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A SIMPLIFIED ALGORITHM FOR THE IDENTIFICATION OF LONGITUDINAL DYNAMICS OF AN AIRCRAFT

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ABSTRACT

This paper presents a simple and efficient identification algorithm. The proposed algorithm can be used in a large class of plants whose dynamics can be fairly approximated by an under-damped second-order model. The presented study, determines the minimum time duration of a step pulse input, in order to produce output response, rich enough to generate the model parameters, within accepted accuracy limits.

The proposed approach can be applied for on-line estimation, of basic model parameters of an aircraft longitudinal dynamics, during flight.

Computer simulation results showed that the proposed simplified technique is capable to determine the unknown model parameters, within acceptable accuracy, and shorter time.

Due to easy application, and great reduction of computation time, which is a major problem in most indirect adaptive control algorithms, the proposed scheme can be used for adaptive control of a large class of real plants under actual practical conditions.

1. INTRODUCTION

In most approaches of empirical model identification, the model is identified by making small step change(s) in the input variable(s) about nominal operating conditions, the resulting dynamic response is used to determine the model parameters [1]. Many advanced statistical methods are available for more complex model structures [2]. These methods have the general concept of estimating the model parameters from the input-output relationships without control. Box and

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MacGregor [3] showed that the process model could be identified when being controlled, but only under specific conditions.

A common step in most of the empirical methods, presented in Literature [1-5], used step change, long-lasting, in the input variable in order to detect the output response. This long-lasting disturbance represents a major problem for some plants, like aircraft for example, whose performance imperfection has to be minimized.

The proposed algorithm is introduced in section-2. Two examples are worked out, using MATLAB-5. Sample of the obtained simulation results is shown in section-3. Analysis of the application of the presented algorithm is discussed in section-4. Finally, conclusions and main references are provided.

2. THE PROPOSED ALGORITHM

The proposed algorithm is concerned with plants, whose available a priori knowledge indicates that their model structure can be fairly represented by an under damped second order system. The previous study of the system should show that:

1- The system is stable
2- The step response reached the steady state without steady state error, from which the system gain could be calculated.
3- The system response is apparently similar to that of an under-damped second order system.

In this case, the system could be described by a second order transfer function::

$$\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\,\omega_n\,s + \omega_n^2} \tag{1}$$

where : ω_n is the natural frequency of the system, ζ is the damping ratio, ($0 < \zeta < 1$), U denotes the input and Y denotes the output, both are expressed in deviation variables.

In most empirical Low-order model identification algorithms, published in Literature [1], the shape of the input perturbation is step- change input. Its magnitude is selected according to the actual plant nature and conditions.

In this identification scheme, a specified type of input perturbation is proposed. The magnitude of the input step pulse and its duration must be small enough to attain plant safety, and to avoid causing severe performance disturbance. Introducing an input step pulse to the physical system; the output response data is collected, every properly chosen time-increment.

The main data to be determined in this algorithm are:

(i) Maximum peak overshoot ratio (Δ_{max}).
(ii) Peak time, t_{p1} [sec.].

From the recorded plant response, the model parameters would be estimated using equations (2) and (3).

$$\zeta = \left| \frac{\ell n\,(\Delta_{max})}{\sqrt{\pi^2 + (\ell n\,\Delta_{max})^2}} \right| \tag{2}$$

$$\omega_n = \frac{\pi}{t_{p1}\sqrt{1-\zeta^2}} \tag{3}$$

The relations (2) and (3) can be derived from the basic characteristics of time-response of a second-order system [6].

The proposed pulse – duration of the input change would be limited nearly, to the value of t_{p1}, depending on the facts reached in this work.

It is important to emphasize, here, that the model developed by this procedure relates the input perturbation to output response. The plant modeled includes all equipment between the input and output. Thus, the obtained empirical model provides the proper information for control analysis, because it includes the actual elements in the existing control loop.

3. SIMULATION RESULTS

Simulation results of this is paper have been obtained using MATLAB-5. First group of results is concerned with the output response of a second order system with parameters (ω_n = 1 [rad. /sec.], ζ = o.3), subjected to a step input pulse with different time-duration (T_{in}). Eight cases from the obtained results are depicted in table 1.

