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## A Preliminary Design for Multistage Axial Flow Compressors

### Part I: Computer Package

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#### ABSTRACT

A computer package is developed to furnish a preliminary design for the multistage axial flow compressors normally found in aero-engines and industrial gas turbines. The inputs to this code is the requested pressure ratio, mass flow rate, rotational speed, isentropic efficiency as well as the inlet total pressure and temperature. The outputs of this package are the number of stages, annulus dimensions at inlet and outlet, and two dimensional mean flow characteristics. Such 2-d results are next used to obtain the 3-d variation from hub-to-tip sections of the blades using different methods. Such analyses provide the velocity triangles over different sections of the blades at the inlet and outlet to successive stages. Consequently, a layout of the whole compressor as well as the degree of blade twists over its height for both stator and rotor blade rows are calculated. To assure the structural integrity of the compressor blades, mechanical design is also employed for calculating both centrifugal tensile and gas bending stresses. Moreover, the initially assumed efficiency is recalculated using available cascade data.

#### KEY WORDS

Multistage Axial Compressor, Two Dimensional, Three Dimensional, Blade Stresses, Efficiency, Computer Package

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**NOMENCLATURE**

$A$	annulus area	$\alpha$	stator angle
$C_a$	axial velocity	$\beta$	rotor angle
$C_{DA}$	annulus drag coefficient	$\psi$	stream potential function
$C_{DP}$	profile drag coefficient	$\phi$	velocity potential function
$C_{DS}$	secondary loss coefficient	$\eta$	efficiency
$C_D$	total drag coefficient	$\lambda$	work- done factor
$C_u$	whirl velocity	$\Lambda$	degree of reaction
$M$	Mach number	$\sigma_{ct}$	centrifugal tensile Stress
$m$	mass flow rate	$\zeta$	hub-tip ratio
$N$	rotational speed ,number of stages	<b>Subscript</b>	
$R$	gas constant	1,2	inlet, outlet
$r$	radius	t	tip
$T_o$	total temperature	m	mean
		r	root

**1 INTRODUCTION**

Multi-stage axial compressors are sited in various applications like:  
 1- Aerospace vehicles : where the axial compressors resemble a basic module of the air breathing engines in aircrafts ,helicopters, unmanned aerospace vehicles and V/STOL aircrafts; kerrebrock[1], Oates[2], Archer[3], and Roskam[4]  
 2- Marine vehicles: including the power plants for submarines, hydrofoil boats, navel surface ships, hovercrafts....etc; Lakshminarayana [5] and Turton [6]  
 3- Industrial Application: including gas turbines and petroleum transmission ;Hill and Peterson [7] and Wilson and Korakianitis [8]. The design process is an iterative procedure among many engineering disciplines like aerodynamics, thermodynamics, heat transfer, material science, structural and stress analyses. In both aeroengines, gas turbines in marine and industrial applications, the overall compression ratio may exceed 30 and the number of Stages may also exceed 15; Walsh and Fletcher [9]. Moreover a layout for a typical turbofan engine is shown in Fig.1.

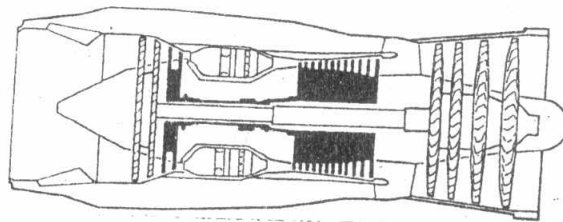


Fig.1 Turbofan Engine

As stated by Wisler [10] from an aerodynamic perspective, the designer of the compressor plays a major role in achieving the design objectives of such aero engines that ensure excellent efficiencies and stall margins. Aero engine manufactures stated that for each 1% improvement in the high-pressure compressor efficiency, a 0.5% improvement in the specific fuel consumption (SFC) is obtained. The main challenges in compressor design; Cumpsty [11], Balje [12] and Wilson, et al. [8] are to:

- Improve compressor performance and ruggedness simultaneously
- Maintain or reduce weight and cost
- Select compressor that optimizes entire engine system. The function of the compressor is to achieve certain overall pressure ratio for a certain mass flow rate at certain rotational speed;

