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Kobry Elkobbah,
Cairo, Egypt

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ANALYTIC SENSITIVITIES IN COMPOSITE WING-SHAPE SYNTHESIS PART ONE: PANEL BUCKLING CONSTRAINTS

M.M. ELNOMROSSY*, M.A. MEHANNA**

ABSTRACT

Equivalent plate modeling techniques based on Ritz analysis with simple polynomials proved to be efficient tools for structural modeling of wings in the preliminary design stage. This paper describes a formulation of wing equivalent plate modeling in which it is simple to obtain analytic, explicit expressions for terms of the stiffness and geometric stiffness matrices without the need to perform numerical integration. The explicit expressions for the terms of the stiffness and geometric stiffness matrices are used for the buckling analysis of trapezoidal fiber composite wing skin panels. The formulation based on Ritz analysis using simple polynomials leads to explicit expressions for the analytic sensitivities of the stiffness and geometric stiffness matrices with respect to layer thickness, fiber orientations, and panel shape. Integration of the resulting buckling analysis based on the equivalent plate approach is discussed. To assess the new capability, test cases were chosen to address the convergence rate with increased polynomial order, the accuracy of analysis results for panels of general shape and the integration of wing box structural analysis with panel buckling analysis for fast structural analysis of airplane wings.

KEY WORDS

Composite wing, equivalent plate, buckling, sensitivity analysis, stiffness matrix, geometric stiffness matrix, finite element method.

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* Staff Member, Aerospace Department, Military Technical College, Cairo
** Engineer, Egyptian Armed Forces.

NOMENCLATURE

[A]: in-plane local stiffness matrix for panel
A^S: cross-sectional area of spar
A^S_1, A^S_2: coefficients of linear cross-sectional area of a spar
A^R: cross-sectional area of rib
A^R_1, A^R_2,: coefficients of linear cross-sectional; area of a rib
[D]: out-of-plane local stiffness matrix of panel
d (x, y): wing depth distribution
F_B (x, y): weight function ensuring zero displacement on panel boundary
[F_1], [F_2], [F_3]: matrices containing admissible functions and their derivatives
f_p (x,y) : admissible functions
H (i): i^{th} coefficient in the polynomial series for wing depth
h (x,y) : total thickness of panel
I_{TR}: integral of a simple polynomial term over the trapezoidal panel area
[K]: stiffness matrix
[K_G]: geometric stiffness matrix
{k}: matrix of function derivatives
$m_{qw,pw}$, $n_{qw,pw}$: powers of x and y for elements of {k}
[N]: 2x2 matrix for in-plane loads
N_x, N_y, N_{xy}: in-plane loads per unit length
[Q_i]: material properties matrix
{q}: vector of generalized displacements for panel
R, S: coefficients of front line or aft line of panel
T^i_k: the i^{th} coefficient in the polynomial series for thickness of the k^{th} layer
t_i (x,y) : thickness of the i^{th} layer
U_i, V_j: shape dependent coefficients of F_B
w (x,y) : panel out-of-plane displacements
x_A, x_F: aft and forward x coordinates of rib edges
x_{FL}, x_{FR}, x_{AL}, x_{AR} : x coordinates of the vertices of a wing trapezoid

ε_x, ε_y, γ_{xy} : skin engineering strains
σ_x, σ_y, σ_{xy} : skin layer stresses
θ : fiber orientation angle
λ : buckling eigenvalues

Subscripts

A : aft (rear)
F : front
L : left
R : right

INTRODUCTION

In the context of airplane preliminary design, as aerodynamic and structural characteristics of an evolving configuration change, optimization with respect to shape is essential. Yet, the automated preliminary design synthesis of wing structures, in which the wing planform shape is varied and control surfaces are sized and moved to their optimal locations, is still a challenge.

The equivalent plate models for low aspect ratio thin wings were used for wing aeroelastic tailoring optimization, parametric studies, and multidisciplinary synthesis, [1-2]. What makes the equivalent plate approach desirable in the context of wing preliminary design and optimization are the ease and speed of data preparation for new configurations, the high computational efficiency and the ease of manipulation of the Ritz displacement functions used.

Equivalent plate models, however, need modification before their true potential can be realized for wing shape optimization. For effective, reliable behavior sensitivity analysis, the ability to calculate analytical derivatives with respect to shape as well as sizing design variables is important. This is even more important when Ritz functions used are simple polynomials. Simple polynomial Ritz functions lead to substantial saving in computing time as well as to numerical ill conditioning of the resulting equations when the order of polynomials is increased. Experience reported in references [3] and [4] shows that useful analysis results can be obtained with an equivalent plate model based on simple polynomial before the static and dynamic solutions become ill conditioned.

Buckling of skin panels in thin-walled structures is one of the most important failure modes to be considered in wing design synthesis. A large number of literature's exist today on the buckling of isotropic and anisotropic panels, [5]. While the study of isolated panels of rectangular shape has reached certain maturity, research addressing the buckling of panels functioning as components in a larger structural system is still evolving. In the complete structure, such as an airplane wing, buckling can occur locally or globally. The need to assess global behavior and local behavior simultaneously, leads to computationally intensive analysis. The complex global-local interactions can also make buckling constraints highly non-linear in terms of the structural design variables. Thus, in the context of airframe structural optimization, proper representation of buckling constraints is a major challenge, and is an area of active research, [6-9].

When wing planform becomes subject to design optimization in addition to sizing of structural members, or when variation of the internal structure is allowed, the panel buckling becomes more complex. In the general case, panels are rarely rectangular. They are usually trapezoidal in shape. Moreover, while in high aspect ratio wings it may be acceptable to assume unidirectional compression on the skin panels, in low aspect ratio wings, panels are usually loaded by in-plane loads in a combined manner. Therefore, it is important to develop an efficient buckling analysis methodology applicable to trapezoidal panels in combined in-plane loading. Such a methodology should be design oriented, i.e., issues of sensitivity analysis and approximation concepts should be addressed.

This paper begins with a formulation of the equivalent plate approach that makes it possible to obtain analytic shape sensitivities of wing box structures.