Table 1. Results of a system with ω_n = 1, ζ= 0.3

Case No.	T_{in} [sec.]	ω_n [rad./sec.]	ζ	t_{p1} [sec.]	Δ_{max} [%]
1	∞	1	0.3	3.29	37.17
2	10	1	0.3	3.29	37.17
3	3.5	1	0.3	3.29	37.17
4	3.3	1	0.3	3.29	37.17
5	3.0	1	0.3	3.09	36.09
6	2.5	1	0.3	2.76	28.53
7	2.0	1	0.3	2.44	13.71
8	1.5	1	0.3	2.14	No Over shoot

Simulation responses of cases no. 1, 3, 6 and 8 from table1, are shown in (Fig. 1a, b, c, d respectively). From the given results, we see that t_{p1}, and Δ_{max} keep their correct values (t_{p1} = 3.29 sec., Δ_{max} = 37.17% determined by case No.1, for unit step input), when ever

$$T_{in} \geq t_{p1}. \tag{4}$$

When the condition (4) is satisfied, we can estimate the parameters from the obtained data, using eqns. (2) , (3) as follows:

$$\zeta = \left| \frac{\ell n\ (0.3717)}{\sqrt{\pi^2 + [\ell n\,(0.3717)]^2}} \right| = 0.3005 \tag{5}$$

$$\omega_n = \frac{\pi}{t_{p1}\sqrt{1-\zeta^2}} = \frac{\pi}{3.29\sqrt{1-(0.3)^2}} = 1.0009 \quad \text{[rad./sec.]} \tag{6}$$

The computed values of model parameters, using our algorithm (From eqns.(5), (6)) are fairly accurate, compared with the assumed system parameters used for simulations ($\omega_n = 1$, $\zeta = 0.3$). In second group of results, simulations are obtained for another second order system (with ω_n = 10 rad. /sec., ζ = 0.5). The results are recorded in table 2.

Table 2. Results of a system with ω_n = 10 , ζ= 0.5

Case No.	T_{in} [sec.]	ω_n [rad./sec.]	ζ	t_{p1} [sec.]	Δ_{max} [%]
1	∞	10	0.5	0.363	16.2
2	4.0	10	0.5	0.363	16.2
3	2.0	10	0.5	0.363	16.2
4	0.8	10	0.5	0.363	16.2
5	0.4	10	0.5	0.363	16.2
6	0.35	10	0.5	0.352	16.099
7	0.3	10	0.5	0.312	13.23
8	0.2	10	0.5	0.239	No Over shoot

Results of table 2, ensure the same condition (4) for accurate identification process. Only cases No. 6 and 8 are selected from the table to be shown in Fig 2-a and 2-b; to avoid repetition.

From the sample of results, depicted in this paper, and many other worked simulations, we can reach the following remarks:

(a) Recording correct values of max. peak overshoot ratio (Δ_{max}), and its time (t_{p1}), from the output response of the system to a step input, are enough to identify the two basic parameters of a second-order model, (namely ζ, ω_n), from the eqns. (2) and (3).

(b) Decreasing the time duration of the input step pulse, up to the correct value of t_{p1} will change the shape of the output response, but still the correct values of (Δ_{max} and t_{p1}) will be kept unchanged, as if the input is a step input. We do mean by correct value of tp1, the value recorded when the input is long-lasting step (with $T_{in} = \infty$).

(c) If the time-duration of the input step pulse, is decreased less than the correct value of t_{p1}; the indicated recorded values for the peak overshoot and the peak-time would have misleading data. No one can rely on such misleading data, for accurate identification process.

(d) In order to minimize the time of identification process, which is substantially necessary in a large class of plants, we can use a step-pulse input with limited pulse width (T_{in}). Value of T_{in} can be chosen not exactly equal to the value of t_{p1}, but rather higher by a small time percent (δ_t) to guarantee accuracy of the obtained identification results. Hence, T_{in} can be chosen as,

$$\delta_t = \frac{T_{in} - t_{p1}}{t_{p1}} \times 100 = 2 - 5\ \% \tag{7}$$

4. APPLICATION ON AIRCRAFT LONGITUDINAL DYNAMICS

Referring to the available references of aircraft dynamics and mechanics of flight (e.g. [7]), and earlier research studies of the author in upgrading of conventional flight control systems [8-13], the model structure of an aircraft can be established. Generally, the longitudinal motion of a conventional airplane can be described by a fourth order transfer function. The fourth-order polynomial of the denominator shows that the free longitudinal motions of aircraft consist of two oscillatory modes. One of these is a relatively well- damped, high frequency oscillation called the "short – period". The other is lightly damped, relatively low-frequency oscillation called the "phugoid".

According to the dominant motion of the aircraft considered, the model of the aircraft longitudinal dynamics can be well approximated by one of the two mentioned modes.

Consider a conventional airplane having specific configuration and characteristics, as detailed in [7], at altitude = 20,000 [ft.] and true air speed = 660 [ft./sec.].

The basic characteristic equations for "phugoid" and "short-period" motions for such aircraft, are well estimated by the following eqns. (8) and (9) respectively;

$$s^2 + 2\,(0.0714)\,(0.0630)\,s + (0.0630)^2 = 0 \tag{8}$$

$$s^2 + 2\,(0.493)\,(4.27)\,s + (4.27)^2 = 0 \tag{9}$$

Here, the values describing the main parameters of the second order models are estimated by

$\zeta = 0.0714$, $\omega_n = 0.0630$ [rad./sec.], for phugoid;

and $\zeta = 0.493$, $\omega_n = 4.27$ [rad./sec.], for short period.

