# Normality tests Procedure with power comparison

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**الملخص**

تستند النماذج الاحصائية المعلمية مثل (الارتباط، الانحدار، t test ، وتحليل التباين، إلخ...)على العديد من الفروض. احد اهم هذه الفروض هو ان تتبع الاخطاء العشوائية او المتغيرات المستقلة للتوزيع الطبيعي. إذا لم يتحقق هذا الفرض ، فإن التحليلات والتفسيرات تكون غير جديرة بالثقه وقد تؤدي الى استنتاجات غير صحيحة. وتوجد طرق عديدة للتحقق من فرض الاعتدالية للبيانات عن طريق الرسومات الإحصائية مثل المدرج التكراري والساق والاوراق و Q–Q plot او قد يتم اجراء الإختبار بإستخدام الطرق الرياضية المعروفة.

الهدف الرئيسى من الورقه البحثية هو دراسة اهم اختبارات التوزيع الطبيعي و عمل مقارنة بين هذه الاختبارات، ففي بعض الاحيان قد تؤدي بعض الاختبارات الى رفض الوزيع الطبيعي بينما تؤدي اختبارات اخرى الى قبول فرض التوزيع الطبيعي، مما يحعل الباحث في حيرة عن حساب هذه الاختبارات، وبالتالي وعن طريق حساب قوة الاختبارات تحت شروط مختلفه. ، تم عمل مقارنة بين هذه الإختبارات لتحديد افضل الاختبارات.

## Abstract

The condition of normality is required for Standard statistical procedures. the results of these methods will be in. improper when the normality in not satisfied. There for, the normality assumption is required before proceeding most statistical analysis. There are many tests available to assess the assumption of normality, these tests do not have the same nature and power to diagnose the departures of data from normality, there for the choice of appropriate test always remain an important key in the assessment of normality

In this article and due to the importance of this subject and wide spread development of normality tests, a comprehensive Power comparison study of existing and new developed tests for normality is proposed. This study addresses the performance of 36 normality tests, for various sample sizes, considering several significance levels and for a number of symmetric and asymmetric distributions. General results for normality testing from this study are discussed according to the nature of alternative distribution.

**Keywords**: Tests for normality; Monte Carlo simulation; Power comparison; normal distribution.

1.Introduction

There is many of statistical models and procedures that depend on the validity of a given data hypothesis, being the normality of the data assumption one of the most commonly found in statistical studies. Statistical procedure like standard errors and consequently, the test statistics computed from such standard errors in parametric statistics such as the t-test, tests for regression coefficients, analysis of variance, and the F-test of homogeneity of variance include the tests that have as an underlying assumption, the distribution of the population from which the sample data was generated to have be normal The normality tests can, therefore, be seen to be of much importance since the acceptance or rejection of the normality assumption of given data set plays central role in numerous research fields. The problem of testing normality has become very importance in both theoretical and empirical research and has led to the development of a large number of goodness-of-fit tests to detect departures from normality. There is nearly 40 different normality tests have been developed [1]. Given the importance of this subject and the extensive development of normality tests over the years, comprehensive characterizations and power comparisons of normality tests have also been the focus of attention, thus helping the analyst in the choice of best tests for this particular needs. normality tests that have been developed are based on different characteristics of the normal distribution, it can be seen from these comparison studies that their power to detect departures from normality can be significantly different depending on the nature of the non-normality.

The easiest way for detecting normality using graphical methods. The normal quantile-quantile plot (Q-Q plot) is the most commonly used and effective diagnostic tool for checking normality of the data [2]. There are other graphical methods that can be used to assess the normality assumption include histogram, box-plot and stem-and-leaf plot. Even though the graphical methods can serve as a useful way in checking normality for sample of n independent observations, they are still not sufficient to provide conclusive evidence that the normal assumption holds. Therefore, to support the graphical methods, more formal methods which are the numerical methods and formal normality tests should be performed before making any conclusion about the normality of the data.

Simulation study is presented herein to estimate the power of 36 tests aiming to assess the validity of the univariate normality assumption of a data set. The selected normality tests include a group of well-established normality tests and more recently developed ones. Section 2 presents general description of the normality tests and grouped in to four general categories, section (2.1) tests based on empirical distribution function section (2.2) tests based on moments section (2.3) tests based on regression and correlation section (2.4) other tests for normality, while Section3 discuses simulation study for power comparison of normality tests by using Monte Carlo computations. The simulation process was carried out using R programming. Section 5 presents the power results of the normality tests for the different alternative distribution. Finally, conclusions and recommendations resulting from the study are provided in Section 6.

## 2. Tests for normality

The normality tests are considered in this study for testing the composite null hypothesis for the case where both location and scale parameters, μ and σ, respectively, are unknown. Normality test formulations differ based on the different characteristics of the normal distribution they focus.

In the this study, it is considered that $x_1, x_2, \ldots, x_n$ represent a random sample of size $n$; $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$ represent the order statistics of that sample; $x, s^2$, the sample mean $(\bar{x})$, variance $(s^2)$ respectively given by

$$\bar{x} = n^{-1}\sum_{i=1}^{n} x_i \quad ; \qquad s^2 = (n-1)^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 ,$$

kewness and kurtosis are respectively given by,

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = \frac{\mu_3}{\sigma^{3/2}} \qquad ; \qquad \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{\mu_4}{\sigma^4}$$

where the $kth$ central moment $\mu_k$ is defined b

$$\mu_k = n^{-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

### 2.1. Empirical Distribution Function Tests

The methodology of the Empirical Distribution Function (EDF) tests in testing normality of data is to compare the empirical distribution function which is estimated based on the data with the cumulative distribution function (CDF) of normal distribution to see if there is a good agreement between them.

#### 2.1.1 The Kolmogorov-Smirnov modified by Lilliefors Test Statistic

Lilliefors [3]. modified Kolmogorov’s test statistic used for testing normality. The KS test is appropriate in a situation where the parameters of the hypothesized distribution are completely known. However, sometimes it is difficult to completely or initially specify the parameters as the distribution is unknown. In this case, the parameters need to be estimated based on the sample data. When the original KS statistic is used in such situation, the results can be misleading where the probability of type I error tend to be smaller than the ones given in the standard table of the KS test Lilliefors [3]. The main variation with the KS test, the parameters for LF test are estimated based on the sample. Therefore, in this situation, the LF test will be preferred over the KS test Oztuna [4].The test statistic is defined as:

$$\mathrm{K} - \mathrm{S} = max_x |\mathrm{F} * (\mathrm{X}) - \mathrm{S_n}(\mathrm{X})|$$

Where($\mathrm{S_n}(\mathrm{X})$) is the sample cumulative distribution function and ($\mathrm{F} * (\mathrm{X})$) is the cumulative distribution function (CDF) of the null distribution. Even though the LF statistic is the same as the KS statistic, the table for the critical values is different which leads to a different conclusion about the normality of a data Mendes & Pala [5]. The normality hypothesis of the data is then rejected for large values of K–S.

### 2.1.2 Anderson-Darling Test Statistic

Anderson-Darling (AD) test is a modification of the Cramer-von Mises (CVM) test. It differs from the CVM test in such a way that it gives more weight to the tails of the distribution [5]. Anderson and Darling [7] defined the statistic for this test as,

$$\mathrm{AD} = \mathrm{n} \int_{-\infty}^{\infty} [F_n(x) - \mathrm{F}^*(\mathrm{x})]^2 \ \psi(\mathrm{F}^*(x)) d(\mathrm{F}^*(\mathrm{x})$$

where $F_n(x)$ ia the empirical distribution function (EDP) ,$\mathrm{F}^*(\mathrm{x})$is the cumulative distribution function of the standard normal distribution and $\psi(\mathrm{x})$ is a weight function given by $\psi(x)$=$[\mathrm{F}^*(\mathrm{x}) - (1 - \mathrm{F}^*(\mathrm{x}))]^{-1}$. To make the computation of this statistic easier, the following formula can be applied Anderson and Darling [8],

$$AD = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)[(\ln(p_i) + \ln(1-(p_{n+1-i})]$$

Where the $p_i$values are given by $\varphi\left(z_{(i)}\right)$ , $z_i = (x_i - \bar{x})/s$.This study used the following modified AD statistic to increase its power given by Stephens [9] which takes into accounts the sample size n,

$$AD^* = AD\left(1 + \frac{0.75}{n} + \frac{2.25}{n^2}\right)$$

The normality hypothesis of the data is then rejected for large values of the test statistic.

### 2.1.3 The Zhang-Wu Test Statistic

Zhang and Wu [10] introduced a new class of EDF test statistics $Z_c$ and $Z_A$ of the general form

$$Z = \int_{-\infty}^{\infty} 2n\left\{F_n(x)\ln\left(\frac{F_n(x)}{F_o(x)}\right) + \left(1 - F_n(x)\right)\ln\left[\frac{\left(1 - F_n(x)\right)}{\left(1 - F_o(x)\right)}\right]\right\} dw(x)$$

Where $F_o(x)$ is a hypothetical distribution function completely specified and w(x) is a weight function. If dw(x) is considered to be $[1/F_o(x)]$. $[1/(1- F_o(x))]dF_o(x)$ and $F_o(x)$is $\Phi$ (x) , the test statistic is obtained by

$$Z_C = \sum_{i=1}^{n}\left[\ln\frac{\left(1/\Phi(z_{(i)}\right) - 1}{(n-0.5)\,/(i-0.75) - 1}\right]^2$$

In the case where dw(x) is considered to be $[1/F_n(x)]$. $[1/(1-F_n(x))]dF_n(x)$ the test statistic ZA is the obtained by

$$Z_A = -\sum_{i=1}^{n}\left[\frac{\ln\Phi\left(z_{(i)}\right)}{n-i+0.5} + \frac{\ln\left[1-\Phi\left(z_{(i)}\right)\right]}{i-0.5}\right]$$

For both tests, the normality hypothesis of the data is rejected for large values of the test statistic.

### 2.1.4 The Glen–Leemis–Barr test Statistic

Glen, Leemis and Barr [42] suggested a test statistic based on the quintiles of the order statistics. This test statistic included in this category because of the relation between the order statistics and the EDF. The Glen–Leemis–Barr test statistic $P_S$ is given by

$$P_S = -n - \frac{1}{n}\sum_{i=1}^{n}\left[(2n+1-2i)\ln\left(p_{(i)}\right) + (2i-1)\ln\left(1-p_{(i)}\right)\right],$$

where $p_{(i)}$ are the elements of the vector $p$ containing the quintiles of the order statistics sorted in ascending order. The elements of $p$ can be obtained by defining vector $u$ with elements sorted in ascending order and given by $u_i = \Phi\,(z_{(i)})$ [11]. Considering that $u_1, u_2, \ldots, u_n$ represent the order statistics of a sample taken from a uniform distribution U(0;1), their quantiles, which correspond to the elements of $p$, can be determined knowing that $u_i$ follows a Beta distribution $\mathrm{B}(i; n-i+1)$. The normality hypothesis of the data is rejected for large values of the test statistic.

### 2.2. Tests using moments

Karl Pearson is credited with having been the first to recognize that deviations in distribution from the normal could, for the most part, be characterized by differences in the third and fourth standardized moments. It follows naturally that formal testing for normality could be accomplished by evaluating sample moments and comparing them to theoretical moments. This required knowledge of the distribution of the sample moments under normality Thode, [12].

## 2.2.1 D'Agostino-Pearson Test Statistic

D'Agostino and Pearson [13] suggested test statistic by combines normalizing transformations of skewness and kurtosis, $Z(\sqrt{b_1})$ and $Z(b_2)$. The test statistics is given by

$$K^2 = [(Z(\sqrt{\beta_1})]^2 + [(Z(\beta_2)]^2$$

where the transformed skewness $Z(\sqrt{\beta_1})$ is obtained by

$$Z(\sqrt{\beta_1}) = \frac{\ln\left(\frac{Y}{c} + \sqrt{(Y/c)^2+1}\right)}{\sqrt{\ln(w)}}, \qquad (1)$$

where

$$Y = \sqrt{\beta_1} \cdot \sqrt{\frac{(n+1)(n+3)}{6(n-2)}};\ w^2 = -1 + \sqrt{2\delta - 1},$$

$$\delta = \frac{3(n^2+27n-70)(n+1)(n+3)}{(n-2)(n+5)(n+7)(n+9)}\ ;\ c = \sqrt{\frac{2}{(w^2-1)}},$$

and the transformed kurtosis $Z(\beta_2)$ is obtained by

$$Z(\beta_2) = \left[\left(1 - \frac{2}{9A}\right) - \sqrt[3]{\frac{1-2/A}{1+y\sqrt{2/(A-4)}}}\right]\sqrt{\frac{9A}{2}}$$

with

$$A = 6 + \frac{8}{\sqrt{\tau}}\left(\frac{2}{\sqrt{\tau}} + \sqrt{1+\frac{4}{\tau}}\right),$$

$$\sqrt{\tau} = \frac{6(n^2-5n+2)}{(n+7)(n+9)}\sqrt{\frac{6(n+3)(n+5)}{n((n-2)(n-3)}}$$

and

$$y = \frac{\beta_2 - 3(n-1)/(n+1)}{24n(n-2)(n-3)/[(n+1)^2(n+3)(n+5)]}$$

The test statistic follows approximately a chi-square distribution with 2 degree of freedom when a population in normally distributed [14]. The normality hypothesis of the data is rejected for large values of the test statistic.

### 2.2.2 Jarque-Bera Test Statistic

In the field of economics, the Jarque–Bera test is a popular goodness-of-fit test. It has been first proposed by Bowman and Shenton [14] but is mostly known from the proposal of Jarque and Bera [15]. The Lagrange multiplier procedure on the Pearson family of distributions is used to obtain tests for normality. The test statistic is given as:

$$JB = n\left[\frac{\beta_1}{6} + \frac{(\beta_2 - 3)^2}{24}\right]$$

The JB statistic is asymptotically chi-squared distributed with two degrees of freedom [14]. The normality hypothesis of the data is rejected for large values of the test statistic

### 2.2.3 The Doornik–Hansen Test Statistic

In order to increase its efficiency various modifications of the Jarque–Bera test have been proposed over the years. A known formulation is that of Doornik and Hansen [16], which suggests the use of the transformed skewness according to Equation (1) and the use of a transformed kurtosis according to the proposal in [17]. The statistic of the Doornik–Hansen test DH is thus given by

$$DH = \left[Z\left(\sqrt{b_1}\right)\right]^2 + [Z_2]^2$$

in which the transformed kurtosis $Z_2$ is obtained by

$$Z_2 = \left[\left(\frac{\xi}{2a}\right)^{\frac{1}{3}} - 1 + \frac{1}{9a}\right](9a)^{\frac{1}{2}}$$

With ξ and a obtained by

$\xi = (b_2 - 1 - b_1)2k$ ;

$$k = \frac{(n+5)(n+7)(n^3+37n^2+11n-313)}{12(n-3)(n+1)(n^2+15n-4)},$$

and

$$a = \frac{(n+5)(n+7)[n^2+27n-70] + b_1(n-7)(n^2+2n-5)}{6(n-3)(n+1)(n^2+15n-4)}$$

According to [16] DH is also approximately chi-squared distributed with two degrees of freedom. and the normality hypothesis of the data is rejected for large values of the test statistic.

### 2.2.4 The Gel–Gastwirth robust Jarque–Bera Test Statistic

Gel and Gastwirth [18] suggested a robust version of the Jarque–Bera test. Since the sample moments are, among other things, known to be sensitive to outliers, Gel and Gastwirth have proposed a modification of JB that uses a robust estimate of the dispersion in the skewness and kurtosis. definitions. The selected robust dispersion measure is the average absolute deviation from the median and leads to the following statistic of the robust Jarque–Bera test RJB given by

$$RJB = \frac{n}{6}\left(\frac{\mu_3}{j_n^3}\right)^2 + \frac{n}{64}\left(\frac{\mu_4}{j_n^4} - 3\right)^2$$

with $J_n$ obtained by

$$j_n = \frac{\sqrt{\pi/2}}{n}\sum_{i=1}^{n}|x_i - M|$$

In which M is the sample median. The normality hypothesis of the data is rejected for large values of the test statistic and, according to Gel and Gastwirth [18], RJB asymptotically follows the chi-square distribution with two degrees of freedom.

### 2.2.5 The Hosking L-moments Based Test Statistic

Hosking [19] suggested the use of linear combinations of the order statistics instead of central moments, termed $L$-moments, which are less affected by sample variability and, therefore, are more powerful to outliers and better for making inferences about an underlying probability distribution. Hosking has shown that the $rth$ order sample L-moment can be estimated by

$$l_r = \sum_{K=O}^{r-1} p^*_{r-1,k} \,.\, b_k \quad ,$$

where $p^*_{r-1,k}$ and $b_k$ are obtained by

$$p^*_{r-1,k} = (-1)^{r-k}\binom{r}{k}\binom{r+k}{k}$$

$$b_k = n^{-1}\sum_{i=1}^{n}\frac{(i-1)(i-2)\ldots(i-k)}{(n-1)(n-2)\ldots(n-k)}(x_i)$$

Hosking [19] defined a new measures of skewness and kurtosis, termed L-skewness $\tau_3$ and L-kurtosis $\tau_4$, based on the second, third and fourth sample L-moments, which have similarities with the corresponding central moments. The new measures of skewness and kurtosis according to Hosking [19] defined as

$\tau_3 = \frac{l_3}{l_2}$

$$\tau_4 = \frac{I_4}{I_2}$$

The value of $\tau_3$ is bounded between −1 and 1 for all distributions and is close to zero for the normal distribution, while the value of $\tau_4$ is ≤ 1 for all distributions and is close to 0.1226 for the normal distribution and Hosking [19] has suggested that normality could be tested based on τ3 and τ4 according to the following statistic

$$T_{Lmom} = \frac{\tau_3 - \mu_{\tau_3}}{Var(\tau_3)} + \frac{\tau_{4-}\mu_{\tau_4}}{Var(\tau_4)} \qquad (2)$$

where $\mu_{\tau_3}$ and $\mu_{\tau_4}$ are the mean of $\tau_3$and $\tau_4$, and $Var(\tau_3)$ and $Var(\tau_4)$ are their correspondingvariances. The values of$\mu_{\tau_3}$, $\mu_{\tau_4}$, $Var(\tau_3)$ and $Var(\tau_4)$ can be obtained by simulation. Nonetheless,$\mu_{\tau_3}$ and $\mu_{\tau_4}$ are expected to be close to 0 and 0.1226 respectively. Hosking [19] provides an approximation for $Var(\tau_3)$. The normalityhypothesis of the data is rejected for large values of $T_{Lmom}$, which is also approximately chi-squareddistributed with two degrees of freedom according to [20].

