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AN ADAPTIVE RECEIVER FOR M-ARY SIGNALS FOR RAPIDLY TIME VARYING FADING CHANNELS

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ABSTRACT

The paper presents an adaptive receiver structure for reception of *M*-ary signals over a rapidly time varying mobile communication channel. The channel impulse response is expanded into a set of basis sequences and a time invariant (TI) expansion parameters. The proposed receiver continuously estimates the time invariant expansion parameters directly within the metric calculation of the log-likelihood function in a recursive manner. The performance of the receiver is evaluated in terms of the misdetection probability and is compared to other receivers. The effects of timing and phase offsets on the performance of receiver are examined by simulation. Finally, the proposed receiver can accommodate the rapidly time varying channel with adequate performance.

KEY WORDS

Wireless Communications, Adaptive Signal Processing, and Fading.

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I. INTRODUCTION

The mobile radio channel is characterized by multipath fading. Multipath is a sever impediment to reliable communications over the fading channel. A moving environment or mobile leads to a rapidly varying channel, where conventional receiver structures which designed as optimal for the additive white Gaussian noise (AWGN) channel, exhibit a limited performance even at high signal to noise ratio. Then, the development of mobile cellular resulted in increasing interest in the study of signal reception in the presence of this rapidly time varying environment. If the channel exhibits intersymbol interference (ISI) in the received data, adaptive equalizers [1-5], that use adaptive algorithms [6,7], are used to recover the received signal and to remove the ISI. When the channel is rapidly varying, tracking these variations become a challenging problem and hence the equalizer may not be able to track the channel. In this paper, such rapidly fading channels are considered, specifically, the channel model described in [8, 9, 10, 11 p.383] is used. In this model, the time varying channel taps are modeled by a finite linear combination of complex exponentials. The basis expansion approach of [8, 10] is used in this paper and the time varying channel coefficients are expanded into a set of basis sequences and expansion parameters. These basis sequences are assumed to be known while the expansion parameters are unknown and needs to be identified. In [8], higher order statistics are used to estimate the expansion parameters based on minimizing a moment matching criterion and use them to estimate the time varying channel coefficients. It is proved that identifiably of the channel can not be achieved from the second order cumulants. It requires fourth order cumulant to identify it under the assumption of linear independence on the basis sequences.

In this paper, an optimal adaptive receiver structure for reception of M-ary signals over a rapidly time varying fading channel is presented. The receiver is based on maximum likelihood criterion. The channel impulse response is expanded onto a set of basis sequences and a time invariant (TI) expansion parameters. The proposed receiver continuously estimates the time invariant expansion parameters directly within the metric calculation of the log-likelihood function. The recursive least square (RLS) approach is used to perform this estimation. The performance of the receiver is demonstrated in the simulation section and is compared to other receivers. Also, the effects of timing and phase offsets are examined by simulation.

The paper is organized as follows. In section II, the proposed receiver is derived. In section III, the results of the computer simulation to demonstrate the performance of the receiver are provided. Conclusions are provided in section IV.

II. MATHEMATICAL DERIVATION OF THE PROPOSED RECEIVER

A. Problem Statement and System Model

A data sequence is transmitted over a rapidly time varying channel using one out of *M* signals. The sampled signals at time instant *k* is denoted by $s_i(k)$; $i = 1,2,...,M$.

The discrete equivalent (or combined) impulse response of the channel including transmitter filter is denoted by $c(k,l)$ where *l* represents the channel memory ($l = 0,1,...,L-1$). The unknown channel taps are correlated even if the scatters in the physical channel are uncorrelated. The additive noise is assumed to be white Gaussian noise. The received signal is filtered and sampled at a rate of f_s. The problem is formulated as follows. Given the discrete received signal $r(k)$, the receiver must decide which signal $s_i(k)$ has been sent where $s_i(k)$ is the sampled transmitted signal. That is, the hypotheses H_i; $i = 1,2,...,M$, over the observation interval N_s, are given by