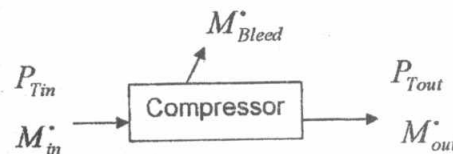


Fig.2 Input and output for a compressor

Compressors are designed by the indirect method that is, first one defines the flow path (casing and hub) radii and the number of stages. Thus the thermodynamic, aerodynamic properties and work input at each axial location (i.e., stage) along the flow path are defined which provide the air angles (velocity triangles) at the mean section of blades; Cumpsty [11]. An iterative analysis is conducted in which the continuity, momentum, energy and state equations are satisfied at each station. Although the blades and their shape are not yet defined, their presence is recognized by work input, turning ability and loss. Next, three-dimensional analysis is used to design the blades and vanes to achieve the desired thermodynamic and aerodynamic properties. Radial equilibrium equation is employed for such 3-D variation of air angles in the radial direction from hub-to-tip. Several methods are employed; namely, free vortex, exponential, first power and constant reaction; Horlock [13] and Johnsen and Bullock [14]. Most researchers devoted their analyses whether simplified or sophisticated models to the design of a single blade row, a single stage or one and half stage; Hanlon [15], Lakshminarayana [5] as well as the Von Karman Institute lecture series [16]. If the radial velocities are negligible and stream surfaces are cylindrical, the development of a cylindrical section would result in a rectilinear cascade of infinite number of blades. Then, for incompressible and irrotational flow, the cascade flow is expressed by the following equations

$$\nabla^2 \psi \text{ And } \nabla^2 \phi = 0$$

where  $\psi$  and  $\phi$  are the stream and velocity potential functions these equations may be solved via conformal mapping transformation; Horlock [13] and Johnsen and Bullock [14]. Alternatively, methods of surface singularity, frequently known as the Panel Method are employed; Minassian [17] and McFarland [18]. In the present day turbomachinery, the three dimensional effects are hardly negligible and their incorporation into design and analysis is essential for accurate prediction of the performance of these machinery. In brief, these 3-D treatments can be classified as

axisymmetric or nonaxisymmetric solution Lakshminarayana [5].

Axisymmetric solution:

- Simplified radial equilibrium analysis (SRE)
- Actuator disk theories (ADT)
- Passage averaged equations (PAE)

Nonaxisymmetric solutions:

- Lifting line and lifting surface approach
- Quasi-three-dimensional methods
- numerical solutions of exact equations (Potential, Euler and Navier-stokes)

In the presents work the simplified radial equilibrium analysis (SRE) is employed for each stage. The procedure followed here is symbolically illustrated by the algorithm shown in Fig.3 together with the input and some of the outputs. This work presents a computer package for both aerodynamic and mechanical design of the multi-stage axial compressor. In fact the computer package developed here can provide complete details for the preliminary design of multistage axial compressor. Based on the requested pressure ratio, flowing mass flow rate, and rotational speed, a complete design for the compressor is attained. Drawings for the compressor layout as well as the stator and rotor geometry for each stage are available. The stresses as well as the associated factor of safety for blades of each blade row are calculated upon blade material selection. Check for the stage efficiency previously assumed is also performed. In part (II) of this work, an application for this package is tested in a preliminary design of the high pressure compressor in the famous turbofan engine CF6. Few publications treating the design of a multistage turbomachines are available. As an example, the work by Lewis [19-21] present a computer program for the design of a multi-stage axial turbines. In the succeeding sections the design steps are thoroughly described and applied to high pressure compressor (HPC) of the aircraft engine CF6.

## DESIGN PROCEDURE

The preliminary design steps for an axial compressor suitable for turbofan and turbojet engines will be outlined here; Saravananamuttoo [23]. It will be assumed that the compressor has no inlet guide vanes so the flow enters axially the first stage of compressor. The complete design process for the compressor will encompass the following steps:

- (1) Calculation of the annulus dimensions at inlet and outlet.
- (2) Determination of number of stages, using an assumed efficiency.
- (3) Calculation of the air angles for each stage at the mean radius.
- (4) Determination of the variation of the air angles from root to tip using different methods; Free vortex, First power ... etc.
- (5) Investigation of compressibility effect.
- (6) Selection of compressor blading, using experimentally obtained cascade data.
- (7) Check on efficiency previously assumed, using the cascade data.
- (8) Check on the factor of safety for the prevailing state of stress.

