Robust Feedback Linearization Controller for Small UAV

Ahmed Abdelkader Wasfy*, Ziad Mohamed Ahmed**, and Mohamed Hussien Salah
MTC, Egypt, *Ahmedwasfi20@yahoo.com, **Zeadmohamed13@gmail.com
Supervisor: Ehab Safwat, Ahmed Taimour Hafez, Ahmed M. Kamel
MTC, Egypt, e.khattab@mtc.edu.eg

Abstract*– Fixed-wing unmanned aerial vehicles (UAVs) have become increasingly important in military, civil, and scientific sectors. The aerodynamic model is characteristic of nonlinearity and that the linear equation design approach based on small perturbations of linear system has failed to meet the design requirements cause the study of the dynamic inversion control. Nonlinear dynamic inversion is a straightforward technique for designing control laws for nonlinear systems. Moreover, it guarantees high levels of safety and performance in sever flight conditions in comparison with the linearized control systems. This paper focuses on the use of nonlinear dynamic inversion in the design of a flight control system for traditional Unmanned Aircraft Vehicles (UAVs). The two-timescale assumption that separates the fast dynamics which are the three angular rates of the aircraft from the slow dynamics which include the Angle of Attack (AoA), side-slip angle, and bank angle. A dynamic inversion control law is designed for the fast variables using the deflection of aerodynamic control surfaces as inputs. Next, dynamic inversion is applied to the control of the slow states using the outputs of the fast loop as inputs. Simulation results for the nonlinear flight control system are given to illustrate the effectiveness of the technique.

Keywords—Dynamic inversion, UAV, Control law, non-linearity.

I. INTRODUCTION

Flying scale models are often considered as an additional tool in aircraft design more often and simple UAV is used for a growing number of applications such as surveillance and monitoring [1]. One of the design objectives of these UAVs is the cost reduction. It is also mandatory to fly automated or at least have sufficient stability augmentation [2]. UAVs will routinely operate in urban environments for a range of applications and around large infrastructure for data gathering or sapling extending range and endurance. It is a well-known fact that aircrafts model is characterized by a tight coupling between the structural, aerodynamic and propulsion systems. Moreover, the estimation of the various aerodynamic parameters, is extremely difficult. What with increasingly complex missions of UAV and the non-linearity characteristic of dynamic model, linear equation design approach based on small perturbations of linear system has failed to meet the design requirements. Consequently, a robust control system is required in order to achieve stable flights in most outdoor conditions.

Since decades the automatic control occupies increasingly a significant place in aeronautics, many works discussed various aspects related to the control systems automation. The purpose of the controller is to ensure that the output closely tracks the input while optimizing the controller parameters for achieving the best performance, focusing on the responsibilities of the controller to calculate the control commands while keeping the system overshoot, settling time, and steady-state error within defined design margins based on the system requirements. Also, to meet increasing demands on performance and reliability of UAVs, nonlinear control techniques are often utilized. These techniques are actively being studied to handle nonlinear aerodynamics and kinematic effects, actuator saturation, rate limitations, modeling uncertainty, and time-varying dynamics.

Nonlinear control methods have been developed to overcome the shortcomings of linear design approaches. NDI approach is one of the best known and used successfully to address some practical control problems as it is based on algebraically transform a nonlinear system dynamic into a fully or partly linear one so that linear control techniques can be applied. It differs from the conventional linearization using Jacobian technique [3], which is achieved by exact state transformation and feedback under the assumption of a correct onboard dynamic model, instead of a linear approximation of the dynamics around trim points [4], [5].

The main premise of NDI is the time consequence between two different controllers, that together comprise the dynamic inversion controller. The first controller depending on the nonlinear model by canceling the unwanted nonlinear terms and transform the complex nonlinear system to a first-order linear dynamics. The second part of the controller is concerning the performance of the tracking response, therefore, it is linear with control commands forcing the system to behave in the required manner [2]. The linear controller is designed to improve the system robustness against any model mismatch [4]. NDI has gained considerable popularity to address some practical control problems include the helicopters control [7], satellites [8], industrial robots [9], and biomedical devices [6]. The literature survey of the NDI outlined below.

This paper discusses the powerful usage of nonlinear dynamic inversion as a common approach of deriving control laws for a small UAV. The high nonlinearity, multiple control surface and multiple control objectives for longitudinal, lateral and directional maneuvering are the challenges opposes flight control system while the simulation results demonstrate the effectiveness of the designed control law using dynamic inversion.