$$H_i : r(k) = \sum_{l=0}^{L-1} s_i(k-l)c(l,k) + n(k), \tag{1}$$

where *L* is the channel memory length and *n(k)* are independent and identically distributed (i.i.d) complex valued zero mean Gaussian noise samples with known variance σ_n^2. The optimum maximum likelihood (ML) receiver chooses the hypothesis H_i with the largest likelihood function, but it requires perfect knowledge of the channel time varying coefficients, $c(k,l)$. These coefficients are usually modeled as Gaussian random process. However, a more precise description of the time variations of the channel coefficients can be provided for the multipath channels, which have small number of reflectors. For example, for constant vehicle velocity, the mobile radio channel is almost periodically varying when the multipath delays change linearly with time due to the carrier modulation inherent in the transmitted signal [8], [12], and [11, p. 383]. Its time varying coefficients can be expressed as a combination of exponentials whose frequency depends on the carrier frequency and the vehicle speed [10]. In this paper, we consider the channels, which their time varying coefficients $c(k,l)$ can be approximated by a linear combination of a finite number of basis sequences $f_n(k)$:

$$c(l,k) = \sum_{n=1}^{N} \theta_{nl} f_n(k) \tag{2}$$

where θ_{nl} are non-random expansion parameters and $f_n(k)$ are basis sequences. For fast mobile radio channels, these basis sequences are expressed as [8]:

$$f_n(k) = \exp\{j\alpha_n k\} \tag{3}$$

where α_n are some frequencies; $n = 1,2,...,N$. These frequencies are assumed to be known and estimation of these frequencies are found in [8]. Practical values are used for these basis sequences in the simulation section.

B. Recursive Evaluation of the ML Metric

In this subsection, the log-likelihood metric for the received signal is derived and evaluated recursively. Using (1), (2), and (3), $r(k)$ can be expressed as:

$$r(k)=\sum_{l=0}^{L-1}\sum_{n=1}^{N}\theta_{nl}\,s_i(k-l)\,f_n(k)+n(k), \qquad i=1,2,\ldots,M \tag{4}$$

Let us define the following vectors and matrices:

$$\Theta_l=[\theta_{1l},\theta_{2l},\ldots,\theta_{Nl}]^T \tag{5}$$

and

$$\mathbf{H}_{l,k}^{(i)}=\big[f_1(k)s_i(k-l) \quad f_2(k)s_i(k-l) \quad \ldots \quad f_N(k)s_i(k-l)\big] \tag{6}$$

where i is referred to the hypothesized signal. Let the coefficients Θ_l be assembled into the $(N\times L)\times 1$ unknown vector $\mathbf{A}$:

$$\mathbf{A}=[\Theta_0^T\ \Theta_1^T\ \ldots\ \Theta_{L-1}^T]^{T'} \tag{7}$$

and also

$$\mathbf{H}_k^{(i)}=[\mathbf{H}_{0,k}^{(i)}\ \mathbf{H}_{1,k}^{(i)}\ \ldots\mathbf{H}_{L-1,k}^{(i)}] \tag{8}$$

where the superscript T' denotes matrix transposition. Using the above definitions, we can rewrite (4) in the following representation;

$$r(k)=\mathbf{H}_k^{(i)}\,\mathbf{A}+n(k) \tag{9}$$

Let $\mathbf{r}=[r(1),r(2),\ldots,r(N_s)]^{T'}$ denotes the noisy received signal vector and $\mathbf{s}_i=[s_i(1),s_i(2),\ldots,s_i(N_s)]^{T'}$ denotes the i[th] transmitted signal vector. Since the observation noise is assumed to be white Gaussian, then the probability density function (PDF) of the received signal vector **r** under hypothesis H_i, conditioned on the channel parameters vector **A**, can be written as:

$$f(H_i,\mathbf{r}/\mathbf{A})=\frac{1}{\left(\pi\sigma_n^2\right)^{N_s}}\exp\left\{\frac{-1}{\sigma_n^2}\sum_{k=1}^{N_s}\left|r(k)-\mathbf{H}_k^{(i)}\,\mathbf{A}\right|^2\right\} \tag{10}$$

For equi-probable messages, the relevant conditional log-likelihood function (LLF) under hypothesis H_i may be written as:

$$\Delta_{N_s}(H_i;\mathbf{r}/\mathbf{A})=\sum_{k=1}^{N_s}\left|r(k)-\mathbf{H}_k^{(i)}\,\mathbf{A}\right|^2; \qquad i=1,\ldots,M \tag{11}$$