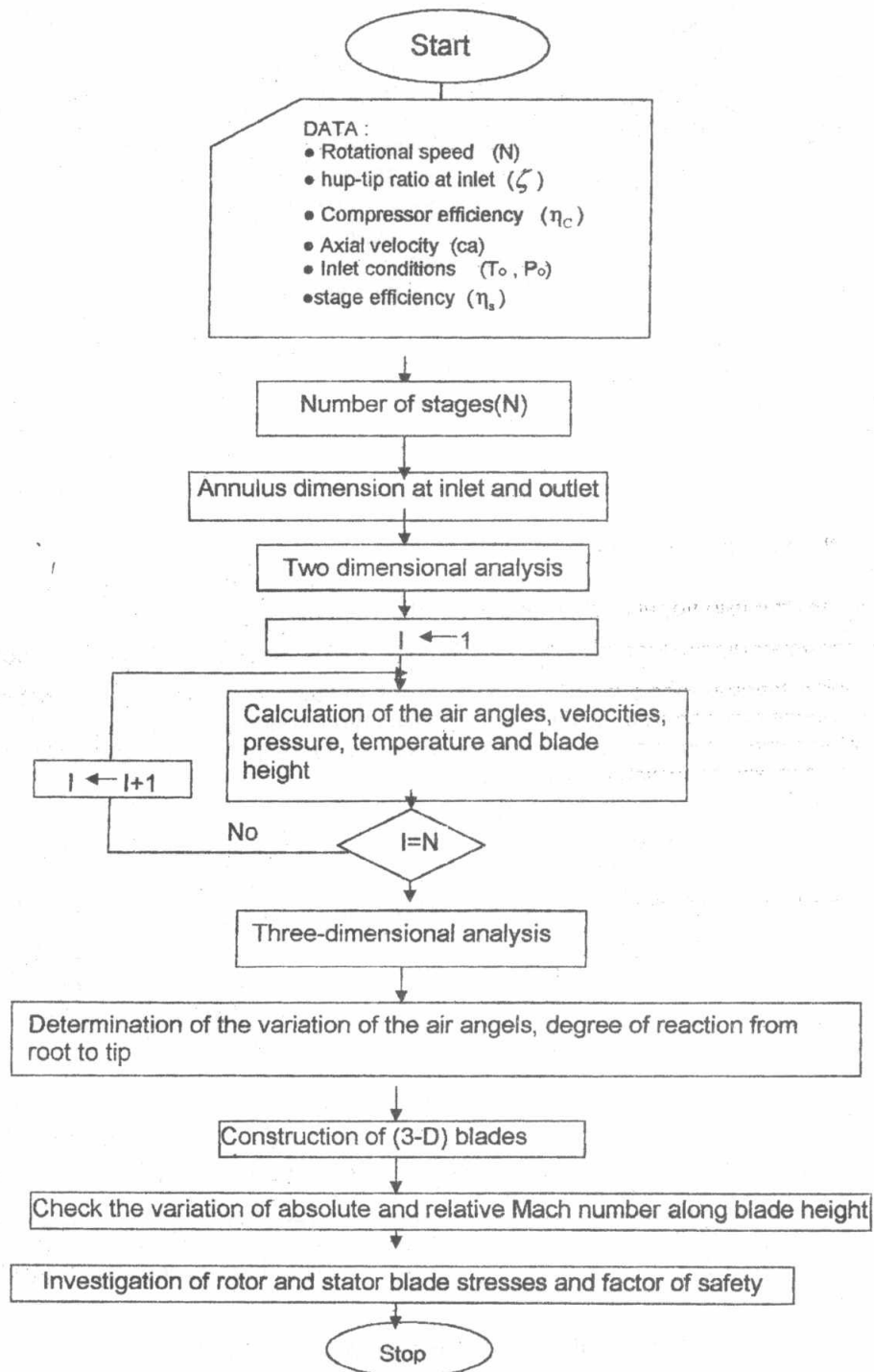


Fig.3 Design procedure

1 Calculation of annulus dimensions.

Previous experience suggests the following recommended values for some design parameters.

- A tip speed  $U_t$  of around 350 m/s will lead to acceptable stresses
- The axial velocity  $C_a$  could range from 150 to 200 m/s.
- The hub-tip ratio at entry may vary between 0.4 and 0.6,

From known values of axial velocity, mass flow rate and inlet conditions we can determine the annulus dimensions as follows:-

From the continuity equation, the tip radius at inlet section is given by:

$$r_t^2 = \left( \frac{m \cdot R T_{o1}}{c_a p_{o1}} \right) \frac{1}{(1-\zeta^2) \left( 1 - \frac{c_a^2}{2c_p T_{o1}} \right)^{\frac{1}{\gamma-1}}} \tag{1}$$

with the rotational speed expressed as: 
$$N = \frac{U_t}{2\pi r_t} \tag{2}$$

then, the hub-tip ratio ( $\zeta$ ) is systematically varied from 0.4 to 0.6 and the corresponding values of both  $r_t$  and  $N$  are calculated. Comparing the dimensions and rotational speed of both the compressor and turbine we arrive at the best values for speed and tip radius that satisfy such matching between compressor and turbine at this stage it is appropriate to check the Mach relative to the rotor tip at inlet to the compressor which is expressed as

$$M_{r_{max}} = \frac{W_t}{a} = \frac{\sqrt{U_t^2 + c_a^2}}{\gamma R T_1} \tag{3}$$

