A New Soft Approach for Statistical Hypothesis Testing Problems With Data Fusion Using Neyman Pearson Criterion

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Abstract

Problem of statistical hypothesis testing arises in the context of digital communication and detection systems. These systems employ several receivers/sensors to observe a source of message information/decisions produces a bit 0 or 1. The observed decisions are reported to a data fusion processor which is responsible for combining the received decisions from the various sensors into a final global decision. This approach is called hard-decision approach. The alternative approach for decision fusion is the soft decision approach where each sensor report a measure of uncertainty or confidence value for each hypothesis to the data fusion processor. The soft decision approach has the advantage of better performance over a comparable hard-decision approach. This paper proposes a new soft decision approach in statistical hypothesis testing problems with data fusion using Neyman Pearson criterion. The performance of the proposed approach is evaluated using Monte Carlo simulations and compared to that of a hard-decision approach. The proposed approach is simple and shows better performance.

1. Introduction

There is an increasing interest in simultaneously employing multiple sensors for observing a source of digital information. In military application, this problem arises in detection of targets. The basic goal of such multiple sensor systems is to improve system performance, for example, reduce the probability of error. This can be achieved by integrating the information obtained from the various sensors. There are two major options for hard-decision in multisensor distributed communication/detection systems. The first option is centralized option (centralized detection system) where all sensors observations are transmitted to a central processor to derive a global decision. This requires transmission of all sensors observations without delay, which requires a large communication bandwidth. The second option is decentralized option (decentralized detection

system with fusion) where the signal processing is distributed among the sensors and the fusion processor. The sensors are allowed to derive local decisions; then the fusion processor is responsible for integrating the received decisions from the various sensors into a final global decision.

In contrast to the soft decision approach, which allows the information to be integrated over a wide range of signal level, the hard-decision approach does not provide any information to the fusion processor for signals below the decision threshold. Several work has been reported to explore the fusion of hard and soft sensors decisions. Tenney and Sandell [5] have the pioneering effort in extending the Bayesian decision theory to the case of distributed sensors. Z. Chair et al. [6] derived the data fusion structure to be used at the fusion center which minimize the overall probability of error. Thomopoulos et al. [8] derived the optimum fusion rule for the fusion of hard and semisoft (quality identified in a single bit) using Neyman Pearson criterion. E. Waltz [3] showed that the soft-decision has provided time and range improvements over a comparable hard-decision system.

This paper proposes a soft decision approach based on fuzzy logic techniques. The proposed fuzzy decision approach does not require prior statistical knowledge of the sensing process (conditional probability matrix). The optimum fusion rule using the proposed approach is derived. The performance of the proposed approach is evaluated and compared to the performance of the hard-decision approach. The proposed approach provides detection probability improvement over a comparable hard-decision system, thus it reduces the performance loss between the centralized and the decentralized (hard-decision) approaches.

2. Review of Centralized and Decentralized Approaches

In the centralized approach, all sensor observations are transmitted to a central processor in order to derive a global decision u_0. No local decisions are made by the sensors. Under each hypothesis, the sensors observations have known joint probability densities $P(y_1, y_2, \ldots\ldots y_n | H_0)$ and $P(y_1, y_2, \ldots\ldots y_n | H_1)$, where $y_i s, i = 1, 2, \ldots\ldots, n$ are random vectors representing the sensors observations. The crux of the centralized hypothesis testing problem is to derive a decision strategy of the form:

$$u_o = \begin{cases} 0 & H_o \text{ is declared to have been detected} \\ 1 & H_1 \text{ is declared to have been detected,} \end{cases} \tag{1}$$

where u_0 depends on the observations. According to Neyman-Pearson criterion, it is required to find a decision strategy expressed as a density function $P(u_0 | y_1, y_2, \ldots, y_n)$ which maximizes the global detection probability (GDP) for a desired global false alarm probability ($GFAP$) where

$$GFAP = \Pr\{u_0 = 1| H_0\}, \tag{2}$$

$$GDP = \Pr\{u_0 = 1| H_1\} . \tag{3}$$

The solution of the centralized problem is [5]:

(a) deterministic, so that the decision rule is a function

$$\gamma(\mathbf{y_1}, \mathbf{y_2}, \ldots \mathbf{y_n}) \longrightarrow \{0,1\}, \tag{4}$$

where $u_0 = i$ is interpreted as choosing H_i, and

(b) given by a Likelihood Ratio Test

$$\gamma(\mathbf{y_1}, \mathbf{y_2}, \ldots \mathbf{y_n}) = \begin{cases} 0 \text{ if } \mathbf{Lr}(\mathbf{y_1}, \mathbf{y_2}, \ldots \mathbf{y_n}) < \mathbf{t_0} \\ 1 \text{ if } \mathbf{Lr}(\mathbf{y_1}, \mathbf{y_2}, \ldots \mathbf{y_n}) \ge \mathbf{t_0}, \end{cases} \tag{5}$$

where

$$Lr(y_1, y_2, \ldots y_n) = \frac{P(y_1, y_2, \ldots y_n | H_1)}{P(y_1, y_2, \ldots y_n | H_0)}, \tag{6}$$

(c) the threshold t_0 is determined according to the desired $GFAP$.

We now consider the structure of the decentralized detection system. This approach greatly reduces channel capacity for two reasons. First, a report of a decision is a simpler message than a sensor observation, and second, most observations need not be reported at all since they don't correspond to a detection. In this approach, a number of sensors n receive and process the observations y_i's to generate the sensor decisions u_i, $i = 1, 2, 3, \ldots, n$ with $u_i = 1$ decide target present and $u_i = 0$ decide target absent. The Likelihood ration test ($LHRT$) can be determined according to equation (6) and compared to a threshold using equation (5). Equation (6) represents the ratio between the joint probability densities under both hypothesis. The plot of the hard-decision versus the $LHRT$ for a given threshold t_0 is shown in Fig. 1. The optimum data fusion structure using Neyman Pearson criterion is derived in [2], [6] and [8]. The individual

decisions are weighted according to the false alarm and detection probabilities of each sensor (pf_i, pd_i). The optimum data fusion structure is given by:

$$u_0 = \begin{cases} 0 \text{ if } \sum_{i=1}^{n} a_i u_i < T_0 \\ 1 \text{ if } \sum_{i=1}^{n} a_i u_i \geq T_0, \end{cases} \tag{7}$$

where

$$a_i = \frac{pd_i(1 - pf_i)}{pf_i(1 - pd_i)}, \tag{8}$$

and the threshold T_0 is determined from the desired *GFAP*. When all sensors are similar and have a common operating point (pf, pd), all the coefficients a_i's in (8) are equal, hence the optimum data fusion rule reduces to:

$$u_0 = \begin{cases} 0 \text{ if } \sum_{i=1}^{n} u_i < k \\ 1 \text{ if } \sum_{i=1}^{n} u_i \geq k, \end{cases} \tag{9}$$

where k is a positive integer. For $k = 1$, (9) reduces to an *OR* fusion rule while for $k = n$, it becomes an *AND* fusion rule. The *GFAP* and *GDP* corresponding to (9) are given by:

$$GFAP = \sum_{i=k}^{n} c_i^n \, pf^i \, (1 - pf)^{n-i}, \tag{10}$$

$$GDP = \sum_{i=k}^{n} c_i^n \, pd^i \, (1 - pd)^{n-i}, \tag{11}$$

where

$$c_i^n = \frac{n!}{i!\,(n-i)!} \tag{12}$$

Thus, for every desired value of *GFAP*, there is an optimum integer k that maximizes the *GDP*.

3. Proposed Fuzzy Logic Decision approach

We assume that there are n detectors with statistically independent observations $y_i, i = 1, \ldots, n$. Instead of reporting the sensors hard- decisions to the fusion center, each sensor is allowed to derive a soft-decision μ_i by defining a fuzzy set A_i in X as a set of ordered pairs :

$$\mathbf{A_i} = \{(\mathbf{x}, \mu_{Ai}(\mathbf{x})) \,|\, \mathbf{x} \in \mathbf{X} \}, i = 1, 2, \ldots, n, \tag{13}$$

where $\mu_{Ai}(x)$ is called the membership function or grade of membership of x in A_i which maps X to the membership space M in the interval $[0,1]$. If M contains only the values of 0 and 1, A_i is nonfuzzy set and $\mu_{Ai}(x)$ is identical to a nonfuzzy set (hard decision set). If $\mu_{Ai}(x)$ is greater than 0.5, the sensor will

favor hypotheses H_1 and the corresponding hard decision will be $\mu_i = 1$. If $\mu_{A_i}(x)$ is less than 0.5, the sensor is more likely to favor hypotheses H_0 and the corresponding hard decision is $u_i = 0$. Thus the relation between the hard decision u_i and the soft decision μ_i is then given by:

$$u_i = \begin{cases} 1 & \text{if } \mu_i \geq 0.5 \\ 0 & \text{if } \mu_i < 0.5 \end{cases} \quad . \tag{14}$$

In many cases, it is convenient to express the membership function of a fuzzy subset in terms of a standard function with adjustable parameters. Our human expertise contains two heuristics :

1- as the difference between the Likelihood function and the threshold increases, the corresponding confidence (the grade of membership) of the decision increases and vise versa,

2- if the Likelihood function is equal to the threshold a value of 0.5 is a suitable value of the membership function in this case.