For a given **A,** the optimum ML receiver chooses the hypothesis $\hat{H}$ that maximizes

$f(H_i;\mathbf{r}/\mathbf{A})$ or, equivalently, minimizes $\Delta_{N_s}(H_i;\mathbf{r}/\mathbf{A})$ that is:

$$\hat{H}=H_i:\Delta_{N_s}(H_i;\mathbf{r}/\mathbf{A}) < \min_{q\neq i} \Delta_{N_s}(H_q;\mathbf{r}/\mathbf{A}) \qquad (12)$$

An estimate of **A** is required to evaluate (12). The evaluation of (12) and the estimation of **A** can be performed recursively in time as follows: Let $\hat{\mathbf{A}}_k^{(i)}$ denotes the estimation of the channel parameters vector under hypothesis *i* and at time step *k*. Then by substitution of $\hat{\mathbf{A}}_k^{(i)}$ into (11), we have the log-likelihood function $\Delta_{N_s}(H_i;\mathbf{r}/\hat{\mathbf{A}}_k^{(i)})$. In fact, if the equation:

$$\Delta_m(H_i;\mathbf{r}/\hat{\mathbf{A}}_k^{(i)})=\sum_{k=1}^{m}\left|r(k)-\mathbf{H}_k^{(i)}\hat{\mathbf{A}}_k^{(i)}\right|^2 \qquad (13)$$

is defined for $m=1,2,...,N_s$, we easily obtain the recursion

$$\Delta_m(H_i;\mathbf{r}/\hat{\mathbf{A}}_k^{(i)})=\Delta_{m-1}(H_i;\mathbf{r}/\hat{\mathbf{A}}_k^{(i)})+\left|r(m)-\mathbf{H}_m^{(i)}\hat{\mathbf{A}}_m^{(i)}\right|^2 \qquad (14)$$

where $\Delta_{m-1}(\mathbf{r}/\hat{\mathbf{A}}_k^{(i)})$ represents the evaluation of the LLF within the interval $k=1,2,...,m-1$. This formula suggests a recursive solution for the minimization of (11). A separate channel parameters vector estimate $\hat{\mathbf{A}}_k^{(i)}$ is created for each hypothesis using the observations and the hypothesized signal.

C. Estimation of the Channel Parameters Vector

Possible approaches to a sample-by-sample estimation of the channel parameters vector are gradient based methods (like LMS) and recursive least square based methods. It is important to observe that the true channel parameters vector is time invariant, so the task of the adaptive algorithm in the proposed approach is to converge to the channel parameters as opposed to tracking them. The covariance form of the least square (CRLS) approach [13] is chosen to perform this estimation due to its fast convergence. Applying this form results in the following algorithm:

$$\hat{\mathbf{A}}_k^{(i)}=\hat{\mathbf{A}}_{k-1}^{(i)}+\mathbf{K}_k^{(i)}[r(k)-\mathbf{h}_k^{(i)T'}\hat{\mathbf{A}}_{k-1}^{(i)}] \qquad (15)$$

where

$$\mathbf{K}_k^{(i)}=\mathbf{P}_{k-1}^{(i)}\mathbf{h}_k^{(i)}\left[\mathbf{h}_k^{(i)T'}\mathbf{P}_{k-1}^{(i)}\mathbf{h}_k^{(i)}+\frac{1}{w_k}\right]^{-1} \qquad (16)$$

is the weighting vector and,

$$\mathbf{P}_k^{(i)}=\left(\mathbf{I}-\mathbf{K}_k^{(i)}\ \mathbf{h}_k^{T'}\right)\mathbf{P}_{k-1}^{(i)} \qquad (17)$$