### 2.2.6 The Hosking test based on trimmed L-moments Test Statistic

Although L-moments show some robustness towards outliers in the data, as previously referred, they may still be affected by extreme observations Elamir and Seheult [21]. A robust generalization of the sample L-moments has therefore, been formulated by Elamir and Seheult [21].and that leading to the development of trimmed L-moments. The suggested formulation for the trimmed L-moments allows for both symmetric and asymmetric trimming of the smallest and largest sample observations.

Considering an integer symmetric trimming level $t$ ,Elamir and Seheult [21]. have shown that the $r$th order sample trimmed L-moment $l_r^{(t)}$ can be estimated by

$$l_r^{(t)} = \frac{1}{r}\sum_{i=t+1}^{n-t}\left\{\frac{\sum_{k=0}^{r-1}\left[(-1)^k\binom{r-1}{k}\binom{i-1}{r+t-1-k}\binom{n-i}{t+k}\right]}{\binom{n}{r+2t}}\right\}(x_i)$$

Elamir and Seheult [21] also define new measures of skewness and kurtosis Based on the second, third and fourth sample trimmed L-moments, termed TL-skewness $\tau_3^{(t)}$ and TL-kurtosis $\tau_4^{(t)}$ , given by

$$\tau_3^{(t)} = \frac{l_3^{(t)}}{l_2^{(t)}},$$

and

$$\tau_4^{(t)} = \frac{l_4^{(t)}}{l_2^{(t)}}$$

Based on these new measures and similar to the statistic given by Eq (2), the following statistic is considered,

$$T_{Lmom}^{(t)} = \frac{\tau_3^{(t)} - \mu_{\tau_3}^{(t)}}{Var\left(\tau_3^{(t)}\right)} + \frac{\tau_4^{(t)} - \mu_{\tau_4}^{(t)}}{Var\left(\tau_4^{(t)}\right)},$$

where, for a selected trimming level $t$ , $\mu_{\tau_3}^{(t)}$ and $\mu_{\tau_4}^{(t)}$ are the mean of $\tau_3^{(t)}$ and $\tau_4^{(t)}$ and $Var\left(\tau_3^{(t)}\right)$and $Var\left(\tau_4^{(t)}\right)$ are their corresponding variances. As for the previous test, the values of $\mu_{\tau_3}^{(t)}$, $\mu_{\tau_4}^{(t)}$ ,$Var\left(\tau_3^{(t)}\right)$ and $Var\left(\tau_4^{(t)}\right)$can be obtained by simulation. In this study three versions of this test are considered, which correspond to symmetric trimming levels of 1, 2 and 3. For each test, the normality hypothesis of the data is rejected for large values of the statistic $T_{Lmom}^{(t)}$ .

### 2.2.7 Bontemps-Meddahi Tests Statistics

Bontemps and Meddahi [22] have suggested a family of normality tests based on moment conditions known as Stein equations and their relation with Hermit polynomials. By using the generalized method of moments approach associated with Hermite polynomials the test statistics are developed, which leads to test statistics that are robust against parameter uncertainty. The general model of the test family is thus given by

$$BM_{3-p} = \sum_{k=3}^{p} \left\{ \frac{1}{\sqrt{n}} \sum_{i=1}^{n} H_k(Z_i) \right\}^2 ,$$

where $z = (x_i - \bar{x})/s$ and $H_k(.)$represents the $kth$ order normalized Hermite polynomial. The general expression given by

$$\forall i > 1, \ \ H_i(\mu) = \frac{1}{\sqrt{i}}\left[\mu \,.\, H_{i-1}(\mu) - \sqrt{i-1}H_{i-2}(\mu)\right] \ \ , H_o(\mu) = 1, \ \ H_1(\mu) = \mu \ \ (3)$$

Different tests can be obtained by assigning different values of $p$, which represents the maximum order of the considered normalized Hermite polynomials in the expression above. In this study two different tests are considered in this work with $p$ = 4 and $p$ = 6 ; these tests are termed $BM_{3-4}$ and $BM_{3-6}$ respectively. According to Bontemps and Meddahi [22]; the general $BM_{3-p}$ family of tests asymptotically follows the chi-square distribution with $p-$ 2 degree of freedom and the hypothesis of normality is rejected for large values of the test statistic.

### 2.2.8 The Brys–Hubert–Struyf MC–LR Test Statistic

Brys, Hubert and Struyf [23] have suggested a test statistic based on robust measures of skewness and tail weight. The robust measure of skewness is the medcouple MC [24,25] defined as

$$\text{MC} = \underset{x_{(i)} \le m_F \le x_{(j)}}{\text{med}} h(x_{(i)}, x_{(j)})$$

Where med stands for the median, $m_F$ is the sample median and the kernel function $h$ is given by

$$h(x_{(i)}, x_{(j)}) = \frac{(x_{(i)} - m_f) - (m_f - x_{(j)})}{x_{(i)} - x_{(j)}}$$

For the case where$x_{(i)} = x_{(j)} = m_f$, $h$is then set by

$$h(x_{(i)}, x_{(j)}) = \begin{cases} 1 & i > j \\ 0 & i = j \\ -1 & i < j \end{cases}$$

The left medcouple (LMC) and the right medcouple (RMC) are the considered robust measures of left and right tail weight [26], respectively, and are defined by

$$\text{LMC} = -\text{MC}(x < m_f) \; and \; \text{RMC} = \text{MC}(x > m_f),$$

The test statistic $T_{MC-LR}$ is then defined by

$$T_{MC-LR} = n(\omega - \acute{\omega})^t . V^{-1} . (\omega - \acute{\omega})$$

in which $\omega$ is set as $[MC, LMC, RMC]^t$ , and $\omega$ and V are obtained based on the influence function of the estimators in $\omega$ [25,26]. For the case of a normal distribution, $\omega$ and V are defined as [23]

$$\omega = [0, 0.199, 0.199]^t \; ; \; V = \begin{bmatrix} 1.25 & 0.323 & -0.323 \\ 0.323 & 2.62 & -0.0123 \\ -0.323 & -0.0123 & 2.62 \end{bmatrix}$$

According to Brys, Hubert and Struyf [23], it is suggested that $T_{MC-LR}$ approximately follows the chi-square distribution with three degrees of freedom and the normality hypothesis of the data is rejected for large values of test statistic.

### 2.2.9 Bonett-Seier Test Statistic

Bonett and Seier [27] have introduced a modified measure of kurtosis for testing normality, which is based on a modification of aproposal by Geary [28]. The test statistic of the new kurtosis measure $T_w$ is thus given by:

$$T_w = \frac{\sqrt{n+2}\,.\ (\in -3)}{3.54}$$

where

$$\in = 13.29\left[\ln\sqrt{\beta_2} - \ln\left(n^{-1}\sum_{i=1}^{n}|x_i - \bar{x}|\right)\right]$$

The normality hypothesis is rejected for both small and large values of $T_w$using a two sided test [27], it is suggested that $T_w$ approximately follows a standard normal distribution.

### 2.2.10 The Cabaña-Cabaña Test Statistic

Cabaña and Cabaña [29] have suggested four families of normality tests based on transformed empirical processes. Two tests families are of the Kolmogorov–Smirnov type while the other two are of the Cramér–von Mises type. One family of each type of test focuses on changes on skewness and the other one is sensitive to changes in kurtosis. According to Cabaña and Cabaña [29], the power of the Kolmogorov–Smirnov type tests is seen to be very similar to that of the Cramér–von Mises type tests. Therefore, only the Kolmogorov–Smirnov type tests were selected in this study, as their implementation complexity is relatively lower than that of the Cramér–von Mises type tests.

Based on the definition of approximate transformed estimated empirical processes (ATEEP) sensitive to changes in skewness or kurtosis the test statistics introduced The proposed ATEEP sensitive to changes in skewness is defined as:

$$\omega_{s\&l}(x) = \phi(x)\,.\overline{H}_3 - \phi(x)\,.\sum_{j=1}^{l}\frac{1}{\sqrt{j}}H_{J-1}(x)\,.\overline{H}_{j+3}$$

where $l$ is a dimensionality parameter, $\phi(x)$ is the probability density function of the standard normal distribution, Hj $(\cdot)$ represents the $j$th order normalized Hermite polynomial given by Equation (3) and $\overline{H}_j$ is the $j$th order normalized mean of the Hermite polynomial defined as

$$\overline{H}_j = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} Hj(x_i)$$

The proposed ATEEP sensitive to changes in kurtosis is defined as:

$$\boldsymbol{\omega_{k\&l}(x) = -\phi(x)\,.\overline{H}_3 + [\phi(x) - x.\phi(x)].\overline{H}_4 - \phi(x)\,.\sum_{j=2}^{l}\left(\sqrt{\frac{j}{j-1}}H_{j-2}(x)\,.H_j(x)\right).\overline{H}_{j+3}}$$

According to Cabaña and Cabaña [29], the dimensionality parameter $l$ ensures that the test is consistent against alternative distributions differing from the normal distribution having the same mean and variance in at least one moment of order not greater than $l + 3$. The Kolmogorov–Smirnov type test statistics sensitive to changes in skewness and in kurtosis, $T_{s\&l}$and $T_{K\&l}$ respectively, are defined as

$$T_{s\&l} = \max|\boldsymbol{\omega_{s\&l}(x)}| \quad and \quad T_{k\&l} = \max|\boldsymbol{\omega_{k\&l}(x)}|.$$

According to Cabaña and Cabaña [29], For both cases, the normality hypothesis of the data is rejected for large values of the test statistic.

### 2.2.11 Desgagn_e-Lafaye-de-Micheaux omnibus and directional Test Statistic

Desgagné and Lafaye de Micheaux [30] introduced two types of test statistics. The. The first test is based on 2nd-power skewness and kurtosis, which are interesting alternatives to the classical Pearson's skewness and kurtosis, called 3rd-power skewness and 4th-power kurtosis. This test is based on the dependency between $B_2$ and $K_2$ in small samples, because the $\chi^2$distribution results from sum of squares of two independent standard normal. The proposed statistic to test the composite hypothesis of normality, for finite sample sizes $n \geq 10$, is denoted by $Z_{APD}$ and given by

$$Z_{APD} = Z^2(B_2) + Z^2\left(K_2 - {B_2}^2\right),$$

where $Z(B_2) = \frac{n^{1/2} B_2}{[(3-8/\pi)(1-1.9/n)]^{1/2}}$ ,

$$Z\left(K_2 - {B_2}^2\right) = \frac{n^{1/2}\left[\left((K_2 - {B_2}^2)^{1/3}\right) - \left((2 - \log 2 - \gamma)/2\right)^{1/3}(1 - 1.026/n)\right]}{\left[72^{-1}\left((2 - \log 2 - \gamma)/2\right)^{-4/3}(3\pi^2 - 28)(1 - 2.25/n^{.8})\right]^{1/2}}$$

$$B_2 = \frac{1}{n}\sum_{i=1}^{n} Z_i^2 \, sign(Z_i),$$

$$K_2 = \frac{1}{n}\sum_{i=1}^{n} Z_i^2 \, log|Z_i|, \qquad (4)$$

and

$$\gamma = -\Psi(1) = 0.577215665 \qquad (5)$$

The null hypothesis is rejected if $X_{APD}$ is larger than the chi-squared quantile $\chi^2_{2,\alpha}$ at a significance level of α.

Second test introduced by Desgagné and Lafaye de Micheaux [30] when it is known that the distribution of the random variable is symmetric by using a directional test and thus increasing the power. Consider a directional test based on sample 2nd-power kurtosis. To test the composite hypothesis of normality, for finite sample sizes $n \geq 10$, is denoted by $z_{\text{EPD}}$ and given by

$$Z_{\text{EPD}} = \frac{n^{1/2}\left[\frac{((2K_2)^{\alpha_n}-1)}{\alpha_n} + \frac{(2-\log 2-\gamma)^{-0.06}-1}{0.06} + (1.23/n^{.95})\right]}{\left[\frac{(2-\log 2-\gamma)^{-2.12}(3\pi^2-28)}{2} - (3.78/n^{0.733})\right]^{1/2}}$$

where $K_2$ and $\gamma$ is given in equation (4), (5) and

$$\alpha_n = -0.06 + 2.1/n^{0.67}$$

The directional test follows a normal distribution with N( 0,1) (Desgagné and Lafaye de Micheaux [30] .The null hypothesis is rejected if $|Z_{\text{EPD}}|$ is larger than the normal quintile $Z_{\alpha/2}$, at a significance level of $\alpha$.

## 2.3 Tests based on Regression and correlation

. Regression or correlation tests are based on measures of linear correlation in probability plots. In contrast to probability plots, regression tests are formal procedures which can be used to objectively assess normality. However, the features of a set of data which cause the non-normality cannot be determined solely on the basis of the test. It is therefore recommended that a test for normality, be it regression test or other type of test, be done in conjunction with a raw data plot and a probability plot.

### 2.3.1 Shapiro-Wilk Test Statistic

Shapiro and Wilk [31] test was originally restricted for sample size of less than 50. This test was the first test that was able to detect departures from normality due to either skewness or kurtosis, or both Althouse [32].

If a set of observations $x_i$ come from a normal distribution, then on a normal probability plot

$$x_i = \mu + \sigma z_i$$

If we denote the expected value of the $i$th order statistic by E( $x_{(i)}$) = $w_i$ and **V** is the covariance matrix of the order statistics , $x_{(1)}, x_{(2),} \dots, x_{(n(}$, then the best linear unbiased estimate of $\sigma$ is obtained from the generalized least squares regression of the sample order statistics on their expected values, which is (up to a constant)

$$b = a' x$$

The original Shapiro-Wilk test statistic [31] is defined as,

$$W = \frac{K\sigma^2}{(n-1)s^2} = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2},$$

where$x_{(i)}$is the $i^{th}$ order statistic, $\bar{x}$ is the sample mean,

$$a_i = \frac{m' V^{-1}}{(m' V^{-1} V^{-1} m)^{1/2}}$$

in which **m** and **V** are the mean vector and covariance matrix of the order statistics of the standard normal distribution.

The value of W lies between zero and one. Small values of W lead to the rejection of normality whereas a value of one indicates normality of the data. SW test was modified by Royston [33] to broaden the restriction of the sample size to 2000 and algorithm AS181 was then provided [34]. Later, Royston [35] observed that Shapiro-Wilk's approximation for the weights a used in the algorithms was in adequate for n $>$ 50 . He then gave an improved approximation to the weights and provided algorithm AS R94 Royston [35] which can be used for any n in the range $3 \leq n \leq 5000$. This study used the algorithm AS R94 Royston [35].

### 2.3.2 Shapiro-Francia Test Statistic

Since explicit values of **m** and **V** are not readily available and the computation of $\boldsymbol{V^{-1}}$ is time consuming for large samples, Shapiro and Francia [36] suggested an approximation to the Shapiro-Wilk W-test. Let $x_{1,}x_2, x_{3,\ldots,}x_n$ be a random sample to be tested for departure from normality, ordered $x_{(1)} < x_{(2)} < \cdots < x_{(n)}$, and let *m*' denote the vector of expected values of standard normal order statistics. The test statistic is defined as:

$$W' = \frac{[\sum_{i=1}^{n} m_i\, x_i]^2}{[\sum_{i=1}^{n} m_i{}^2][\sum_{i=1}^{n}(x_i - \bar{x})^2]}$$

The $W'$ equals the product-moment correlation coefficient between the $x_i$ and the $m_i$, and therefore measures the straightness of the normal probability plot $x_i$; small values of *W*' indicate non-normality.

Shapiro-Francia test is particularly useful than the Shapiro-Wilk test especially for large samples where explicit values of ***m*** and ***V*** utilized in the Shapiro-Wilk test are not readily available and the computation of $\boldsymbol{V^{-1}}$ is time consuming. The normality hypothesis of the data is rejected for small values of the test.

### 2.3.3 The Rahman-Govindarajulu modification of the Shapiro–Wilk Test Statistic

Rahman and Govindarajulu [37] have proposed a modification to the Shapiro–Wilk test, here on termed $W_{RG}$.

According to these proposals, each element $a_i$of the new vector of weights becomes

$$a_i = -(\mathrm{n}+1)(n+2)\phi(m_i)[m_{i-1}\phi(m_{i-1}) - 2m_i\phi(m_i) + m_{i+1}\phi(m_{i+1})]$$

where it is assumed that $\mathrm{m}_0\phi(\mathrm{m}_0) = \mathrm{m}_{\mathrm{n}+1}\phi(\mathrm{m}_{\mathrm{n}+1}) = 0$. with this modification, the new test statistic $W_{RG}$ assigns larger weights to the extreme order statistics than the original W test, which has been seen to result in higher power against short tailed alternative distributions Rahman and Govindarajulu [37].The normality hypothesis of the data is rejected for small values of $W_{RG}$.

