile if the input u_k = 0.48125, the second and third rules are fired.

5. EXPERIMENTAL RESULTS

We assume a TWS radar system assigned to track a high performance target flying with expected ground speed of 250 m/sec and its maneuverability can reach 13g. Calculating the corresponding angular turn rate, we find it to be in the range [-30°,30°]. Therefore, we can define the range of e_k/R_k (=u_k)to be [-0.5774,0.5774] where R_k is the distance that a target can fly during one scan period T. For a TWS radar system, the antenna scan rate varies from 12 to 20 rpm; i.e., the scan period can be defined in the range [3,5] seconds. For a high-performance target flying with a speed 250 m/sec, the error e_k can be defined as shown in Table 6.
Applying the FLPF, we get the defuzzified results shown in Table 7, where the empty cells define a membership function that was not triggered. Moreover, applying the dynamic tuning of the particles number explained in the previous section, we get the defuzzified output using centroid method shown in Table 8. When we combine the results using union operator, we get an aggregated result as summarized in graphically in Figure 9 using union operator and in Fig.10 using intersection operator.
Fig.11 shows the performance of the proposed tracking a high-performance target moving with constant speed for a while then performing a maneuver with 13g. It can be seen from the figure that our proposed algorithm tracked the target successfully. Moreover, using the appropriate particles number reduces the computational load in case of tracking the target while moving with a constant speed.

6. CONCLUSION

In this paper, we introduced a new technique to tune particles number dynamically using a fuzzy-logic-based framework. Membership functions have been chosen to include the whole angular turn rate of interest assuming a TWS radar tracking a high-performance target. Instead of choosing a fixed number of particles, the dynamic tuning of particles number provide a small particles number for a non-maneuvering target; meanwhile it assigns a larger number for a maneuvering target according to the measured residual error.

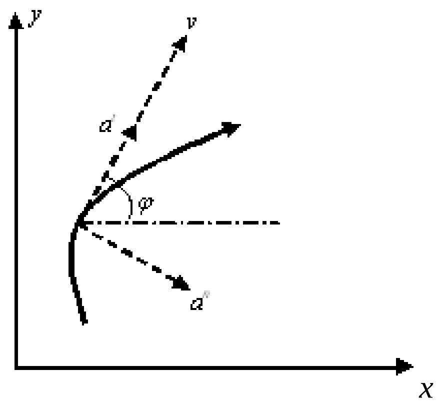

Fig.1. Geometry of 2-D target motion trajectory

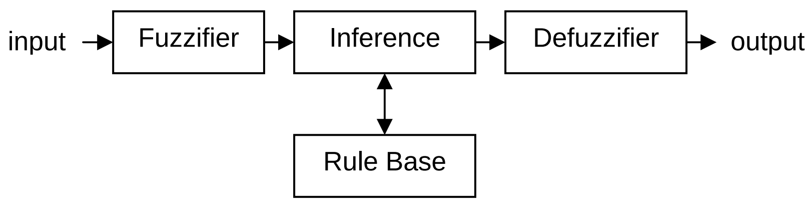

Fig.2. Fuzzy logic system block diagram

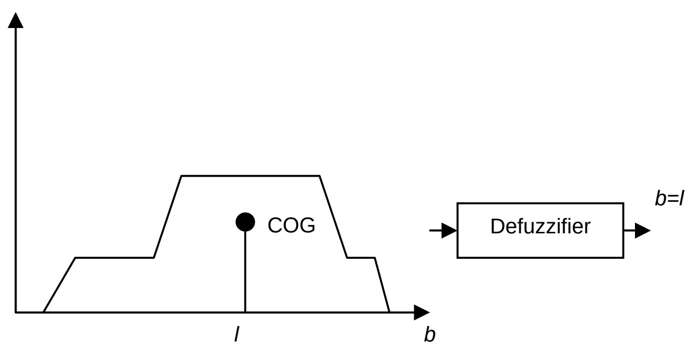

Fig.3. COG defuzzifier

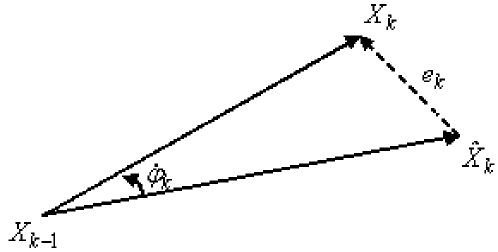

Fig.4. Graphical representation of residual error between expected and actual target position