is the estimation error covariance matrix. The factor w_k is a weighting factor, which is set to one for equal weights. It is noted that, $\hat{\mathbf{A}}_k^{(i)}$ is the estimate of the channel parameters vector at time k under hypothesis i and the term $\mathbf{h}_k^{(i)T'}\hat{\mathbf{A}}_{k-1}^{(i)}$ in (15) is a prediction of the actual measurement $r(k)$ under hypothesis i. The above algorithm updates $\hat{\mathbf{A}}_k^{(i)}$ by iteratively adding an adjustment term. The adjustment term is given by a vector of weights $\mathbf{K}_k^{(i)}$, which is multiplied by the error $[r(k)-\mathbf{h}_k^{(i)T'}\hat{\mathbf{A}}_{k-1}^{(i)}]$ to determine the parameter change. The value of this error is small when there is a matching between the observation $r(k)$ and the term $\mathbf{h}_k^{(i)T'}\hat{\mathbf{A}}_{k-1}^{(i)}$ (i.e. when the observation $r(k)$ contains the correct hypothesis) and it is large when there is a mismatch between them. The estimation algorithm is initialize either using an initial guess $\hat{\mathbf{A}}_0^{(i)}$ for the channel parameters vector (since the channel parameters vector is time invariant) or using the following step [13]

$$\mathbf{P}_0^{(i)^{-1}} = \frac{1}{a^2}\mathbf{I}_{N\times L} + \mathbf{h}_0^{(i)}\, w_0\, \mathbf{h}_0^{(i)T'} \tag{18}$$

and

$$\hat{\mathbf{A}}_0^{(i)} = \mathbf{P}_0^{(i)}\left[\frac{1}{a}\boldsymbol{\varepsilon} + \mathbf{h}_0^{(i)}\, w_0\, r_0\right] \tag{19}$$

where a is a very large number, $\mathbf{I}_{N\times L}$ is the identity matrix with dimension $(N\times L)\times(N\times L)$ and $\boldsymbol{\varepsilon}=col(\varepsilon,\varepsilon,...,\varepsilon)$ is a $(N\times L)\times 1$ column vector and its elements ε is a very small number.

D. Structure of the Proposed Receiver

The structure of the receiver is shown in Fig.1. As explained above, the decision of the receiver is based on the evaluation of the function, $q_i = \Delta_{N_s}(H_i;\mathbf{r}/\hat{\mathbf{A}}_k^{(i)})$, then the receiver executes the following algorithm for each hypothesis ; $i = 1,2,...,M$:

(1) Start with an initial estimate for the channel parameters vector $\hat{\mathbf{A}}_0^{(i)}$ using (19).

(2) Use the observation and the hypothesized signal to find $\hat{\mathbf{A}}_1^{(i)}$.

(3) Substitute $\hat{\mathbf{A}}_1^{(i)}$ in (13) and then find $\Delta_1(H_i;\mathbf{r}/\hat{\mathbf{A}}_1^{(i)})$.

(4) Update the estimate of the channel parameters vector to find $\hat{\mathbf{A}}_2^{(i)}$ and then use it to evaluate $\Delta_2(H_i;\mathbf{r}/\hat{\mathbf{A}}_2^{(i)})$ using (14).

(5) Repeat steps (2) to (4) until all the data samples have been processed (i.e. when $k = N_s$) and then obtain q_i.

(6) Compare among q_i; i=1, 2,..., M and determine i that corresponds to the minimum value of q.

III. Computer Simulations and Results

In this section, the performance of the receiver is evaluated for the mobile radio-fading channel defined in (2) using Mont Carlo simulation. The channel has two time varying taps, each one is a linear combination of three basis sequences denoted by $f_{1,k}=1$, $f_{2,k}=\exp\{j\pi k/60\}$, and $f_{3,k}=\exp\{j\pi k/100\}$. These bases simulate a realistic situation for a 900 MHz carrier frequency, bit rate around 20 Kb/s and a vehicle speed of 100 Km/h. The values of expansion parameters θ_{nl} are given in Table I [8] are chosen so that the fading channel passes through the minimum and non-minimum phase regions. The input to the channel is a QPSK signal. The generated signal has independent and identically distributed symbols. The frame length is 512 samples. A white Gaussian noise is simulated and added to the signal at the input of the receiver. The signal to noise ratio is defined as SNR=10log(E_b/N_o) where $N_o/2$ is the noise power spectral density and E_b is the energy per bit. It is assumed that no phase and time offsets in the carrier. However, the effects of these offsets are studied in this section.

