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VIBRATION ATTENUATION IN A PERIODIC ROTATING TIMOSHENKO BEAM

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ABSTRACT

In this study we investigate the effect of abrupt geometric discontinuities on the vibration of a Timoshenko rotating beam. The beam model is created using finite element code developed on MATLAB. Stop and pass bands are identified using periodic analysis for single cell. The results are verified using published data. Numerical results indicate the effectiveness of such structure configuration on vibration attenuation.

KEY WORD:
Passive Damping, Periodic Structures, Rotating Structures and Thick Beams

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1. INTRODUCTION

The term “Periodic Structure” is used to describe structures that consist of a set of identical parts, cells, connected together. Periodic Structures have drawn the attention of researchers since the mid-sixties [1]-[9] because of their high ability to attenuate vibrations. Meanwhile, special attention is given to rotating beams [10]-[23] as rotating beams have wide range of engineering applications. Special attention is given to short beams where rotary inertia is taken into consideration [12], [14], [16], [20], [21] and [23] . In this paper, we will present an attempt to demonstrate the ability of geometrical periodicity of a rotating Timoshenko beam to attenuate vibrations.

2. Periodic Rotating Timoshenko Beam

2.1 Displacement field

Figure 1 shows the configuration of a periodic rotating Timoshenko cantilever beam. The beam of length (*L*) is connected to a rigid hub of radius (*a*) and rotates about the hub axis with angular velocity (*Ω*). The beam itself consists of geometrically identical cells. Each cell consists of two elements as shown in Figure 2. Each element is three nodded elements with four degrees of freedom per node.
The total deflection of the element as shown in Figure 3 at location (*x*) in z-direction can be expressed by:

$$w(x) = w_b(x) + w_s(x) \tag{1}$$

Where the subscripts b and s denotes the bending and shear deformations in *xz* plane respectively. Both deformations are assumed to be fifth order polynomials. They are similar in nature but different in nodal displacements. The elements degrees of freedom can be expressed by:

$$w_i(x) = \lfloor N_1(x) \quad N_2(x) \quad N_3(x) \quad N_4(x) \quad N_5(x) \rfloor \times \lfloor w_{i1} \quad w'_{i1} \quad w_{i2} \quad w'_{i2} \quad w_{i3} \quad w'_{i3} \rfloor^T$$

Or: (2)

$$w_i(x) = \lfloor N(x) \rfloor \{w_i^e\}$$

Where *N(x)* is the shape function and w_i denotes either w_b or w_s.

2.2 Total system energy

2.2.1 Strain energy

The system strain energy due to bending deformation and rotary inertia can be expressed as follows:

$$U = \frac{1}{2}\int_0^l \left\{ EI_{yy}\left(\frac{\partial^2 w_b}{\partial x^2}\right)^2 + GAK_s\left(\frac{\partial^2 w_s}{\partial x^2}\right)^2 \right\} dx + \frac{1}{2}\int_0^l f_c(x)\left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)^2 dx \tag{3}$$

Where *E* and *G* are Young's modulus elasticity and modulus of rigidity, K_S is the shear factor, I_{yy} and *A* are the moment of inertia of cross-section and its area. $f_c(x)$ is the centrifugal force due to rotation. The centrifugal force will be discussed in details later in this section.

2.2.2 Kinetic energy

The element kinetic energy (*T*) is given by:

$$T = \frac{1}{2}\int_0^l \rho I_{yy}\left(\frac{\partial^2 w_b}{\partial x \partial t}\right)^2 dx + \frac{1}{2}\int_0^l \rho A\left(\frac{\partial w_b}{\partial t} + \frac{\partial w_s}{\partial t}\right)^2 dx \tag{4}$$

Where ρ is the mass density.

2.2.3 External Work

The external work due to externally applied force (*F*) is given by:

$$W = \int_0^l wF dx \tag{5}$$

2.3 Hamilton's Principle

According to Hamilton's principle, the first variation of the system total energy equals to zero.

$$\delta\Pi = \int_{t_1}^{t_2} (U - T - W)dt = 0 \tag{6}$$

2.3.1 Stiffness matrix

The stiffness matrix can be derived by taking the first variation for the first integral of strain energy given by equation (3):

$$[K] = \begin{bmatrix} [KB] & [0] \\ [0] & [KS] \end{bmatrix}, \quad [KB] = \int_0^l EI_{yy}\{N_{xx}\}\lfloor N_{xx} \rfloor dx, \quad [KS] = \int_0^l GAK_s\{N_{xx}\}\lfloor N_{xx} \rfloor dx \tag{7}$$

where the subscript *x* denotes differentiation once with respect to *x*.