Such values would be drastically violated, when the flight attitude parameters (e.g. altitude and speed), are greatly changed.

4.1 Identification Algorithm Procedure:

In order to identify the approximate longitudinal dynamic model parameters of an aircraft, at certain flight attitude, the following procedure is proposed:

(1) Allow the aircraft to reach steady-state in its flight conditions (constant altitude and speed).

(2) Introduce a single step change of appropriate magnitude, in the input variable (δ_e : is the input elevator deflection, made by the control stick).

(3) Collect the input and output response data, until the first peak over shoot occurs (Record the values of Δ_{max} , t_{p1},). Aircraft pitch angle (θ) is the output

(4) Let the input elevator deflection return to zero, just after measuring t_{p1} by a small time – increment δ_t such that $\delta_t = \dfrac{T_{in} - t_{p1}}{t_{p1}} \times 100 = 2 - 5\ [\%]$.

(5) Perform calculations of the estimated parameters (ζ , ω_n) of the model using eqns. (2) and (3).

The simulation results of longitudinal dynamics of an aircraft, approximated by the short period oscillatory model (as in eqn. (9)), are similar to the shown results of Fig. 2. These results are not shown here, for space limitations and to avoid repetition.

4.2 Input step Magnitude Analysis:

The accuracy of the model obtained depends on the magnitude of the input change. Naturally, the larger the input step, the more accurate the modeling results, but the larger the disturbance to the aircraft flight. Also, the shorter time duration of the input change, the less disturbance to the aircraft flight, which is a vital requirement.

The output change, related to the input change, cannot be too small, otherwise it would be hardly detected from the output noise. Noise in the measured output of the aircraft is most Likely to happen due to many sources, such as;

(i) Discrepancies of engine components [13].
(ii) Wind-gust external disturbances [14].
(iii) Sensors non-idealities [1].

There is a rough guideline, for modeling of dynamic processes, states that the signal-to-noise ratio should be at least 5 (see [1]). From the analysis discussed so far, and the a prioi knowledge of the aircraft system, a compromised value for magnitude of input change is selected.

5. CONCLUSIONS

Simulation results showed that using an input pulse for second-order model identification instead of the step-input, is absolutely successful in attaining the same accuracy, within minimum identification time.

The proposed algorithm can be applied easily, not only to aircraft longitudinal dynamics, but also to a large category of plants and processes whose dominant dynamics can be well approximated by second order oscillatory model.

It is worth emphasizing that the vast majority of control strategies are based on empirical models; thus, the method introduced in this paper can be of great practical importance.

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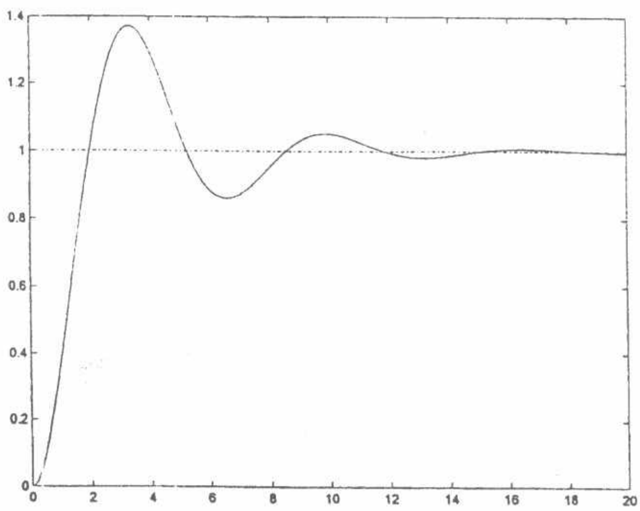

Fig. (1a). Time Response for Case-1 of Table-1.

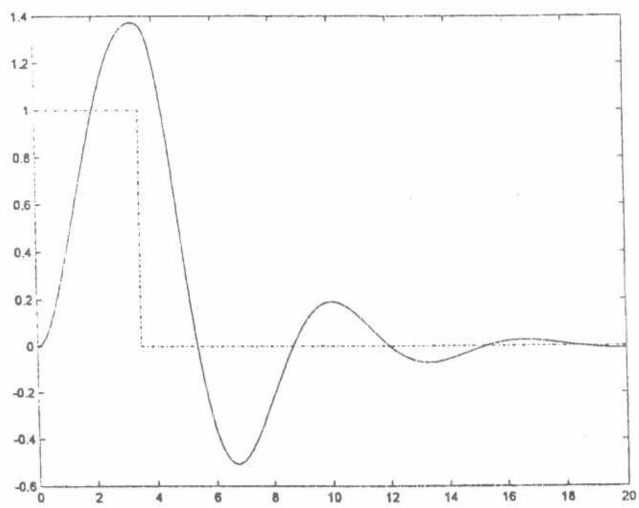

Fig. (1b). Time Response for Case-3 of Table-1.

