

Soft Sets and Soft Topological Spaces via its Applications

R. Mareay^a, T. Medhat^{b,*} and Manal. E. Ali^c

^aDepartment of Mathematics, Faculty of Science, Kafrelsheikh University, Kafrelsheikh, 33516, Egypt.

^bDepartment of Electrical Engineering, Faculty of Engineering, Kafrelsheikh University, Kafrelsheikh, 33516, Egypt.

^cDepartment of Physics and Engineering Mathematics, Faculty of Engineering, Kafrelsheikh University, Kafrelsheikh, 33516, Egypt.

*Corresponding Author: T. Medhat, E-mail: tmedhatm@eng.kfs.edu.eg

Abstract: The soft set is used in a variety of disciplines. It is a tool for handling ambiguous, uncertain, and indeterminate data. Numerous academics have introduced and researched the notion of soft sets in several domains, including game theory, operation research, probability, and decision-making. The concepts of soft sets and soft topological spaces are introduced in this study. This article introduces the definitions of the soft topology and discusses its foundations and associated characteristics. Examples from the real-world have been provided to assist explain some of the traits of this field. These methods have shown to be quite beneficial in many applications.

Keywords: Soft sets; Soft topological space; Pre-open soft set; Reduct-soft set; Decision making; Rough set.

1. Introduction

Researchers in a variety of fields, including engineering, health, sociology, economics, and environmental science, deal with the difficulties of modelling ambiguous data on a regular basis. Classical techniques are not always successful in these areas because the uncertainties that arise in these fields might be of a variety of different sorts and sizes. There are various approaches to modelling uncertainty that are less well-known and less usually beneficial. While probability theory, rough sets [12,23], fuzzy sets [27] are well-known and frequently helpful techniques. Those who disagree with Molodtsov point out that each of those ideas has its own set of challenges. Molodtsov [20] originally presented the concepts of soft sets as a generic tool of mathematics for handling uncertain items. Since then, it has been widely used. In recent years [4,5,6,11,21,22], there has been an increasing amount of research on the properties and uses of soft set theory.

By combining ideas from the study of fuzzy sets, some novel of soft set theory applications has been created recently [1,7,14,15,17,18,25]. When it comes to

decision-making problems involving rough sets, Maji and colleagues [16] describe how they used rough set techniques to solve a problem of decision-making and then applied soft set theory to it, and they have also published extensive theoretical research on soft sets [17,22]. The Authors in [9] provide a unique concept of the reduction of soft set parameterizations, and this concept is contrasted with the similar reduction notion of attributes in rough set theory. When Aktas and colleagues [2] defined a group, they incorporated soft set algebraic structures into the concept of a group. Soft topology, soft-open set and soft-closed set, were proposed in [3,6,19]. Kandil and colleagues [10] studied the relationships between several forms of soft topological space subsets.

Pre-operator on soft topological space, pre-open soft set, pre-closed soft set, pre-interior, and pre-closure are all ideas that are introduced in this article. These principles will be shown with the help of certain instances. We demonstrate a real-world application of the theory of soft set, using the rough set technique, through a case study. The structure of this article is as follows: Section 2 provides a quick overview of the

fundamental concept of the theory of soft set, soft topology, and theory of rough set. Section 3 presents the definition of soft topological space and pre-open soft set. In Section 4, we present an application of the theory of soft set in decision-making problem. Finally, Section 5 provides the conclusion.

2. Preliminaries

This section introduces the fundamental notions of soft sets that can be found in [17,20], as well as some important definitions on rough mathematics that can be found in [23]. In this work, U denotes a universe, M denotes a collection of parameters, $P(U)$ denotes the set of U , and $A \subseteq M$ denotes the set of A parameters.

2.1 Soft sets and soft topology

Definition 1. A soft set K_A on universe U is defined as the following ordered pairs:

$$K_A = \{(m, k_A(m)): m \in M, k_A(m) \in P(U)\}$$

where $k_A: M \rightarrow P(U)$ such that $k_A(m) = \emptyset$ if $m \notin A$. Here, k_A is called an approximation function of soft set K_A , and $S(U)$ stands for the set of all soft sets over U .