According to heuristics 1 and 2, a suitable membership function can be defined as:

$$\mu_A(x;\, \alpha, \beta, \gamma) = \begin{cases} 0 & \text{if } x \leq \alpha \\ 2\left[\dfrac{x-\alpha}{\gamma-\alpha}\right]^2 & \text{if } \alpha \leq x \leq \beta \\ 1 - 2\left[\dfrac{x-\alpha}{\gamma-\alpha}\right]^2 & \text{if } \beta \leq x \leq \gamma \\ 1 & \text{if } x \geq \gamma \end{cases} \tag{15}$$

where x represents the likelihood ratio, β represents the sensor threshold. The actual values of γ and α depend on the expected signal range. The assigned membership function is shown in Fig. 2.

Let $\mathbf{u}$ be the vector formed of the sensors hard-decisions, i.e.,

$$\mathbf{u} = (u_1, u_2, u_3, \ldots\ldots, u_n) \, . \tag{16}$$

The Likelihood function of the fusion center is given by

$$Lr(\mathbf{u}) = \frac{P(\mathbf{u}|H_1)}{P(\mathbf{u}|H_0)} = \frac{P(u_1, u_2, \ldots\ldots, u_n|H_1)}{P(u_1, u_2, \ldots\ldots, u_n|H_0)} \quad . \tag{17}$$

From the independent assumptions of the observations, we can write

$$Lr(\mathbf{u}) = \frac{P(\mathbf{u}|H_1)}{P(\mathbf{u}|H_0)} = \prod_{i=1}^{n} \frac{P(u_i|H_1)}{P(u_i|H_0)} \quad . \tag{18}$$

Equivalently, we can write

$$P(\mathbf{u}|H_1) = \prod_{S+} P(u_i = 1|H_1) \prod_{S-} P(u_i = 0|H_1) \, , \tag{19}$$

$$P(\mathbf{u}|H_0) = \prod_{S+} P(u_i = 1|H_0) \prod_{S-} P(u_i = 0|H_0), \tag{20}$$

where $S+$ is the set of all i such that $u_i = 1 (\mu_i \geq 0.5)$, $S-$ is the set of all i such that $u_i = 0 (\mu_i < 0.5)$, and

$$\mathbf{P(u_i = 1 | H_1)} = \mathbf{P(\mu_i \geq 0.5 | H_1)} = \mathbf{pd_i},$$

$$\mathbf{P(u_i = 0 | H_1)} = \mathbf{P(\mu_i < 0.5 | H_1)} = \mathbf{1 - pd_i},$$

$$\mathbf{P(u_i = 1 | H_0)} = \mathbf{P(\mu_i \geq 0.5 | H_0)} = \mathbf{pf_i}, \tag{21}$$

$$\mathbf{P(u_i = 0 | H_0)} = \mathbf{P(\mu_i < 0.5 | H_0)} = \mathbf{1 - pf_i}$$

and pf_i and pd_i are the false alarm and the detection probabilities of the ith sensor respectively. The corresponding log Likelihood ratio test is

$$Log\,Lr(\mathbf{u}) = \sum_{S+} \log\frac{pd_i}{pf_i} + \sum_{S-} \log\frac{1 - pd_i}{1 - pf_i}. \tag{22}$$

Therefore the data fusion rule can be expressed as

$$u_0 = \begin{cases} 0 & \text{if } \sum_{i=1}^{n} b_i\,\mu_i < \log\lambda_0 \\ 1 & \text{if } \sum_{i=1}^{n} b_i\,\mu_i \geq \log\lambda_0, \end{cases} \tag{23}$$

where $\log\lambda_0$ is determined according to the desired *GFAP* and the optimum coefficients $b_i, i = 1, 2, ..., n$ are given by

$$\mathbf{b_i} = \begin{cases} \log\dfrac{\mathbf{pd_i}}{\mathbf{pf_i}}\square & \text{if } \mu_i \geq 0.5 \\[3mm] \log\dfrac{\mathbf{1 - pd_i}}{\mathbf{1 - pf_i}}\square & \text{if } \mu_i < 0.5. \end{cases} \tag{24}$$