### 2.3.4 The Filliben correlation Test Statistic

Filliben [38] described the probability plot correlation coefficient r as a test for normality. The correlation coefficient is defined between the sample order statistics and the estimated median values of the theoretical order statistics.

Considering that $m_1$,$m_2$, . . . ,$m_n$represent the estimated median values of the order statistics from a uniform distribution U(0;1), each $m_i$ is obtained by

$$m_i = \begin{cases} 1 - 0.05^{(1/n)} & i = 1 \\ \dfrac{(i - 0.3172)}{(n + 0.365)} & 1 < i < n \\ 0.05^{(1/n)} & i = n \end{cases}$$

Upon which the estimated median values of the theoretical order statistics can be obtained using the transformation $M_{(i)} = \Phi^{-1}(m_{(i)})$ The correlation coefficient r is then defined as

$$r = \frac{\sum_{i=1}^{n} x_{(j)} \ M_{(j)}}{\sqrt{\sum_{i=1}^{n} {M_{(i)}}^2} \ \sqrt{(n-1) \,.\, s^2} \,.}$$

Leading to the rejection of the normality hypothesis of the data for small values of $r$.

### 2.3.5 The Chen–Shapiro Test Statistic

Chen and Shapiro [39] introduced an alternative test statistic CS based on normalized spacings and defined as

$$\mathrm{CS} = \frac{1}{(n-1)\, s} \sum_{i=1}^{n=1} \frac{x_{(i+1)} - x_{(i)}}{M_{(i+1)} - M_{(i)}}$$

in which $M_{(i)}$is the $i$th quintile of a standard normal distribution obtained by

$$M_{(i)} = \Phi^{-1}\left[\left(i - 0.375\big/_{n} + 0.25\right)\right]$$

Because of a close relation between CS and the Shapiro–Wilk test their performance is expected to be similar also. The normality hypothesis of the data is rejected for small values of CS.

### 2.3.6 The D'Agostino Test Statistic

D'Agostino [40] proposed the D test statistic as an extension of the Shapiro–Wilk test. The D'Agostino proposal eliminates the need to define the vector of weights a of the Shapiro–Wilk test statistic and is obtained by

$$D = \frac{\sum_{i=1}^{n}(i - (n+1)/2) .\, x_{(i)}}{n^2 .\, \sqrt{\mu_2}}$$

The normality hypothesis of the data is rejected for both small and large values of D using a two-sided test.

### 2.3.7 The Zhang Test Statistic

Zhang [41] introduced the $Q$ test statistic based on the ratio of two unbiased estimators of standard deviation, $q_1$ and $q_2$ the test statistic given by

$Q = \ln(q_1/q_2)$.

where

$$q_1 = \sum_{i=1}^{n} a_i x_{(i)} \quad , \qquad q_2 = \sum_{i=1}^{n} b_i x_{(i)}$$

and the $i$th order linear coefficients $a_i$ and $b_i$ result from

$a_i = [(u_i - u_1)(n-1)]^{-1}$, for $i \neq 1$ ; $a_i = \sum_{i=2}^{n} a_i$

$$b_i = \begin{cases} -b_{n-i+1} = [(u_i - u_{i+4})(n-4)]^{-1} & i = 1,2,\ldots,4 \\ (n-4)^{-1}.[(u_i - u_{i+4})^{-1} - (u_{i-4} - u_i)^{-1}] & i = 5,\ldots,n-4 \end{cases}$$

where the $i$th expected value of the order statistics of a standard normal distribution, $u_i$ , is defined by

$$M_{(i)} = \Phi^{-1}\left[\left(i - 0.375 / n + 0.25\right)\right]$$

According to Zhang [41] $Q$ is less powerful against negatively skewed distributions. Therefore, Zhang has also proposed the alternative statistic Q* by switching the $i$th order statistics $x_{(i)}$ in $q_1$ and $q_2$ by

$$x^*_{(i)} = -x_{(n-i+1)}$$

Based on the definition of both $Q$ and $Q*$, the normality hypothesis of the data is rejected for both small and large values of the statistic using a two-sided test.

In addition to these two tests, Zhang [41] has also proposed a joint test $Q - Q*$, stemming from the fact that $Q$ and Q$*$ are approximately independent. Therefore, for the case of the joint test Q−Q∗, the normality hypothesis of the data is rejected at the significance level α when rejections obtained for either one of the two individual tests for a significance level of α/2.

According to Zhang [41], both Q and Q∗ approximately follow a normal distribution. However, Hwangand Wei [42] have proven otherwise and stated that the performance of these tests is better when based on their empirical distribution. Since the joint test has shown to be more powerful than the individual tests Hwangand Wei [42], the joint test Q−Q∗ is the primary choice for the current study. Nonetheless, the Q test is also included for comparison purposes.

### 2.3.8 The del Barrio-Cuesta-Albertos-Matrán-Rodríguez-Rodríguez quantile correlation Test Statistic

A novel approach for normality testing, based on the L2-Wasserstein distance, has been proposed by del Barrio, Cuesta- Albertos, Matránand Rodríguez-Rodríguez [43] The BCMR test statistics is defined by

$$\mathrm{BCMR} = \frac{m_2 - \left[\sum_{i=1}^{n} x_{(i)} \cdot \int_{(i-1)/n}^{i/n} \Phi^{-1}(t)dt\right]^2}{\mu_2}$$

where, according to del Barrio, Cuesta- Albertos, Matránand Rodríguez- Rodríguez [43], the numerator represents the squared $L_2$-Wasserstein distance. The normality hypothesis of the data is rejected for large values of the test statistic.

### 2.3.9 The $\boldsymbol{\beta_3^2}$ Coin Test Statistic

Coin [44] has proposed a normality test based on a polynomial regression focused on detecting symmetric non-normal alternative distributions. According to Coin [44], the analysis of standard normal Q–Q plots of different symmetric non-normal distributions suggests that fitting model of the type

$$\mathcal{Z}_i = \beta_1 . \alpha_i + \beta_3 . \alpha_i^3$$

where $\beta_1$ and $\beta_3$ are fitting parameters and $\alpha_i$ represent the expected values of standard normal order statistics, leads to values $\beta_3$ different from zero when in presence of symmetric non-normal distributions. Therefore, Coin (2008) suggests the use of $\beta_3^2$ as a statistic for testing normality thus rejecting the normality hypothesis of the data for large values of $\beta_3^2$ . As suggested by Coin [44], the values of $\alpha_i$ are obtained using the approximations provided by Royston [45].

## 2.4. Other tests

There is other tests for normality which not based on EDF tests or kurtosis , skewness tests or correlation and regretion tests.

### 24.1 The Epps–Pulley Test Statistic

Epps and Pulley [45] have proposed a test statistic TEP based on the following weighted integral

$$T_{EP} = \int_{-\infty}^{\infty} \left|\phi_n(t) - \hat{\phi}_0(t)\right|^2 dG(t),$$

where $\phi_n(t)$ is the empirical characteristic function given by

$$\phi_n(t) = n^{-1} \sum_{j=1}^{n} e^{-itx_j}$$

and $\hat{\phi}_0(t)$ is the sample estimate of the characteristic function of the normal distribution given by

$$\hat{\phi}_0(t) = e^{-it\bar{x}-0.5m_2 t^2}$$

and G(t) is an adequate function chosen according to several considerations Epps and Pulley [44]. By setting d G(t) = g(t)dt and selecting

$$g(t) = \sqrt{\mu_2/2\pi} \,.\, e^{(-0.5\mu_2 t^2)},$$

the following statistic can be obtained as

$$T_{EP} = 1 + \frac{n}{\sqrt{3}} + \frac{2}{n}\sum_{k=2}^{n}\sum_{j=1}^{k-1} e^{\left(-(x_j - x_k)^2\right)/(2\mu_2)} - \sqrt{2}\sum_{j=1}^{n} e^{\left(-(x_j-\bar{x})^2\right)/\,(4\mu_2)}$$

for which the normality hypothesis of the data is rejected when large values of $T_{EP}$ are obtained. To simplify the use of this test by eliminating the need for tables of percentage points of $T_{EP}$,an approximation to the limit distribution of $T_{EP}$ has been presented by Henze [46].

### 2.4.2 The Martinez–Iglewicz Test Statistic

Martinez and Iglewicz [48]. have proposed a normality test based on the ratio of two estimators of variance, where one of the estimators is the robust biweight scale estimator $S_b^2$

$$S_b^2 = \frac{n\sum_{|\hat{Z}_i|<1}(x_i - M)^2\left(1 - \tilde{Z}_i^2\right)^4}{\left[\sum_{|\hat{Z}_i|<1}\left(1 - \tilde{Z}_i^2\right)\left(1 - 5\tilde{Z}_i^2\right)\right]^2}$$

where M is the sample median, $\tilde{Z}_i$= ($x_i$−M)/(9A), with A being the median of |$x_i$ −M|, and when$|\hat{Z}_i|$> 1, $\hat{Z}_i$ is set to 0. The Martinez–Iglewicz test statistic then is given by

$$I_n = \frac{\sum_{i=1}^{n}(x_i - M)^2}{(n-1)\,.S_b^2}$$

for which the normality hypothesis of the data is rejected for large values of $I_n$.

### 2.4.3 Gel-Miao-Gastwirth Test Statistic

Gel, Miao and Gastwirth. [49]. have proposed a directed normality test, which focuses on detecting heavier tails and outliers of symmetric distributions. The test is based on the ratio of the standard deviation and the robust measure of dispersion $j_n$ as defined in the expression

$$j_n = \frac{\sqrt{\pi/2}}{n}\sum_{i=1}^{n}|x_i - M| \quad ,$$

where $M$ is the sample median. The test statistic is thus given by

$$R_{SJ} = \frac{s}{Jn}$$

and should tend to one under a normal distribution. The normality hypothesis is rejected for large values of the $R_{sj}$, and the statistic $\sqrt{n}\,(R_{sj} - 1)$ is asymptotically normally distributed [49]. However, it has been empirically found that rejecting the normality hypothesis using a two-sided test extends the range of application of this test, namely to light tailed distributions, without a significant reduction of its power towards heavy-tailed distributions. Given its enhanced behavior, the two-sided test is the primary choice for the current study. Nonetheless, a detailed power comparison of the two-sided test with the one-sided test, hereon termed $R_{\mathrm{sJ},1}$, is also presented.

### 2.4.4 Spiegelhalter Test Statistic

Using the logic that the combination of a good test for short tails and good test for long tails would result in a good test for an unspecified symmetric alternative, Spiegelhalter [50]. used a combination of the most powerful location and scale invariant (MPLSI) tests to obtain a test that would be useful under more general conditions. He defined a test statistic against symmetric alternatives as

$$T_S = \frac{(\lambda_U^* + \lambda_D^*)}{\lambda_N^*} \quad ,$$

where $\lambda_U^* = (n^2 - n)^{-1}(x_{(n)} - x_{(1)})^{-n+1}$ ;

$$\lambda_D^* = [\gamma(x_m)B_n]^{-n+1},$$

And $x_m$ is the sample median,

$$\gamma(\tilde{\theta}) = \sum_{i=1}^{n} |x_i - \tilde{\theta}|,$$

and for $n$ odd,

$$B_n = \left[\sum \frac{\gamma^{n-1}(x_m)\gamma^{-n+1}(x_{(i)})}{(2i-n)(n+2-2i)}\right]^{\frac{-1}{n-1}}$$

for $n$ even

$$B_n = \left[\frac{(n-1)(x_{(n_2)} - x_{(n_1)})}{2\gamma(x_m)} + 0.5 + \sum_{i \neq n_1, n_2} \frac{\gamma^{n-1}(x_{m)}\gamma^{-n+1}(x_{(i)})}{(2i-n)(n+2-2i)}\right]^{\frac{-1}{n-1}}$$

$$\lambda_N^* = (1/2)n^{-\frac{1}{2}}\pi^{-\frac{n}{2}+\frac{1}{2}}\Gamma\left(\frac{n}{2} - \frac{1}{2}\right)\left[\sum (x_i - \bar{x})^2\right]^{-\frac{n}{2}+\frac{1}{2}},$$

Due to the complexity of calculating $\lambda_D^*$ , however, Spiegelhalter [50]. examined a simplified approximation of $T_S$ substituting Geary's test for that component,

$$T_S^{`} = [(C_n v)^{-n+1} + a(1)^{-n+1}]^{1/n-1}$$

where

$$C_n = \frac{(n!)^{\frac{1}{n}-1}}{2n} \ ,$$

$$v = \frac{x_n - x_1}{s},$$

$$a(1) = \gamma\ (\bar{x}) / \ n\hat{\sigma}$$

and the Geary's test statistic $a$ is the ratio of the mean deviation to standard deviation, given by

$$a = \sum_{i=1}^{n} |x_i - \bar{x}| \ / n\sqrt{\mu_2}$$

The normality hypothesis of the data is rejected for large values of the test statistic.

## 3 Comparative study

The first part of the simulation study involved the generation of 10000 random samples from the standard normal distribution for the different sample sizes. Each sample generated was then tested for normality and the type I error rate, that is the rate of rejection of the hypothesis of normality of the data, was then recorded at specified significance level ($\alpha = .05$ ,0.1).Two levels of the significance were considered to investigate the effect of the significance level on the power of the test. The probability of this type I error rate of the test should be bounded upwards by the chosen level of significance; otherwise the test cannot be used for the given purpose. On the other hand, a test with type I error rate far smaller or greater than the chosen α is an indicative of a test with low and high power respectively.

In the second part of the simulation study, Monte Carlo procedures were used to evaluate the power of data was generated from several alternative non-normal distributions as highlight in chapter 2.These include symmetric short and long tailed distributions such as Laplace(3,1) , Uniform(0,1), Beta(0.25,0.25) , Beta(1.5,1.5), Student-t(df=5) , Student-t(df=8), Student- Cauchy(0,7) ; asymmetric short and long tailed distributions such as Beta (3,1), $\chi^2$(df=5) , $\chi^2$(df=15) , Gamma(3,4), Weibull(15,3) , Exponential(5) lognormal(1,1) , Gumbel(0,1).These distributions were selected to cover various standardized skweness $\sqrt{\beta_1}$ and kurtosis $\beta_2$ values.

The study is carried out for four sample sizes (n = 10, , n = 20, , n = 50, n = 100 and) and considering significance levels α of 0.10, 0.05.Although critical values or limiting distributions of the tests statistics are available for some of the tests considered herein, critical values for each sample size under consideration were, nonetheless, derived empirically for each test for the considered nominal significance levels, before carrying out the power study. These critical values were based on 1,0000 samples drawn from the standard normal distribution. In addition to the referred critical values, the values of $\mu_{\tau 3}, \mu_{\tau 4}$,Var(τ3) and Var(τ4), for the Hosking L-moments based test, and the values of $\mu_{\tau 3}^{(t)}, \mu_{\tau 4}^{(t)}$, $\mathrm{Var}(\tau_3^{(t)})$ and $\mathrm{Var}(\tau_4^{(t)})$, for the Hosking trimmed L-moments based test, were also determined for each sample size by simulation from 10,000 samples drawn from the standard normal distribution. For the latter test, the parameters were obtained for each of the previously referred trimming levels t of 1, 2 and 3. The values resulting from this empirical evaluation the values of $\mu_{\tau 3}$ and of $\mu_{\tau 3}^{(t)}$ for the different trimming levels are very close to zero, and are considered to be zero in the subsequent power study.

