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The Friedmann Robertson Walker Space Time In Lyra Geometry

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IN THE GRAVITATIONAL theory based on Lyra's geometry, we study the Friedmann Robertson Walker (FRW) cosmological model in the presence of perfect fluid. Exact solutions of the Einstein equations are obtained and discussed in the following cases: (i) $\beta(t)$ (the time component of the displacement vector introduced by Lyra) is a constant, (ii) $\beta(t)$ is time dependent and q (the deceleration parameter of the model) is a constant and (iii) $\beta(t)$ and q are functions of time t. The thermodynamic functions of the universe are calculated and studied for the cases $k = 0,1,-1$ flat, open, and closed universe, respectively.The entropy of the universe is obtained as a constant which means we deal with a stage of evolution in which the universe is adiabatic. The distance modulus for our model is calculated at different values of the red shift and compared with the results obtained in supernovae la. The physical and geometrical properties of the obtained models are discussed.

Keywords: Lyra geometry; The Friedmann Robertson Walker Cosmological Model; Entropy, Energy momentum tensor; Thermodynamic functions.
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1 Introduction

In 1918, Einstein [1] proposed his general theory of relativity which is a geometrizing of the gravitation. Weyl [2] tried to generalize the theory of relativity by introducing a scalar field φ for geometrizing the electromagnetic and gravitation both. The scalar field introduced by Weyl make the length transfer of the vector is non-integrable under parallel transport which means that the spectral lines emitted by atoms will not remains constant but would depend on their past histories. So, the theory was not taken seriously at that time and criticized by Einstein [3]. In 1951, Lyra [4] suggested a modification of Riemannian geometry, by introducing a gauge function $x^{\circ}(x^i)$ into the structureless manifold which removes the non-integrability condition of the length of a vector under parallel transport and a cosmological constant is naturally introduced from the geometry. In Lyra geometry unlike Weyl the connection is metric preserving as in Riemannian.

In the description of the present state of the universe, the Fridmann Robertson Waker space time describe a homogenous and isotropic distribution of its matter. Partridge and Wilkinson [5] and Ehlers et al. [6] pointed out that spatially homogeneous and isotropic universes can be described by FRW model.

In the modified theories of general relativity, FRW space time was studied in different aspects. Taser [7] investigated conformally symmetric FRW metric in $f(R,T)$ gravity. Induced Brownian motion by the FRW cosmological model in the presence of a cosmic string was investigated by Mota and de Mello [8]. Stavrinos et al. [9] studied Friedmann-like Robertson-Walker model in generalized metric space-time with weak anisotropy. Paris and Visser [10] investigated minimal conditions for the creation of a FRW universe from a bounce. The behavior of FRW cosmological models in scalar-tensor gravity was analyzed by Kolitch and Eardley [11]. Holden et al. [12] presented a phase-plane analysis of FRW cosmologies in Brans-Dicke gravity. Isochronous cosmological solutions of the FRW model were investigated by Chen et al. [13]. Aref'eva et al. [14] studied dynamics in nonlocal linear models in the FRW metric. Goswami [15] presented FRW space-time in the perspective of the latest developments

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begun by Perlmutter and Riess in cosmology. Goswami et al. [16] studied FRW accelerating universe with interactive dark energy. Faraoni and Cooperstock [17] studied the total energy of open FRW universes. Upadhyay [18] studied field-dependent symmetries in FRW models. Kim et al. [19] studied FRW models that do not require zero active mass. Boyanovsky et al. [20] studied scalar field dynamics in FRW spacetimes. Van den Hoogen et al. [21] investigated scaling solutions in Robertson Walker-spacetimes. Melia [22] derived the FRW metric coefficients from the general form of the spherically symmetric line element and demonstrated that, because the co-moving frame also happens to be in free fall, the symmetries in FRW metrices are valid only for a medium with zero active mass. Chen et al. [23] studied the periodic solutions with the equal period for the FRW model. Ibanez and Verdaguer [24] studied by dimensional reduction the solutions of Einstein equations in a five-dimensional vacuum. They derived two metrics which can be interpreted as finite perturbations on FRW models. Stewart [25] discussed the mathematical principles that depend on the theory of gauge-invariant perturbations of homogeneous isotropic cosmological models. Lewis et al. [26] studied efficient computation of cosmic microwave background anisotropies in closed FRW models. Finite exact perturbations of FRW cosmologies constructed with the inverse scattering technique of Belinskii and Zakharov were studied by Diaz et al. [27]. The late-time evolution of FRW models with a perfect fluid matter source and a scalar field arising in the conformal frame of $f(R)$ theories nonminimally coupled to matter was studied by Miritzis [28]. Das and Singh [29] studied the polytropic gas dark energy model and new-age graphic dark energy model in the flat FRW universe and establish a correspondence between them for the scalar fields. Singh and Singh [30] studied Robertson Walker models in Einstein's theory with cosmological terms and in Lyra's geometry. By considering a time-dependent displacement field and variable deceleration parameter, Pradhan et al. [31] studied FRW models in Lyra's geometry and obtained a new class of exact solutions for exponential, polynomial and sinusoidal universe respectively. Singh [32] studied a spatially homogeneous and isotropic Robertson Walker model with zero curvature of the universe on Lyra's geometry.