The paper’s structure is as follows: Section II presents the nonlinear modeling of the UAV. Section III discusses the derivation of the control law using dynamic inversion, then the simulation results are presented. Finally, the conclusions of this paper are given in Section V.

.

II. AIRPLANE MATHEMATICAL MODELING

The first step to develop a 6-DoF nonlinear flight simulation model for a UAV is to develop the mathematical model that describes its dynamics and its surroundings. It includes the development of the airplane Equations of Motion (dynamic model), development of the mathematical representation for the aerodynamic forces and moments, and development of the equations describing the variations in air temperature, pressure and density with altitude (atmospheric model).

A. Dynamic Model

The dynamic model contains the nonlinear differential equations describing the motion of the airplane. The Equations of Motion (EoM) of the airplane are developed assuming the airplane as a rigid body, the earth is flat, non-rotating, the airplane mass is constant during any particular dynamic analysis, and the X-Z plane is a plane of symmetry. The force and moment equations are derived from Newton’s second law and then represented in airplane body axes. For the trajectory equations, since airplane position updates usually occur in earth axes, then airplane linear velocities must be converted into linear position rates in the earth axes. This is achieved by applying a transformation from body axes to earth axes. The standard six degrees of freedom nonlinear differential equations for a conventional fixed-wing airplane can be summarized in the state space vector form as follows:

$$\dot{\vec{V}}_b = \vec{\omega}_b X \vec{V}_b + B\vec{g}_o + \vec{F}_b \Big/ m \tag{1}$$

$$\dot{\vec{\omega}}_b = I_B^{-1}\left[\vec{\omega}_b \otimes I_B \vec{\omega}_b + \vec{M}_b\right] \tag{2}$$

$$\dot{\vec{\phi}}_b = \varepsilon \vec{\omega}_b \tag{3}$$

$$\dot{\vec{r}}_e = B^T \vec{V}_b \tag{4}$$

where ε is the transformation matrix of angular velocity to Euler angle rates, B is the transformation matrix form local horizon to body axes, $\vec{\omega}_b = \left[p,q,r\right]^T$, $\vec{V}_b = \left[u,v,w\right]^T$ $\vec{\varphi}_b = \left[\varphi,\theta,\psi\right]^T$, $\vec{r}_e = \left[x,y,h\right]^T$, $I_b = \left[I_x, I_y, I_z, I_{xz}\right]^T$. These equations represent the core of the airplane mathematical model [11]. They are in the state space form which $\dot{X} = f\left(x,\mathrm{u}\right)$ where, $u = \left[\delta_a,\delta_e,\delta_r,\delta_t\right]$ is the airplane control input vector and is the airplane states vector $x = \left[u,v,w,p,q,r,\varphi,\theta,\psi,x,y,h\right]^T$.

B. Aerodynamic Forces and Moments

Aerodynamic loads are classified into longitudinal and lateral loads. For the longitudinal aerodynamics, the longitudinal forces and moment are affected by the angle of attack AoA, elevator deflection δ_e , pitch rate q ,the derivative of the AoA $\dot{\alpha}$. The effects of the $q,\dot{\alpha}$ are generally appear in lift and pitching moment [12], while they are assumed equal zero for the drag. So, the lift L drag D, and pitching moment m may be written as:

$$L = 0.5\rho V_T^2 S C_L\left(\alpha, q, \delta_e, \dot{\alpha}\right) \tag{5}$$

$$Y = 0.5\rho V_T^2 S C_Y\left(\beta, p, r, \delta_a, \delta_r\right) \tag{6}$$

$$D = 0.5\rho V_T^2 S C_D\left(\alpha, \delta_e\right) \tag{7}$$

$$l = 0.5\rho V_T^2 S c C_l\left(\beta, p, r, \delta_a, \delta_r\right) \tag{8}$$

$$m = 0.5\rho V_T^2 S b C_l\left(\alpha, q, \delta_e, \dot{\alpha}\right) \tag{9}$$

$$n = 0.5\rho V_T^2 S c C_n\left(\beta, p, r, \delta_a, \delta_r\right) \tag{10}$$

where ρ is the air density, V_T is the total velocity, S is the wing plan form area, c is the wing mean aerodynamic chord, C_L, C_D, C_m are the lift, drag and pitching moment coefficients respectively, C_l, C_m, C_n are roll, pitching, yaw moment coefficients respectively. A summary of the measured geometric data is shown in Table I.