Table A1. Simulated power for symmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Laplace (3,1) | K-S | EDF tests | 14.07 | 22.6 | 42.31 | 70.07 | 22.61 | 33.01 | 55.97 | 80.79 |
| | AD* | | 16.44 | 27.2 | **54.19** | 82.43 | 24.24 | 37.96 | 64.95 | 88.36 |
| | $Z_c$ | | 15.58 | 24.68 | 45.8 | 69.44 | 21.79 | 32.05 | 54.01 | 76.66 |
| | $Z_A$ | | **17.12** | 25.89 | 45.16 | 68.2 | **25.73** | 36.36 | 57.15 | 78.31 |
| | $P_S$ | | 16.53 | **27.47** | 54.84 | **83.09** | 24.51 | **38.46** | **65.32** | **88.67** |
| | $K^2$ | Moment tests | 18.96 | 29.1 | 49.7 | 71.62 | 27.47 | 39.63 | 60.63 | 81.39 |
| | JB | | 18.75 | 30.82 | 56.11 | 80.28 | 27.69 | 41.3 | 65.94 | 85.85 |
| | $DH$ | | 19.01 | 32.05 | 57.53 | 80.26 | 27.7 | 41.58 | 67.97 | 87.70 |
| | RJB | | **20.83** | **36.47** | **66.4** | 89.09 | **30.1** | **47.71** | **75.76** | 93.58 |
| | $T_{Lmom}$ | | 20.08 | 32.96 | 61.80 | 86.8 | 27.75 | 42.45 | 70.77 | 91.48 |
| | $T_{Lmom}^{(1)}$ | | 11.39 | 20.16 | 41.39 | 71.09 | 18.02 | 28.94 | 52.14 | 78.96 |
| | $T_{Lmom}^{(2)}$ | | 6.67 | 13.38 | 29.53 | 54.02 | 12.89 | 20.92 | 39.16 | 63.88 |
| | $T_{Lmom}^{(3)}$ | | 4.99 | 10.18 | 22.01 | 41.54 | 9.810 | 16.81 | 31.37 | 51.91 |
| | $BM_{3-4}$ | | 18.79 | 30.7 | 56.06 | 80.19 | 27.83 | 41.15 | 65.88 | 85.84 |
| | $BM_{3-6}$ | | 19.05 | 32.45 | 60.53 | 83.96 | 27.59 | 40.98 | 69.76 | 90.06 |
| | $T_{MC-LR}$ | | 4.77 | 5.62 | 6.990 | 11.98 | 9.370 | 11.54 | 14.56 | 21.1 |
| | $T_w$ | | 13.56 | 28.35 | 63.51 | **90.57** | 20.50 | 38.23 | 71.99 | **94.39** |
| | $T_{MC-LR} - T_w$ | | 10.64 | 21.66 | 50.47 | 83.09 | 16.31 | 29.15 | 59.06 | 87.66 |
| | $T_{s\&5}$ | | 18.21 | 27.12 | 41.91 | 56.86 | 26.84 | 36.99 | 53.45 | 68.55 |
| | $T_{k\&5}$ | | 11.93 | 29.17 | 60.5 | 85.04 | 15.83 | 34.35 | 67.67 | 90.3 |
| | $Z_{\mathrm{APD}}$ | | 16.92 | 31.59 | 61.37 | 86.85 | 25.9 | 42.52 | 70.73 | 91.63 |
| | $Z_{\mathrm{EPD}}$ | | 14.75 | 28.74 | 61.46 | 89.12 | 21.67 | 38.17 | 71.26 | 93.15 |
| | $W$ | R and D tests | 15.71 | 26.23 | 52.51 | 79.32 | 22.93 | 35.42 | 62.07 | 86.13 |
| | $W'$ | | **18.84** | 31.73 | 59.29 | 84.00 | 26.68 | 41.67 | 69.63 | 89.8 |
| | $W_{RG}$ | | 11.95 | 15.55 | 25.97 | 49.01 | 17.34 | 21.55 | 33.58 | 57.89 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $D$ | | 15.11 | 28.64 | 61.12 | **87.62** | 23.08 | 38.24 | 70.4 | **92.4** |
| $r$ | | 19.06 | **32.44** | 60.27 | 84.73 | **27.1** | **42.43** | **70.44** | 90.29 |
| $cs$ | | 15.66 | 24.98 | 48.3 | 75.55 | 22.8 | 33.91 | 57.9 | 82.38 |
| $Q$ | | 12.63 | 18.36 | 27.63 | 37.06 | 19.45 | 26.52 | 37.24 | 46.59 |
| Q−Q* | | 12.86 | 17.17 | 27.28 | 36.57 | 20.25 | 25.85 | 35.89 | 46.33 |
| BCMR | | 16.73 | 27.89 | 54.44 | 80.51 | 24.39 | 37.72 | 64.05 | 87.15 |
| $\beta_3^2$ | | 16.52 | 30.25 | **61.7** | 86.21 | 23.67 | 38.84 | 69.67 | 90.87 |
| $T_{EP}$ | Other tests | 16.89 | 25.99 | 51.23 | 78.74 | 24.84 | 36.62 | 62.61 | 86.52 |
| $I_n$ | | 19.25 | 37.34 | 69.04 | 91.26 | 30.23 | 48.28 | 77.36 | 94.63 |
| $R_{sj}$ | | **20.71** | **38.98** | **73.04** | **94.5** | **30.58** | **50.45** | **82.81** | **97.18** |
| $T_S^{`}$ | | 13.8 | 29.61 | 69.5 | 94.38 | 20.92 | 40.07 | 79.86 | 97.05 |

Table A2. Simulated power for symmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Uniform (0,1) | K-S | EDF tests | 5.87 | 10.34 | 24.59 | 58.1 | 12.24 | 19.75 | 41.11 | 75.96 |
| | AD* | | 7.31 | 17.28 | 55.92 | 94.9 | 15.22 | 30.5 | 72.34 | 98 |
| | $Z_c$ | | **8.91** | **23.33** | **83.14** | **99.96** | **18.92** | **42.23** | **93.69** | **100** |
| | $Z_A$ | | 5 | 13.79 | 78.91 | 99.95 | 11.69 | 28.49 | 89.83 | 99.98 |
| | $P_S$ | | 6.93 | 16.9 | 56.5 | 95.22 | 14.91 | 30.09 | 72.63 | 98.06 |
| | $K^2$ | Moment tests | 2.52 | 14.68 | **77.96** | **99.67** | 8.19 | 29.34 | **88.32** | **99.91** |
| | JB | | 1.83 | 0.38 | 1.56 | 77.19 | 4.5 | 3.96 | 52.13 | 98.31 |
| | $DH$ | | 6.4 | 10.52 | 47.07 | 95 | 11.68 | 20.55 | 69.12 | 98.94 |
| | RJB | | 1.6 | 0.24 | 0.01 | 1.21 | 3.74 | 0.93 | 0.14 | 74.18 |
| | $T_{Lmom}$ | | 5.7 | 20.12 | 70.08 | 97.62 | 14.37 | 34.09 | 81.3 | 99.02 |
| | $T_{Lmom}^{(1)}$ | | 3.19 | 6.47 | 25.24 | 59.26 | 7.88 | 15.83 | 39.05 | 71.35 |
| | $T_{Lmom}^{(2)}$ | | 3.84 | 4.32 | 11.57 | 28.99 | 9.24 | 11.44 | 22.24 | 41.98 |
| | $T_{Lmom}^{(3)}$ | | 5.69 | 4.37 | 7.76 | 16.61 | 10.41 | 10.03 | 15.87 | 27.25 |
| | $BM_{3-4}$ | | 1.85 | 0.38 | 1.36 | 76.27 | 4.48 | 2.97 | 50.28 | 98.26 |
| | $BM_{3-6}$ | | 2.36 | 4.24 | 45.47 | 93.55 | 8.01 | 22.21 | 76.45 | 98.98 |
| | $T_{MC-LR}$ | | 10.67 | 14.19 | 19.17 | 31.33 | 17.97 | 23.11 | 28.29 | 42.85 |
| | $T_w$ | | 10.41 | 26.19 | 62.1 | 94.05 | 18.41 | 37.92 | 74.85 | 97.18 |
| | $T_{MC-LR} - T_w$ | | 4.66 | 9.61 | 37.35 | 80.69 | 10.12 | 18.3 | 52.31 | 88.25 |
| | $T_{s\&5}$ | | 2.23 | 1.23 | 2.75 | 15.28 | 5.14 | 4.26 | 12.21 | 42.95 |
| | $T_{k\&5}$ | | **14.03** | 25.23 | 68.33 | 98.07 | **26.01** | **44.81** | 87.46 | 99.66 |
| | $Z_{APD}$ | | 9.44 | 20.3 | 62.67 | 96.31 | 15.9 | 31.24 | 76.01 | 98.61 |
| | $Z_{EPD}$ | | 12.2 | **31.27** | 77.36 | 98.78 | 19.9 | 43.56 | 85.23 | 99.54 |
| | $W$ | R and D tests | 7.94 | 20.37 | 74.51 | 99.57 | 17.11 | 36.77 | 87.83 | 99.96 |
| | $W'$ | | 4.57 | 7.96 | 45.84 | 96.17 | 10.4 | 18.99 | 67.25 | 99.06 |
| | $W_{RG}$ | | **13.45** | **38.9** | **93.24** | **100** | **24.03** | **55.61** | **97.65** | **100** |
| | $D$ | | 3.95 | 8.86 | 56.63 | 95.61 | 8.24 | 16.19 | 69.03 | 97.64 |
| | $r$ | | 4.13 | 6.62 | 41.26 | 94.84 | 9.57 | 16.53 | 62.23 | 98.54 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $cs$ | | 8.1 | 22.82 | 79.79 | 99.85 | 17.42 | 39.38 | 90.82 | 99.99 |
| $Q$ | | 7.23 | 14.2 | 55.27 | 95.92 | 13.6 | 26.25 | 71.64 | 98.68 |
| Q−Q* | | 7.4 | 14.56 | 56.09 | 95.93 | 13.96 | 26.6 | 70.58 | 98.68 |
| BCMR | | 7.13 | 16.67 | 68.04 | 99.23 | 15.37 | 32.74 | 84.1 | 99.9 |
| $\beta_3^2$ | | 8.73 | 31.12 | 90.92 | 99.97 | 17.7 | 47.03 | 96.14 | 99.99 |
| $T_{EP}$ | Other tests | 5 | 12.21 | 53.38 | **93.71** | 12.7 | 28.56 | 72.73 | 97.99 |
| $I_n$ | | 2.39 | 0.44 | 0 | 0 | 4.7 | 0.94 | 0.01 | 0 |
| $R_{sj}$ | | 1.5 | 0.16 | 0 | 0 | 3.1 | 0.47 | 0 | 0 |
| $T_S^{`}$ | | **14.45** | **41.05** | **90.12** | 99.69 | **23.32** | **53.7** | **93.92** | **99.85** |

Table A3. Simulated power for symmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Beta (0.25,0.25) | K-S | EDF tests | 39.05 | 76.44 | 99.76 | **100** | 54.42 | 87.49 | 99.97 | **100** |
| | AD* | | | 96.2 | | | | | | |
| | | | 63.39 | | **100** | **100** | 77.15 | 98.55 | **100** | **100** |
| | $Z_c$ | | **75.51** | **99.28** | **100** | **100** | **87.25** | **99.83** | **100** | **100** |
| | $Z_A$ | | 58.76 | 98.28 | **100** | **100** | 77.04 | 99.56 | **100** | **100** |
| | $P_S$ | | 62.33 | 96.16 | **100** | **100** | 76.47 | 98.55 | **100** | **100** |
| | $K^2$ | Moment tests | 21.19 | 81 | 98.69 | 17.3 | 42.27 | 92.4 | 98.69 | 17.3 |
| | JB | | 5.15 | 1.98 | 98.61 | **100** | 17.13 | 75.11 | **100** | **100** |
| | $DH$ | | 57.58 | 90.48 | **100** | **100** | 68.16 | 95.8 | **100** | **100** |
| | RJB | | 5 | 1.34 | 0.18 | 99.97 | 8.42 | 3.11 | 69.37 | **100** |
| | $T_{Lmom}$ | | **65.42** | **98.07** | **100** | **100** | **81.15** | **99.23** | **100** | **100** |
| | $T_{Lmom}^{(1)}$ | | 16.58 | 76.57 | 99.83 | **100** | 37.03 | 86.76 | 99.93 | **100** |
| | $T_{Lmom}^{(2)}$ | | 7.17 | 44.9 | 96.22 | 99.99 | 15.73 | 62.59 | 98.25 | **100** |
| | $T_{Lmom}^{(3)}$ | | 8.33 | 24.28 | 85.42 | 99.6 | 14.05 | 42.13 | 91.84 | 99.81 |
| | $BM_{3-4}$ | | 5.18 | 1.94 | 98.11 | **100** | 13.74 | 64.5 | **100** | **100** |
| | $BM_{3-6}$ | | 11.37 | 79.17 | **100** | **100** | 51.91 | 95.78 | **100** | **100** |
| | $T_{MC-LR}$ | | 64.22 | 86.84 | 99.17 | 99.98 | 72.09 | 91.07 | 99.53 | 99.99 |
| | $T_w$ | | 41.5 | 83.75 | 99.83 | **100** | 52.5 | 89.18 | 99.93 | **100** |
| | $T_{MC-LR} - T_w$ | | 31.38 | 85.42 | 99.96 | **100** | 45.51 | 90.97 | 99.98 | **100** |
| | $T_{s\&5}$ | | 7 | 6.42 | 42.82 | 95.89 | 13.09 | 19.41 | 75.89 | 99.95 |
| | $T_{k\&5}$ | | 37.45 | 75.74 | 98.96 | **100** | 57.03 | 86.01 | 99.61 | **100** |
| | $Z_{APD}$ | | 59.72 | 92.78 | 100 | **100** | 69.98 | 96.35 | **100** | **100** |
| | $Z_{EPD}$ | | 47.01 | 87.14 | 99.86 | **100** | 55.9 | 91.24 | 99.91 | **100** |
| | $W$ | R and D tests | 70.57 | 98.65 | **100** | **100** | 84.06 | 99.67 | **100** | **100** |
| | $W'$ | | 53.46 | 93.98 | **100** | **100** | 70.74 | 97.89 | **100** | **100** |
| | $W_{RG}$ | | **82.58** | **99.78** | **100** | **100** | **90.57** | **99.92** | **100** | **100** |
| | $D$ | | 9.6 | 6.66 | 5.36 | 5.96 | 16.88 | 12.96 | 11.78 | 14.03 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $r$ | | 50.54 | 92.68 | **100** | **100** | 68.01 | 97.35 | **100** | **100** |
| $cs$ | | 71.25 | 98.84 | **100** | **100** | 84.34 | 99.73 | **100** | **100** |
| $Q$ | | 47.76 | 73.66 | 98.69 | **100** | 56.48 | 81.86 | 99.51 | **100** |
| Q−Q* | | 47.59 | 73.81 | 99.03 | **100** | 57 | 82.01 | 99.49 | **100** |
| BCMR | | 67.33 | 97.96 | 100 | **100** | 81.7 | 99.51 | **100** | **100** |
| $\beta_3^2$ | | 55.09 | 94.95 | 100 | **100** | 66.96 | 96.9 | **100** | **100** |
| $T_{EP}$ | Other tests | 36.65 | 86.99 | **100** | **100** | 61.29 | 95.74 | 100 | **100** |
| $I_n$ | | 20.45 | 9.98 | 1.27 | 0.07 | 23.49 | 11.03 | 1.38 | 0.08 |
| $R_{sj}$ | | 4.95 | 1.17 | 0.03 | 0 | 7.42 | 1.84 | 0.05 | 0 |
| $T_S^{`}$ | | | | | | | | | |
| | | **78.62** | **99.46** | **100** | **100** | **86.1** | **99.72** | **100** | 100 |

Table A4. Simulated power for symmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Beta (1. 50,1.50 ) | K-S | EDF tests | 4.52 | 6.37 | 11.51 | 25.47 | 9.57 | 13.26 | 21.73 | 41.9 |
| | AD* | | 4.89 | 7.84 | 22.38 | 58.28 | 10.76 | 16.32 | 37.22 | 73.73 |
| | $Z_c$ | | **5.41** | **8.33** | **38** | 88.76 | **12.19** | **20.45** | **61.06** | **96.81** |
| | $Z_A$ | | 3.55 | 4.71 | 31.2 | **89.09** | 8.25 | 12.42 | 50.76 | 95.69 |
| | $P_S$ | | 4.68 | 7.59 | 22.72 | 59.22 | 10.44 | 16.13 | 37.49 | 74.1 |
| | $K^2$ | Moment tests | 1.84 | 5.72 | 42.18 | **89.19** | 5.85 | 13.99 | 58.34 | **95.53** |
| | JB | | 1.61 | 0.38 | 0.11 | 22.05 | 4.74 | 2.3 | 16.86 | 73.4 |
| | $DH$ | | 4.09 | 3.71 | 13.99 | 56.97 | 7.92 | 8.81 | 30.61 | 79.95 |
| | RJB | | 1.57 | 0.41 | 0.02 | 0.02 | 4.00 | 1.22 | 0.14 | 25.00 |
| | $T_{Lmom}$ | | 3.31 | 7.49 | 32.19 | 72.16 | 9.13 | 16.47 | 47.19 | 83.48 |
| | $T_{Lmom}^{(1)}$ | | 3.91 | 4.2 | 10.11 | 25.72 | 8.74 | 10.64 | 19.61 | 38.32 |
| | $T_{Lmom}^{(2)}$ | | 4.29 | 3.99 | 5.96 | 12.38 | 9.58 | 9.53 | 13.2 | 21.4 |
| | $T_{Lmom}^{(3)}$ | | 4.82 | 4.14 | 4.81 | 8.22 | 9.63 | 9.51 | 11.32 | 15.5 |
| | $BM_{3-4}$ | | 1.63 | 0.38 | 0.1 | 21.22 | 4.75 | 2.04 | 15.54 | 72.84 |
| | $BM_{3-6}$ | | 2.04 | 1.73 | 13.35 | 52.44 | 6.38 | 10.46 | 39.39 | 81.03 |
| | $T_{MC-LR}$ | | 7.18 | 8.86 | 9.16 | 13.38 | 13.58 | 15.82 | 16.09 | 21.51 |
| | $T_w$ | | 7.27 | 15.29 | 34.93 | 71.14 | 13.99 | 23.89 | 49.04 | 81.31 |
| | $T_{MC-LR} - T_w$ | | 4.27 | 5.73 | 16.51 | 44.08 | 9.36 | 12.04 | 27.9 | 58.73 |
| | $T_{s\&5}$ | | 1.99 | 0.79 | 0.97 | 3.66 | 5.25 | 3.41 | 4.71 | 15.53 |
| | $T_{k\&5}$ | | **9.78** | 12.55 | 35 | 76.86 | **19.19** | **26.98** | **61.27** | 93.93 |
| | $Z_{APD}$ | | 5.73 | 9.23 | 29.16 | 70.12 | 10.78 | 16.73 | 44.17 | 82.59 |
| | $Z_{EPD}$ | | 8.24 | **16.9** | **47.1** | 85.07 | 14.7 | 26.83 | 59.94 | 92.41 |
| | $W$ | R and D tests | 4.9 | 7.92 | 30.05 | 80.23 | 11.35 | 17.99 | 50.65 | 91.78 |
| | $W'$ | | 3.22 | 3.02 | 11.32 | 53.03 | 7.44 | 8.41 | 25.25 | 72.90 |
| | $W_{RG}$ | | **7.93** | **17.64** | **59.91** | **96.83** | **15.89** | **31.35** | **76.47** | 99.28 |
| | $D$ | | 4.41 | 8.59 | 44.94 | 89.14 | 8.66 | 15.18 | 58.94 | 94.2 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $r$ | | 3.1 | 2.63 | 9.17 | 47.29 | 6.99 | 7.28 | 21.55 | 67.7 |
| $cs$ | | 5.14 | 9.08 | 36.53 | 86.45 | 11.46 | 19.84 | 56.75 | 94.77 |
| $Q$ | | 4.86 | 6.18 | 20.96 | 62.62 | 9.94 | 13.75 | 36.61 | 79.72 |
| Q−Q* | | 4.46 | 6.57 | 21.88 | 62.84 | 9.71 | 13.66 | 36.21 | 79.79 |
| BCMR | | 4.51 | 6.38 | 24.55 | 74.33 | 10.29 | 15.36 | 44.3 | 88.43 |
| $\beta_3^2$ | | 5.08 | 13.1 | 57.34 | 95.89 | 12.13 | 24.5 | 72.75 | **98.7** |
| $T_{EP}$ | Other tests | 3.82 | 6.03 | 21.41 | **58.77** | 9.43 | 15.82 | 40.15 | 77.89 |
| $I_n$ | | 2.18 | 0.48 | 0 | 0 | 4.79 | 1.09 | 0.04 | 0 |
| $R_{sj}$ | | 1.66 | 0.48 | 0.02 | 0 | 3.91 | 1.22 | 0.05 | 0 |
| $T_S^`$ | | **8.32** | **17.64** | **46.44** | 73.5 | **15.51** | **27.85** | **57.08** | **80.17** |