In this paper, we study the FRW cosmological model with perfect fluid with different form of the Lyra term and deceleration parameter. In section 2 we drive the field equations and the conservation equations of the energy momentum tensor. Solution of the field equations in the case of constant displacement vector is introduced in section 3. In section 4, we introduce the solution with β and q are constants while in section 5 we investigate solution in the case of $\beta(t)$ and $q(t)$ are functions of the time t. In section 6, conclusion remarks are indicated.

2 The FRW metric and field equations

The FRW metric takes the form:

$$ds^2 = dt^2 - \frac{M^2(t)}{(1+\frac{1}{4}kr^2)^2}[dr^2 + r^2 d\theta^2 + r2 sin2\theta d\phi 2. \quad (1)$$

Einstien field equations on Lyra geometry as obtained by Sen [33] has the form:

$$G_{\mu\nu} + \frac{3}{2}\phi_\mu\phi_\nu - \frac{3}{4}g_{\mu\nu}\phi_\alpha\phi^\alpha = -\chi T_{\mu\nu}, \quad (2)$$

$T_{\mu\nu}$ is the energy momentum tensor which in the presence of a perfect fluid reads as:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}. \quad (3)$$

ϕ_α is a displacement vector with one component in the form:

$$\phi_\alpha = (\beta(t), 0,0,0). \quad (4)$$

For the metric (1) the field equations (2) read as:

$$3(\frac{k}{M^2} + \frac{\dot{M}^2}{M^2}) - \frac{3}{4}\beta^2 = \chi\rho, \quad (5)$$

$$\frac{k}{M^2} + \frac{\dot{M}^2}{M^2} + \frac{2\ddot{M}}{M} + \frac{3}{4}\beta^2 = -\chi p. \quad (6)$$

Conservation equation $T^{\nu}_{\mu;\nu}$ leads to:

$$\dot{\rho} + 3(\rho + p)\frac{\dot{M}}{M} = 0. \quad (7)$$

Conservation of the left hand side of (2) yields:

$$\frac{3}{2}\beta\dot{\beta} + \frac{9}{2}\beta^2\frac{\dot{M}}{M} = 0, \quad (8)$$

where overhead dot denotes derivatives with respect to t.

Solution of the field equations (5), (6) with two conditions (7) , (8) are not possible in general. So, we consider the following cases:

3 Solution of the field equations with β is constant

For $\beta = \beta_\circ$(constant), equation (8) has a solution in the form $M(t) = c_1(constant)$. From (5) and (6), the pressure and density are constants and given by $\rho = \frac{1}{\chi}[\frac{3k}{c_1^2} - \frac{3\beta_\circ^2}{4}]$ and $p = -\frac{1}{\chi}[\frac{k}{c_1^2} + \frac{3\beta_\circ^2}{4}]$. The change on the entropy of the universe reduces to $\frac{dS}{dt} = 0$. [34], [35], [36], [37], [38], that is $\boldsymbol{S} = C_1(constant)$, which means that we deal with an adiabtic process.