Example 1.

Consider the universe $U = \{b_1, b_2, b_3, b_4, b_5, b_6\}$ of six buildings and $M = \{m_1, m_2, m_3, m_4, m_5\}$ is the set of parameters refer to expensive, beautiful, sunny, cheap, and modern, respectively. Let k_A be a mapping from the parameters to the buildings. For instance, $k_A(m_1)$ means the expensive building i.e,

$$k_A(m_1) = \{b \in U: b \text{ is expensive building}\}.$$

Suppose that $A \subseteq M$ such that $A = \{m_1, m_2, m_3\}$ and $k_A(m_1) = \{b_1, b_2\}$, $k_A(m_2) = \{b_3, b_4, b_5\}$ and $k_A(m_3) = U$.

Then the soft set K_A can be written as following:

$$K_A = \{(m_1, \{b_1, b_2\}), (m_2, \{b_3, b_4, b_5\}), (m_3, U)\}$$

Definition 2. Let $K_A \in S(U)$. If $k_A(m) = \emptyset \forall m \in M \rightarrow K_A = K_\emptyset$ and it is called an empty soft set.

Definition 3. Let $K_A \in S(U)$. If $k_A(m) = U \forall m \in A \rightarrow K_A$ is A -universal soft set. If $A = M \rightarrow K_M$ stands for M -universal soft set.

Definition 4. Let $K_A, K_B \in S(U)$. Then K_A is soft subset of K_B , denoted by $K_A \subseteq K_B$, if

- (i) $A \subseteq B$.
- (ii) $k_A(m) \subseteq k_B(m) \forall m \in M$.

In this instance, K_B is a soft superset of K_A , and K_A is a soft subset of K_B .

Definition 5. Over a universe U , two soft subsets K_A and K_B are said to be soft equal if K_A and K_B are a soft subset of each other.

Definition 6. Let $K_A, K_B \in S(U)$. Then, the soft intersection $K_A \tilde{\cap} K_B$, the soft union $K_A \tilde{\cup} K_B$, and the soft difference $K_A \tilde{\setminus} K_B$ of K_A and K_B are defined by the following approximation functions:

$$k_{A \tilde{\cap} B}(m) = f_A(m) \cap k_B(m),$$

$$k_{A \tilde{\cup} B}(m) = f_A(m) \cup k_B(m),$$

$$k_{A \tilde{\setminus} B}(m) = f_A(m) \setminus k_B(m),$$

respectively, and the soft complement K_A^c of K_A can be defined as the following approximation function:

$$k_A^c(m) = K_A^c(m)$$

where $k_A^c(m)$ is the complement of $k_A(m)$.

Definition 7. [8] Let $K_A \in S(U)$. The soft power set of K_A is given by

$$\tilde{P}(K_A) = \{K_{A_i}: K_{A_i} \subseteq K_A, i \in I \subseteq N\}$$

and the cardinality of it is defined by

$$|\tilde{P}(K_A)| = 2^{\sum_{x \in M} |k_A(x)|}$$

where $|k_A(x)|$ stands for the cardinality of $k_A(x)$.

Definition 8. Let $K_A \in S(U)$. A soft topology $\tilde{\tau}$ on K_A , is a collection of soft subsets of K_A that achieves these properties:

- (i) $K_\emptyset, K_A \in \tilde{\tau}$,
- (ii) $\{K_{A_i} \subseteq F_A: i \in I \subseteq N\} \subseteq \tilde{\tau} \Rightarrow \tilde{\cup}_{i \in I} K_{A_i} \in \tilde{\tau}$,
- (iii) $\{K_{A_i} \subseteq K_A: 1 \leq i \leq n, n \in N\} \subseteq \tilde{\tau} \Rightarrow \tilde{\cap}_{i=1}^n K_{A_i} \in \tilde{\tau}$.

The pair $(K_A, \tilde{\tau})$ stands for a soft topological space.

Definition 9. Let $(K_A, \tilde{\tau})$ be a soft topological space. Then x is called an open soft set if $x \in \tilde{\tau}$. The set of

members of $\tilde{\tau}$ over U are open soft sets and denoted by $OS(U)$.