TABLE I
GEOMETRIC CHARACTERISTICS OF THE UTILIZED UAV

Geometric characteristics	Wing	H. Tail	V. Tail
Span [m]	1.554	0.54	1.554
Root Chord [m]	0.28	0.202	0.28
Tip Chord [m]	0.28	0.10	0.28
Quarter chord sweep angle	0^o	16^o	42^o
Leading edge sweep angle	0^o	20^o	45^o
Root chord incidence angle	2.0^o	0^o	0^o
Twist angle	0^o	0^o	0^o
Dihedral angle	2.0^o	0^o	0^o

III. DYNAMIC INVERSION CONTROL LAW

The Flight control laws are necessary for stabilizing or augment the stability of UAV configurations that would otherwise be aerodynamically unstable or nearly unstable [7]. In other cases, the control laws efficiently utilize multiple control actuation devices, including aerodynamic surfaces and thrust vectoring, to achieve the highest levels of aircraft performance. In every case, the control laws augment aircraft to achieve satisfactory handling characteristics. Because of the multivariable nature of the flight control laws, there are some challenges like the multiple control surfaces, multiple sensors, multiple disturbances internal or external, multiple control objectives for lateral, longitudinal and directional maneuvering and finally the unmodeled dynamics which called multiple uncertainties [3], [6]. To maintain high maneuverability, a nonlinear dynamic inversion controller is designed. The rotation is achieved by actuating the control surfaces such as elevators, ailerons, and a rudder. Attitude control is generally conducted using two loops the first rotation control loop for fast-time scale state dynamics, and attitude control loop for slow-time scale state dynamics. The two-time scaled DI controller block diagrams is shown in Fig. 1 [1]. DI approach is used to achieve $\alpha_d, \beta_d, \varphi_d$ that reduce the controller order to simplifies the design process.

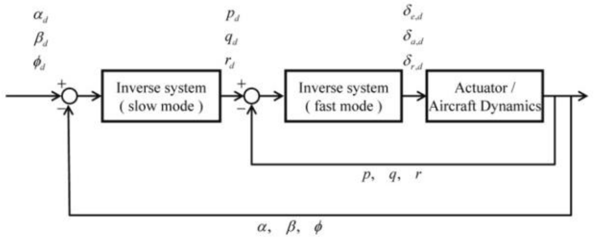

Fig. 1 Two-time scaled dynamic inversion controller

The inner loop corresponds to the fast state dynamics p, q, r are controlled by $\delta_a, \delta_e, \delta_r$. The outer loop corresponds to the slow-state dynamics α, β, φ which are controlled by angular body rates. Regarding the outer loop, the transient dynamics of the fast states occur so quickly that they have a negligible effect on the slow states. In the inner loop, slow time scale variables such as V, α, β are assumed to remain constant. The fast-time scale controller aims to maintain p_d, q_d, r_d close to their outer loop values.

A. Inner Fast Loop

The inner closed loop system bandwidth must be faster than the successive outer loop with ratio between 2: 5 to ensure replacing of the inner loop with unity during the execution of the outer fast loop [4], [11]. The fast loop dynamics body rates are controlled by the control surface commands. The desired closed-loop fast dynamics used is:

$$\dot{p}_d = \omega_p \left(p_c - p \right) \tag{11}$$

$$\dot{q}_d = \omega_q \left(q_c - q \right) \tag{12}$$

$$\dot{r}_d = \omega_r \left(r_c - r \right) \tag{13}$$

where $\omega_p, \omega_q, \omega_r$ are the band width frequency set as high as they can be without exciting structural modes or being subject to the bandwidth limitations of the control actuators. The frequency of the roll, pitch and yaw channels is set to be 10 [Hz] to ensure the avoiding of coupling between inner and outer loop moreover [13], these values are designed either by plotting the Bode plot or by plotting the fast Fourier Transform of the inner loop fast states with Chirp signal as control inputs [14]. The roll, pitch and yaw moments coefficients are as follow:

$$C_l = C_{l_\beta}\beta + C_{l_r}\left(\frac{rb}{2V_T}\right) + C_{l_p}\left(\frac{pb}{2V_T}\right) + C_{l_{\delta_a}}\delta_a + C_{l_{\delta_r}}\delta_r \tag{15}$$

$$C_n = C_{n_\beta}\beta + C_{n_r}\left(\frac{rb}{2V_T}\right) + C_{n_p}\left(\frac{pb}{2V_T}\right) + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r \tag{16}$$

$$C_l = C_{mo} + C_{m_\alpha}\alpha + C_{m_q}\left(\frac{cq}{2V_T}\right) + C_{m_{\dot\alpha}}\dot{\alpha} + C_{m_{\delta_e}}\delta_e \tag{17}$$

Submitting with (14) and (11) into the moment (2), with some mathematical manipulations. The following inner loop control laws are obtained:

$$\begin{bmatrix} \delta_a \\ \delta_e \\ \delta_r \end{bmatrix} = G(x)^{-1} + \left(\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} - \begin{bmatrix} f_p(x) \\ f_q(x) \\ f_r(x) \end{bmatrix} \right) \tag{18}$$

where $f_p(x), f_q(x), f_r(x)$ are the nonlinear terms of the moment (2) which is independent of $\delta_a, \delta_e, \delta_r$. $G(x)^{-1}$ is the pseudo inverse of the control matrix that includes the variables related to the control surface commands.