The enthalpy (H), the Helmholtz free energy (F) and the Gibbs free energy (G) are constants and read as [34], [35], [36], [37]:

$$\boldsymbol{H} = \frac{32\sqrt{\frac{r^4 \sin[\theta]^2 c_1^6}{(4+kr^2)^6}}(4k-3c_1^2\beta_\circ^2)}{\chi c_1^2}, \quad \boldsymbol{F} = \frac{48\sqrt{\frac{c_1^6 r^4 \sin[\theta]^2}{(4+kr^2)^6}}(4k-c_1^2\beta_\circ^2)}{\chi c_1^2}, \tag{9}$$

$$\boldsymbol{G} = \frac{32\sqrt{\frac{r^4 \sin[\theta]^2 c_1^6}{(4+kr^2)^6}}(4k-3c_1^2\beta_\circ^2)}{\chi c_1^2}. \tag{10}$$

Halford [39] has pointed out that if we put $\beta_\circ^2 = \frac{-4\Lambda}{3}$, we can achieve almost a complete equivalence between Lyra cosmological theory and the relativistic theory. For the case of stiff matter solutions with a cosmological constant in general relativity theory, Beesham [40] has shown that the corresponding equations are equivalent in both geometries if we set $\frac{3}{8}\beta^2 = -\Lambda$. Further, the vacuum solutions are also identical to the corresponding general relativity vacuum solutions with a cosmological constant. Our solution obtained in this section is consistant with Halford condition [39].

4 Solution of the field equations with time dependent displacement vector and constant deceleration parameter

For $\beta(t)$ is time dependent and q is constant, from (8) we get:

$$\beta(t) = \frac{m_1}{M^3} \tag{11}$$

where m_1 is a constant. The deceleration parameter is given by:

$$q = -\frac{R\ddot{R}}{\dot{R}^2}, \tag{12}$$

where R is the average scale factor and reads as:

$$R^3 = \sqrt{-g} = \frac{r^2 \sin[\theta]}{\left(1+\frac{kr^2}{4}\right)^3}[M(t)]^3 \tag{13}$$

From (12) we get:

$$R = K(c_1 t + c_2)^{\frac{1}{1+q}}, \tag{14}$$

By comparing (13) and (14) we can take:

$$M(t) = (c_1 t + c_2)^{\frac{1}{1+q}}, \tag{15}$$

where $K(r,\theta) = \frac{\sqrt[3]{r^2 \sin[\theta]}}{\left(1+\frac{kr^2}{4}\right)}$, c_1 and c_2 are constants.

From (5) and (6) we get:

$$p = \frac{\frac{4(-1+2q)c_1^2}{(1+q)^2(tc_1+c_2)^2} - 4k(tc_1+c_2)^{-\frac{2}{1+q}} - 3(tc_1+c_2)^{-\frac{6}{1+q}}m_1^2}{4\chi}, \tag{16}$$

$$\rho = \frac{3\left(\frac{4c_1^2}{(1+q)^2(tc_1+c_2)^2} + 4k(tc_1+c_2)^{-\frac{2}{1+q}} - (tc_1+c_2)^{-\frac{6}{1+q}}m_1^2\right)}{4\chi}, \tag{17}$$

and (11) reduces to

$$\beta(t) = m_1(tc_1+c_2)^{-\frac{3}{1+q}}. \tag{18}$$

For the model (1), the volume element $V = \sqrt{\frac{r^4 \mathrm{Sin}[\theta]^2 (tc_1+c_2)^{\frac{6}{1+q}}}{\left(1+\frac{kr^2}{4}\right)^6}}$, the Hubble parameter $H = \frac{c_1}{(1+q)(tc_1+c_2)}$, the expansion scalar $\theta = \frac{3c_1}{(1+q)(tc_1+c_2)}$, the non zero components of the shear tensor (σ_i^j) are $\sigma_1^1 = \sigma_2^2 = \sigma_3^3 = -\frac{c_1}{(1+q)(tc_1+c_2)}$ and the shear $\sigma = \sqrt{\frac{3c_1^2}{2(1+q)^2(tc_1+c_2)^2}}$.

The thermodynamic functions of the universe read as: $\boldsymbol{S} = C_1$

$$\boldsymbol{H} = \frac{32\sqrt{\frac{r^4 \sin[\theta]^2 (tc_1+c_2)^{\frac{6}{1+q}}}{(4+kr^2)^6}}\left(\frac{4c_1^2}{(1+q)(tc_1+c_2)^2} + 4k(tc_1+c_2)^{-\frac{2}{1+q}} - 3(tc_1+c_2)^{-\frac{6}{1+q}}m_1^2\right)}{\chi}, \tag{19}$$