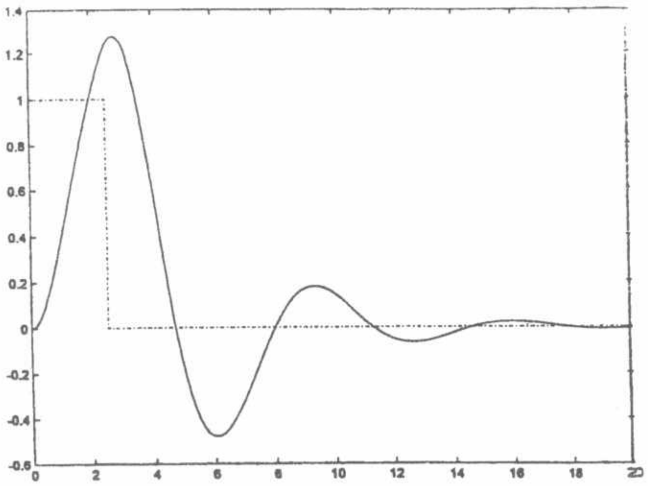

Fig. (1c). Time Response for Case-6 of Table-1.

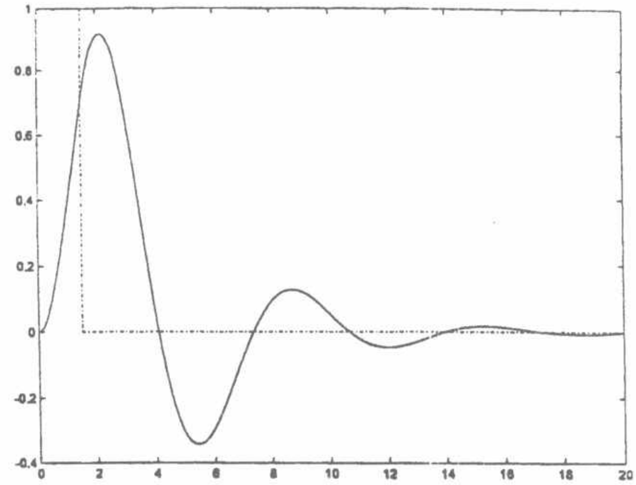

Fig. (1d). Time Response for Case-8 of Table-1

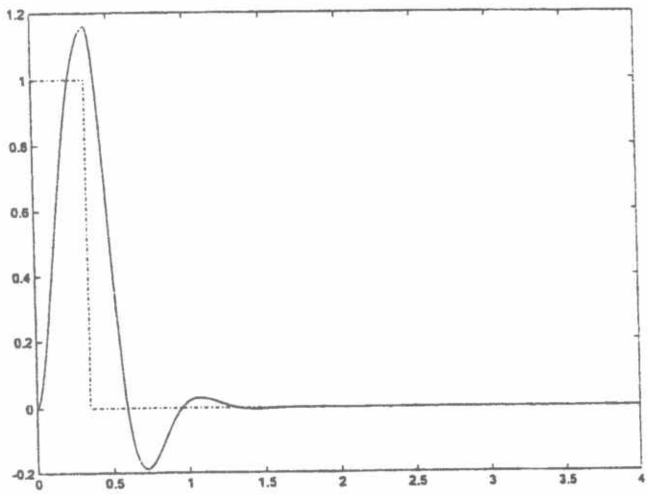

Fig. (2a). Time Response for Case-6 of Table-2.

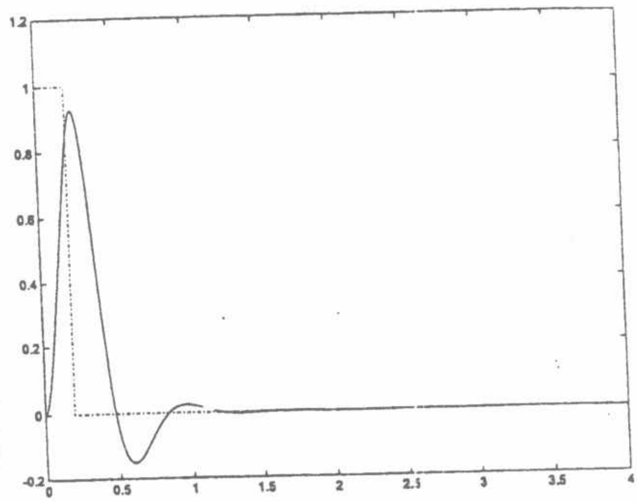

Fig. (2b). Time Response for Case-8 of Table-2.

Note: ----------- represents input step

——— represents output response