Table A5. Simulated power for symmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Student -t (5) | K-S | EDF tests | 9.36 | 13.25 | 20.52 | 32.82 | 15.71 | **36.3** | 30.64 | 44.44 |
| | AD* | | 11.13 | 16.62 | 30.09 | 47.88 | 17.78 | 24.81 | 39.82 | 57.82 |
| | $Z_c$ | | 11.72 | 18.86 | **36.06** | **54.04** | 17.26 | 25.32 | 42.5 | **60.48** |
| | $Z_A$ | | **12.48** | **19.19** | 33.56 | 49.63 | **19.38** | 27.39 | **42.98** | 59.69 |
| | $P_S$ | | 11.23 | 16.81 | 30.37 | 48.39 | 17.85 | 24.89 | 39.99 | 58.09 |
| | $K^2$ | Moment tests | 13.64 | 22.21 | 39.35 | 57.43 | 20.45 | 30.29 | 48.45 | 66.8 |
| | JB | | 13.54 | 22.97 | 43.91 | 64.45 | 20.63 | 31.35 | 52.19 | 71.28 |
| | $DH$ | | 12.91 | 22.48 | 43.41 | 63.69 | 19.62 | 30.21 | 52.22 | 71.96 |
| | RJB | | **13.78** | **24.08** | **45.97** | **67.56** | **21.17** | **33.14** | **55.07** | **75.07** |
| | $T_{Lmom}$ | | 12.59 | 20.26 | 35.56 | 53.41 | 19.41 | 27.82 | 43.82 | 62.57 |
| | $T_{Lmom}^{(1)}$ | | 6.72 | 9.10 | 13.50 | 20.26 | 11.75 | 15.23 | 20.85 | 28.26 |
| | $T_{Lmom}^{(2)}$ | | 4.69 | 6.49 | 8.59 | 11.23 | 10.08 | 12.25 | 15.08 | 18.26 |
| | $T_{Lmom}^{(3)}$ | | 4.64 | 5.65 | 7.07 | 8.52 | 9.12 | 10.83 | 12.76 | 14.28 |
| | $BM_{3-4}$ | | 13.63 | 22.96 | 43.92 | 64.45 | 20.77 | 31.25 | 52.16 | 71.28 |
| | $BM_{3-6}$ | | 13.25 | 22.88 | 44.13 | 64.29 | 20.17 | 29.45 | 52.01 | 71.78 |
| | $T_{MC-LR}$ | | 4.82 | 4.83 | 5.55 | 5.77 | 9.3 | 9.77 | 9.91 | 11.27 |
| | $T_w$ | | 9.72 | 16.55 | 35.13 | 57.72 | 15.11 | 23.88 | 43.86 | 66.88 |
| | $T_{MC-LR} - T_w$ | | 8.15 | 13.88 | 27.05 | 47.38 | 13.85 | 19.9 | 33.99 | 54.5 |
| | $T_{s\&5}$ | | 13.12 | 20.95 | 34.46 | 45.92 | 20.14 | 28.76 | 43.61 | 55.21 |
| | $T_{k\&5}$ | | 9.08 | 20.55 | 44.48 | 66.2 | 13.65 | 25.19 | 50.64 | 72.9 |
| | $Z_{APD}$ | | 11.35 | 21.14 | 40.27 | 61.81 | 18.64 | 28.97 | 49.3 | 69.57 |
| | $Z_{EPD}$ | | 10.48 | 18.71 | 39.09 | 63.49 | 16.29 | 25.82 | 48.47 | 70.85 |
| | $W$ | R and D tests | 11.35 | 18.5 | 35.58 | 55.63 | 17.62 | 26.02 | 43.71 | 63.92 |
| | $W'$ | | **12.83** | 21.61 | **41.31** | 62.23 | 19.56 | 29.39 | 50.69 | 70.05 |
| | $W_{RG}$ | | 9.32 | 12.00 | 19.26 | 29.43 | 14.48 | 17.92 | 24.43 | 35.35 |
| | $D$ | | 11.07 | 18.83 | 39.48 | **63.03** | 17.27 | 26.21 | 48.48 | **71.27** |
| | $r$ | | 12.99 | **22.17** | 42.1 | 63.08 | **19.86** | **29.78** | **51.24** | 70.61 |
| | $cs$ | | 11.44 | 17.78 | 32.99 | 51.68 | 17.57 | 25.12 | 40.89 | 59.33 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $Q$ | | 9.48 | 13.76 | 23.86 | 33.36 | 15.76 | 21.61 | 31.17 | 41.26 |
| Q−Q* | | 9.55 | 13.62 | 24.01 | 32.67 | 15.68 | 21.09 | 31.05 | 41.05 |
| BCMR | | 12.12 | 19.42 | 37.15 | 57.61 | 18.32 | 27.33 | 45.76 | 65.68 |
| $\beta_3^2$ | | 10.56 | 18.94 | 40.4 | 64.27 | 16.06 | 25.62 | 48.33 | 71.43 |
| $T_{EP}$ | Other tests | 11.93 | 17.45 | 30.94 | 48.61 | 18.46 | 25.74 | 40.4 | 59.56 |
| $I_n$ | | 12.51 | **24.04** | **47.94** | 71.22 | **20.61** | **32.76** | **57.23** | **78.86** |
| $R_{sj}$ | | **12.72** | 22.55 | 43.49 | 66.39 | 20.04 | 31.37 | 54.73 | 75.8 |
| $\grave{T}_S$ | | 9.77 | 17.19 | 40.69 | **65.32** | 15.3 | 24.68 | 51.84 | 75.21 |

Table A6. Simulated power for symmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Student-t (8) | K-S | EDF tests | 6.84 | 8.56 | 10.48 | 14.54 | 12.84 | 15.04 | 18.1 | 23.15 |
| | AD* | | 8.06 | 10.23 | 14.93 | 22.19 | 14 | 17.27 | 22.98 | 31.71 |
| | $Z_c$ | | 8.48 | 11.78 | **20.14** | **31.04** | 13.95 | 18.02 | 26.37 | **37.91** |
| | $Z_A$ | | **9.03** | **12.34** | 18.04 | 26.15 | **15.14** | **19.66** | **26.42** | 35.86 |
| | $P_S$ | | 8.17 | 10.26 | 15.07 | 22.58 | 14.13 | 17.3 | 23.06 | 32.06 |
| | $K^2$ | Moment tests | **9.89** | 14.46 | 22.24 | 33.69 | 16.15 | 21.86 | 30.53 | 43.57 |
| | JB | | 9.7 | 14.96 | 25.43 | 39.85 | 16.47 | 22.68 | 33.81 | 48.21 |
| | $DH$ | | 9.46 | 14.64 | 24.41 | 38.25 | 15.4 | 21.16 | 33.3 | 48.34 |
| | RJB | | 9.74 | **15.73** | **26.34** | **41.74** | 16.43 | **23.46** | **35.56** | **51.69** |
| | $T_{Lmom}$ | | 9.08 | 12.52 | 18.33 | 26.88 | 15.39 | 19.32 | 25.89 | 36.25 |
| | $T_{Lmom}^{(1)}$ | | 5.66 | 6.85 | 8.03 | 11.14 | 10.83 | 12.79 | 14.02 | 17.28 |
| | $T_{Lmom}^{(2)}$ | | 4.89 | 5.87 | 6.17 | 7.96 | 10.76 | 10.82 | 12.24 | 12.91 |
| | $T_{Lmom}^{(3)}$ | | 5.03 | 5.43 | 5.76 | 6.60 | 9.6 | 10.26 | 11.25 | 11.9 |
| | $BM_{3-4}$ | | 9.77 | 14.97 | 25.44 | 39.8 | **16.58** | 22.62 | 33.78 | 48.24 |
| | $BM_{3-6}$ | | 9.63 | 14.5 | 24.78 | 39.44 | 16.01 | 20.89 | 33.07 | 48.01 |
| | $T_{MC-LR}$ | | 4.68 | 4.81 | 5.28 | 4.8 | 9.24 | 9.92 | 10.32 | 9.54 |
| | $T_w$ | | 7.15 | 10.29 | 17.01 | 29.36 | 12.68 | 16.75 | 25.03 | 38.95 |
| | $T_{MC-LR} - T_w$ | | 6.68 | 9.16 | 13.2 | 21.73 | 11.87 | 14.63 | 19.79 | 29.09 |
| | $T_{s\&5}$ | | 9.69 | 13.58 | 19.96 | 26.52 | 16.05 | 20.58 | 28.23 | 36.05 |
| | $T_{k\&5}$ | | 6.97 | 13.44 | 25.18 | 40.66 | 11.47 | 17.88 | 30.9 | 48.62 |
| | $Z_{\text{APD}}$ | | 8.35 | 13 | 21.85 | 34.61 | 14.68 | 20.34 | 30.02 | 43.94 |
| | $Z_{\text{EPD}}$ | | 7.7 | 11.62 | 20.2 | 35.17 | 13.39 | 18.2 | 28.15 | 43.89 |
| | $W$ | R and D tests | 8.25 | 11.21 | 19.00 | 30.55 | 13.98 | 18.32 | 26.59 | 39.34 |
| | $W'$ | | 9.38 | 13.55 | **23.2** | 36.52 | 15.24 | 21.01 | 32.12 | 46.21 |
| | $W_{RG}$ | | 6.98 | 7.94 | 9.11 | 11.63 | 11.65 | 13.04 | 13.85 | 16.36 |
| | $D$ | | 7.43 | 11.36 | 20.6 | 34.92 | 13.46 | 18.11 | 28.5 | 44.48 |
| | $r$ | | **9.5** | **13.82** | 23.84 | **37.27** | **15.33** | **21.31** | **32.63** | **46.9** |
| | $cs$ | | 8.29 | 10.79 | 17.3 | 27.12 | 13.85 | 17.85 | 24.39 | 34.86 |
| | $Q$ | | 7.31 | 10.09 | 14.77 | 19.99 | 12.8 | 17.1 | 21.63 | 27.17 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Q−Q∗ | | 7.36 | 9.15 | 14.47 | 20.27 | 13.47 | 16.29 | 21.79 | 28.13 |
| BCMR | | 8.79 | 11.92 | 20.16 | 32.3 | 14.62 | 19.46 | 28.22 | 41.34 |
| $\beta_3^2$ | | 7.87 | 11.96 | 21.51 | 36.13 | 12.89 | 17.99 | 29.13 | 45.14 |
| $T_{EP}$ | Other tests | 8.83 | 10.68 | 15.49 | 22.69 | 14.85 | 17.78 | 23.71 | 33.45 |
| $I_n$ | | 9.03 | **15.3** | **27.43** | **44.67** | **16.46** | **22.89** | **36.97** | **55.94** |
| $R_{sj}$ | | **9.09** | 14.00 | 23.4 | 38.03 | 15.71 | 21.91 | 34.05 | 50.34 |
| $T_S^{`}$ | | 7.23 | 10.61 | 21.58 | 38.33 | 12.44 | 17.01 | 31.58 | 49.73 |

Table A7. Simulated power for symmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Cauchy (o,7) | K-S | EDF tests | 57.91 | 74.21 | 99.27 | **100** | 65.23 | 87.99 | 99.6 | **100** |
| | AD* | | 61.25 | 78.34 | **99.65** | **100** | 68.08 | 90.98 | 99.81 | **100** |
| | $Z_c$ | | 58.3 | 73.92 | 99.34 | **100** | 63.4 | 86.74 | 99.53 | **100** |
| | $Z_A$ | | 61.14 | 76.42 | 99.44 | **100** | 67.91 | 89.51 | 99.66 | **100** |
| | $P_S$ | | **61.44** | **78.55** | 99.65 | **100** | **68.38** | **91.14** | **99.82** | **100** |
| | $K^2$ | Moment tests | 59.44 | 75.58 | 99.27 | **100** | 67.58 | 89.32 | 99.61 | **100** |
| | JB | | 59.29 | 76.46 | 99.55 | **100** | 67.14 | 89.99 | 99.72 | **100** |
| | $DH$ | | 62.62 | 78.35 | 99.6 | **100** | 69.27 | 90.5 | 99.81 | **100** |
| | RJB | | **64.4** | 81.2 | 99.77 | **100** | **71.52** | **92.91** | **99.9** | **100** |
| | $T_{Lmom}$ | | 64.36 | 80.98 | 99.8 | **100** | 70.41 | 92.29 | 99.86 | **100** |
| | $T_{Lmom}^{(1)}$ | | 27.92 | 50.04 | 96.88 | 99.97 | 35.94 | 71.31 | 97.85 | 99.99 |
| | $T_{Lmom}^{(2)}$ | | 9.99 | 25.92 | 81.05 | 98.48 | 17.32 | 46.24 | 86.5 | 99.10 |
| | $T_{Lmom}^{(3)}$ | | 4.92 | 13.2 | 56.59 | 87.71 | 9.88 | 28.64 | 65.93 | 91.98 |
| | $BM_{3-4}$ | | 59.32 | 76.23 | 99.54 | **100** | 67.07 | 89.94 | 99.72 | **100** |
| | $BM_{3-6}$ | | 61.35 | 78.98 | 99.72 | **100** | 68.43 | 91.01 | 99.8 | **100** |
| | $T_{MC-LR}$ | | 11.51 | 14.25 | 34.58 | 64.18 | 18.77 | 29.58 | 47.66 | 74.83 |
| | $T_w$ | | 50.92 | 73.39 | **99.81** | **100** | 57.35 | 89.2 | 99.84 | **100** |
| | $T_{MC-LR} - T_w$ | | 42.52 | 66.99 | 99.6 | **100** | 47.69 | 84.76 | 99.74 | **100** |
| | $T_{s\&5}$ | | 58.99 | 73.03 | 98.33 | 99.99 | 65.89 | 86.44 | 99.04 | **100** |
| | $T_{k\&5}$ | | 51.16 | 85.96 | 99.75 | **100** | 54.45 | 88.08 | 99.81 | **100** |
| | $Z_{\text{APD}}$ | | 60.93 | **88.61** | 99.76 | **100** | 68.47 | 91.87 | 99.84 | **100** |
| | $Z_{\text{EPD}}$ | | 54.36 | 86.46 | 99.78 | **100** | 60.52 | 89.74 | 99.85 | **100** |
| | $W$ | R and D tests | 59.47 | 86.09 | 99.61 | **100** | 65.71 | 89.28 | 99.73 | **100** |
| | $W'$ | | 62.9 | 88.71 | 99.71 | **100** | 69.7 | 91.83 | 99.79 | **100** |
| | $W_{RG}$ | | 52.99 | 78.17 | 98.47 | 99.99 | 58.37 | 81.51 | 98.94 | **100** |
| | $D$ | | 59.24 | 87.93 | 99.74 | **100** | 65.92 | 90.92 | **99.85** | **100** |
| | $r$ | | **63.2** | **89.04** | 99.72 | **100** | **69.97** | **92.02** | 99.8 | **100** |
| | $cs$ | | 59.45 | 85.4 | 99.51 | **100** | 65.6 | 88.55 | 99.67 | **100** |
| | $Q$ | | 42.1 | 60.04 | 79.28 | 89.69 | 49.65 | 66.07 | 83.14 | 91.36 |
| | Q−Q∗ | | 41.83 | 58.11 | 79.2 | 89.11 | 49.29 | 64.48 | 82.8 | 90.98 |
| | BCMR | | 60.95 | 87.03 | 99.62 | **100** | 67.08 | 90.26 | 99.74 | **100** |
| | $\beta_3^2$ | | 55.68 | 86.61 | **99.76** | **100** | 61.65 | 89.55 | 99.82 | **100** |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $T_{EP}$ | Other tests | 60.07 | 86.22 | 99.66 | **100** | 66.77 | 89.76 | 99.76 | **100** |
| $I_n$ | | 64.86 | 90.95 | 99.83 | **100** | **72.65** | 93.64 | 99.91 | **100** |
| $R_{sj}$ | | **64.96** | **91.6** | **99.87** | **100** | 72.44 | **94.01** | **99.93** | **100** |
| $T_S^{`}$ | | 51.67 | 86.27 | 99.84 | **100** | 58.35 | 89.88 | 99.9 | **100** |