It also shows how this formulation leads to the expressions of stiffness and geometric stiffness terms. The equations for panel buckling analysis are derived. Analytic sensitivities with respect to panel shape, thickness and fiber orientations are derived. Test cases and results of numerical evaluations concluded the work.

MODELING USING SIMPLE POLYNOMIALS

Wings can move along the fuselage, change sweep, chord and span. Spars and ribs move also as the internal structural layout is changed. Wings are made of collections of trapezoidal areas. A typical wing box trapezoid is shown in Fig.1. Its planform shape is defined by its left and right spanwise coordinates y_L, y_R and the x coordinates of its four edges x_{FL}, x_{FR}, x_{AL}, x_{AR}. Its depth is defined by a simple polynomial in x and y in the form:

$$d(x,y) = \sum_{i=1}^{N} H(i)\, x^{mi}\, y^{ni} \qquad (1)$$

where H(i) are coefficients and m_i, n_i are exponents of x and y terms in the polynomial series.

Figure. 1 also shows spars and ribs in the trapezoid. Spar geometry is defined by four shape variables, namely the (x,y) coordinates of the left and right edges in addition to the depth distribution that determines flange vertical distance from the midplane. Ribs, running parallel to the x-axis, are defined geometrically by three design variables each x_F, x_A and y_{RIB}.

The skins are made of N_L unidirectional composite layers, and the thickness of each layer is described by a polynomial in x and y of the form:

$$t_i(x,y) = \sum_{k=1}^{N_i} T_k^i x^{m_k^t} y^{n_k^t} \qquad i = 1,\ldots,N_L \qquad (2)$$

The powers m^t_k and n^t_k are x and y powers of the k^{th} term of the thickness series for the i^{th} layer. The coefficients T^i_k define the thickness of the i^{th} layer and they serve as sizing design variables for that layer. The exponents m^t_k and n^t_k are preassigned and can be selected to generate a complete polynomial or to represent a product of a polynomial in x by a polynomial in y, [4]. The overall thickness of skin panels is given by:

$$h(x,y) = \sum_{i=1}^{NL} t_i(x,y) = \sum_{i=1}^{NL} \sum_{K=1}^{Ni} T_k^i x^{m_k^t} y^{n_k^t} \qquad (3)$$

Flange areas for spars and ribs are allowed to very linearly. In the case of a spar, the flange area A^S is defined in terms of the spanwise coordinate y, and in the case of a rib, the flange area A^R is a function of x:

$$A^S(y) = A_1^S + A_2^S y \tag{4}$$

$$A^R(x) = A_1^R + A_2^R x \tag{5}$$

The coefficients A^R_i and A^S_i serve as sizing type design variables for the flange areas.

Admissible Functions

Fig.1 shows a trapezoidal panel defined by coordinates of its vertices in the x, y axes. The subscripts L and R denotes left and right sides, respectively. The subscripts F and A denote front and aft lines, respectively, and x_F and x_A are the x coordinates of the forward and rear points on a line parallel to the sides of the panel. Based on Fig.1, the equations for points on the front and rear lines can be written as:

$$x_F(y) = \frac{x_{FL} y_R - x_{FR} y_L}{y_R - y_L} + \left(\frac{x_{FR} - x_{FL}}{y_R - y_L}\right) y = R_F + S_F y \tag{6}$$

$$x_A(y) = \frac{x_{AL} y_R - x_{AR} y_L}{y_R - y_L} + \left(\frac{x_{AR} - x_{AL}}{y_R - y_L}\right) y = R_A + S_A y \tag{7}$$

The following function satisfies the zero displacement boundary conditions on the circumference of the panel:

$$F_B(x, y) = [x - S_F y - R_F][x - S_A y - R_A][y - y_L][y - y_R] \tag{8}$$

Using Eqs. (6) and (7) and expanding in terms of x and y yields:

$$F_B(x, y) = (U_1 + U_2 y + U_3 y^2)(V_1 + V_2 x + V_3 y + V_4 x^2 + V_5 xy + V_6 y^2) \tag{9}$$

and, using index notation:

$$F_B(x, y) = \sum_{i=1}^{3} \sum_{j=1}^{6} U_i V_j x^{m_j^v} y^{n_i^u + n_j^v} \tag{10}$$

where the constants U_i and V_j are given in terms of panel vertex coordinates by:

$$\begin{aligned} &U_1 = y_L y_R \quad &; \quad &U_2 = -(y_L + y_R) \quad &; \quad &U_3 = 1 \\ &V_1 = R_A R_F \quad &; \quad &V_2 = -(R_A + R_F) \quad &; \quad &V_3 = (R_A S_F + R_F S_A) \\ &V_4 = 1 \quad &; \quad &V_5 = -(S_A + S_F) \quad &; \quad &V_6 = S_F S_A \end{aligned} \tag{11}$$

Powers of x and y corresponding to the constants U_i and V_j in Eq.(10) are given in Table 1.

Multiplying the weight function $F_B(x,y)$ by a general simple polynomial series, admissible functions for the simply supported trapezoidal panel are obtained:

$$w(x,y) = F_B(x,y)\sum_{p=1}^{N_w} q_p x^{m_p^w} y^{n_p^w} \tag{12}$$

where the coefficients q_p are the generalized displacements, and w(x,y) is the vertical displacement of the panel. Substituting the expression for $F_B(x,y)$ from Eq.(10), it is possible to write:

$$w(x,y) = \sum_{p=1}^{N_w} q_p \sum_{i=1}^{3} \sum_{j=1}^{6} U_i V_j x^{(m_j^v + m_p^w)} y^{(n_i^u + n_j^v + n_p^w)} \tag{13}$$

The admissible functions are thus expressed in terms of simple polynomials, where the p^{th} admissible function is given by:

$$f_p(x,y) = \sum_{i=1}^{3} \sum_{j=1}^{6} U_i V_j x^{(m_j^v + m_p^w)} y^{(n_i^u + n_j^v + n_p^w)} \tag{14}$$

The unknown elastic panel deflection w(x,y) is approximated by a series of admissible functions:

$$w(x,y) = \sum_{p=1}^{N_w} f_p(x,y) q_p = [F_1(x,y)]\{q\} \tag{15}$$

J	V_j	m^v_j	n^v_j	I	U_i	n^u_i
1	V_1	0	0	1	U_1	0
2	V_2	1	0	2	U_2	1
3	V_3	0	1	3	U_3	2
4	V_4	2	0	-	-	-
5	V_5	1	1	-	-	-
6	V_6	0	2	-	-	-

Table 1: Constant U_i and V_j and their corresponding powers of x and y associated with the admissible displacement series function.