2.3.2 Rotation-induced stiffness matrix

The rotation-induced stiffness is the added stiffness due to centrifugal force. It can be derived by taking the first variation of the second integral of strain energy in equation (3):

$$[s] = \left[\int_0^l f_c(x)\{N_x\}\lfloor N_x \rfloor dx\right] \tag{8}$$

2.3.3 Mass matrix

The Mass matrix can be evaluated by taking the first variation of the kinetic energy, given by equation (4), and integrating by parts once:

$$[M]=\begin{bmatrix}[MS]+[MB] & [MB]\\ [MB] & [MB]\end{bmatrix},\quad [MS]=\int_0^l \rho I_{yy}\{N_x\}\lfloor N_x\rfloor dx,\ [MB]=\int_0^l \rho A\{N\}\lfloor N\rfloor dx \tag{9}$$

2.3.4 Force vector

The vector of externally applied force can be expressed by:

$$\{F\}=\int_0^l F\{N\}dx \tag{10}$$

2.3.5 Element matrix equation

Finally the element matrix equations can be expressed by:

$$\begin{bmatrix}[MS]+[MB] & [MB]\\ [MB] & [MB]\end{bmatrix}\begin{Bmatrix}\ddot{w}_b\\ \ddot{w}_s\end{Bmatrix}+\left(\begin{bmatrix}[KB] & [0]\\ [0] & [KS]\end{bmatrix}+\Omega^2\begin{bmatrix}[s] & [s]\\ [s] & [s]\end{bmatrix}\right)\begin{Bmatrix}w_b\\ w_s\end{Bmatrix}=\{F\} \tag{11}$$

2.4 Centrifugal Force

The centrifugal force induced by rotation at station (x) within the i^{th} element, measured from its left end can be expressed as follows:

$$f_c(x)=\int_{a+x_i+x}^{a+L}\rho A\zeta d\zeta \tag{12}$$

Where (a) is the hub radius, (x_i) is the distance from the beam root to the left end of the element. (ζ) is a local coordinate parallel to x-coordinate and is measured from the general station (x) as shown in Figure 1.

Since the beam is not uniform, the integration in equation (12) should be rewritten as:

$$f_c(x)=f_0(x)+f_1 \tag{13}$$

Where:

$$f_0(x)=-\rho A_i\left((a+x_i)x+0.5x^2\right),\quad f_1=\rho A_i\left((a+x_i)L_i+\frac{L_i^2}{2}\right)+\sum_{j=i+1}^{n}\rho A_j\left((a+x_j)L_j+\frac{L_j^2}{2}\right) \tag{14}$$

Where ρA_j , x_j and L_j is the mass per unit length of the j^{th} element, distance from root to its left end and its length respectively.

2.5 Periodic Analysis

When a wave faces abrupt change in geometry and/or material properties, part of it reflected. This reflection is destructive in some frequency bands called stop bands. In order to locate these stop bands, transfer matrix analysis is used to formulate an input/output relation between forces and displacements at left (node 1) and right (node 5) ends of the cell. See Figure 2.

$$\lfloor K_{dynamic} \rfloor \{w\}_{cell} = \{F\}_{cell}\,,\ \ [K_{dynamic}] = \left(K + \Omega^2 S - \omega^2 M\right) \tag{15}$$

By condensing the internal nodes, the above relation can be rewritten as:

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} w_1 \\ w_5 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_5 \end{Bmatrix} \tag{16}$$

where w_1, F_1, w_5 and F_5 are the displacements and forces at nodes *1* and *5* respectively. Rewriting the above equation in form of input/output relation:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} w_1 \\ F_1 \end{Bmatrix} = \begin{Bmatrix} w_5 \\ F_5 \end{Bmatrix} \tag{17}$$

Assume:

$$\begin{Bmatrix} w_5 \\ F_5 \end{Bmatrix} = e^{\mu} \begin{Bmatrix} w_1 \\ -F_1 \end{Bmatrix} \tag{18}$$

Substituting equation (18) into (17):

$$[T] \begin{Bmatrix} w_1 \\ F_1 \end{Bmatrix} = e^{\mu} \begin{Bmatrix} w_1 \\ F_1 \end{Bmatrix},\ [T] = \begin{bmatrix} T_{11} & T_{12} \\ -T_{21} & -T_{22} \end{bmatrix} \tag{19}$$

The above eigenvalue problem can be solved for propagation factor μ, which is, generally, a complex number its real part represents the boundaries of the pass/stop bands, and the imaginary part gives the attenuation value. Since the transfer matrix *[T]* varies from one cell to another, the location of stop bands will vary to. Thus average will be taken for all cells.