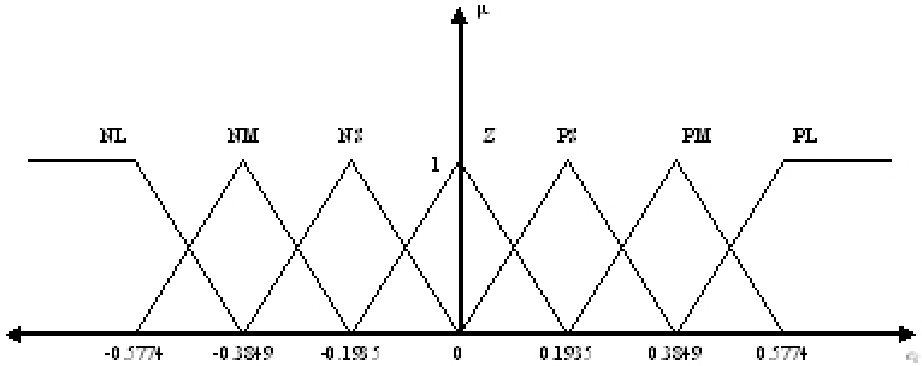

Fig.5. Fuzzy membership functions for the input space with seven partitions for the input variable, u_k

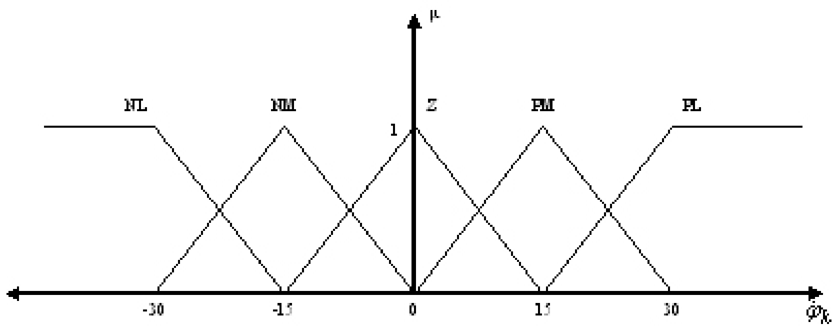

Fig.6. Fuzzy membership functions for the output space with five partitions for the input variable, $\dot{\varphi}_k$

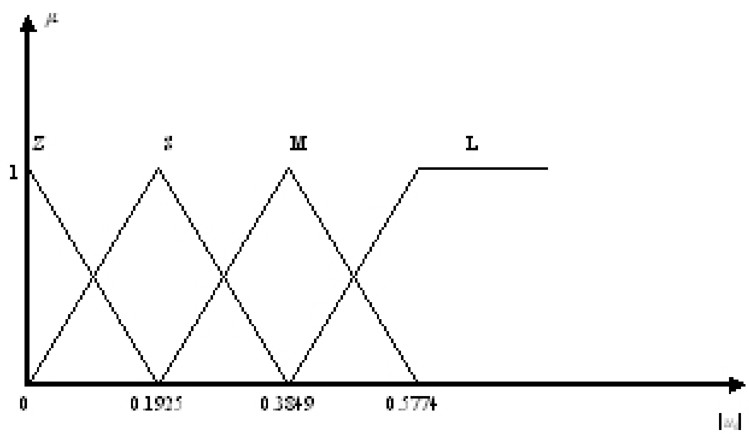

Fig.7. Fuzzy membership functions for the input space with four partitions for the input variable, $|u_k|$

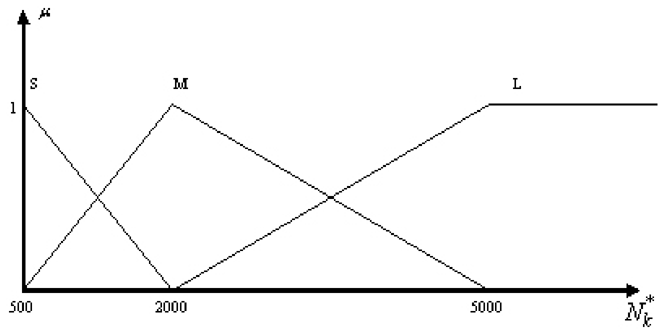

Figure 8. Fuzzy membership functions for the output space with four partitions for the input variable, N_k^*