refer to figure (4) illustrating the velocity triangle at inlet

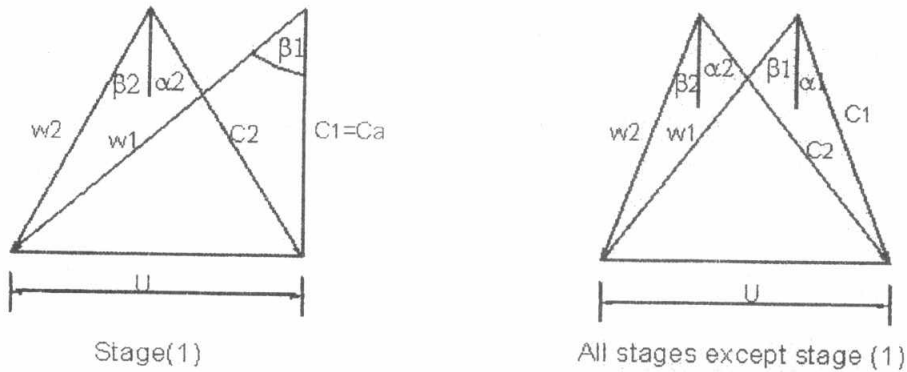


Fig.4 Typical velocity triangle at mean section

To estimate the annulus dimensions at exit from the compressor assuming the mean radius and axial velocity are constant for all the compressor stages and the polytropic efficiency of all stages  $\eta_s$  is 0.9, then from the overall pressure ratio of the compressor the outlet annulus area denoted by subscript(2) will be given by

$$A_2 = \frac{m \cdot R T_{o_2}}{c_a P_{o_2}} \frac{1}{\left(1 - \frac{c_a^2}{2c_p T_{o_2}}\right)^{\frac{1}{1-\gamma}}} \quad (4)$$

where  $T_{o_2} = T_{o_1} \pi^{\frac{\gamma}{\eta_s}}$  (5)

Thus

$$h_2 = \frac{A_2}{2\pi r_m}, r_{r2} = r_m + \frac{h}{2}, r_{r_2} = r_m - \frac{h}{2}$$

• Estimation of number of stages

Assuming polytropic efficiency of 0.9, the stagnation temperature rise through the compressor is given by the relation:  $\Delta T_{o, \text{comp}} = T_{o_2} - T_{o_1}$

refer also to figure(4) The stage temperature rise  $\Delta T_{o_s}$  can vary widely

between 10 to 30 K for subsonic stages and may be 45 K, or higher in high-performance transonic stages. Throughout the compressor the temperature rise equation is given by

$$\Delta T_{o_s} = \lambda U_m C_a (\tan \beta_1 - \tan \beta_2) / 2C_p \quad (6)$$

For the first stage  $W_1 = C_a / \cos \beta_1, \tan \beta_1 = (U/C_a)$  (7)

Applying de Haller equation for diffusion criterion  $W_2/W_1 \geq 0.72$ , then (8)

$$W_2 = 0.72 W_1 \quad \text{and} \quad \cos \beta_2 = C_a / W_2 \quad (9)$$

Using this deflection and neglecting the work-done factor  $\lambda$  in equation a crude estimate for the number of stage will be given by: (10)

$$N = (T_{o_2} - T_{o_1}) / \Delta T_{o_s}$$

2 Stage-by-stage design:

2.1 Mean section (2-D)

In this step, we evaluate the air angles for each stage at the mean radius i.e.the mean flow. Taking the following points into account

1- The work done factor  $\lambda$  will have the values of 0.98 for the first stage, 0.93 for the second stage, 0.88 for the third stage and 0.83 for namely the remaining stages.

2- The diffusion can readily be checked using the de-Haller number;  $W_2/W_1$  which must not be less than 0.72.

3- The degree of reaction is calculated for the first stage, while it is assumed for all the remaining stages to have values less than that calculated in the first stage and not less than 0.5.

4- The outlet conditions of any stage is assumed to be equal to the inlet condition of the succeeding stage, i.e.

$$(P_{o_j} = P_{i_{j+1}}) \text{ and } (T_{o_j} = T_{i_{j+1}}), \alpha_{3j} = \alpha_{i_{j+1}}$$

5- The axial velocity at the mean section ( $C_{am}$ ) is assumed constant and denoted ( $c_a$ ). Also, since the mean radius is constant, then the rotational speed is also constant and denoted by ( $U$ ).

### 2.1.1 Stage 1:

Given the values of:

$(P_{o1})_{st1}$ ,  $(T_{o1})_{st1}$ ,  $(\Delta T_o)_{st1}$ ,  $(U_m)_{st1}$ ,  $(C_{a1m})_{st1}$ ,  $C_p$ ,  $\eta_{st1}$  and  $\lambda$ , then

$$(\Delta C_{um})_{st1} = \frac{C_p (\Delta T_o)_{st1}}{\lambda U_m}$$

Since there are no inlet guide vanes then  $(\alpha_{1m})_{st1} = \text{zero}$  and  $(C_{u1m})_{st1} = \text{zero}$

$$(C_{u2m})_{st1} = (\Delta C_{um})_{st1} \quad (11)$$

$$(\alpha_{2m})_{st1} = \tan^{-1} \frac{(C_{u2m})_{st1}}{C_a} \quad (12)$$

$$(\beta_{1m})_{st1} = \tan^{-1} \frac{(U_m)_{st1}}{C_a} \quad (13)$$

$$(\beta_2)_{st1} = \tan^{-1} \frac{(U_m)_{st1} - (C_{u2m})_{st1}}{C_a} \quad (14)$$