The inner loop simulation focuses on the ability of the nonlinear dynamic inversion control laws to degrade the coupling between roll and yaw channels. In addition to that, the performance of tracking sever flight maneuvers is analyzed by drawing the 3D trajectory by initially starting with positions x=y=0, h=200 [m].

- Roll rate commands while regulating other channels.

The conventional configuration of the utilized UAV is suffering from the existence of a coupling between the roll and yaw planes. Consequently, the excitation roll rate affect the yaw rate channel. In contrast, the pitch rate appeared to be blind in responding to the roll rate commands but because of the initial trim value of pitch angle, it forces the pitch rate non-zero negative. UAV roll rate follows the desired command with efficient transient response in addition to minimum steady state error which confirms the flight controller ability to respond to sever roll maneuver as shown in the upper of Fig. 2. Coupling between Yaw and roll channels is negligible as shown in the lower of Fig. 2. There is no coupling between the longitudinal plane and lateral one as shown in the middle of Fig. 2 that represents the pitch rate response. The flight control system drives the UAV with a powerful performance in tracking sharp roll maneuvers while the overall stability is guaranteed as shown in Fig. 3.

- Yaw rate commands while regulating other channels.

With the same previous tracking roll rate commands, the yaw rate time domain analysis highlights the performance of tracking sharp variations commands from positive to negative as shown in the lower of Fig. 4. One of the nonlinear flight controller’s problems is the coupling effect between lateral states, however the decoupling is one of the pivotal assumptions of the linear control theory even it usually deceives the system by decreasing its robustness against the model uncertainties which disappear while using the nonlinear dynamic inversion technique. The roll rate response is a witness for the effectiveness of the DI method for decreasing the coupling as shown in the upper of Fig. 4. Again, the longitudinal plane represented in pitch rate q ensures that there is no coupling as shown in the middle of Fig. 4. The trajectory of the UAV is pointing out to the potential of the flight control system to do maneuvers with very high performance. Even though the UAV loses little altitude while yawing around z-axis but after finishing the maneuver the flight controller forces the vehicle to increase its speed with small pitch angle to return again to the initial altitude h = 200 [m] as shown in Fig. 5.

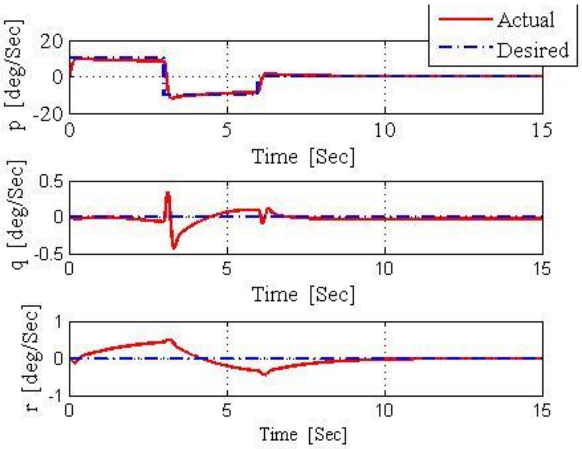

Fig. 2 Roll rate excitation with coupling effect in yaw and pitch rates

- Pitch rate commands while regulating other channels

The pitch rate q seems to be a good tracking system for the desired input which appears in the time domain analysis of the pitch rate response as shown in the middle of Fig. 6, taking into consideration the initial negative value of pitch angle which initially forces the transient response to be delayed. Commands in longitudinal channels have no effect in the lateral channels, consequently the roll and yaw rates don’t suffer from coupling as in the previous lateral channels. The roll and yaw rate responses can be neglected as shown in the upper and lower of Fig. 6 respectively. The trajectory of the UAV illustrated in Fig. 7 is completing the effectiveness of the designed fast inner loop flight simulation model in responding to the sharp maneuvers in all channels. Although increasing the pitch angle to very high value is critical because of the stalling effect, but the pitch channel controller is robust enough to respond to the pitch maneuver without losing the initial altitude or going into the stall mode.