Definition 10. Let $(K_A, \tilde{\tau})$ be a soft topological space. Then a soft set K_A is closed soft set in U , if the relative complement K_A^c is open soft set. $CS(U)$ stands for The set of all closed sets over U .

2.2 Concepts of Rough Set Theory

Definition 11. An information system described as $S = (U, A, \rho, V)$, where U stands for the universe which is a non-empty finite set of objects, and A stands for the attribute which is a finite collection of non-empty set of characteristics. Each attribute $a \in A$ may be thought of as a function ρ that translates items of U to a set V_a , where the set V_a is referred to as the attribute's value set. i.e., $\rho: U \times A \rightarrow V_a$.

Definition 12. The indiscernibility relation for a subset $E \subseteq A$ is:

$$I(E) = \{(x, y) \in U^2: a(x) = a(y), \forall a \in E\}$$

, or

$$I(E) = \bigcap_{a \in E} I(\{a\}),$$

The equivalence class of $I(E)$ is defined by:

$$[x]_E = \{y \in U: (x, y) \in I(E)\},$$

$U/I(E)$ stands for the all equivalence classes family of E , where:

$$U/I(E) = \{[x]_E: \forall x \in U\},$$

Definition 13. Let $E \subseteq A$, $a \in E$, then a is unnecessary attributes in E if:

$$U/I(E) = U/IND(E - \{a\}),$$

And $RED(E) = E - \{a\}$ is called the reduct of E . The minimal reduct of E is called the set M iff:

- (i) $U/I(M) = U/I(E)$.
- (ii) $U/I(M) \neq U/IND(M - \{a\}), \forall a \in M$

The core is the collection of all knowledge characteristics that cannot be deleted during the reduction process:

$$CORE(E) = \bigcap RED(E),$$

3. Soft Topological Spaces and Pre-open Soft Set

Definition 14. Let $(K_A, \tilde{\tau})$ be a soft topological space. A mapping $int(cl): S(U) \rightarrow S(U)$ is a pre-open operation on $OS(U)$ if $K_A \subseteq int(cl(K_A)), \forall K_A \in OS(U)$. The collection of pre-open soft set is identified by $POS(U) = \{K_A: K_A \subseteq int(cl(K_A)), K_A \in S(U)\}$. Also, the pre-closed soft set is the complement of pre-open soft set is called. i.e., $PCS(U) = \{K_A^c: F_A \text{ is pre-open soft set}, K_A \in S(U)\}$ is the family of all pre-closed soft sets.

Theorem 1. Let $(K_A, \tilde{\tau})$ be a soft topological space and $cl(int): S(U) \rightarrow S(U)$ be an operation on $OS(U)$. Then

- (i) The arbitrary union of pre-open soft set is pre-open soft set.
- (ii) The arbitrary intersection of pre-closed soft set is pre-closed soft set.

Proof.

- (i) Let $K_{Ai} \in POS(U), i \in I \Rightarrow K_{Ai} \subseteq int(cl(K_{Ai}))$. It follows that $\bigcup_i K_{Ai} \subseteq \bigcup_i int(cl(K_{Ai})) \subseteq int(cl(\bigcup_i K_{Ai}))$. Hence, $\bigcup_i K_{Ai} \in POS(U), \forall i \in I$.
- (ii) Immediate

Remark 1. The soft intersection of two pre-open soft sets does not necessarily have to be a pre-open soft set, as shown in the example below.

Example 2. Let $U = \{b_1, b_2, b_3\}, M = \{m_1, m_2, m_3\}, K_A = \{(m_1, \{b_1, b_3\}), (m_2, \{b_2, b_3\})\}, K_{A_1} = \{(m_1, \{b_1, b_3\})\}, K_{A_2} = \{(m_2, \{b_2, b_3\})\}$. Then $\tilde{\tau} = \{K_\emptyset, K_A, K_{A_1}, K_{A_2}\}$ is the soft topological space over U . The two soft sets K_{A_3} and K_{A_4} where:

$K_{A_3} = \{(m_1, \{b_1, b_2\}), (m_2, \{b_1\})\}, K_{A_4} = \{(m_1, \{b_2, b_3\}), (m_2, \{b_1\})\}$, are pre-open sets but the intersection $K_{A_3} \tilde{\cap} K_{A_4} = \{(m_1, \{b_2\}), (m_2, \{b_1\})\}$ is not pre-open.