Table B1. Simulated power for Asymmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Beta (3,1) | K-S | EDF tests | 13.5 | 28.27 | 63.86 | 95.51 | 23.12 | 41.01 | 77.88 | 98.53 |
| | AD* | | 18.46 | 41.09 | 88.3 | 99.93 | 29.8 | 55.88 | 94.33 | 99.99 |
| | $Z_c$ | | **21.06** | **48.92** | 96.38 | **100** | **33.96** | 66.73 | 98.95 | **100** |
| | $Z_A$ | | 20.64 | 52.6 | **98.57** | **100** | 32.33 | **67.79** | **99.51** | **100** |
| | $P_S$ | | 18.06 | 40.26 | 87.51 | 99.89 | 29.49 | 54.98 | 93.67 | 99.97 |
| | $K^2$ | Moment tests | 12.35 | 21.26 | 51.98 | 97.29 | 20.97 | 35.33 | 79.5 | 99.8 |
| | JB | | 13.35 | 22.88 | 61.62 | 98.56 | 23.99 | 43.8 | 90.2 | 99.91 |
| | $DH$ | | 14.54 | 34.63 | 88.79 | 99.91 | 23.34 | 49.08 | 94.77 | 100 |
| | RJB | | 11.02 | 17.31 | 39.49 | 88.06 | 19.23 | 29.82 | 65.47 | 99.54 |
| | $T_{Lmom}$ | | 12.32 | **39.83** | **91.35** | **99.95** | 23.9 | **56.8** | **95.77** | **99.99** |
| | $T_{Lmom}^{(1)}$ | | 6.93 | 18.99 | 58.53 | 92.92 | 12.79 | 31.88 | 71.63 | 96.59 |
| | $T_{Lmom}^{(2)}$ | | 5.34 | 11.51 | 35.97 | 72.01 | 10.98 | 20.85 | 50.50 | 82.00 |
| | $T_{Lmom}^{(3)}$ | | 5.29 | 8.08 | 24.77 | 52.66 | 10.42 | 15.84 | 37.38 | 65.36 |
| | $BM_{3-4}$ | | 13.47 | 23.13 | 62.08 | 98.56 | 23.98 | 43.33 | 90.24 | 99.91 |
| | $BM_{3-6}$ | | 14.84 | 31.55 | 77.22 | 98.94 | **27.11** | 51.58 | 91.79 | 99.89 |
| | $T_{MC-LR}$ | | 13.55 | 23.88 | 41.97 | 73.6 | 22.48 | 34.84 | 53.86 | 82.06 |
| | $T_w$ | | 7.52 | 9.75 | 11.45 | 16.18 | 13.58 | 16.74 | 18.88 | 24.73 |
| | $T_{MC-LR} - T_w$ | | 4.28 | 6.56 | 22.8 | 54.65 | 8.56 | 12.82 | 34.21 | 65.81 |
| | $T_{s\&5}$ | | **15.49** | 30.66 | 74.57 | 97.86 | 24.01 | 44.62 | 86 | 99.4 |
| | $T_{k\&5}$ | | 10.73 | 12.92 | 16.37 | 18.68 | 19.67 | 23.06 | 28.17 | 32.99 |
| | $Z_{APD}$ | | 15.37 | 36.47 | 85.93 | 99.78 | 25.98 | 51.92 | 92.95 | 99.95 |
| | $Z_{EPD}$ | | 9.47 | 11.27 | 11.42 | 12 | 15.88 | 18.1 | 17.93 | 18.93 |
| | $W$ | R and D tests | 20.7 | 48.9 | 95.74 | **100** | 33.1 | 65.1 | 98.51 | **100** |
| | $W'$ | | 18.04 | 40.62 | 91.16 | **100** | 28.85 | 56.06 | 96.45 | **100** |
| | $W_{RG}$ | | 22.26 | 54.37 | 97.28 | **100** | 34.57 | **69.22** | 98.88 | **100** |
| | $D$ | | 9.33 | 12.96 | 21.17 | 34.53 | 16.41 | 20.97 | 31.52 | 46.31 |
| | $r$ | | 17.63 | 39.18 | 90.18 | 99.99 | 28.09 | 54.59 | 95.74 | **100** |
| | $cs$ | | 20.72 | **49.75** | 96.37 | **100** | 33.1 | 65.97 | 98.71 | **100** |
| | $Q$ | | 9.94 | 9.88 | 10.37 | 9.34 | 16.35 | 17.74 | 19.74 | 18.88 |
| | Q−Q* | | **23.07** | 62.79 | **99.65** | **100** | **34.96** | 76.82 | **99.88** | **100** |
| | BCMR | | 20.47 | 46.98 | 94.98 | **100** | 32.18 | 63.31 | 98.12 | **100** |
| | $\beta_3^2$ | | 5.9 | 7.62 | 17.37 | 34.01 | 12.08 | 15.25 | 27.58 | 49.02 |
| | $T_{EP}$ | Other tests | **18.34** | **41.48** | **87.6** | **99.78** | **30.46** | **57.37** | **93.86** | **99.96** |
| | $I_n$ | | 12.38 | 18.49 | 29.37 | 45.57 | 20.69 | 27.65 | 40.49 | 58.05 |
| | $R_{sj}$ | | 7.78 | 9.13 | 8.29 | 8.21 | 13.48 | 14.29 | 14.07 | 14.34 |
| | $T_S^{`}$ | | 10.77 | 17.07 | 21.96 | 18.67 | 18.91 | 26.68 | 30.31 | 24.37 |

Table B2. Simulated power for asymmetric distributions

| Distribution | Tests | Type of the test | α =0.05 n =10 | α =0.05 n =20 | α =0.05 n =50 | α =0.05 n =100 | α =0.1 n =10 | α =0.1 n =20 | α =0.1 n =50 | α =0.1 n =100 |
|---|---|---|---|---|---|---|---|---|---|---|
| **(5)$\chi^2$** | K-S | EDF tests | 13.14 | 27.99 | 58.22 | 88.99 | 21.58 | 40.09 | 71.16 | 94.40 |
| | AD* | | 17.65 | 38.61 | 80.29 | 98.56 | 26.92 | 51.04 | 88.27 | 99.48 |
| | $Z_c$ | | **19.96** | 44.27 | 88.91 | 99.75 | 29.22 | 57.45 | 94.91 | 99.94 |
| | $Z_A$ | | 19.87 | **46.32** | **92.49** | **99.96** | **29.93** | **58.54** | **96.32** | **99.98** |
| | $P_S$ | | 17.39 | 38.15 | 79.05 | 98.35 | 26.92 | 50.48 | 87.24 | 99.25 |
| | $K^2$ | Moment tests | 16.76 | 34.17 | 70.47 | 96.87 | 25.05 | 45.28 | 82.91 | 99.35 |
| | JB | | 17.76 | 36.04 | 75.3 | 98.34 | 27.65 | 51.47 | 90.01 | 99.61 |
| | $DH$ | | 15.6 | 36.83 | 85.68 | **99.51** | 23.44 | 48.11 | 92.44 | **99.89** |
| | RJB | | 15.92 | 31.96 | 66.61 | 95.13 | 24.37 | 43.75 | 80.67 | 99.07 |
| | $T_{Lmom}$ | | 14.66 | 37.56 | 84.2 | 99.29 | 23.29 | 50.47 | 90.76 | 99.78 |
| | $T^{(1)}_{Lmom}$ | | 7.63 | 17.97 | 49.38 | 84.78 | 13.20 | 28.79 | 62.16 | 90.69 |
| | $T^{(2)}_{Lmom}$ | | 5.34 | 10.82 | 29.52 | 60.72 | 10.70 | 18.92 | 42.55 | 71.98 |
| | $T^{(3)}_{Lmom}$ | | 5.2 | 8.25 | 20.49 | 43.42 | 9.72 | 15.38 | 31.44 | 55.42 |
| | $BM_{3-4}$ | | 17.92 | 36.18 | 75.6 | 98.35 | 27.75 | 51.32 | 90.03 | 99.61 |
| | $BM_{3-6}$ | | **18.67** | 39.01 | 79.21 | 97.98 | 28.67 | 52.75 | 89.77 | 99.55 |
| | $T_{MC-LR}$ | | 8.07 | 13.79 | 25.44 | 50.01 | 14.96 | 22.91 | 36.87 | 62.02 |
| | $T_w$ | | 7.92 | 12.87 | 17.4 | 25.78 | 13.48 | 20.22 | 24.6 | 33.86 |
| | $T_{MC-LR} - T_w$ | | 5.84 | 9.18 | 19.43 | 40.86 | 10.43 | 14.74 | 28.49 | 51.48 |
| | $T_{s\&5}$ | | 18.62 | **42.96** | **86.86** | 99.4 | **29.78** | **57.4** | **93.95** | 99.81 |
| | $T_{k\&5}$ | | 11.54 | 22.9 | 41.7 | 61.75 | 16.9 | 27.85 | 47.77 | 68.06 |
| | $Z_{APD}$ | | 15.19 | 37.08 | 82.8 | 99.18 | 24.38 | 50.12 | 90.2 | 99.76 |
| | $Z_{EPD}$ | | 10.64 | 17.95 | 28.65 | 44.67 | 16.74 | 25.28 | 36.87 | 51.99 |
| | $W$ | R and D tests | 19.45 | **44.4** | 89.07 | 99.74 | 28.98 | 56.7 | 94.43 | 99.92 |
| | $W'$ | | 19.04 | 41.4 | 85.42 | 99.54 | 28.19 | 53.37 | 92.02 | 99.79 |
| | $W_{RG}$ | | 18.21 | 42.8 | 88.19 | 99.77 | 27.23 | 55.89 | 93.7 | 99.97 |
| | $D$ | | 13.23 | 24.7 | 47.5 | 73.27 | 20.27 | 33.08 | 57.1 | 79.57 |
| | $r$ | | 18.95 | 40.8 | 84.71 | 99.47 | 27.8 | 52.7 | 91.31 | 99.71 |
| | $cs$ | | 19.41 | **44.4** | **89.43** | **99.8** | 28.92 | **57.04** | **94.62** | 99.93 |
| | $Q$ | | 12.38 | 33.6 | 87.09 | **99.8** | 21.09 | 49.02 | 93.84 | **99.98** |
| | Q–Q* | | 14.2 | 23.1 | 41.74 | 57.72 | 22.1 | 32.93 | 52.12 | 68.69 |
| | BCMR | | **19.49** | 43.7 | 88.32 | 99.7 | **29.02** | 56.4 | 94.01 | 99.91 |
| | $\beta_3^2$ | | 6.89 | 9.69 | 12.17 | 15.82 | 12.58 | 16.29 | 18.92 | 22.92 |
| | $T_{EP}$ | Other tests | **19.67** | **41.9** | **83.71** | **98.92** | **29.41** | **55.1** | **90.98** | **99.73** |
| | $I_n$ | | 15 | 31.3 | 57.95 | 82.13 | 24.15 | 40.66 | 67.11 | 87.58 |
| | $R_{sj}$ | | 12.1 | 21.5 | 34.44 | 52.38 | 19.39 | 28.6 | 44.16 | 62.23 |
| | $T_S^{`}$ | | 9.07 | 14.8 | 21.83 | 32.07 | 15.67 | 23.57 | 31.25 | 42.3 |

Table B3. Simulated power for asymmetric distributions

| Distribution | Tests | Type of the test | α =0.05 n =10 | α =0.05 n =20 | α =0.05 n =50 | α =0.05 n =100 | α =0.1 n =10 | α =0.1 n =20 | α =0.1 n =50 | α =0.1 n =100 |
|---|---|---|---|---|---|---|---|---|---|---|
| **(15)$\chi^2$** | K-S | EDF tests | 7.9 | 12.17 | 22.75 | 41.39 | 14.04 | 20.61 | 34.68 | 55.79 |
| | AD* | | 8.75 | 15.41 | 32.82 | 61.2 | 15.26 | 24.53 | 44.98 | 71.83 |
| | $Z_c$ | | 9.53 | 17.9 | 41.34 | 71.34 | 16.05 | 27.55 | 53.69 | 81.86 |
| | $Z_A$ | | **9.61** | **18.43** | **43.57** | **75.81** | **16.61** | **28.03** | **55.44** | **84.5** |
| | $P_S$ | | 8.75 | 15.19 | 32.22 | 59.77 | 15.24 | 24.21 | 44.19 | 70.26 |
| | $K^2$ | Moment tests | 9.89 | 16.24 | 33.59 | 59.45 | 16.02 | 24.68 | 45.12 | 74.79 |
| | JB | | 9.97 | 16.94 | 35.71 | 64.54 | 16.73 | 26.8 | 50.51 | 79.13 |
| | $DH$ | | 8.05 | 14.69 | 37.57 | 70.58 | 14.27 | 22.85 | 50.38 | 81.67 |
| | RJB | | 9.33 | 15.46 | 31.49 | 57.73 | 15.41 | 23.93 | 43.96 | 74.31 |
| | $T_{Lmom}$ | | 8.04 | 15.55 | 35.69 | 65.95 | 14.17 | 24.17 | 47.33 | 76.68 |
| | $T_{Lmom}^{(1)}$ | | 6.24 | 8.72 | 18.5 | 35.98 | 11.08 | 15.59 | 28.24 | 48.26 |
| | $T_{Lmom}^{(2)}$ | | 5.08 | 6.6 | 11.8 | 21.72 | 10.42 | 13.04 | 20.38 | 32.22 |
| | $T_{Lmom}^{(3)}$ | | 5.12 | 6.09 | 9.17 | 15.81 | 9.96 | 11.51 | 16.57 | 24.46 |
| | $BM_{3-4}$ | | **10.05** | 16.98 | 35.91 | 64.7 | 16.87 | 26.75 | 50.65 | 79.25 |
| | $BM_{3-6}$ | | 10.04 | 17.47 | 35.23 | 59.75 | 16.89 | 26.13 | 49.42 | 75.4 |
| | $T_{MC-LR}$ | | 5.47 | 7.06 | 9.84 | 15.95 | 10.5 | 13.47 | 16.99 | 24.93 |
| | $T_w$ | | 6.12 | 8.01 | 9.19 | 10.9 | 11.38 | 14.25 | 14.86 | 17.1 |
| | $T_{MC-LR} - T_w$ | | 5.07 | 6.44 | 8.25 | 11.78 | 10.07 | 11.57 | 14.21 | 19.35 |
| | $T_{s\&5}$ | | 9.69 | **19.41** | **44.98** | **76.54** | **17.42** | **29.84** | **59.3** | **86.14** |
| | $T_{k\&5}$ | | 7.52 | 11.59 | 19.03 | 26.48 | 12.57 | 16.66 | 24.71 | 33.28 |
| | $Z_{\text{APD}}$ | | 8.35 | 15.85 | 36.41 | 67.56 | 14.91 | 24.92 | 48.98 | 78.63 |
| | $Z_{\text{EPD}}$ | | 7.35 | 9.98 | 13.52 | 17.16 | 12.81 | 16.2 | 20.11 | 24.2 |
| | $W$ | R and D tests | 9.02 | **17.97** | **41.48** | 72.93 | 15.92 | 27.25 | 53.47 | 82.36 |
| | $W'$ | | 9.68 | 17.45 | 38.9 | 69.5 | 15.94 | 25.97 | 50.41 | 79.15 |
| | $W_{RG}$ | | 8.52 | 16.27 | 36.08 | 68.13 | 14.32 | 25.19 | 48.33 | 78.46 |
| | $D$ | | 7.56 | 11.51 | 18.46 | 26.84 | 13.5 | 18.04 | 26.59 | 35.1 |
| | $r$ | | **9.78** | 17.32 | 38.5 | 68.74 | 15.92 | 25.69 | 49.49 | 78.33 |
| | $cs$ | | 9.1 | 17.71 | 41.3 | **73.39** | 16.03 | 27.16 | **53.53** | **82.43** |
| | $Q$ | | 5.91 | 10.94 | 26.76 | 50.87 | 11.33 | 19.66 | 39.51 | 64.75 |
| | Q−Q* | | 8.89 | 12.43 | 20.63 | 27.65 | 15.64 | 20.28 | 29.46 | 37.99 |
| | BCMR | | 9.45 | 17.96 | 40.85 | 72.11 | **16.14** | **27.29** | 52.92 | 81.6 |
| | $\beta_3^2$ | | 5.59 | 6.39 | 7.88 | 8.32 | 10.63 | 11.74 | 14.19 | 14.47 |
| | $T_{EP}$ | Other tests | **9.42** | **17.3** | **37.83** | **66.28** | **16.68** | **27.19** | **49.87** | **77.11** |
| | $I_n$ | | 8.31 | 14.49 | 25.24 | 38.24 | 15.14 | 21.77 | 33.69 | 48.16 |
| | $R_{sj}$ | | 7.65 | 10.91 | 14.12 | 18.92 | 13.49 | 16.92 | 22.17 | 27.97 |
| | $T_S^{`}$ | | 6.23 | 8.29 | 10.43 | 13.25 | 11.38 | 14.72 | 17.93 | 20.6 |