BUCKLING ANALYSIS

Based on the principle of virtual work, a variational equation for a symmetrically layered composite plate under the action of internal bending moment and transverse shear forces in the absence of transverse load or initial deformation is:

$$\iint \{w_{,xx}\, w_{,yy}\, 2w_{,xy}\}[D]\delta\begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix} dxdy + \iint \{w_{,x}\, w_{,y}\}\begin{bmatrix} N_x N_{xy} \\ N_{xy} N_y \end{bmatrix}\delta\begin{Bmatrix} w_{,x} \\ w_{,y} \end{Bmatrix} dxdy = 0 \qquad (16)$$

where the vertical displacement due to load is w(x,y). The matrix [D] is the 3x3 local transverse stiffness matrix. The unknown elastic displacement w(x,y) is approximated by a series of admissible functions, Eq.(15). The first derivatives, then, can be expressed as:

$$\begin{Bmatrix} w_{,x} \\ w_{,y} \end{Bmatrix} = \begin{bmatrix} f_{1,x} f_{2,x} \ldots\ldots f_{N,x} \\ f_{1,y} f_{2,y} \ldots\ldots f_{N,y} \end{bmatrix}\begin{Bmatrix} q_1 \\ q_2 \\ \ldots\ldots \\ q_N \end{Bmatrix} = [F_2]\{q\} \qquad (17)$$

and the second derivatives are given by:

$$\begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ w_{,xy} \end{Bmatrix} = \begin{bmatrix} f_{1,xx} f_{2,xx} \ldots\ldots f_{N,xx} \\ f_{2,yy} f_{2,yy} \ldots\ldots f_{N,yy} \\ 2f_{1,xy}\, 2f_{2,xy} \ldots 2f_{N,xy} \end{bmatrix}\begin{Bmatrix} q_1 \\ q_2 \\ \ldots\ldots \\ q_N \end{Bmatrix} = [F_3]\{q\} \qquad (18)$$

The matrices $[F_1]$, $[F_2]$, and $[F_3]$ are all functions of x and y. Using Eqs. (17) and (18) to express the virtual displacements and their derivatives in terms of virtual generalized displacements, Eq.(16) leads to:

$$\{q\}^T \iint [F_3]^T[D][F_3]dxdy\{\delta q\} + \{q\}^T \iint [F_2]^T[N][F_2]dxdy\{\delta q\} = 0 \qquad (19)$$

The matrix equation for the linear buckling analysis of the panel is thus:

$$[\,[K] + \lambda[K_G]]\{q] = 0 \qquad (20)$$

where the stiffness matrix is:

$$[K] = \iint [F_3]^T[D][F_3]dxdy \qquad (21)$$

and the geometric stiffness matrix is:

$$[K_G] = \iint [F_3]^T[N][F_3]dxdy \qquad (22)$$

The scalar λ is used as a scaling parameter increasing or decreasing the given in-plane loads N_{ij} simultaneously, to determine whether the panel is stable or unstable. Eq.(20) is a generalized linear eigenvalue problem. Since the stiffness and geometric stiffness matrices are real and symmetric, the eigenvalues are real. The buckling constraint for the panel is in the form:

$$1-\lambda_{min} \le 0 \qquad (23)$$

assuming that the given in-plane loads have to be increased to reach instability.

STIFFNESS MATRIX

Polynomial description of skin layer thickness in terms of the x-y coordinates was given in Eqs.(2) and (3). This thickness distribution is for the entire wing box, or segments of the wing box. The panels are those trapezoidal skin segments defined by the supporting internal spar-rib array, as shown in Fig.1. For each panel containing N_L layers of fibers, the in-plane stiffness matrix [A] can be expressed in terms of individual layer thickness and fiber orientation angles as:

$$[A]=\sum_{i=1}^{N_L}\{[Q_0]+[Q_1]\cos2\theta_i+[Q_2]\cos4\theta_i+[Q_3]\sin2\theta_i[Q_4]\sin4\theta_i\}t_i(x,y) \qquad (24)$$

where the matrices $[Q_0]$ to$[Q_4]$ depend on material invariants as given in Ref.10. Let a material and fiber orientation dependent matrix $[Q(\theta_i)]$, a 3x3 matrix, be defined as:

$$[Q(q_i)]=\{[Q_0]+[Q_1]\cos2\theta_i+[Q_2]\cos4\theta+[Q_3]\sin2\theta_i+[Q_4]\sin4\theta_i\} \qquad (25)$$

The in-plane stiffness matrix [A] can now be expressed in terms of the sizing design variables T^i_k and fiber orientations, as a polynomial:

$$[A]=\sum_{i=1}^{N_L}\sum_{k=1}^{N_i}[Q(\theta_i)]x^{m_k^{il}}y^{n_k^{il}}T_k^i \qquad (26)$$

For unidirectional, orthotropic, or quasihomogeneous laminates, Ref.10, the in-plane and bending stiffness matrices are related through:

$$[D]=[A]\frac{h^2}{12} \qquad (27)$$

Using Eq.(2) to express h^2 in terms of sizing design variables, double summation is needed. The indices l1 and l2 are used for summation of polynomial terms associated with each layer, as follows:

$$h(x,y)=\sum_{i=1}^{N_L}\sum_{l1=1}^{N_{l1}}T_{l1}^{i1}x^{m_{l1}^{il1}}y^{n_{l1}^{il1}}=\sum_{i=1}^{N_L}\sum_{l2=1}^{N_{l2}}T_{l2}^{i2}x^{m_{l2}^{i2}}y^{n_{l2}^{il2}} \qquad (28)$$