Definition 15. Let $(K_A, \tilde{\tau})$ be a soft topological space over U and $x_m \in S(U)$. Then

(i) x_m is called a pre-interior soft point of K_A if there exists $K_{A_i} \in POS(U)$ such that $x_m \in K_{A_i} \subseteq K_A$.
 $PS(int(K_A))$ stands for the set of all pre-interior soft points of K_A .

Therefore

$$PS(int(K_A)) = \bigcup \{K_{A_i} : K_{A_i} \subseteq K_A, K_{A_i} \in POS(U)\}.$$

(ii) x_m is called a pre-closure soft point of K_A if $K_A \cap K_E \neq \emptyset, \forall K_E \in PCS(U)$.
 $PS(cl(K_A))$ stands for the set of all pre-closure soft points of K_A .

Therefore

$$PS(cl(K_A)) = \bigcap \{K_E : K_E \in PCS(U), K_A \subseteq K_E\}.$$

Theorem 2. Let (K_A, τ) be soft topological space over U , $int(cl): S(U) \rightarrow S(U)$ and $K_A, K_E \in S(U)$. In that case, the operator $PS(int)$ has the following conditions satisfied:

- (i) $PS(int(\tilde{U})) = \tilde{U}$ and $PS(int(\tilde{\emptyset})) = \tilde{\emptyset}$.
- (ii) $PS(int(K_A)) \subseteq F_A$.
- (iii) $PS(int(K_A))$ is the largest pre-open soft set contained in K_A .
- (iv) If $K_A \subseteq K_E$, then $PS(int(K_A)) \subseteq PS(int(K_E))$.
- (v) $PS(int(PS(int(K_A)))) = PS(int(K_A))$.
- (vi) $PS(int(K_A)) \cup PS(int(K_E)) \subseteq PS(int(K_A \cup K_E))$
- (vii) $PS(int(K_A) \cap K_E) \subseteq PS(int(K_A)) \cap PS(int(K_E))$

Proof. Immediate.

Theorem 3. Let (K_A, τ) be a soft topological space over U , $int(cl): S(U) \rightarrow S(U)$ and $K_A, K_E \in S(U)$. In that case, the operator $PS(cl)$ has the following conditions satisfied:

- (i) $PS(cl(\tilde{U})) = \tilde{U}$ and $PS(cl(\tilde{\emptyset})) = \tilde{\emptyset}$.
- (ii) $PS(cl(K_A)) \subseteq F_A$.
- (iii) $PS(cl(K_A))$ is the smallest pre-open soft set contains K_A .

(iv) If $K_A \subseteq K_E$, then $PS(cl(K_A)) \subseteq PS(cl(K_E))$.

(v) $PS(cl(PS(cl(K_A)))) = PS(cl(K_A))$.

(vi) $PS(cl(K_A)) \cup PS(cl(K_E)) \subseteq PS(cl(K_A \cup K_E))$.

(vii) $PS(cl(K_A) \cap K_E) \subseteq PS(cl(K_A)) \cap PS(cl(K_E))$.

Proof. Immediate.

4. A Decision-Making Application of the Theory of Soft Set

Molodtsov [20] provides several applications of soft sets in various fields, as: probability, game theory, and operations research, etc. In this part, we offer a preliminary method to applying the theory of soft sets to a decision-making issue [24], by the help of the techniques of rough set.

Example 3. Let $U = \{0,1,2,3,4,5,6,7,8,9\}$ be the set of numbers from 0 to 9, $E = \{a, b, c, d, e, f, g\}$ be the LEDs of the seven-segment display which gives the numbers from 0 to 9 as shown in Figure 1:

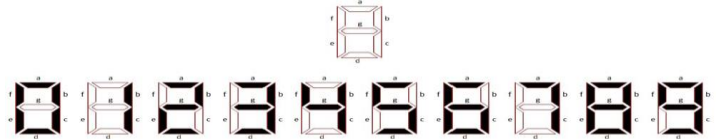


Figure 1: Seven-segment display

The problem in this application is how to detect the number from 0 to 9 with the minimum number of LEDs.