**De-Haller number**

$$\frac{W_{2m, st1}}{W_{1m, st1}} = \frac{\cos(\beta_{1m})_{st1}}{\cos(\beta_{2m})_{st1}} \quad (15)$$

$$(W_{1m})_{st1} = \frac{c_a}{\cos(\beta_{1m})_{st1}} \quad (16)$$

$$(W_{2m})_{st1} = \frac{C_a}{\cos(\beta_{2m})_{st1}} \quad (17)$$

$$(C_{2m})_{st1} = \frac{c_a}{\cos(\alpha_{2m})_{st1}} \quad (18)$$

$$\Lambda = 1 - \frac{(C_{u2m})_{st1} + (C_{u1m})_{st1}}{2U} \quad (19)$$

$$P_{-3} = \left( \frac{\eta_{st} (\Delta T_o)_{st1}}{(T_{o1})_{st1}} + 1 \right)^{\frac{\gamma}{\gamma-1}} (P_{o1})_{st1} \quad (20)$$

$$(T_{o3})_{st1} = (T_{o1})_{st1} + (\Delta T_o)_{st1} \quad (21)$$

from equations (11) through (21) we obtain the Outputs:  $(\alpha_{2m})_{st1}$ ,  $(\beta_{1m})_{st1}$ ,  $(\beta_{2m})_{st1}$ ,  $(C_{u2m})_{st1}$ ,  $(W_{1m})_{st1}$ ,  $(W_{2m})_{st1}$ ,  $(C_{1m})_{st1}$ ,  $(C_{2m})_{st1}$



**2.1.2 STAGE 2:**

Given the values of:

$(P_{o1})_{st2}$ ,  $(T_{o1})_{st2}$ ,  $(\Delta T_o)_{st2}$ ,  $U$ ,  $C_a$ ,  $C_p$ ,  $\eta_{st2}$  and  $\Lambda$ .

Then assume  $\Lambda$  and use the following relation to obtain the angles  $(\beta_{1m})_{st2}$ ,  $(\beta_{2m})_{st2}$ ,  $(\alpha_{1m})_{st2}$  and  $(\alpha_{2m})_{st2}$ , the relative velocities  $(W_{1m})_{st2}$  and  $(W_{2m})_{st2}$  and the absolute velocities  $(C_{1m})_{st2}$  and  $(C_{2m})_{st2}$ ; refer to figure (4)

$$\tan \beta_{1m.st2} = \frac{1}{2c_a} \left[ 2U\Lambda + \frac{c_p \Delta T_{os2}}{\lambda U} \right] \quad (22)$$

$$\tan \beta_{2m.st2} = \frac{1}{2c_a} \left[ 2U\Lambda - \frac{c_p \Delta T_{os2}}{\lambda U} \right] \quad (23)$$

$$\tan \alpha_{1m.st2} = \frac{U}{c_a} - \tan \beta_{1m.st2} \quad (24)$$

$$\tan \alpha_{2m.st2} = \frac{U}{c_a} - \tan \beta_{2m.st2} \quad (25)$$

Using similar relations as in stage 1, we obtain the velocities  $c_{1m}$ ,  $c_{2m}$ ,  $w_{1m}$ ,  $w_{2m}$  as well as the outlet pressure and temperature of stage 2

**2.1.3 Stage (i)**

For any stage other than the first and last one, given the outlet values of the previous stage  $(i-1)$ , and assuming the values of  $\Lambda (\geq 0.5)$ , we apply the following relations to obtain the angles  $\beta_1$  and  $\beta_2$

$$(\tan(\beta_{1m})_i - \tan(\beta_{2m})_i) = \frac{C_p (\Delta T_o)_i}{\lambda_i U c_a} \quad (26)$$

$$(\tan(\beta_{1m})_i + \tan(\beta_{2m})_i) = \frac{2U}{C_a} \Lambda_i \quad (27)$$

next, if  $\Lambda = 0.5$ , then  $\alpha_{1i} = \beta_{2i}$ ,  $\alpha_{2i} = \beta_{1i}$

$$\text{if } \Lambda \neq 0.5, \text{ then use the relation } \tan \alpha_{1i} = \frac{U}{C_a} - \tan \beta_{1i} \quad (28)$$

$$\tan \alpha_{2i} = \frac{U}{C_a} - \tan \beta_{2i} \quad (29)$$

$$\text{Moreover, for any value of } \Lambda, \text{ then } T_{o3i} = T_{o1i} + \Delta T_{os_i} \quad (30)$$

$$P_{o3i} = P_{o1i} \left[ 1 + \frac{\eta_{S_i} \Delta T_{os_i}}{T_{o1i}} \right]^{\frac{\gamma}{\gamma-1}} \quad (31)$$

de-Haller number for both stator and rotor blade rows is checked and must be equal to or greater than 0.72