The bending stiffness matrix [D] can now be written as:

$$[D]=\frac{1}{12}\sum_{i=1}^{N_L}[Q(\theta_i)]\sum_{i1=1}^{N_L}\sum_{i2=1}^{N_L}\sum_{k=1}^{N_i}\sum_{l1=1}^{N_{i1}}\sum_{l2=1}^{N_{i2}}\{T_k^i T_{l1}^{i1} T_{l2}^{i2} x^{(m_k^{ii}+m_{l1}^{i2}+m_{l2}^{i2})} y^{(n_k^{ii}+n_{l1}^{i1}+n_{l2}^{i2})}\} \qquad (29)$$

The dependence of [D] on the sizing design variables and fiber orientations is now expressed in explicit form. With the polynomial admissible functions presented in Eqs.(15) and (14), and the stiffness matrix based on Eq.(21), expressions for terms of the stiffness matrix in polynomial form can now be derived. The elements of the $[F_3]$ matrix are all polynomials and the q,p element of $[F_3]$ can be written as:

$$F_{3_{q,p}}=\sum_{i=1}^{3}\sum_{j=1}^{6}F_{ij}^{qp}x^{m_{ij}^{qp}}y^{n_{ij}^{qp}} \qquad (30)$$

where the coefficients F^{qp}_{ij} and the corresponding powers of x and y are given in Table.2.

q row of $[F_3]$	F^{qp}_{ij} coefficient	m^{qp}_{ij} power of x	n^{qp}_{ij} power of y
1	$U_iV_j(m^v_j+m^w_p)(m^v_j+m^w_p-1)$	$m^v_j+m^w_p-2$	$n^u_i+n^v_j+n^w_p$
2	$U_iV_j(n^u_i+n^v_j+n^w_p)(n^u_i+n^v_j+n^w_p-1)$	$m^v_j+m^w_p$	$n^u_i+n^v_j+n^w_p-2$
3	$U_iV_j(m^v_j+m^w_p)(n^u_i+n^v_j+n^w_p)$	$m^v_j+m^w_p-1$	$n^u_i+n^v_j+n^w_p-1$

Note: If any powers of x and y is less than zero, the element is set to zero

Table 2: Coefficients and powers of polynomial terms in the $[F_3]$ matrix.

Eq.(30) together with Eq.(29) are substituted into the expression for the stiffness matrix. Then, the r,sth term of the stiffness matrix is:

$$K_{rs}=\sum_{a=1}^{3}\sum_{b=1}^{3}\iint(F_{3ar}D_{ab}F_{3bs})dxdy \qquad (31)$$

and the final expression for the K_{rs} element in polynomial form is:

$$K_{rs}=\sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{i=1}^{3}\sum_{j=1}^{6}\sum_{i=1}^{N_L}\sum_{i1=1}^{N_L}\sum_{i2=1}^{N_L}\sum_{k=1}^{N_i}\sum_{l1=1}^{N_{i1}}\sum_{l2=1}^{Ni2} F \qquad (32)$$

and

$$F=\frac{1}{12}F_{ij}^{ar}F_{ij}^{bs}Q_{ab}(\theta_i)T_k^i T_{l1}^{i1} T_{l2}^{i2}\iint x^{m_{rs}}y^{n_{rs}}dxdy \qquad (32a)$$

where the powers of x and y terms in the area integral are:

$$m_{rs} = mf_{ij}^{ar} + mf_{ij}^{bs} + m_k^{ti} + m_{l1}^{tl1} + m_{l2}^{tl2}$$
$$n_{rs} = nf_{ij}^{ar} + nf_{ij}^{bs} + n_k^{ti} + n_{l1}^{tl1} + n_{l2}^{tl2} \quad (33)$$

All elements of the stiffness matrix are, thus, linear combinations of integrals over the panel's area. Note the explicit dependence on thickness coefficients and fiber orientations. Dependence on panel shape is more complex. The coefficients U_i and V_j determine the F coefficients in Table.2. In addition, the area integrals depend on the shape of the panel and the limits of integration change when the planform shape in changing. The integration can be carried out analytically as shown in Ref.11.

GEOMETRIC STIFFNESS MATRIX

the geometric stiffness matrix in the buckling analysis formulation for skin panels is given in Eq.(22), and it depends on the matrix $[F_2]$, containing derivatives of the admissible functions, and the matrix [N] containing the in-plane loads.

In-plane loads from wing box stress analysis

For preliminary design purposes, if the skin panels are small relative to the wing, buckling evaluation may be accurate enough if average N_x, N_y and N_{xy} are used for the panel buckling analysis. The in-plane stress resultants are assumed constant throughout the panel. This simplifies the integration and makes it possible to use interaction formulas for fast approximate buckling analysis, Ref.9.

When an equivalent plate modeling approach is used for the wing box analysis, the in-plane skin stresses are obtained from the wing generalized displacements calculated in the wing box stress analysis stage. In this formulation, admissible functions for the wing box analysis are given as polynomials in x and y. In this case, the transverse displacement of the wing is:

$$w^*(x,y) = \sum_{i=1}^{\bar{N}_w} x^{\bar{m}_i} y^{\bar{n}_i} \bar{q}_i \quad (34)$$

where a bar will associate variables with the wing box analysis. In Eq.(34), the powers and the number of terms are known from the admissible series used for the wing box displacement solution. The coefficients q's are the generalized wing box displacements.