4. Computer Simulation and Examples

We assume the case of n-identical sensors with Gaussian distributed observations with mean value s; i.e.

$$\mathbf{P(y_i | H_\theta)} = \frac{1}{\sqrt{2\pi}} e^{-(y_i - \theta s_i)^2/2}, \theta = 0, 1, s_i > 0. \tag{25}$$

The Neyman-Pearson test, utilizing all of the received observations y_i's in case of the centralized detection system will have the form

$$u_0 = \begin{cases} 0 & \text{if } \sum_{i=1}^{n} y_i < T \\ 1 & \text{if } \sum_{i=1}^{n} y_i \geq T. \end{cases} \tag{26}$$

To achieve a desired *GFAP*, a threshold of

$$T = \sqrt{n}\,\phi^{-1}(GFAP), \tag{27}$$

is needed at the fusion center, where the ϕ function is defined as

$$\phi(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \, e^{-z^2/2} \, dz .$$ (28)

The corresponding GDP is given by

$$GDP = \phi\left(\frac{T - ns}{\sqrt{n}}\right) .$$ (29)

The decision rules of the sensors in case of the decentralized detection systems are given by

$$u_i = \begin{cases} 0 & \text{if } Lr(y_i) = \dfrac{p(y_i|H_1)}{p(y_i|H_0)} < \eta_i \\[2mm] 1 & \text{if } Lr(y_i) = \dfrac{p(y_i|H_1)}{p(y_i|H_0)} \geq \eta_i . \end{cases}$$ (30)

The corresponding false alarm and detection probabilities are

$$\mathbf{pf_i} = \phi(\eta_i),$$
$$\mathbf{pd_i} = \phi(\eta_i - s_i),$$ (31)

where η_i is the ith detector threshold and is determined according to the sensor false alarm probability.

The common signal to noise ratio (SNR) of the sensors is evaluated as

$$SNR = \frac{[E_1\{y_i\} - E_0\{y_i\}]^2}{Var_0\{y_i\}} = s_i^2, \; i = 1, \ldots, n,$$ (32)

where $E_\theta\{y_i\} = E\{y_i|H_\theta\}, \theta = 0,1$ and $Var_0\{y_i\} = Var\{y_i|H_0\}$. (33)

The form of the membership of Eq.(13) is considered in the simulation. The parameter β, $\beta = (\alpha + \gamma)/2$, is the crossover point. The values of α and γ are taken to be

$$\alpha = \beta - 3\sigma,$$
$$\gamma = \beta + 3\sigma,$$ (34)

where σ is the standard deviation of the noise. The fusion center performance is described as the receiver operating characteristic (ROC), which plots the detection probability versus the false alarm probability. Fig. 3 compares the global performance improvement in the centralized and the decentralized schemes in case of five identical sensors with Gaussian distributed observations and $0\,dB$ per sensor observations. Fig. 3 also shows the common sensors ROC. The global performance improvement of data fusion systems (centralized or decentralized) over the individual sensor's ROC is obvious. The performance loss due to the decentralized approach compared to the centralized approach is also obvious. Fig. 4 depicts the same plots using the proposed fuzzy decision approach.