Table B4. Simulated power for asymmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Gamma(3,4) | K-S | EDF tests | 12.72 | 23.67 | 50.42 | 81.74 | 20.59 | 34.92 | 64.01 | 90.48 |
| | AD* | | 15.5 | 31.99 | 71.07 | 96.38 | 24.86 | 44.34 | 81.37 | 98.28 |
| | $Z_c$ | | 17.63 | 37.68 | 81.99 | 99.09 | 26.75 | 50.16 | 90.05 | 99.81 |
| | $Z_A$ | | **17.81** | **39.81** | **86.39** | **99.71** | **27.35** | **51.59** | **92.02** | **99.92** |
| | $P_S$ | | 15.38 | 31.37 | 69.98 | 95.85 | 24.77 | 43.81 | 80.3 | 97.93 |
| | $K^2$ | Moment tests | 15.8 | 30 | 62.93 | 93.98 | 23.61 | 40.58 | 76.87 | 98.48 |
| | JB | | 16.51 | 31.63 | 68.05 | 96.31 | 25.68 | 45.96 | 84.55 | 99.21 |
| | $DH$ | | 14.2 | 31.18 | 77.8 | 98.56 | 21.44 | 41.9 | 86.73 | 99.62 |
| | RJB | | 14.83 | 27.75 | 60.16 | 91.81 | 22.53 | 39.1 | 74.75 | 97.96 |
| | $T_{Lmom}$ | | 13.44 | 30.94 | 75.79 | 97.75 | 21.73 | 44.04 | 84.5 | 99.06 |
| | $T_{Lmom}^{(1)}$ | | 7.18 | 15.05 | 41.69 | 76.65 | 12.87 | 24.46 | 54.82 | 84.93 |
| | $T_{Lmom}^{(2)}$ | | 5.21 | 9.45 | 24.52 | 52.17 | 10.63 | 17.09 | 36.6 | 63.99 |
| | $T_{Lmom}^{(3)}$ | | 5.17 | 7.15 | 17.06 | 36.13 | 9.77 | 13.56 | 27.13 | 48.01 |
| | $BM_{3-4}$ | | 16.64 | 31.78 | 68.46 | 96.35 | 25.87 | 45.94 | 84.63 | 99.23 |
| | $BM_{3-6}$ | | 17.05 | 33.64 | 71.21 | 95.46 | 26.7 | 46.88 | 83.81 | 98.63 |
| | $T_{MC-LR}$ | | 7.40 | 11.18 | 20.94 | 40.35 | 13.74 | 19.3 | 31.32 | 52.78 |
| | $T_w$ | | 7.75 | 10.97 | 15.43 | 21.91 | 13.64 | 17.89 | 22.29 | 29.62 |
| | $T_{MC-LR} - T_w$ | | 5.70 | 8.16 | 16.18 | 33.09 | 10.43 | 13.29 | 24.23 | 43.08 |
| | $T_{s\&5}$ | | **17.09** | **37.43** | **80.76** | **98.63** | **27.45** | **51.64** | **90.06** | **99.65** |
| | $T_{k\&5}$ | | 11.21 | 19.26 | 35.91 | 54.38 | 16.16 | 24.47 | 42.38 | 61.03 |
| | $Z_{\text{APD}}$ | | 14.01 | 31.47 | 74.7 | 97.68 | 22.53 | 43.92 | 84.2 | 99.1 |
| | $Z_{\text{EPD}}$ | | 10.43 | 15.24 | 24.3 | 37.49 | 16.45 | 22.19 | 32.65 | 45.61 |
| | $W$ | R and D tests | 17.02 | **37.56** | 81.89 | 99.05 | **26.42** | **49.93** | 89.41 | 99.78 |
| | $W'$ | | 17.12 | 35.45 | 77.71 | 98.4 | 25.78 | 47.03 | 86.28 | 99.41 |
| | $W_{RG}$ | | 16.04 | 35.35 | 79.87 | 99 | 24.83 | 47.97 | 88.03 | 99.7 |
| | $D$ | | 11.97 | 20.73 | 40.5 | 63.11 | 18.76 | 28.76 | 50.00 | 70.72 |
| | $r$ | | 16.99 | 34.99 | 76.97 | 98.22 | 25.52 | 46.28 | 85.63 | 99.32 |
| | $cs$ | | 17.04 | 37.31 | **82.19** | **99.22** | 26.37 | 49.85 | **89.83** | **99.81** |
| | $Q$ | | 10.22 | 26.94 | 75.98 | 98.63 | 18.14 | 41.56 | 86.98 | 99.63 |
| | Q−Q∗ | | 13.33 | 20.24 | 36.02 | 52.08 | 20.9 | 29.77 | 46.87 | 63.32 |
| | BCMR | | **17.42** | 37.06 | 80.94 | 98.94 | 26.36 | 49.63 | 88.9 | 99.75 |
| | $\beta_3^2$ | | 6.70 | 7.98 | 11.35 | 13.26 | 12.42 | 14 | 17.64 | 20.89 |
| | $T_{EP}$ | Other tests | **17.41** | **35.79** | **76.19** | **97.22** | **27.2** | **48.66** | **84.92** | **98.9** |
| | $I_n$ | | 13.85 | 26.6 | 51.19 | 74.59 | 22.79 | 36.07 | 60.51 | 81.82 |
| | $R_{sj}$ | | 11.31 | 17.92 | 29.24 | 43.69 | 17.87 | 25.37 | 38.82 | 53.68 |
| | $T_S^{`}$ | | 8.31 | 11.85 | 19.02 | 27.57 | 15.00 | 19.88 | 28.43 | 36.78 |

Table B5. Simulated power for asymmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Weibull (15,3) | K-S | EDF tests | 8.1 | 13.86 | 26.56 | 48.11 | 15.01 | 22.32 | 38.34 | 61.87 |
| | AD* | | 9.57 | 17.28 | 37.04 | 66.84 | 16.34 | 27.16 | 49.16 | 76.42 |
| | $Z_c$ | | 10.52 | 20.37 | 45.84 | 75.51 | 17.3 | 30.16 | 57.41 | 84.42 |
| | $Z_A$ | | **10.83** | 21.03 | **47.44** | **78.49** | **17.93** | **30.91** | **58.76** | **85.63** |
| | $P_S$ | | 9.49 | 17.11 | 36.43 | 65.44 | 16.34 | 26.95 | 48.34 | 75.05 |
| | $K^2$ | Moment tests | 10.78 | 19.98 | 38.9 | 67.54 | 16.86 | 28.62 | 50.49 | 79.8 |
| | JB | | 10.89 | 20.65 | 41.58 | 71.85 | 18.13 | 30.98 | 56.4 | 83.34 |
| | $DH$ | | 8.97 | 17.44 | 42.66 | 75.19 | 15.46 | 26.02 | 55.33 | 84.89 |
| | RJB | | 10 | 19.03 | 37.42 | 66.7 | 16.77 | 27.79 | 50.83 | 80.04 |
| | $T_{Lmom}$ | | 9.07 | 17.87 | 40.82 | 71.39 | 15.7 | 27 | 52.05 | 81.15 |
| | $T_{Lmom}^{(1)}$ | | 5.83 | 9.87 | 21.67 | 42.82 | 11.27 | 17.28 | 31.82 | 54.57 |
| | $T_{Lmom}^{(2)}$ | | 4.87 | 7.34 | 13.78 | 27.04 | 10.68 | 13.54 | 22.46 | 37.5 |
| | $T_{Lmom}^{(3)}$ | | 5.15 | 5.84 | 10.69 | 18.51 | 9.97 | 11.64 | 18.28 | 28.25 |
| | $BM_{3-4}$ | | 10.94 | 20.72 | 41.86 | 71.9 | 18.23 | 30.96 | 56.57 | 83.43 |
| | $BM_{3-6}$ | | 10.85 | 20.94 | 41.49 | 67.93 | **18.27** | 29.67 | 54.56 | 80.33 |
| | $T_{MC-LR}$ | | 5.74 | 7.85 | 10.7 | 17.63 | 10.84 | 14.34 | 18.69 | 27.23 |
| | $T_w$ | | 6.77 | 8.57 | 10.04 | 14.03 | 11.91 | 15.03 | 16.22 | 21.23 |
| | $T_{MC-LR} - T_w$ | | 5.54 | 7.12 | 9.5 | 15.15 | 10.86 | 12.82 | 15.83 | 22.55 |
| | $T_{s\&5}$ | | **11.56** | **22.09** | **48.84** | **79.4** | 18.19 | **31.49** | **60.84** | **86.63** |
| | $T_{k\&5}$ | | 7.49 | 13.43 | 25.22 | 40.21 | 13.21 | 19.68 | 32.95 | 48.75 |
| | $Z_{APD}$ | | 8.82 | 18.13 | 41.44 | 73.37 | 15.87 | 28.11 | 53.75 | 82.41 |
| | $Z_{EPD}$ | | 7.45 | 11 | 15.27 | 22.95 | 12.84 | 17.32 | 22.58 | 31.06 |
| | $W$ | R and D tests | 10.14 | 20.2 | **46.18** | 77.35 | 17.21 | 30.08 | **57.56** | **85.03** |
| | $W'$ | | 10.5 | **20.38** | 44.35 | 75.07 | **17.25** | 29.25 | 55.77 | 83.22 |
| | $W_{RG}$ | | 9.23 | 17.4 | 38.64 | 70.7 | 15.81 | 26.35 | 50.12 | 79.8 |
| | $D$ | | 7.97 | 12.95 | 22.2 | 35.24 | 13.76 | 20.45 | 30.47 | 44.77 |
| | $r$ | | **10.54** | 20.3 | 44.15 | 74.6 | 17.12 | 29.17 | 55.06 | 82.56 |
| | $cs$ | | 10.14 | 19.88 | 45.5 | **77.38** | 17.2 | 29.88 | 57.14 | 84.83 |
| | $Q$ | | 10.11 | 15.38 | 24.58 | 34.03 | 16.71 | 23.65 | 34.44 | 44.82 |
| | Q−Q* | | 6.72 | 10.98 | 27.12 | 46.09 | 12.81 | 19.22 | 36.96 | 57.86 |
| | BCMR | | 10.52 | 20.34 | 45.73 | 76.72 | 17.39 | **30.31** | 57.49 | 84.86 |
| | $\beta_3^2$ | | 6.28 | 7.56 | 9.02 | 10.97 | 11.39 | 13.14 | 14.88 | 17.9 |
| | $T_{EP}$ | Other tests | **10.43** | **19.57** | **42.18** | **71.54** | **17.9** | **30.08** | **54.75** | **81.25** |
| | $I_n$ | | 9.13 | 17.76 | 31.01 | 48.86 | 16.84 | 25.79 | 40.34 | 59.14 |
| | $R_{sj}$ | | 8.21 | 12.98 | 18.55 | 27.01 | 14.59 | 20.04 | 27.09 | 37.39 |
| | $T_S^`$ | | 6.83 | 8.74 | 12.34 | 18.91 | 12.24 | 15.5 | 20.27 | 27.96 |

Table B6. Simulated power for asymmetric distributions

| Distribution | Tests | Type of the test | α =0.05 n =10 | α =0.05 n =20 | α =0.05 n =50 | α =0.05 n =100 | α =0.1 n =10 | α =0.1 n =20 | α =0.1 n =50 | α =0.1 n =100 |
|---|---|---|---|---|---|---|---|---|---|---|
| Exponential (5) | K-S | EDF tests | 29.2 | 59.84 | 95.89 | 99.99 | 41.02 | 71.94 | 98.48 | 99.99 |
| | AD* | | 41.07 | 77.47 | 99.71 | **100** | 53.22 | 85.87 | 99.91 | **100** |
| | $Z_c$ | | 45.18 | 83.64 | 99.98 | **100** | 57.91 | 91.44 | **99.99** | **100** |
| | $Z_A$ | | **46.1** | **86.82** | **99.99** | **100** | **58.49** | **92.64** | **99.99** | **100** |
| | $P_S$ | | 40.48 | 76.72 | 99.67 | **100** | 53.05 | 85.36 | 99.89 | **100** |
| | $K^2$ | Moment tests | 31.45 | 59.52 | 95.72 | 99.99 | 42.33 | 71.82 | 99.18 | **100** |
| | JB | | 34.24 | 64.38 | 97.94 | **100** | 48.89 | 81.05 | 99.79 | **100** |
| | $DH$ | | 35.06 | 74.24 | 99.71 | **100** | 46.03 | 82.79 | 99.92 | **100** |
| | RJB | | 30.12 | 58.06 | 93.52 | 99.99 | 41.34 | 70.23 | 98.04 | **100** |
| | $T_{Lmom}$ | | 31.76 | **77.28** | **99.85** | 100 | 45.98 | **86.59** | **99.96** | **100** |
| | $T_{Lmom}^{(1)}$ | | 12.18 | 43.06 | 91.11 | 99.9 | 19.82 | 56.92 | 95.39 | 99.97 |
| | $T_{Lmom}^{(2)}$ | | 6.47 | 23.83 | 70.17 | 97.14 | 12.52 | 35.53 | 80.73 | 98.58 |
| | $T_{Lmom}^{(3)}$ | | 5.54 | 14.48 | 51.59 | 87.76 | 9.94 | 24.18 | 64.06 | 92.79 |
| | $BM_{3-4}$ | | 34.59 | 64.66 | 98.01 | **100** | 48.7 | 80.75 | 99.79 | **100** |
| | $BM_{3-6}$ | | 37.39 | 72.77 | 99.22 | **100** | 52.27 | 84.05 | 99.8 | **100** |
| | $T_{MC-LR}$ | | 19.69 | 38.44 | 70.73 | 95.89 | 30.01 | 50.98 | 79.87 | 97.7 |
| | $T_w$ | | 12.69 | 21.42 | 37.08 | 57.61 | 18.75 | 29.8 | 45.26 | 65.03 |
| | $T_{MC-LR} - T_w$ | | 7.84 | 20.27 | 63.35 | 94.24 | 12.45 | 28.73 | 73.5 | 96.78 |
| | $T_{s\&5}$ | | **37.6** | 75.13 | 99.59 | **100** | **52.75** | 85.98 | 99.89 | **100** |
| | $T_{k\&5}$ | | 21.34 | 41.15 | 71.89 | 91.46 | 26.57 | 46.66 | 77.02 | 93.91 |
| | $Z_{APD}$ | | 34.4 | 73.94 | 99.57 | **100** | 47.23 | 83.35 | 99.88 | **100** |
| | $Z_{EPD}$ | | 18.29 | 31.43 | 56.16 | 80.09 | 25.12 | 39.58 | 63.82 | 84.59 |
| | $W$ | R and D tests | 44.8 | 83.53 | 99.98 | **100** | 57.23 | 90.91 | 99.99 | **100** |
| | $W'$ | | 42.26 | 79.26 | 99.9 | **100** | 54.16 | 87.6 | 99.99 | **100** |
| | $W_{RG}$ | | 44.76 | 84.53 | 99.94 | **100** | 56.68 | 91.36 | **100** | **100** |
| | $D$ | | 27.4 | 52.55 | 88.34 | 99.06 | 36.6 | 61.98 | 92.03 | 99.49 |
| | $r$ | | 41.79 | 78.53 | 99.87 | **100** | 53.58 | 86.93 | 99.98 | **100** |
| | $cs$ | | **44.82** | 83.89 | 99.98 | **100** | **57.28** | 91.14 | 99.99 | **100** |
| | $Q$ | | 39.91 | **87.52** | **99.99** | **100** | 53.61 | **93.97** | **100** | **100** |
| | Q−Q* | | 25.53 | 41.59 | 68.85 | 85.63 | 34.51 | 51.64 | 77.74 | 92.07 |
| | BCMR | | 44.53 | 82.74 | 99.97 | **100** | 56.98 | 90.48 | 99.99 | **100** |
| | $\beta_3^2$ | | 11.08 | 15.97 | 24.72 | 35.74 | 17.48 | 23.09 | 32.44 | 43.94 |
| | $T_{EP}$ | Other tests | **41.79** | **77.56** | **99.54** | **100** | **54.57** | **86.51** | **99.85** | **100** |
| | $I_n$ | | 32.83 | 59.92 | 90.91 | 99.38 | 44.13 | 69.08 | 94.19 | 99.66 |
| | $R_{sj}$ | | 24.23 | 43.14 | 70.85 | 90.39 | 32.56 | 51.52 | 77.88 | 93.52 |
| | $T_S^{`}$ | | 16.69 | 27.28 | 46.26 | 65.7 | 26.11 | 39.18 | 57.18 | 73.82 |