In equivalent plate using wing structural analysis based on classical plate theory, Refs. 4 and 5, the vertical displacement under loading is w*(x,y), and using Kirchoff's kinematics for a wing, the engineering strains in the x and y directions are given by:

$$\begin{aligned} \bar{\varepsilon}_x &= -z w^{*}_{,xx} \\ \bar{\varepsilon}_y &= -z w^{*}_{,yy} \\ \bar{\gamma}_{xy} &= -2z w^{*}_{,xy} \end{aligned} \qquad (35)$$

Let the wing depth be given by d*(x,y). Then, when skins are thin compared to the depth, they can be assumed located at z = ±d*(x,y)/2. Focusing on the upper skin, in-plane strains are:

$$\begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_y \\ \bar{\gamma}_{xy} \end{Bmatrix} = \frac{\bar{d}(x,y)}{2} \begin{Bmatrix} \bar{w}_{,xx} \\ \bar{w}_{,yy} \\ 2\bar{w}_{,xy} \end{Bmatrix} \qquad (35a)$$

Now, in a polynomial based formulation for the wing box, the depth of the wing is given in polynomial form:

$$\bar{d}(x,y) = \sum_{ih=1}^{N_L} \bar{H}_{ih}\, x^{mh_{ih}}\, y^{nh_{ih}} \qquad (36)$$

Since the displacement w*(x,y) and the depth d*(x,y) are polynomial, it is evident that skin strains due to wing deformation are polynomial too. There are a total of N_L layers, each with fiber orientation θ, and thickness as described by the polynomial, Eq.(2). The in-plane skin stresses in each layer are obtained from:

$$\begin{Bmatrix} \bar{\sigma}_{xx} \\ \bar{\sigma}_{yy} \\ \bar{\sigma}_{xy} \end{Bmatrix} = [Q(\theta_i)] \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\varepsilon}_{xy} \end{Bmatrix} \qquad (37)$$

The in-plane loads can now be defined in terms of wing box displacements by integration through the thickness of the panel:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = -\frac{\bar{d}(x,y)}{2} [A] \begin{Bmatrix} \bar{w}_{,xx} \\ \bar{w}_{,yy} \\ \bar{w}_{,xy} \end{Bmatrix} \qquad (38)$$

Now, the in-plane loads, Eq.(38), are expressed in terms of the generalized displacements {q*} calculated for the wing box solution, as follows:

$$N_x = -\frac{\bar{d}(x,y)}{2}\sum_{qw=1}^{3}\sum_{pw=1}^{N_w} A_{1,qw} w_{qw,pw} x^{\bar{m}_{wq,pw}} y^{\bar{n}_{qw,pw}} \bar{q}_{pw} \tag{39}$$

$$N_y = -\frac{\bar{d}(x,y)}{2}\sum_{qw=1}^{3}\sum_{pw=1}^{N_w} A_{2,qw} w_{qw,pw} x^{\bar{m}_{wq,pw}} y^{\bar{n}_{qw,pw}} \bar{q}_{pw} \tag{40}$$

$$N_{xy} = -\frac{\bar{d}(x,y)}{2}\sum_{qw=1}^{3}\sum_{pw=1}^{N_w} A_{3,qw} w_{qw,pw} x^{\bar{m}_{wq,pw}} y^{\bar{n}_{qw,pw}} \bar{q}_{pw} \tag{41}$$

Now, the polynomial expression for the wing depth can be substituted into Eqs.(39), (40) and (41). The general expression for terms of the in-plane loads is:

$$N_{pp,qq} = -\frac{1}{2}\sum_{ih=1}^{N_h}\sum_{qw=1}^{3}\sum_{pw=1}^{N_w} A_{ppp,qw} w_{qw,pw} H_{ih} x^{(m_{ih}+\bar{m}_{qw,pw})} y^{(n_{ih}+\bar{n}_{qw,pw})} \bar{q}_{pw} \tag{42}$$

The indices pp and qq can be either 1 or 2. Note:

if pp = 1 and qq = 1, then $A_{ppp,qw} = A_{1,qw}$
if pp = 2 and qq = 2, then $A_{ppp,qw} = A_{2,qw}$
when pp ≠ qq, $A_{ppp,qw} = 2A_{3,qw}$

The polynomial expression for the [A] matrix is now used:

$$N_{pp,qq} = -\frac{1}{2}\sum_{ih=1}^{N_h}\sum_{qw=1}^{3}\sum_{pw=1}^{N_w}\sum_{i=1}^{N_L}\sum_{k=1}^{N_l} \{Q_{ppp,qw}(q_i) w_{qw,pw}$$

$$*.H_{ih} T_k^i \bar{q}_{pw} x^{(m_{ih}+nt_k^i+\bar{m}_{qw,pw})} y^{(n_{ih}+nt_k^i+\bar{n}_{qw,pw})} \tag{43}$$

where the indices ppp means the following:

if pp = 1 and qq = 1, then $Q_{ppp,qw} = Q_{1,qw}$
if pp = 2 and qq = 2, then $Q_{ppp,qw} = Q_{2,qw}$
when pp ≠ qq, $Q_{ppp,qw} = 2Q_{3,qw}$

This equation shows how the in-plane loads depend on the wing box solution, the depth of the wing, the thickness coefficients for layers in the panel, material properties and fiber orientations.

Matrix formulation

Based on the Ritz polynomial functions, the matrix $[F_2]$, needed for evaluation of the geometric stiffness matrix, can be written in polynomial form. Thus, the element q,p of the matrix $[F_2]$ is of the form:

$$F_{2_{(q,p)}} = \sum_{i=1}^{3}\sum_{j=1}^{6} \bar{F}_{2_{i,j}}^{q,p} x^{(mf2_{i,j}q,p)} y^{(nf2_{i,j}q,p)} \tag{44}$$

The coefficients and powers of x and y defining the elements of the matrix $[F_2]$ are given in Table.3.

	$F_2{}^{*q,p}{}_{i,j}$	$mf_2{}^{q,p}{}_{i,j}$	$nf_2{}^{q,p}{}_{i,j}$
q=1, row1 of $[F_2]$	$U_iV_j(m^w{}_p+m^v{}_j)$	$m^w{}_p+m^v{}_j-1$	$n^w{}_p+n^u{}_i+n^v{}_j$
q=2, row2 of $[F_2]$	$U_iV_j(n^w{}_p+n^u{}_i+n^v{}_j)$	$m^w{}_p+m^v{}_j$	$n^w{}_p+n^u{}_i+n^v{}_j-1$

Table 3: Coefficients and powers of the $[F_2]$ matrix

Elements of the matrix $[K_G]$ can now be expressed in polynomial form as follows:

$$K_{G_{r,s}} = \iint \sum_{a=1}^{2} \sum_{b=1}^{2} F_{2a,r} N_{a,b} F_{2b,s} dxdy \tag{45}$$