4.1 A Soft Set Tabular Representation

Lin [13] and Yao [26] introduced tabular representation of soft sets. We provide a binary table-based roughly equivalent model. For this, take into account the soft set (F, E) based on the set E of the seven-segment display's LEDs. We may tabulate this soft set, as seen in Table 1. This method of representation will be beneficial for computer storage of soft sets. This style is called soft information systems.

If $h_i \in f(e)$ then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries in Table 1:

Consequently, a soft set may be offered in an information system where a set of parameters is used in

place of the set of characteristics. For example, let $A, B \subseteq E$, where $A = \{a, b, c\}, B = \{d, e, f\}$. Then the soft sets F_A, F_B can be defined as:

$$F_A = \{(a, \{0, 2, 3, 5, 6, 7, 8, 9\}), (b, \{0, 1, 2, 3, 4, 7, 8, 9\}), (c, \{0, 1, 3, 4, 5, 6, 7, 8, 9\})\},$$

$$F_B = \{(d, \{0, 2, 3, 5, 6, 8, 9\}), (e, \{0, 2, 6, 8\}), (f, \{0, 4, 5, 6, 8, 9\})\}.$$

Also, we can define soft topological space and pre-soft topological space.

Definition 16. The decision value C_i of an object $h_i \in U$ is given by

$$C_i = \sum_j h_{ij}$$

where h_{ij} is the entry in the table of soft information system.

Table 1: Seven-segment display via soft set

U/A	a	b	c	d	e	f	g	Decision value
0	1	1	1	1	1	1	0	$C_0 = 6$
1	0	1	1	0	0	0	0	$C_1 = 2$
2	1	1	0	1	1	0	1	$C_2 = 5$
3	1	1	1	1	0	0	1	$C_3 = 5$
4	0	1	1	0	0	1	1	$C_4 = 4$
5	1	0	1	1	0	1	1	$C_5 = 5$
6	1	0	1	1	1	1	1	$C_6 = 6$
7	1	1	1	0	0	0	0	$C_7 = 3$
8	1	1	1	1	1	1	1	$C_8 = 7$
9	1	1	1	1	0	1	1	$C_9 = 6$

4.2 Reduction of Soft Sets

Let (F, E) be a soft set, $A \subseteq E$. Then F_A be a soft subset of (F, E) . The soft set F_Q is called the reduct-soft set of the soft set F_A if Q is a reduct of A . Alternatively a reduct-soft set F_Q of the soft set F_A is the essential part which give the same decision as the the soft set F_A .

The core soft set is the intersection of the reduct-soft set, where $C = \tilde{\cap}$ reduct A .

4.3 Algorithm for Decision-making in Seven-segment display

Step1: Input the soft set (F, E)

Step2: Input the set $A \subseteq E$ of parameters of LEDs which display the numbers form 0 to 9.

Step3: Find all reduct-soft set of (F, A) .

Step4: Find the core of all reduct.

Step5: Calculate the decision value in each case.

Example 4. For Example 3, the reduct-soft sets and core-soft sets can be derived as shown in the following: Tables 2,3,4 and Figures 2,3,4:

Table 2: Reduct-soft set without LED c

U/A	a	b	d	e	f	g	Decision value
0	1	1	1	1	1	0	$C_0 = 5$
1	0	1	0	0	0	0	$C_1 = 1$
2	1	1	1	1	0	1	$C_2 = 5$
3	1	1	1	0	0	1	$C_3 = 4$
4	0	1	0	0	1	1	$C_4 = 4$
5	1	0	1	0	1	1	$C_5 = 4$
6	1	0	1	1	1	1	$C_6 = 5$
7	1	1	0	0	0	0	$C_7 = 2$
8	1	1	1	1	1	1	$C_8 = 6$
9	1	1	1	0	1	1	$C_9 = 5$