**(2.1.4) Final stage(n)**

- At the entry to the final stage the total pressure and temperature i.e.  $(P_{o1})_{st,n}$  and  $(T_{o1})_{st,n}$  are given from the previous stage.
- The outlet condition from the compressor i.e. the compressor delivery pressure and temperature  $(P_{o2})_{comp}$  and  $(T_{o2})_{comp}$  are also known from previous calculations. The last stage characteristics will be calculated by the following equations:

$$(T_{o1})_{st,n} = (T_{o3})_{st,n-1}, (P_{o1})_{st,n} = (P_{o3})_{st,n-1}, (P_{o3})_{st,n} = (P_{o2})_{comp}$$

$$\frac{P_{o3}}{P_{o1}} = \frac{r_c}{p_{o1, st, n}} \tag{32}$$

Where  $r_c$  is the compressor overall pressure ratio

Then:  $(\Delta T_o)_{st,n} = ((T_{o1})_{st,n} / \eta_{st}) [((P_{o3})_{st,n} / (P_{o1})_{st,n})^{\gamma-1/\gamma} - 1]$  (33)  
 from  $\Delta T_{o,n}$  and  $\Lambda = 0.5$ , all the angles, velocities and de-Haller number can be calculated

**(2.2) Three-dimensional flow**

The assumption of two-dimensional flow is quite reasonable for stages having hub-tip ratio greater than about 0.8 where radial movement of the fluid is ignored. But in the case of low hub-tip ratio such as (aero-engine compressors) values as low as 0.4 are used for the first stage and higher values in the later stages. Thus, the annulus will have substantial taper in this case and the streamlines will not lie on a surface of revolution parallel to the axis of the rotor. Under these conditions the flow must have a radial component of velocity. With a low hub-tip ratio, the variation in blade speed from root to tip is large and this will have a major effect on the shape of the velocity triangle and the resulting air angles. For high efficiency it is essential that the blade angles match the air angles closely at all radii, and the blade must therefore be twisted from root to tip to suit the changing air angles. The force balance between the inertia and pressure forces on an element in the spacing between any successive blade rows yields the following equation.

$$\frac{1}{r} \frac{dp}{dr} = \frac{C_u^2}{r} \tag{34}$$

From the energy equation, the enthalpy variation employing equation (32) is expressed by the following relation (known as the radial equilibrium equation); Vavra

$$\frac{dh_o}{dr} = C_a \frac{dc_a}{dr} + C_u \frac{dC_u}{dr} + \frac{C_u^2}{r} \tag{35}$$

Now we have three unknowns ( $h_o$ ,  $C_a$ ,  $C_u$ ) and one equation (radial equilibrium equation). To solve this equation we assume arbitrarily radial variation of any two variables and appropriate variation of the third can be determined. Here we will assume that:

a - The total enthalpy does not change with the radius  
 b-The swirl velocity at the inlet and the exit to the rotor has the following general variation with radius:

$$Cu_1 = aR^n - \frac{b}{R} \quad \text{and} \quad Cu_2 = aR^n + \frac{b}{R} \quad (36)$$

where  $R = \frac{r}{r_m}$

We shall consider the three special cases for n

- a) n = -1
- b) n = 0
- c) n = 1

the general form of the degree of reaction; Saravanamuttoo[23] is expressed as.

$$\Lambda = 1 + \frac{Ca_1^2 - Ca_2^2}{2U(Cu_2 - Cu_1)} - \frac{Cu_2 + Cu_1}{2U} \quad (37)$$

The variation of axial velocity ( $C_a$ ), swirl velocity ( $C_u$ ) for both inlet to rotor (state 1) and outlet (state2) in the radial direction is listed in table (1) for the three different types of blading.

Table (1),Formulae for the variables  $Cu_1, Cu_2, Ca_1, Ca_2, \Lambda$  as function of R for different types of blading

n	Blading	$C_a$	$\Lambda$	$C_u$
-1	Free vortex	$Ca_1 = Ca_{1m}$ $Ca_2 = Ca_{2m}$ $Ca_1 = Ca_2$	$\Lambda = 1 - \frac{a}{U_m R^2}$ $\Lambda = 1 - \frac{(1 - \Lambda_m)}{R^2}$	$Cu_1 = \frac{a - b}{R}$ $Cu_2 = \frac{a + b}{R}$
0	Exponential	$Ca_1^2 - Ca_{1m}^2 = -2[a^2 \ln R + \frac{ab}{R} - ab]$ $Ca_2^2 - Ca_{2m}^2 = -2[a^2 \ln R - \frac{ab}{R} + ab]$ $Ca_{1m} = Ca_{2m}$ $Ca_2^2 - Ca_1^2 = 4ab(\frac{1}{R} - 1)$	$\Lambda = 1 + \frac{a}{U_m} - \frac{2a}{U_m R}$ $\Lambda = 1 + (1 - \frac{2}{R})(1 - \Lambda_m)$	$Cu_1 = a - \frac{b}{R}$ $Cu_2 = a + \frac{b}{R}$
1	First Power	$Ca_1^2 - Ca_{1m}^2 = -2[a^2(R^2 - 1) - 2ab \ln R]$ $Ca_2^2 - Ca_{2m}^2 = -2[a^2(R^2 - 1) + 2ab \ln R]$ $Ca_2^2 - Ca_1^2 = -8ab \ln R$	$\Lambda = 1 + \frac{2a \ln R}{U_m} - \frac{a}{U_m}$ $\Lambda = 1 + (2 \ln R - 1)(1 - \Lambda_m)$	$Cu_1 = aR - \frac{b}{R}$ $Cu_2 = aR + \frac{b}{R}$

### 3. Rotor blade stress

There are three main sources of stress; Mattingly [24]

1- Centrifugal tensile stress the largest, but not necessarily the most important it is a Steady stress.