The indices r and s identify terms in the panel Ritz displacement series for the panel. Using polynomial expressions for $[F_2]$ and $[n]$, the r,s element of $[K_G]$ is written as:

$$K_{G_{r,s}} = -\frac{1}{2}\sum_{a=1}^{2} \sum_{b=1}^{2}\sum_{i=1}^{3} \sum_{j=1}^{6} \sum_{ih=1}^{N_h} \sum_{qw=1}^{3} \sum_{pw=1}^{N_w} \sum_{it=1}^{N_L} \sum_{k=1}^{N_g} \overline{F} \tag{46}$$

and

$$\overline{F} = \overline{F}_{2i,j}^{a,r} \overline{F}_{2i,j}^{b,s} Q_{ppp,qw}(q_{it}) w_{qw,pw} H_{ih} T_k^{it} \overline{q}_{pw} \iint x^{(mG_{r,s})} y^{(nG_{r,s})} dxdy \tag{46a}$$

where

$$mG_{r,s} = mf2_{i,j}^{a,r} + mf2_{i,j}^{b,s} + m_{ih} + mt_k^{it} + \overline{m}_{qw,pw}$$

$$nG_{r,s} = nf2_{i,j}^{a,r} + nf2_{i,j}^{b,s} + n_{ih} + nt_k^{it} + \overline{n}_{qw,pw} \tag{47}$$

The index ppp used in $Q_{ppp,qw}(\theta_{it})$ is defined as in Eq.(43). As in the case of the stiffness matrix , the geometric stiffness matrix is represented as summation of surface integrals of polynomial terms calculated over the area of the panel.

ANALYTIC SENSITIVITIES

Stiffness matrix sensitivities

With the explicit expression of stiffness matrix elements in terms of thickness, fiber orientations and shape of the panel, available in the form of Eq.(32), it is straightforward to obtain sensitivities analytically. Three types of sensitivities can be evaluated:

1) Stiffness sensitivities with respect to thickness design variable
Planform shape variables and orientation angles are fixed in this case. The thickness coefficients appears in the expression for the stiffness matrix, Eq.(32), explicitly in a triple summation over the indices k, l1, l2. Sensitivity is then obtained by direct differentiation, noting that if the design variable involved in T^q_r, then the partial derivative of T^l_r with respect to T^q_r is equal to one only when l=q and k=r; otherwise the derivative is zero.

2) Stiffness sensitivities with respect to fiber orientation
Now, the planform and the thickness are fixed. The angle θ_l represents the orientation of fibers in the i^{th} layer, and the stiffness matrix K_{rs} depends on θ_l through elements of the matrix $[Q(\theta_l)]$, Eq.(25). The derivative of each term in the stiffness matrix will be calculated in the following manner. In the summation over l=1 to N_L, in Eq.(32), all matrices $[Q(\theta_l)]$ are set to zero, except the matrix $[Q(\theta_l)]$ corresponding to the θ_l variable considered. This particular $[Q(\theta_l)]$ is replaced in Eq.(32) by the following expression:

$$\frac{\partial Q(\theta_l)}{\partial \theta_i} = -2[Q_1]\sin 2\theta_i - 4[Q_2]\sin 4\theta_i + 2[Q_3]\cos 2\theta_i + [Q_4]\cos 4\theta_i \qquad (48)$$

Eq.(32) is, thus, used for the sensitivity of the stiffness term, with the derivative Eq.(48) replacing $[Q(\theta_l)]$.

3) Stiffness sensitivities with respect to panel planform variables
Thickness coefficients and orientation angles are held fixed. The terms K_{rs} of the stiffness matrix depend on the planform through matrix $[F_3]$, Eq.(30), and Table.2. Coefficients U_i and V_j are defined through expressions (11). If x is any planform design variable, then:

$$\frac{\partial F^{qp}_{ij}}{\partial x} = \left(\frac{\partial U_i}{\partial x} V_j + U_i \frac{\partial V_j}{\partial x}\right).(\text{integers of Table2}) \qquad (49)$$

Though Eqs.(11), the terms U_i and V_j are given explicitly in terms of the panel shape design variables. Analytic sensitivities of U_i and V_j are obtained by direct differentiation. The derivatives of the terms $F^{qp}ij$ can now be prepared. The derivatives of the surface integrals are also prepared in similar manner. After collecting all the information necessary, the derivative of the term K_{rs} is calculated as follows:

$$\frac{\partial K_{G_{rs}}}{\partial x} = \sum_{a=1}^{3}\sum_{b=1}^{3}\sum_{i=1}^{3}\sum_{j=1}^{6}\sum_{i=1}^{3}\sum_{j=1}^{6}\sum_{i=1}^{N_L}\sum_{i1=1}^{N_L}\sum_{i2=1}^{N_L}\sum_{k=1}^{N_t}\overline{F}_x \qquad (50)$$

and

$$\overline{F}_x = \sum_{i1=1}^{N_{t1}}\sum_{i2=1}^{N_{t2}} \frac{1}{12}\{\overline{F}_{x1} + \overline{F}_{x2}\} \qquad (50a)$$

$$\overline{F}_{x1} = [\frac{\partial F_{ij}^{qr}}{\partial x} F_{ij}^{bs} + F_{ij}^{ar}\frac{\partial F_{ij}^{bs}}{\partial x}] Q_{ab}(q_i) T_k^i T_{i1}^{i1} T_{i2}^{i2} I_{TR}(m,n) \qquad (50b)$$

$$\overline{F}_{x2} = F_{ij}^{ar} F_{ij}^{bs} Q_{ab}(q_i) T_k^i T_{i1}^{i1} T_{i2}^{i2} \frac{\partial I_{TR}(m,n)}{\partial x} \qquad (50c)$$

where

$$I_{TR}(m,n) = \iint_{area} x^m y^n dxdy$$

Geometric stiffness matrix sensitivities

1) <u>Geometric stiffness matrix sensitivities with respect to thickness design variables</u>