3. NUMERICAL RESULTS

The finite element model described above has been developed on MATLAB 7.0. Herein after, some numerical results are listed for comparison purposes. However, no test data are published for rotating periodic Timoshenko beam. Thus, comparison will be accomplished on two steps. The first, comparing the natural frequencies

3.1 Uniform Rotating Timoshenko Beam

Stafford and Giurgiutiu [10] derived semi-analytical methods to evaluate natural frequencies of rotating Timoshenko beam. Table 1 contains comparison between the current finite element model and reference [10]. These non-dimensional natural frequencies are calculated for beam with characteristics stated in Table 2. From Table 1, the finite element model showed good agreement.

Table 1: Non-dimensional natural frequencies of uniform rotating Timoshenko beam

Mode	Exact Ref. [10]	Current Model
1	6.8509	6.9301
2	19.6787	19.839
3	38.5758	38.679
4	56.295	55.229

Table 2: Characteristics of uniform rotating Timoshenko beam

L	*EI*	ρI	*E/G*	ν Passion Ratio	*L/R* (length-to-radius of gyration)	$\sqrt{\alpha}=\sqrt{\frac{\rho AL^2}{EI}}\Omega$ Non-dim rotational speed
1	1	1	2.6	0.3	10	6

3.2 Periodic Rotating Timoshenko Beam

For the sake of comparison, reference [8] is utilized where periodic rotating Euler Bernoulli beam had been analysed both numerically and experimentally. The parameters of the selected beam can be found in Table 3. Figure 4 shows the tip response of the selected beam in comparison with a similar plain beam of thickness 1 mm subjected to rotation speed or 5 revolutions per second (300 *rpm*). It is clear from the figure that tip response is attenuated when average attenuation factor is nonzero. The current model showed good agreement with data published in reference [8].

Table 3: Characteristics of periodic rotating beam

Material (Aluminum)			Beam dim. (cm)				Thin part		Thick Part	
E GPa	ν	ρ Kg/m^3	Hub radius	Length	width	Cell Length	Length cm	Thickness mm	Length cm	Thickness mm
71	0.3	2700	5	45	3.6	11.25	6.25	1	5	3

Figure 5 shows the same beam when it is subjected to rotation speed of 3000rpm. From the figure it is noticed the attenuation factor is getting wider with less attenuation value.

4. CONCLUSIONS

In this study, a finite element model for a periodic rotating Timoshenko beam has been presented. From the results shown, the model proved that geometrical periodicity has high ability to attenuate vibrations in some frequency bands showing good agreement with published data. Also, increasing the rotation speed broaden the stop bands with less attenuation value. Proper design can achieve high attenuation value for some target frequency bands. Also, the selected 3-node 4 DOF/node element showed good results.

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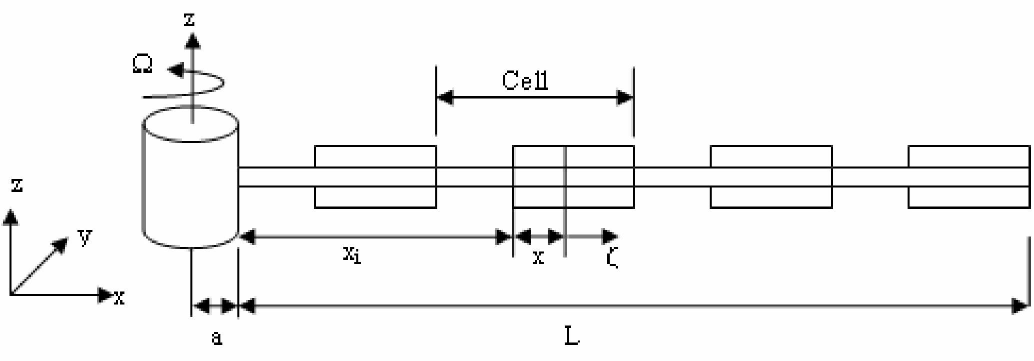