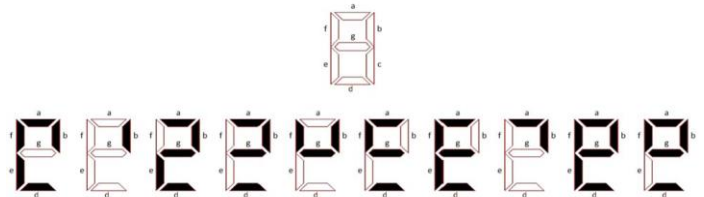


Figure 2: Seven-segment display without LED c

Remark 2. As we can see from the previous illustration, the sets $\{a, b, d, e, f, g\}$ and $\{a, b, c, e, f, g\}$ are two reduct-soft set of $A = \{a, b, c, d, e, f, g\}$ and the core-soft set of A is $\{a, b, e, f, g\}$. Instead of 7 LEDs, just 5 LEDs are required to detect any number. For decision-making, if someone wants to detect number nine, he needs 4 LEDs = $\{a, b, g, f\}$ to be “On” and 1 LED = $\{e\}$ to be “off”.

Table 3: Reduct-soft set without LED d

U/A	a	b	c	e	f	g	<i>Decision value</i>
0	1	1	1	1	1	0	$C_0 = 5$
1	0	1	1	0	0	0	$C_1 = 2$
2	1	1	0	1	0	1	$C_2 = 4$
3	1	1	1	0	0	1	$C_3 = 4$
4	0	1	1	0	1	1	$C_4 = 4$
5	1	0	1	0	1	1	$C_5 = 4$
6	1	0	1	1	1	1	$C_6 = 5$
7	1	1	1	0	0	0	$C_7 = 3$
8	1	1	1	1	1	1	$C_8 = 6$
9	1	1	1	0	1	1	$C_9 = 5$

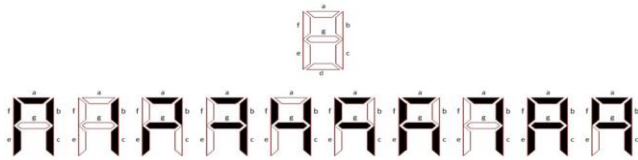


Figure 3: Seven-segment display without LED d

Table 4: Core-soft set without LEDs c and d

U/A	a	b	e	f	g	<i>Decision value</i>
0	1	1	1	1	0	$C_0 = 4$
1	0	1	0	0	0	$C_1 = 1$
2	1	1	1	0	1	$C_2 = 4$
3	1	1	0	0	1	$C_3 = 3$
4	0	1	0	1	1	$C_4 = 3$
5	1	0	0	1	1	$C_5 = 3$
6	1	0	1	1	1	$C_6 = 4$
7	1	1	0	0	0	$C_7 = 2$
8	1	1	1	1	1	$C_8 = 5$
9	1	1	0	1	1	$C_9 = 4$

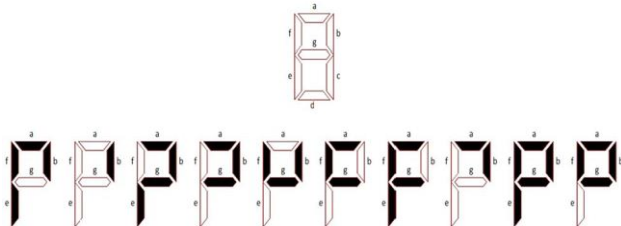


Figure 4: Seven-segment display without LEDs c and d

5. Conclusion

Molodtsov was the first to develop the notion of the theory of soft set, and he offered a lot of applications in many sectors. Topology is a significant branch of mathematics. In this work, we define soft topological space using the concepts of pre-open soft set, and we demonstrate the features of soft topological space that are associated with it. The rough approach of Pawlak is used to apply soft sets to a decision-making issue. In the future, we will investigate further characteristics of soft topological spaces as well as their use in decision-making processes.

References:

[1] B. Ahmad, A. Kharal, On fuzzy soft sets, *Advances in Fuzzy Systems* (2009), 1-6.

[2] H. Aktas, N. Cagman, Soft sets and soft groups, *Information Sciences* 1(77)(2007), 2726-2735.