The design variable in this case is the k^{th} coefficient in the polynomial thickness series for the i^{th} layer. Examination of Eqs.(46) and (47) reveals that the geometric stiffness matrix is explicitly linear in the thickness coefficients. It is also dependent on those coefficients through the wing box solution unless it is assumed that in-plane loads do no change. Differentiation of Eq.(46) leads to:

$$\frac{\partial K_{G_{rs}}}{\partial T_k^{it}} = -\frac{1}{2}\sum_{a=1}^{2}\sum_{b=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{6}\sum_{i=1}^{3}\sum_{j=1}^{6}\sum_{ih=1}^{N_h}\sum_{qw=1}^{3}\sum_{pw=1}^{N_w}\overline{F}_T \qquad (51)$$

And

$$\overline{F}_T = \sum_{R=1}^{N_L}\sum_{S=1}^{N_t} \overline{F}_{2ij}^{ar}\overline{F}_{2ij}^{bs} Q_{ppp,qw}(\theta_i) w_{qw,pw} H_{ih} [G_{ks}^{itR}\overline{q}_{pw} + T_S^R \frac{\partial \overline{q}_{pw}}{\partial T_k^{it}}] I_{TR}(mG_{rs}, nG_{rs}) \qquad (51a)$$

where $G^{it,R}{}_{ks}$ is equal to 1 when it = R and k = S. Otherwise, it is zero.

2) <u>Geometric stiffness matrix sensitivities with respect to fiber orientation</u>

Layer orientations affect the geometric stiffness matrix through the material matrices $Q_{ppp,qw}(\theta_{it})$ and the wing box generalized displacements {q*}. The analytic sensitivity with respect to fiber orientation in a given layer is:

$$\frac{\partial K_{G_{rs}}}{\partial \theta_{it}} = -\frac{1}{2}\sum_{a=1}^{2}\sum_{b=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{6}\sum_{i=1}^{3}\sum_{j=1}^{6}\sum_{ih=1}^{N_h}\sum_{qw=1}^{3}\sum_{pw=1}^{N_w}\overline{F}_{\theta} \qquad (52)$$

and

$$\overline{F}_q = \sum_{it1=1}^{N_L}\sum_{k=1}^{N_s}\overline{F}^{ar}_{2ij}\overline{F}^{bs}_{2ij}\left[\frac{\partial Q_{ppp,qw}(\theta_{itt})}{\partial \theta_{it}}\overline{q}_{pw} + Q_{ppp,qw}(\theta_{itt})\frac{\partial \overline{q}_{pw}}{\partial \theta_{itt}}\right] w_{qw,pw} H_{ih} T^{it}_{k} I_{TR}(mG_{rs}, nG_{rs})$$

3) <u>Geometric stiffness matrix sensitivities with respect to planform variables</u>
Thickness coefficients and orientation angles are held fixed. The geometric stiffness matrix $[K_G]$ depends on the planform variables through Ui and Vj terms in F^{*ar}_{2ij} and F^{*bs}_{2ij}. There is also a dependence on the area integrals (I_{TR}), since those are evaluated over the planform shape of the panel. Thus,

$$\frac{\partial K_{G_{rs}}}{\partial x} = -\frac{1}{2}\sum_{a=1}^{2}\sum_{b=1}^{2}\sum_{i=1}^{3}\sum_{j=1}^{6}\sum_{i=1}^{3}\sum_{j=1}^{6}\sum_{ih=1}^{N_h}\sum_{qw=1}^{3}\sum_{pw=1}^{N_w}\overline{F}_{PV} \qquad (53)$$

and

$$\overline{F}_{PV} = \sum_{it=1}^{N_L}\sum_{k=1}^{N_u}(Q_{ppp,qw}(q_{it}) w_{qw,pw} H_{ih} T^{it}_{k})\{\overline{F}_{PV1} + \overline{F}_{PV2}\} \qquad (53a)$$

$$\overline{F}_{PV1} = \left[\frac{\partial \overline{F}^{ar}_{2ij}}{\partial x}\overline{F}^{bs}_{2ij} + \overline{F}^{ar}_{2ij}\frac{\partial \overline{F}^{bs}_{2ij}}{\partial x}\right]\overline{q}_{pw} I_{TR}(m,n) \qquad (53b)$$

$$\overline{F}_{PV2} = \overline{F}^{ar}_{2ij}\overline{F}^{bs}_{2ij}\left[\overline{q}_{pw}\frac{\partial T_{TR}(m,n)}{\partial x} + \frac{\partial \overline{q}_{pw}}{\partial x} I_{TR}(m,n)\right. \qquad (53c)$$

where x represents any of the planform variables, and the powers of the integrands in $I_{TR}(m,n)$ are $m = mG_{rs}$ and $n = nG_{rs}$. In Eq.(53), it is assumed that overall wing planform is fixed, and panels are changing shape and location due to moving of control surfaces, ribs and spars. If overall planform shape of wing is changing, then derivatives of the wing depth coefficients with respect to the shape design variables must be added, since the wing depth is defined in global x,y coordinates.