The performance comparison among the proposed receiver, the effect of model mis-match and the reference receiver is evaluated in terms of misdetection probability versus signal to noise ratio (SNR) at the input of the receiver. The results of comparison are shown in Fig.2. This figure shows that the proposed receiver has adequate performance and allows an effective tracking of the channel. The lower curve in Fig.2 represents the unrealistic case where the channel is assumed to be known to the receiver. Hence, this curve can be considered as a lower bound or a reference receiver for comparison purposes of the other receivers. Fig. 2 also shows that the degradation in the misdetection probability that results when an incorrect channel model is adopted. In this case, a random disturbance is added to the real and imaginary parts of the channel time varying coefficients to accounts for mis-match model. That is:

$$c(l,k)=\sum_{n=1}^{N}\theta_{nl}\, f_n(k)+e_l(k)$$

The disturbance $e_l(k)$ is generated as an i.i.d. Gaussian random variables with standard deviation 0.2. Then, the proposed algorithm is applied and the misdetection probability is evaluated. The simulated results indicate that the anticipated increase in misdetection probability under this mismatch condition is actually not significant at low SNR. This is because the additive noise dominated the performance at low range of SNR. As SNR increases, the influence of the additive noise decreases, and the model mis-match becomes the dominant source of degradation, causing deterioration in misdetection probability.

Figs. 3 and 4 demonstrate the effects of timing and phase offsets on the performance of the receiver respectively. The figures are plotted for SNR=0, 5, and 9 dB. These figures shows that the receiver is able to detect the signal reliably when phase or timing offset is small. When phase or timing offset increases, the receiver performance degrades rapidly. The reason for this degradation is that: the increase in these offsets causes increase in the residual error in estimation of the channel parameters vector $\mathbf{A}_k$ (since the estimation of $\mathbf{A}_k$ depends on the observations which is shifted in time or phase) and this is introduces an error in the log-likelihood metric calculation given by (18), accordingly, the misdetection probability degrades rapidly. Figures 3 and 4 also show that, there is a range in which the effect of phase or time offset can be neglected and the misdetection probability in this range is small. This range is shown in these figures for SNR=9 dB. This range depends on the SNR (it increases as the SNR increases) and it is up to t_o/T=0.4 for the timing offset and 0.5 rad. for phase offset, where T is the symbol duration.

IV. Conclusion

In this paper, an optimal adaptive receiver for reception of M-ary signals over a rapidly time varying mobile communication channel has been presented. The channel has been expanded into a set of basis sequences and time invariant expansion parameters. This expansion idea provides a helpful tool for addressing problems of such channels. The time invariant expansion parameters of the channel have been estimated continuously and directly within the metric calculation of the log-likelihood function in a recursive manner. The performance of the receiver has been evaluated in terms of the misdetection probability. The receiver provides adequate performance for this type of channel. Also, the effects of timing and phase offsets on the performance of receiver have been studied.

Table I. Basis expansion parameters [8]

θ_{nl}	$n=1$	$n=2$	$n=3$
$l=0$	1	j	2
$l=1$	1	0.5	$-j$

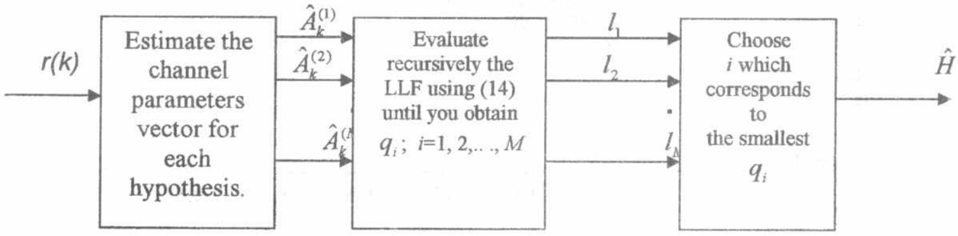

Fig.1. Structure of the proposed adaptive receiver for the rapidly time varying fading channel.