$$\boldsymbol{F} = -\frac{c_1 C_1}{2\pi(1+q)(tc_1+c_2)} + \frac{48\sqrt{\frac{r^4 \sin[\theta]^2 (tc_1+c_2)^{\frac{6}{1+q}}}{(4+kr^2)^6}}\left(\frac{4c_1^2}{(1+q)^2(tc_1+c_2)^2} + 4k(tc_1+c_2)^{-\frac{2}{1+q}} - (tc_1+c_2)^{-\frac{6}{1+q}}m_1^2\right)}{\chi}, \tag{20}$$

$$\boldsymbol{G} = -\frac{c_1 C_1}{2\pi(1+q)(tc_1+c_2)} + \frac{32\sqrt{\frac{r^4 \sin[\theta]^2 (tc_1+c_2)^{\frac{6}{1+q}}}{(4+kr^2)^6}}\left(\frac{4c_1^2}{(1+q)(tc_1+c_2)^2} + 4k(tc_1+c_2)^{-\frac{2}{1+q}} - 3(tc_1+c_2)^{-\frac{6}{1+q}}m_1^2\right)}{\chi}. \tag{21}$$

The behavior of the thermodynamic functions of the universe with the time can be shown as follow. The values of the constants are taken as: $c_1 = 1$, $C_1 = 2$, $c_2 = 1$, $\theta = \frac{\pi}{2}$, $r = 3$, $q = -0.5$, $m_1 = 5$ and $\chi = 8\pi$

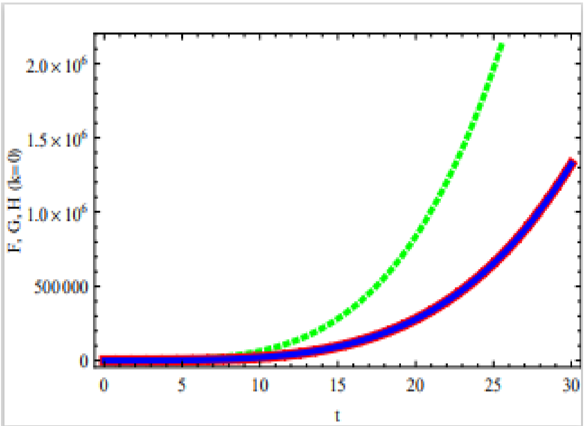

Fig. 1 The Helmholtz F (Green line), Gibbs free energy G (Red line) and the Enthalpy H (Blue line) vs. time t, $0 < t < 30$, ($k = 0$).

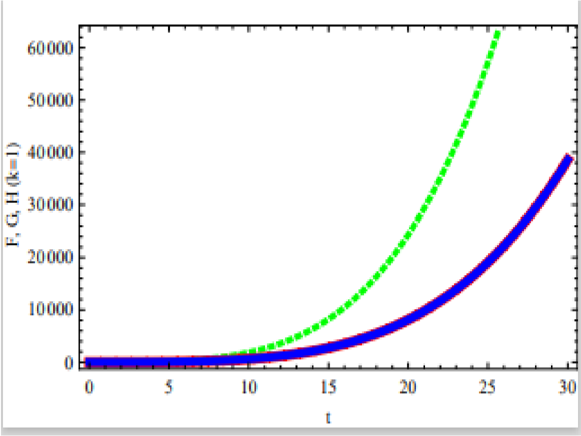

Fig. 2 The Helmholtz F (Green line), Gibbs free energy G (Red line) and the Enthalpy H (Blue line) vs. time t, $0 < t < 30$, $(k = 1)$.

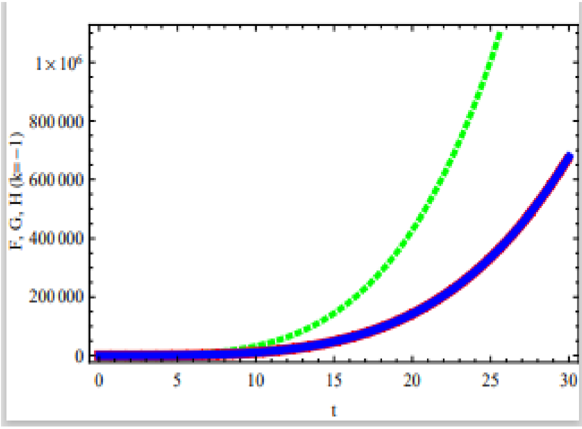

Fig. 3 The Helmholtz F (Green line), Gibbs free energy G (Red line) and the Enthalpy H (Blue line) vs. time t, $0 < t < 30$, $(k = -1)$.