[3] S. Al Ghour, Strong form of soft semiopen sets in soft topological spaces, *Int. J. Fuzzy Log. And Intell. Syst.* 21(2021), 159–168.

[4] S. Al Ghour, Some modifications of pairwise soft sets and some of their related concepts, *Mathematics* 2021, 9, 1781.

[5] M. I. Ali, F. Feng, X. Liu, W. K. Min, M. Shabir, On some new operations in soft set theory, *Computers and Mathematics with Applications* 57(2009), 1547-1553.

[6] T. M. Al-shami, Bipolar soft sets: Relations between them and ordinary points and their applications, *Complexity* 2021,6621854.

[7] N. Cagman, F. Citak, S. Enginoglu, Fuzzy parameterized fuzzy soft set theory and its applications, *Turkish Journal of Fuzzy Systems* 1(1)(2010), 21-35.

[8] N. Cagman, S. Karatas, S. Enginoglu, Soft topology, *Computer and Mathematics with application* 62(2011), 351-358.

[9] D. Chen, E.C.C. Tsang, D.S. Yeung, X. Wang, The parameterization reduction of soft sets and its applications, *Comput. Math. Appl.* 49(2005), 757-763.

- [10] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, A. M. Abd El-latif, gamma-operation and decompositions of some forms of soft continuity in soft topological spaces, *Annals of Fuzzy Mathematics and Informatics* 7(2)(2014), 181-196.
- [11] D. V. Kovkov, V. M. Kolbanov, D. A. Molodtsov, Soft sets theory-based optimization, *Journal of Computer and Systems Sciences International* 46(6)(2007), 872-880.
- [12] E.F. Lashin, A.M. Kozae, A.A. Abo Khadra, T. Medhat, Rough set theory for topological spaces, *International Journal of Approximate Reasoning* 40(1-2)(2005), 35-43.
- [13] T.Y. Lin, A set theory for soft computing, a unified view of fuzzy sets via neighbourhoods, In *Proceedings of 1996 IEEE international Conference on Fuzzy Systems*. New Orleans, LA, September 8-11(1996), 1140-1146.
- [14] P. K. Maji, R. Biswas, A. R. Roy, Intuitionistic fuzzy soft sets, *Journal of Fuzzy Mathematics* 9(3)(2001), 677-691.
- [15] P. K. Maji, R. Biswas, A. R. Roy, Fuzzy soft sets, *Journal of Fuzzy Mathematics* 9(3)(2001), 589-602.
- [16] P.K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, *Comput. Math. Appl.* 44(2002), 1077-1083.
- [17] P.K. Maji, R. Bismas, A.R. Roy, Soft set theory, *Comput. Math. Appl.* 45(2003), 555-562.
- [18] P. Majumdar, S. K. Samanta, Generalized fuzzy soft sets, *Computers and Mathematics with Applications* 59(2010), 1425-1432.
- [19] W.K. Min, On soft generalized closed sets in a soft topological space with a soft weak structure, *Int. J. Fuzzy Log. Intell. Syst.* 20(2020), 119–123.
- [20] D. A. Molodtsov, Soft set theory-first results, *Computers and Mathematics with Applications* 37(1999), 19-31.
- [21] D. Molodtsov, V. Y. Leonov, D. V. Kovkov, Soft sets technique and its application, *Nechetkie Sistemy i Myagkie Vychisleniya* 1(1)(2006), 8-39.
- [22] S. Oztunc, S. Aslan, H. Dutta, Categorical structures of soft groups, *Soft Comput.* 25(2021), 3059–3064.
- [23] Z. Pawlak, Rough sets, *Int. J. Inform. Comput. Sci.* 11(1982), 341-356.
- [24] Z. Pawlak, *Rough Sets: Theoretical aspects of reasoning about data*, Kluwer Academic, Boston, (1991).
- [25] M. Shabir and M. Naz, On soft topological spaces, *Comput. Math. Appl.* 61(2011), 1786-1799.
- [26] Y.Y. Yao. Relational interpretations of neighbourhood operators and rough set approximation operators, *Information Sciences* 111(1-4)(1998), 239-259.
- [27] L.A. Zadeh, *Fuzzy Sets*, *Inform. Control* 8(1965), 338-353.