2-Gas bending stress, fluctuating as the rotor blades pass by the trailing edges of the nozzles.

3-Centrifugal bending stress. When the centroids of the blade cross-section at different radii do not lie on a radial line. Such a stress is small enough to be neglected.

#### 3.1 Centrifugal tensile stress

The maximum value of this stress occurs at the root and is given by

$$(\sigma_{ct})_{\max} = \frac{\rho_b \omega^2}{a_r} \int_r^t ar dr \quad (38)$$

Where  $a$  and  $a_r$  are the cross sectional areas at any radius ( $r$ ) and at root radius,  $\rho_b$  is the blade material density

for uniform cross section, the tensile stress is reduced to

$$(\sigma_{ct})_{\max} = 2\pi N^2 \rho_b A \quad (39)$$

$$\text{Or } (\sigma_{ct})_{\max} = \rho_b U_t^2 [1 - \zeta^2] / 2 \quad (40)$$

Where  $A$  is the annulus area

Normally the rotor blade is usually tapered in chord, and thickness from root to tip such that  $a_t/a_r$  is between  $1/4$  and  $1/3$ . The stress in a tapered blade is given by:  $(\sigma_{ct})_{\max} = \rho_b U_t^2 (1 - \zeta^2) k / 2$

Typical values of  $k$  would range from 0.55 to 0.65 for tapered blade

The first-stage blades, being the longest, are the most highly stressed.

The later stage often appears to be very moderately stressed.

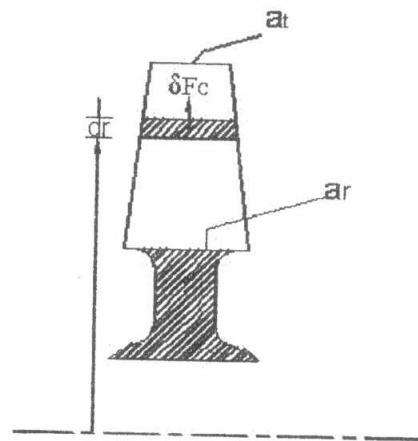


Fig.5 Centrifugal force on a blade element

### 3.2 Gas bending stress

The force arising from the change in angular momentum of the gas in the tangential direction, which produces the useful torque, also produces a gas bending moment about the axial direction

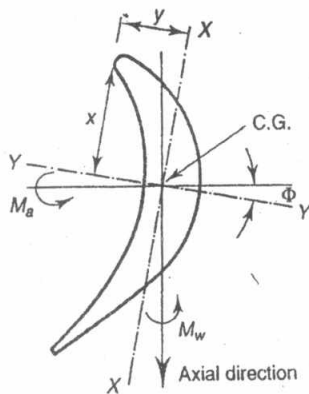
(i.e. when  $Ca_1 \neq Ca_2$ ), and with reaction blading there will certainly be a pressure force in the axial direction  $[(p_2 - p_1)2\pi r/n]$  per unit height, so that there will also be a gas bending moment  $M_a$  about the tangential direction; refer to fig(6) Resolving these bending moments into components acting about the principal axes of the blade cross-section, the maximum stresses can be calculated by the method appropriate to asymmetrical section. A useful approximation for preliminary design purposes is provided by

$$\sigma_{gb} = [x(M_a \cos\Phi - M_w \sin\Phi)] / I_{yy} + [y(M_w \cos\Phi + M_a \sin\Phi)] / I_{xx} \quad (41)$$

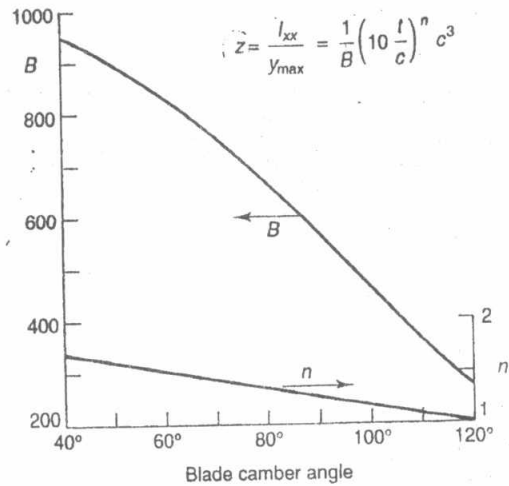
Form the above equation we can calculate the gas bending stress as:

$$(\sigma_{gb}) = m (C_{u2m} - C_{u1m}) h / (2nz c^3) \quad (42)$$

Clearly  $\sigma_{gb}$  is directly proportional the stage work output and blade height, and inversely proportional to the number of blade and section modulus. It is convenient to treat the section modulus as the product  $zc^3$  because  $z$  is largely a function of blade camber angle (i.e. gas deflection) and thickness/chord ratio.