Figure 1: configuration of periodic rotating Timoshenko beam

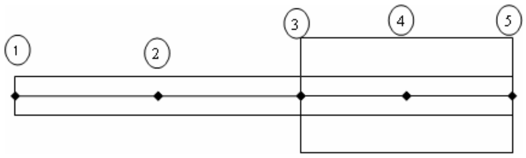

Figure 2: Typical cell configuration

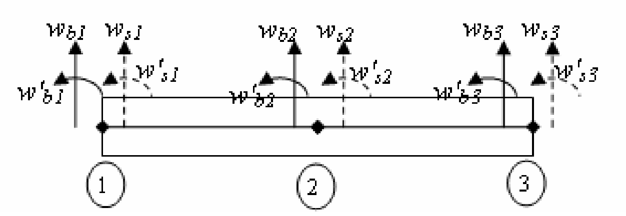

Figure 3: Element degrees of freedom

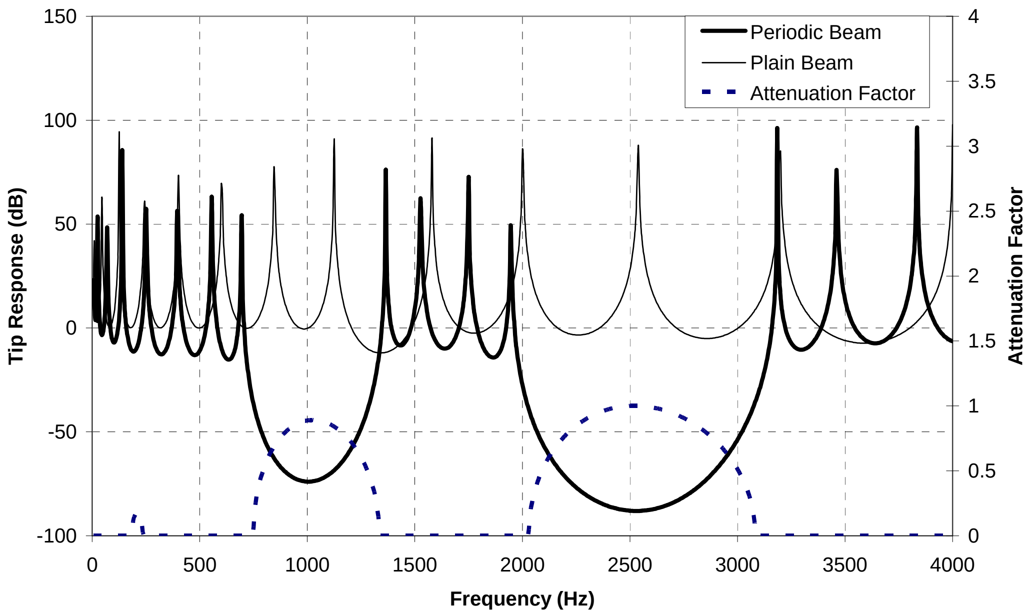

Figure 4: Frequency response of periodic rotating Timoshenko beam (5 rev/sec)

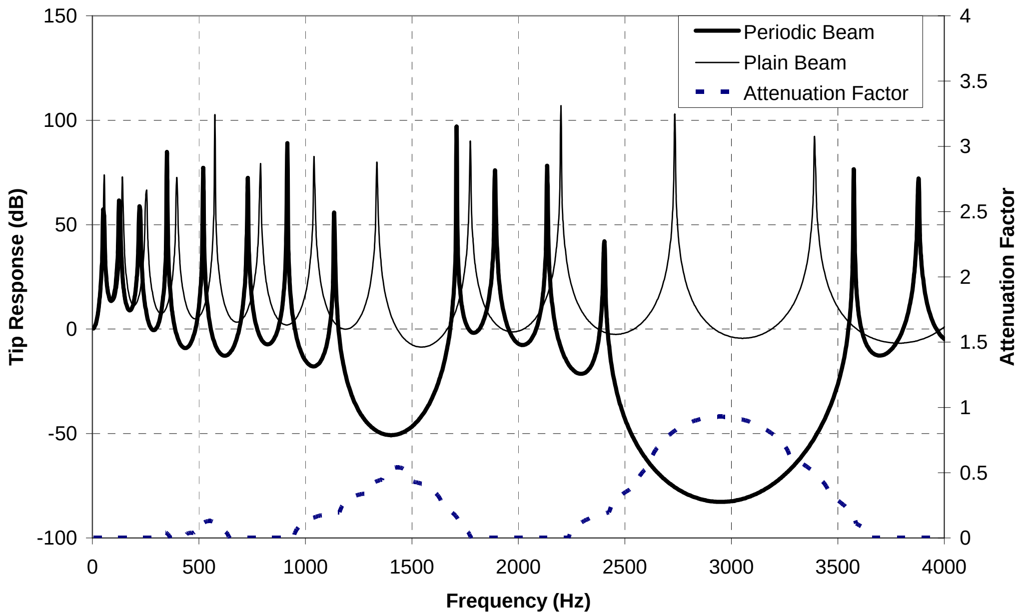

Figure 5: Frequency response of periodic rotating Timoshenko beam (50 rev/sec)