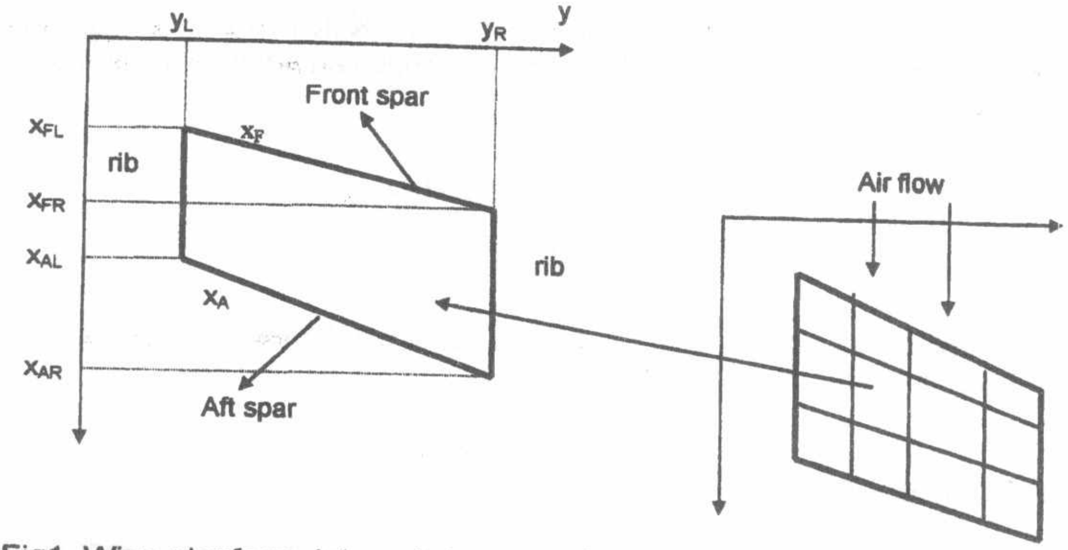

Fig1. Wing planform, internal structure and skin panel geometry

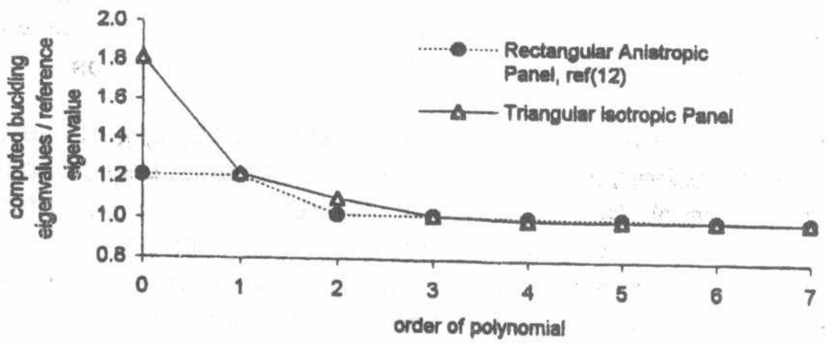

Fig.2 Convergence of critical buckling eigen value for a 30-deg. angle ply rectangular panel in combined in-plane loading, and an isotropic triangular panel under uniform in-plane pressure

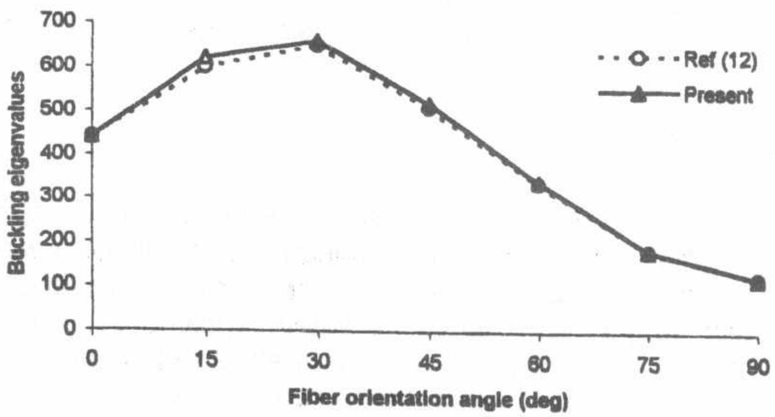

Fig.3 Critical buckling eigenvalues for rectangular angle-ply panel

Eigenvalues sensitivities

Now that the sensitivities of the [K] and $[K_G]$ matrices are available analytically, the sensitivity of buckling eigenvalues and, hence, the sensitivity of buckling constraints is given by:

$$\frac{\partial g}{\partial x}=-\frac{\partial l}{\partial x}=\frac{\{\phi\}^T\left(\frac{\partial [K]}{\partial x}+l\frac{\partial [K_G]}{\partial x}\right)\{\phi\}}{\{\phi\}^T[K_G]\{\phi\}} \qquad (54)$$

This analytic sensitivity can be used to construct direct, reciprocal, or hybrid approximations of the constraint.

TEST CASES AND RESULTS

To assess the new capability, test cases were chosen to address the convergence rate with increased polynomial order, the accuracy of analysis results, the reliability of analytic sensitivities and accuracy of finite difference derivatives, and the integration of wing box structural analysis with panel buckling analysis for fast structural analysis of airplane wings.

Analysis results from Ref.12 are used to test accuracy and convergence rate of the present technique in the case of rectangular anisotropic panels. Simply supported, angle ply laminates made of 20 plies of fiber-glass material with a= 5 (in); b= 10 (in), and combined in-plane loading consisting of $N_x=1$, $N_y=1$ and $N_{xy}=1$, are considered. Figure 2 shows convergence of the buckling load (the critical eigenvalue) as a function of the order of polynomial Ritz series (Eqs.12 and 13). Figure 3 shows critical buckling eigenvalues, calculated with $N_w = 4$ (15-term complete polynomial), for cases in which fiber orientation angles varies. Good accuracy and fast convergence are demonstrated. With only 6 terms (order 2 polynomial) the present results are only within 3% error of Ref.12 results. For automated synthesis, where the need to have very fast analysis and sensitivity results is critical, this fast convergence makes it possible to use low-order models without sacrificing accuracy too much.

CONCLUSIONS

An efficient technique for computation of skin panel buckling constraints has been presented, tailored to the needs of multidisciplinary wing optimization. The formulation used, based on Ritz structural analysis using simple polynomials, makes it possible to obtain analytic sensitivities of buckling constraints with respect to shape, as well as sizing and fiber angle design variables. No numerical integration is required. Closed-form expressions for stiffness and geometric stiffness matrix terms make it possible to identify dependence of those matrices on sizing, fiber angles, and shape design variables. Integration with equivalent plate wing structural analysis is natural, and the details has been described.

Numerical test of the new capability demonstrate high accuracy and fast convergence. As the results presented here show, if some accuracy of analytic predictions can be sacrificed for the purpose of multidisciplinary preliminary design synthesis, then panel buckling analysis and sensitivities, using only 3-6 polynomial terms, can be obtained very efficiently. When integrated with wing structural analysis, aeroservoelastic analysis and aerodynamic loads and drag prediction, the present buckling analysis capabilities add an important element to the integrated multidisciplinary design synthesis of actively controlled fiber composite wings.

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