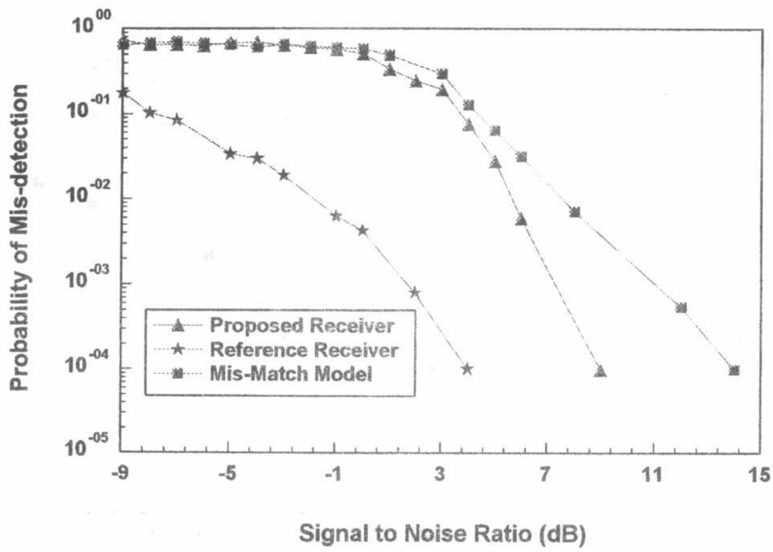

Fig. 2. Probability of misdetection for different receivers

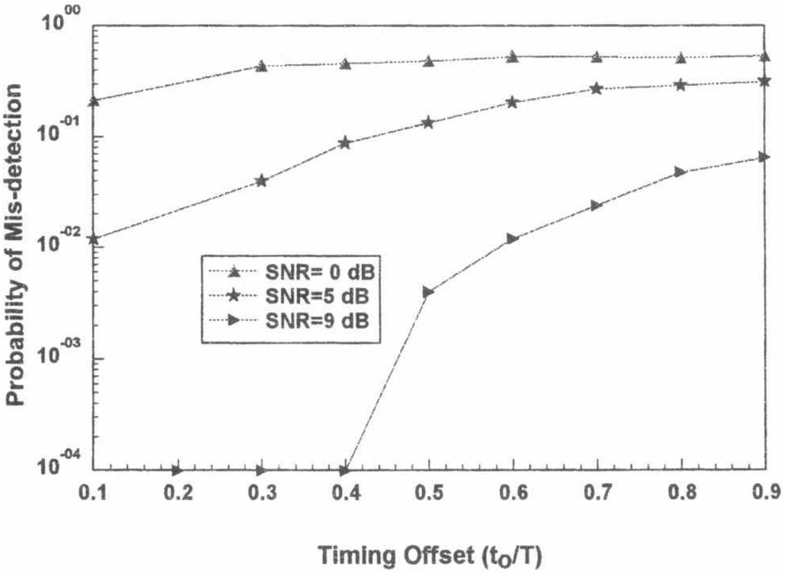

Fig. 3. Effect of timing offset on the performance of the receiver for different signal to noise ratios.

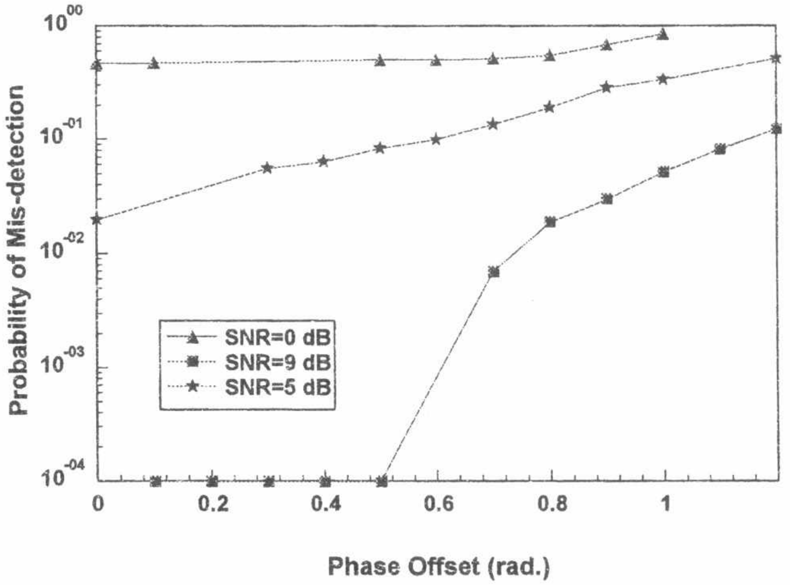

Fig. 4. Effect of phase offset on the performance of the receiver for different signal to noise ratios.

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