(a) Gas bending stress



(b) Approximate rule for section module

Fig 6. Gas bending stress

### 3.3 Centrifugal bending stress

By designing the blade with the centroids of the cross-section slightly off a radial line, it is theoretically possible to design for a centrifugal bending stress which will cancel the gas bending stress. Thus, we will neglect this stress in our analysis.

#### 4 Check on efficiency

After completion of the stage design, it will be necessary to check over the performance, particularly in regard to the efficiency which for a given work input will completely govern the final pressure ratio. This efficiency is dependent on the total drag coefficient for each of the blade rows comprising the stage, three factors must be taken into account when we calculate the total drag coefficient; namely:

1-profile drag coefficient  $C_{DP}$

2-annulus drag coefficient dependent on the relative proportion of the blade row, its influence increasing as the blades become shorter relative to their chord length

$$C_{DA} = 0.020(s/h) \tag{43}$$

3-secondary loss due to trailing vortices and tip clearance

$$C_{DS} = 0.018C_L^2$$

Thus the total drag coefficient  $C_D = C_{DP} + C_{DA} + C_{DS}$  (44)

From consideration of momentum changes and definition of drag force

$$\left( \frac{\bar{w}}{0.5\rho V_1^2} \right) = C_D / \left( \frac{s}{c} \right) \left( \frac{\cos^3 \alpha_m}{\cos^2 \alpha_1} \right) \tag{45}$$

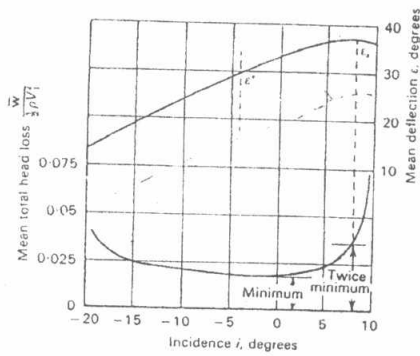
$$\eta_b = 1 - \frac{\bar{w}}{\Delta P_{th}} \tag{46}$$

Figure (7) represents typical cascade characteristics and is employed in the calculation of blade row efficiency  $\eta_b$ . From the stage pressure ratio relation

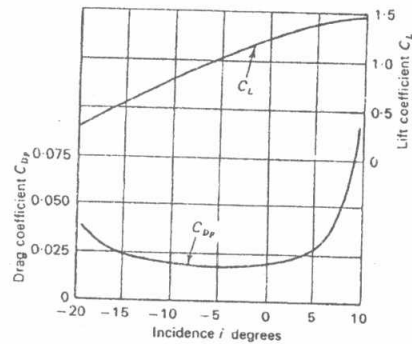
we can arrive after some manipulation that for 50% reaction  $\eta_b \approx \eta_s$ .

For  $\Lambda \neq 0.5$ , then we use the following relation

$$\eta_s = \Lambda \eta_{b, \dots} + (1 - \Lambda) \eta_{b, \dots}$$



(a) loss coefficient



(b) Lift and drag coefficients

Fig.7 Typical Experimental Cascade Data

## 5 Closure to design procedure

The details of the present package have been discussed in the previous sections. Similar packages are available at web sites; for example refer to kurzke [25], Mattingly [26], CADAC [27].

## 7 Conclusions

1-A survey of numerous publications as well as some web sites treating the design of axial compressors in aero engines and/or gas turbines is given.

2-A computer package for the preliminary inverse design of multistage axial compressors is developed.

3-The inputs to this package are the pressure ratio, mass flow rate, rotational speed, isentropic efficiency as well as the total inlet conditions.

4-The output of this package are:

- a- Number of stages
- b- Annulus dimensions at inlet and outlet of the compressor.
- c- Mean section characteristics; namely, flow angles, degree of reaction, total pressure and temperature at each stage.
- d- Three-dimensional variation from hub to tip of all the air properties (flow angles and degree of reaction)
- e- Gas bending and centrifugal tensile stresses of both stators and rotors of each stage.
- f- Checking of the efficiency of each stage
- g- Layout of the compressor
- h- Geometry of the twisted blade for four different three dimensional methods.
- i- The absolute and relative Mach number variation over the blade height at any stage.
- j- The variation of the absolute velocity within the compressor stages.

5-A case study is examined for an axial compressor with certain specified data close to those of the HPC of the CF6 turbofan engine, will be given in part (II) of this paper.

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