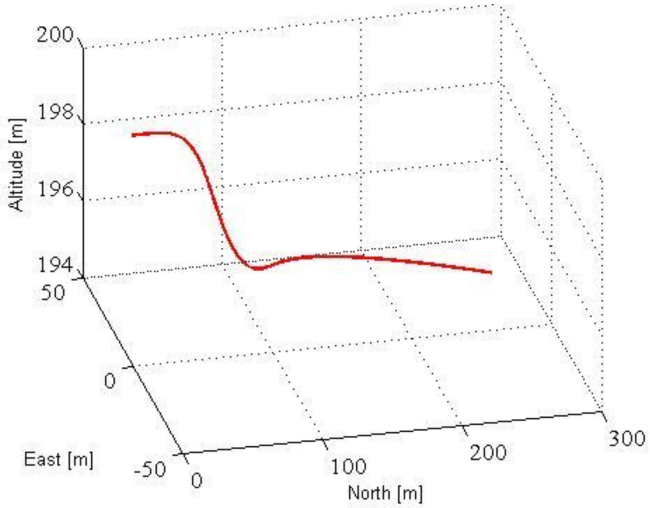

Fig.3 Trajectory of sharp roll maneuver

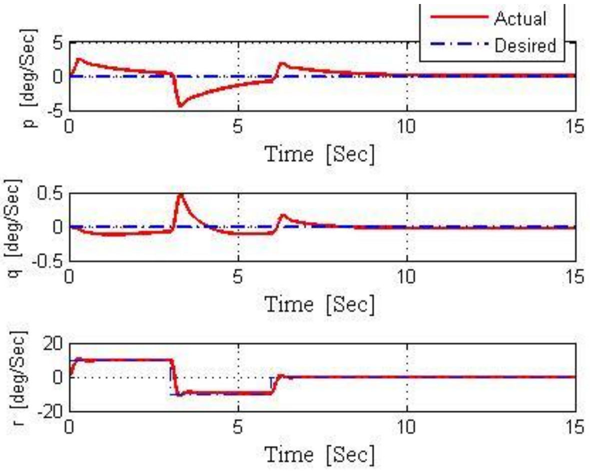

Fig. 4 Yaw rate excitation with coupling effect in roll and pitch rates

V. CONCLUSION

The paper discusses the powerful usage of nonlinear dynamic inversion as a common approach of deriving control laws for a small UAV. The high nonlinearity, multiple control surface and multiple control objectives for longitudinal, lateral and directional maneuvering are the challenges opposes flight control system while the simulation results demonstrate the effectiveness of the designed control law using dynamic inversion.

REFERENCES

[1] R. W. Beard and T. W. Mclain, Small Unmanned Aircraft: Theory and Practice. Princeton University Press, 2012.

[2] A. M. Kamal, Bayoumy and A. M. Elshabka, “Modeling and flight simulation of unmanned aerial vehicle enhanced with fine tuning,” Aerospace Science and Technology, vol. 51, pp. 106–117, 2016.

[3] Y. Sato, T. Yamasaki, H. Takano, and Y. Baba, “Trajectory guidance and control for a small uav,” International Journal of Aeronautical and Space Sciences, vol. 7, no. 2, pp. 137–144, 2006.

[4] J.-J. E. J.-J. E. Slotine and W. Li, Applied nonlinear control. Prentice Hall, 1991.

[5] L. F. Faleiro, The Application of Eigenstructure Assignment to the Design of Flight Control Systems. DLR, 1999.

[6] S. Sieberling, Q. P. Chu, and J. A. Mulder, "Robust flight control using incremental nonlinear dynamic inversion and angular acceleration prediction," Journal of Guidance Control and Dynamics, vol. 33, no. 6, pp. 1732–1742, 2010.

[7] A. Kamal, A. M. Aly, and A. Elshabka, "Modeling, analysis and validation of a small airplane flight dynamics," in AIAA Modeling and Simulation Technologies Conference, 2013.

[8] S. A. Snell, D. F. Nns, and W. L. Arrard, "Nonlinear inversion flight control for a supermaneuverable aircraft," Journal of Guidance Control and Dynamics, vol. 15, no. 4, pp. 976–984, 2012.

[9] D. Boukraa, Y. Bestaoui, and N. Azouz, "3d trajectory tracking for a fixed wing unmanned aerial vehicle using dynamic inversion," IFAC Proceedings Volumes, vol. 40, no. 7, pp. 443–449, 2007.