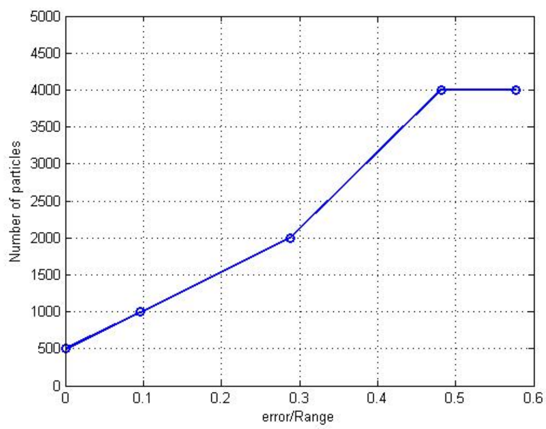

Fig.9. Graphical representation of defuzzified results using union operator

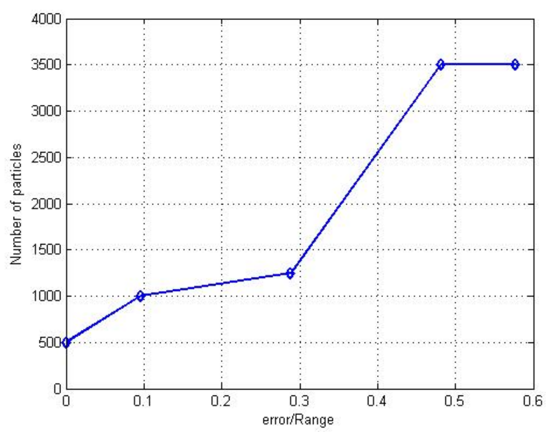

Fig.10. Graphical representation of defuzzified results using intersection operator

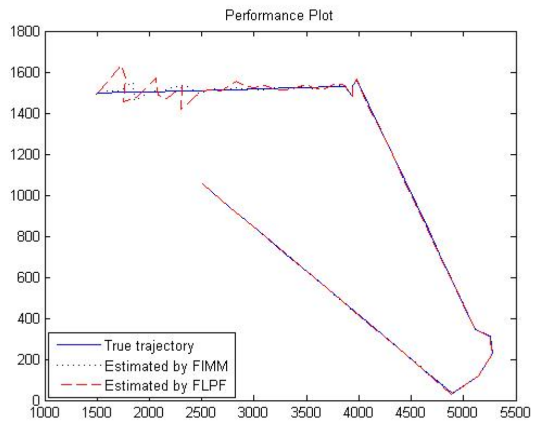

Fig.11. Tracking a high-performance target using FLPF with dynamic tuning of particles number

Table 1. Qualitative statement quantization

PL:	Positive Large
PM:	Positive Medium
PS:	Positive Small
Z:	Zero
NS:	Negative Small
NM:	Negative Medium
NL:	Negative Large

Table 2. Rule Base

1	IF u_k is Z or PL, THEN $\dot{\varphi}_k$ is Z
2	IF u_k is PM, THEN $\dot{\varphi}_k$ is PL
3	IF u_k is PS, THEN $\dot{\varphi}_k$ is PM
4	IF u_k is NS, THEN $\dot{\varphi}_k$ is NM
5	IF u_k is NM, THEN $\dot{\varphi}_k$ is NL
6	IF u_k is Z or NL, THEN $\dot{\varphi}_k$ is Z

Table 3. FAM for the rule base in Table 2

u_k	NL	NM	NS	Z	PS	PM	PL
$\dot{\varphi}_k$	Z	NL	NM	Z	PM	PL	Z

Table 4. Rule Base

1	IF $\lvert u_k \rvert$ is Z or S, THEN N_k^* is S
2	IF $\lvert u_k \rvert$ is M, THEN N_k^* is M
3	IF $\lvert u_k \rvert$ is L, THEN N_k^* is L

Table 5. FAM for the rule base in Table 4

u_k	Z	S	M	L
$\dot{\varphi}_k$	S	S	M	L

Table 6. Scan periods and corresponding residual error

Antenna scan rate	R_k	max (e_k)
12 rpm	1250 m	721 m
20 rpm	750 m	433

Table 7. Defuzzified output using centroid method

u_k	-0.4812	-0.2887	-0.0962	0.0962	0.2887	0.4812
$\dot{\varphi}_k$			0	0		0
					25	25
				15	15	
		-15	-15			
	-25	-25				
	0		0	0		

Table 8. Defuzzified output for dynamic tuning of particles number

$\lvert u_k \rvert$	0.09625	0.28875	0.48125
N_k^*	1000	1000	
		2000	2000
			4000

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