Comparing Fig. 3 and 4, it is clear that the proposed fuzzy decision approach has better performance over the decentralized approach. Thus the fuzzy decision approach reduces the performance loss between the centralized and the decentralized approaches. It is worth noting that data transmission over small communication bandwidth provides system engineering features such as low cost, immunity to jamming, and longer communication range. The performance trade-off between centralized, decentralized, and soft decision approaches allowing us to choose a preferred communication architecture.
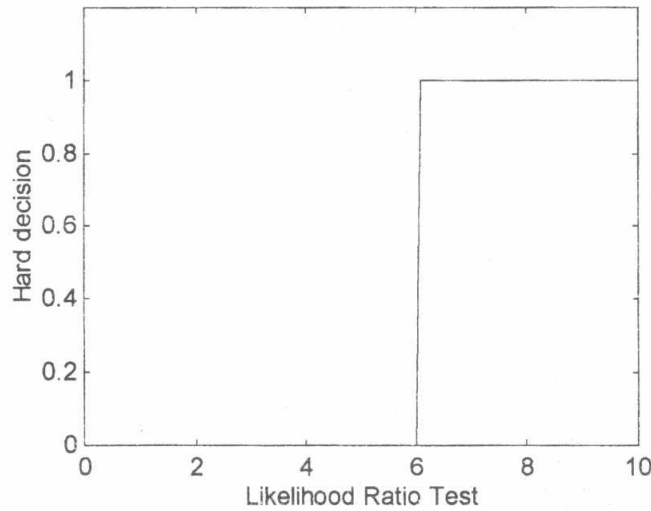
Fig.1 Plot of Hard decision Versus LHR
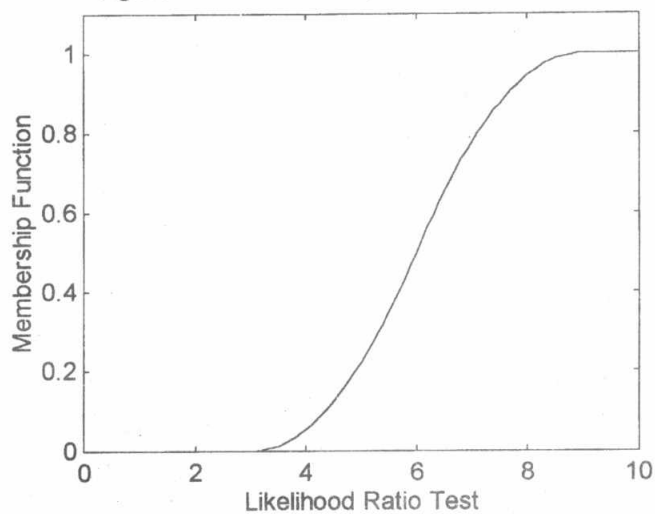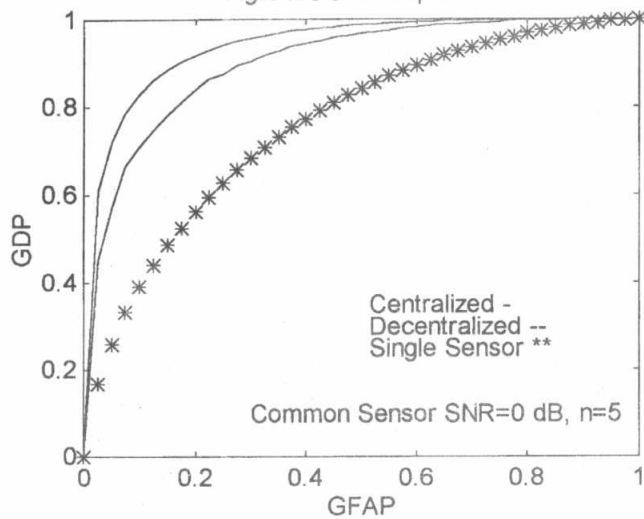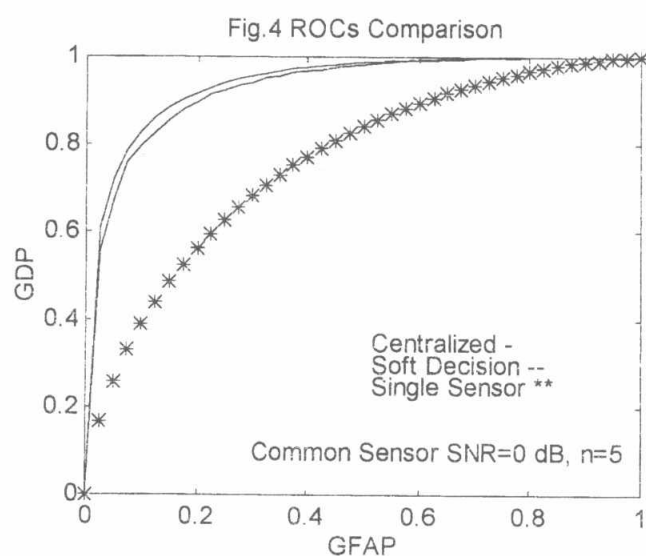
Fig.2 Plot of Membership Function Versus LHR

Fig.3 ROCs Comparison

Centralized -
Decentralized --
Single Sensor **

Common Sensor SNR=0 dB, n=5

Fig.4 ROCs Comparison

Conclusion

In this paper, a fuzzy decision approach in multisensor distributed detection systems has been proposed. The proposed approach has detection probability improvement over a comparable hard-decision approach. we have attempted to obtain and compare the global performance improvement in centralized, decentralized and the proposed fuzzy detection systems in the case five sensors with Gaussian distributed observations. It has been found that the proposed fuzzy decision approach reduces the performance loss between the centralized and the decentralized approaches. The result is important, since it characterizes the performance trade-off between the centralized, decentralized and soft decision approaches, allowing us to choose a preferred communication architecture.

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