Table B7. Simulated power for asymmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Lognormal (1,1) | K-S | EDF tests | 45.41 | 79.01 | 99.51 | **100** | 56.28 | 86.34 | 99.75 | **100** |
| | AD* | | 57.4 | 90.12 | 99.99 | **100** | 67.58 | 93.95 | 99.99 | **100** |
| | $Z_c$ | | 61.26 | 92.69 | **100** | **100** | 71.2 | 96.25 | **100** | **100** |
| | $Z_A$ | | **61.82** | **94.06** | **100** | **100** | **71.9** | **96.8** | **100** | **100** |
| | $P_S$ | | 57.12 | 89.79 | 99.99 | **100** | 67.4 | 93.68 | 99.99 | **100** |
| | $K^2$ | Moment tests | 47.3 | 79.67 | 99.63 | **100** | 57.18 | 86.65 | 99.97 | **100** |
| | JB | | 50.2 | 83 | 99.85 | **100** | 63.99 | 92.3 | **100** | **100** |
| | $DH$ | | 52.32 | 88.57 | **100** | **100** | 61.55 | 93.02 | **100** | **100** |
| | RJB | | 47.12 | 78.83 | 99.45 | **100** | 57.17 | 86.32 | 99.9 | **100** |
| | $T_{Lmom}$ | | 49.23 | **89.95** | 99.99 | **100** | 61.12 | 94.19 | **100** | **100** |
| | $T_{Lmom}^{(1)}$ | | 18.7 | 62.47 | 98.46 | **100** | 28.15 | 73.66 | 99.32 | **100** |
| | $T_{Lmom}^{(2)}$ | | 8.49 | 37.87 | 89.17 | 99.82 | 15.46 | 49.94 | 93.64 | 99.92 |
| | $T_{Lmom}^{(3)}$ | | 5.47 | 22.36 | 74.24 | 97.71 | 10.45 | 33.02 | 82.82 | 98.92 |
| | $BM_{3-4}$ | | 50.35 | 83.15 | 99.85 | **100** | 63.8 | 92.23 | **100** | **100** |
| | $BM_{3-6}$ | | 53.86 | 87.69 | 99.97 | **100** | 67.21 | 93.55 | **100** | **100** |
| | $T_{MC-LR}$ | | 24.3 | 47.94 | 82.72 | 98.9 | 35.08 | 60.8 | 89.59 | 99.54 |
| | $T_w$ | | 21.52 | 43.01 | 77.07 | 95.23 | 27.97 | 50.25 | 81.43 | 96.53 |
| | $T_{MC-LR} - T_w$ | | 14.53 | 42.82 | 90.48 | 99.69 | 19.77 | 50.46 | 93.53 | 99.82 |
| | $T_{s\&5}$ | | **54.26** | 89.06 | **100** | **100** | **67.25** | **94.63** | **100** | **100** |
| | $T_{k\&5}$ | | 34.66 | 64.1 | 93.52 | 99.67 | 39.25 | 68.3 | 95.33 | 99.85 |
| | $Z_{APD}$ | | 51.17 | 88.26 | 99.99 | **100** | 62.39 | 92.93 | **100** | **100** |
| | $Z_{EPD}$ | | 30.12 | 54.51 | 87.91 | 98.75 | 36.73 | 61.13 | 90.99 | 99.1 |
| | $W$ | R and D tests | 60.83 | 92.84 | 100 | **100** | **70.95** | 96.02 | **100** | **100** |
| | $W'$ | | 58.86 | 91.16 | 100 | **100** | 68.63 | 94.79 | **100** | **100** |
| | $W_{RG}$ | | 59.84 | 92.87 | 100 | **100** | 69.56 | 95.94 | **100** | **100** |
| | $D$ | | 44.64 | 76.19 | 98.53 | **100** | 52.79 | 81.73 | 99.18 | **100** |
| | $r$ | | 58.63 | 90.83 | **100** | **100** | 68.16 | 94.56 | **100** | **100** |
| | $cs$ | | **60.98** | **92.85** | **100** | **100** | 70.84 | **96.15** | **100** | **100** |
| | $Q$ | | 50.11 | 91.79 | **100** | **100** | 62.77 | 95.93 | **100** | **100** |
| | Q–Q* | | 40.63 | 64.04 | 90.86 | 98.86 | 49.47 | 72.61 | 94.22 | 99.48 |
| | BCMR | | 60.9 | 92.42 | **100** | **100** | 70.76 | 95.81 | **100** | **100** |
| | $\beta_3^2$ | | 20.8 | 37.23 | 66.88 | 88.4 | 28.24 | 44.43 | 72.41 | 91.28 |
| | $T_{EP}$ | Other tests | **58.52** | **90.22** | **99.98** | **100** | **69.16** | **94.34** | **100** | **100** |
| | $I_n$ | | 50.34 | 81.23 | 99.05 | **100** | 60.61 | 86.25 | 99.5 | **100** |
| | $R_{sj}$ | | 41.11 | 69.22 | 94.78 | 99.79 | 49.56 | 74.65 | 96.32 | 99.86 |
| | $T_S^{`}$ | | 25.43 | 47.24 | 81.22 | 96.59 | 35.48 | 57.41 | 86.54 | 97.68 |

Table B8. Simulated power for asymmetric distributions

| Distribution | Tests | Type of the test | α =0.05 | | | | α =0.1 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | n =10 | n =20 | n =50 | n =100 | n =10 | n =20 | n =50 | n =100 |
| Gumbel (0,1) | K-S | EDF tests | 11.5 | 20.73 | 42.67 | 72.65 | 19.07 | 31.13 | 55.56 | 82.06 |
| | AD* | | 14.2 | 26.96 | 59.22 | 88.82 | 22.13 | 37.59 | 70.17 | 93.25 |
| | $Z_c$ | | 15.47 | 30.62 | 68.03 | 93.07 | 23.25 | 41.43 | 77.56 | 96.17 |
| | $Z_A$ | | **16.06** | **31.92** | **70.31** | **94.44** | **24.26** | **42.53** | **78.98** | **96.75** |
| | $P_S$ | | 14.07 | 26.59 | 58.13 | 88.14 | 22.13 | 37.32 | 69.34 | 92.57 |
| | $K^2$ | Moment tests | 12.35 | 27.58 | 58.89 | 88.4 | 22.03 | 37.68 | 69.88 | 94.19 |
| | JB | | 13.35 | 28.94 | 62.16 | 90.93 | 23.82 | 41.94 | 76.09 | 96.11 |
| | $DH$ | | 14.54 | 26.81 | 65.53 | 93.45 | 19.89 | 36.37 | 76.19 | 96.57 |
| | RJB | | 11.02 | 26.78 | 56.83 | 87.76 | 21.79 | 37.39 | 69.88 | 94.6 |
| | $T_{Lmom}$ | | 12.32 | 26.86 | 63.07 | 91.54 | 20.38 | 37.26 | 73.4 | 95.04 |
| | $T_{Lmom}^{(1)}$ | | 6.93 | 14.08 | 34.4 | 64.82 | 12.85 | 23.01 | 46.15 | 74.47 |
| | $T_{Lmom}^{(2)}$ | | 5.34 | 9.57 | 20.58 | 41.26 | 11.05 | 16.72 | 31.24 | 53.05 |
| | $T_{Lmom}^{(3)}$ | | 5.29 | 7.32 | 14.51 | 28.61 | 9.42 | 13.59 | 23.68 | 38.98 |
| | $BM_{3-4}$ | | 13.47 | 29.06 | 62.41 | 90.97 | 23.89 | 41.92 | 76.17 | 96.14 |
| | $BM_{3-6}$ | | 14.84 | 30.42 | 62.7 | 89.15 | 24.18 | 40.9 | 74.88 | 94.75 |
| | $T_{MC-LR}$ | | 13.55 | 9.15 | 14.87 | 27.62 | 12.25 | 16.47 | 24.18 | 39.46 |
| | $T_w$ | | 7.52 | 11.4 | 19.78 | 30.77 | 13.15 | 18.1 | 26.96 | 39.4 |
| | $T_{MC-LR} - T_w$ | | 4.28 | 9.24 | 17.21 | 32.5 | 10.86 | 14.83 | 24.95 | 41.81 |
| | $T_{s\&5}$ | | **15.49** | **32.93** | **72.07** | **95.42** | **24.87** | **45.68** | **82.3** | **97.93** |
| | $T_{k\&5}$ | | 10.73 | 19.59 | 38.66 | 58.76 | 14.77 | 24.39 | 44.69 | 64.64 |
| | $Z_{APD}$ | | 12.71 | 26.95 | 63.7 | 92.23 | 20.42 | 38 | 74.16 | 95.59 |
| | $Z_{EPD}$ | | 9.7 | 15.54 | 28.2 | 45.05 | 15.34 | 22.39 | 36.13 | 53.19 |
| | $W$ | R and D tests | 15.07 | **30.8** | **68.74** | 93.91 | 23.34 | 41.47 | **77.61** | **96.5** |
| | $W'$ | | **15.63** | 30.43 | 66.47 | 92.96 | 23.15 | 40.59 | 75.69 | 95.84 |
| | $W_{RG}$ | | 13.81 | 26.88 | 62.36 | 91.42 | 21.31 | 37.48 | 72.56 | 94.89 |
| | $D$ | | 11.4 | 19.87 | 40.4 | 63.48 | 17.85 | 27.38 | 49.08 | 71.31 |
| | $r$ | | 15.57 | 30.29 | 66.18 | 92.62 | 22.95 | 40.3 | 75.03 | 95.68 |
| | $cs$ | | 15.16 | 30.31 | 68.62 | **94.08** | 23.29 | 41.24 | 77.49 | 96.42 |
| | $Q$ | | 8.38 | 16.45 | 41.01 | 69.04 | 14.6 | 27.42 | 54.44 | 79.07 |
| | Q−Q* | | 13.06 | 21.1 | 38.49 | 54.19 | 20.61 | 30.48 | 48.92 | 65.34 |
| | BCMR | | 15.52 | 30.61 | 68.21 | 93.71 | **23.68** | **41.71** | 77.5 | 96.3 |
| | $\beta_3^2$ | | 7.46 | 10.03 | 16.55 | 24.24 | 12.66 | 15.93 | 23.48 | 31.77 |
| | $T_{EP}$ | Other tests | **15.7** | **30.06** | **64.75** | **91.36** | **24.31** | **41.64** | **75.22** | **95.33** |
| | $I_n$ | | 13.24 | 25.44 | 50.54 | 75.05 | 21.51 | 34.93 | 59.01 | 81.72 |
| | $R_{sj}$ | | 11.44 | 18.78 | 33.03 | 51.08 | 18.31 | 26.27 | 42.54 | 61.36 |
| | $T_S^{`}$ | | 8.3 | 11.93 | 23.77 | 38.16 | 14.27 | 19.3 | 32.59 | 48.16 |

# 4 Discussion of the results

Since complete lists of the simulated power values of several normality tests for the different sample sizes and significance levels represent a prohibitive amount of data, only a sample of these results, considered to be representative of the general trend of results, is presented, Tables A1 – A7 present the power results for Simulated power for Symmetric distributions, Tables B1–B8 present the power results for Simulated power for Asymmetric distributions.

## 4.1 Symmetric distribution with $\alpha = 0.05$

With Laplace (3, 1) as alternative distribution, RJB and $R_{SJ}$ tests had a power of 20.83%, 20.71% respectively at n = 10 to support that it is the most powerful test under this condition. The least powerful tests under the same condition are $T_{Lmom}^{(3)}$ and $T_{MC-LR}$ with power of 4.99%, 4.77% respectively. With increasing sample size (n) RJB and $R_{SJ}$ tests stay the most powerful tests, with n = 100 RJB, $Z_{\text{EPD}}$, $T_w$, $I_n$, $R_{sj}$, $T_S^{`}$ had power greater than or equal 90%; the $T_{Lmom}^{(3)}$, $T_{MC-LR}$, $Q$, Q–Q∗, tests had power less than 50% at n = 100 or less and they had the least powerful tests.

In the case of a Uniform (٠, 1) as alternative distribution the$T_{k\&5}$, $W_{RG}$, $T_S^{`}$ tests had a power of 14.03%, 13.45%, 14.45% respectively at n = 10 to support that it is the most powerful test under this condition. With increasing sample size $Z_c$ , $W_{RG}$, $T_S^{`}$ , $\beta_3^2$, tests the most powerful tests. With n = 100 about 23 tests had power greater than or equal 90%; the RJB, $I_n$, $Q$,

$R_{sj}$ tests particularly proved to be very bad tests had power less than 2% at n ≤ 100 they had the least powerful tests.

With Beta (0.25, 0.25) as alternative distribution $Z_c$ , $W$ , $W_{RG}$ , $cs$ , $T_S^{`}$ tests had a power more than 70% at n = 10 to support that it is the most powerful test under this condition. With n = 100 about 32 tests had power greater than or equal 95%; $K^2$, D , $I_n$, $R_{sj}$ tests particularly proved to be very bad tests had power less than 20 % at n ≤ 100 they had the least powerful tests.

With Beta (1.5, 1.5) as alternative distribution $T_{k\&5}$ , $Z_{EPD}$ , $W_{RG}$ $T_S^{`}$ tests had a power about 8% or more than at n = 10 to support that it is the most powerful test under this condition. In particular with increasing sample size the most powerful tests are $Z_{c,}$ $Z_A$ , $K^2$, $Z_{\text{EPD}}$ , $W_{RG}$, $D$, BCMR when n = 100 the power for these tests greater than 80%. The least powerful tests are JB, RJB, $T_{Lmom}^{(2)}$ , $T_{Lmom}^{(3)}$ $BM_{3-4}$ , $T_{MC-LR}$, $T_{s\&5}$ , $I_n$, $R_{sj}$ had power less than 20 % at n ≤ 100.

For t(5) and t(8) as alternative distributions all tests were poor in detecting non-normality; even at n = 100, where t = 5 all tests had power less than 70% except $I_n$ had 71.22% and the least powerful tests with power less than 12% are $T_{Lmom}^{(2)}$ ,$T_{Lmom}^{(3)}T_{MC-LR}$. Where t = 8 all tests were very poor in detecting non-normality, the power of all tests less than 50% at n = 100.

With Cauchy (0, 7) as alternative distribution that is symmetric and long-tailed which approximates the normal distribution with undefined kurtosis value; all tests were very high power even at n = 10 the power of all tests is more than 40 % except $T_{Lmom}^{(1)}$ ,$T_{Lmom}^{(2)}T_{Lmom}^{(3)}$ ,$T_{MC-LR}$ had power 27.92% ,٩,٩٩ , 4.92, 11.51 respectively. With increasing sample size the performance of all tests were very high and achieved 100% except $T_{MC-LR}$ the least powerful test with power 64.18% at n=100.

### 4.2 Symmetric distribution with $\alpha = 0.1$

As expected, the power of all tests increased at 10% level of significance in contrast to these at the 5% level. This is because we have a wider range of critical values for non-rejection of the hypothesis of normality thus leading to a higher level of confidence in the results from the tests.

Little variation was observed in the result at the 10% level. For a Uniform (0, 1), n=100 the power of RJB test 74.18% while at 5% level 1.21%. For Beta(0.25,0.25), n=20 the power of JB test 75.11% while at 5% level 1.98.

For student t(5) as alternative distribution the K-S test was the most powerful at n=20 while at the same sample size and 1% level the most powerful test was RJB test.

### 4.3 A Symmetric distribution with $\alpha = 0.05$

In the situation where the alternative distribution is Beta (3, 1), the most powerful tests were $Z_c$ , $Z_A$, $W$, $W_{RG}$, $cs$, Q−Q∗, BCMR all of them achieved power more than 20% at n = 10.while at n= 20 and n=50 the most powerful test was Q−Q∗. In particular with increasing sample size at n =100 most of the tests achieved power more than 95%, except for $T_w$, $Z_{\text{EPD}}$ , $T_{k\&5}$, $Q$, $R_{sj}$, $T_S^{`}$ the least powerful test and the power of these tests were less than 20%.

With $\chi^2$(df-5) distribution as alternative distribution, the most powerful tests in all sample size were $Z_c$ , $Z_A$ , $T_{s\&5}$, $W$, $W'$ , $W_{RG}$ .the least powerful tests were at n= 100 $c$ (25.78) , $c$ (40.86) , $T_{Lmom}^{(3)}$ (43.42) , $T_S^{`}$ (32.07).

With $\chi^2$(df-15) distribution as alternative distribution, the performance of all tests is less than when $\chi^2$(df-5) distribution as alternative distribution, the most and the least powerful tests in all sample size were the same when $\chi^2$(df-5) but with power less.

Regarding to Gamma (3, 4) as alternative distribution, the performance of all tests most of the tests achieved power more than 80% at n= 100, except for $T_w$, $T_{Lmom}^{(3)}$, $\beta_3^2$ the least powerful tests and the power of these tests were about 20% at n =100.

A weibull (15, 3) distribution also showed RJB as the most powerful for sample sizes of $T_{s\&5}$ in all sample size , while the least powerful tests were $T_w$, $T_{MC-LR}$, $T_{MC-LR} - T_w$, $\beta_3^2$ with power about 15% at n =100.

Regarding to Exponential (5) as alternative distribution, the performance of all tests most of the tests achieved power more than 90% at n= 100, except for $\beta_3^2$ the least powerful tests and the power of these test were less than 40% at n =100

In the situation were the alternative distribution is log-normal (1, 1), proved to be one that was easily identified as being non normal by most of the tests. All tests achieved adequate power even small sample size. The power of all tests an n= 100 more than 95% expect $\beta_3^2$ the power of this test under the same condition 91.28% which is the least powerful test.

A Gumbel (0, 1) distribution also showed $Z_A$ and $T_{s\&5}$ as the most powerful tests for all sample sizes, while the least powerful tests were $T_w$, $T_{MC-LR}$, $T_{MC-LR} - T_w$, $\beta_3^2$ with power about 25% at n =100.

### 4.4 A Symmetric distribution with $\alpha = 0.1$

As expected, the power of all tests increased at 10% level of significance in contrast to these at the 5% level. This is because we have a wider range of critical values for non-rejection of the hypothesis of normality thus leading to a higher level of confidence in the results from the tests.

5 Conclusion

A comprehensive power comparison of existing tests for normality has been performed in the presented study.Given the importance of this subject and the wide spread development of normality tests,comprehensive descriptions and power comparison s of such tests are of considerable interest.

Since recent comparison studies do not include several interesting and more recently developed tests, a further comparison of normality tests, which presented herein, is considered to be of foremost interest.

. This study addresses the performance of 36 normality tests, for various sample sizes n, considering several significance levels $\boldsymbol{\alpha}$ and for a number of symmetric, asymmetric distributions.

General recommendations stemming from the analysis of the power of the selected tests indicate the best choices for normality testing are $\boldsymbol{Z_{c,}}$ $\boldsymbol{Z_A}$, $\boldsymbol{Z_{\mathrm{EPD}}}$, $\boldsymbol{W_{RG}}$,and $\boldsymbol{T_S^{`}}$ for Symmetric short - tailed distributions, $\boldsymbol{Z_{c,}}$ $\boldsymbol{Z_A}$ , RJB, *W*', r and $\boldsymbol{I_n}$ Symmetric long - tailed distributions, $\boldsymbol{Z_{c,}}$ $\boldsymbol{Z_A}$,CS, and $\boldsymbol{I_n}$ for Asymmetric short – tailed distributions, and $\boldsymbol{Z_{c,}}$ $\boldsymbol{Z_A}$,$\boldsymbol{w}$, $\boldsymbol{Q}$ and BCMR Asymmetric long – tailed distributions **.**

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