For $k = 0,1,-1$. F, G, and H begin with zero values at the beginning of evolution and increase to reach large values at the end of evolution.

In case of $k = -1$, the values of F, G and H are large comparing with the two cases $k = 0,1$. The two cases $k = 0,1$ nearly gave the same values for F, G, and H (Fig.1, Fig.2 and Fig.3).

5 Distance modulus for supernovae la

The distance modulus for supernovae la reads as:

$$\mu = 5\ln d_l + 25, \quad (22)$$

Here d_l is the luminosity distance given by:

$$d_l = r_1(1+z)R_\circ, \quad (23)$$

where z represent redshift parameter, $R_\circ$ is the present scale factor and r_1 can be obtained from

$$r_1 = \int_t^0 \frac{dt}{R}, \quad (24)$$

from (14), equation (24) reduces to:

$$r_1 = -\frac{q+1}{qkc_1}(c_1 t + c_2)^{\frac{q}{q+1}} + c_3. \quad (25)$$

From the relation $z + 1 = \frac{R_{z=0}}{R}$, where $R_{z=0} = 2.46$ is the present value of the scale factor equation (25) becomes:

$$r_1 = -\frac{q+1}{qkc_1}\left(\frac{2.46}{k(1+z)}\right)^q + c_3. \quad (26)$$

So, equation (23) reads as:

$$d_l = 2.46\left[-\frac{q+1}{qkc_1}\left(\frac{2.46}{k(1+z)}\right)^q + c_3\right](1+z). \quad (27)$$

Finally, the distance modulus μ reads as:

$$\mu = 5\ln(2.46[-\frac{q+1}{qkc_1}(\frac{2.46}{k(1+z)})^q + c_3](1+z)) + 25. (28)$$

In the following we makes a comparison between the values of the distance modulus obtained for supernovae la and the values obtained for our model

Redshift (z)	Supernovae la (μ)	Our model (μ')
0.0132	30.09 : 30.61	30.43
0.0133	32.32 : 32.76	32.6
0.0134	32.88 : 33.32	33.3
0.0264	33.73 : 34.13	34.1
0.0274	34.23 : 34.43	34.4
0.0284	34.38 : 34.84	34.7
0.0295	34.65 : 34.99	34.9
0.0398	35.01 : 35.35	35.3
0.0409	35.68 : 36	36
0.0399	35.74 : 36.08	36.01
0.0519	35.93 : 36.25	36.2
0.0721	36.16 : 36.5	36.48
0.0619	36.18 : 36.52	36.5
0.0813	36.59 : 36.91	36.9

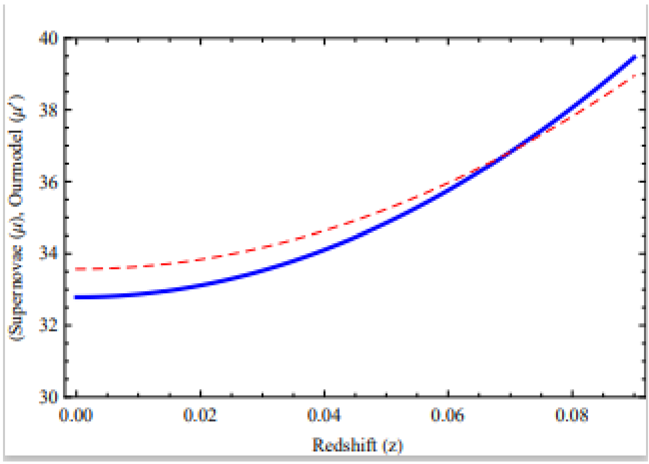

Fig. 4 The distance modulus μ' of our universe (dashed line) and the distance modulus μ of Supernova la μ (thick line).

For the interval $0 < z < 0.07$, the curve of μ' is large than the curve of ⍰.

For $z > 0.07, \mu > \mu'$. At $\mu \simeq 0.07$, we obtained an identical value between μ and μ' (Fig. 4).

6 Solution of the field equations with displacement vector and deceleration parameter are functions of time

For β and q are time dependent functions, we can obtain the solution as follow: If we consider q in the form: $q = -kt + m - 1$.

Equation (8) gives:

$$\beta = \frac{m_1}{M^3}, \quad (29)$$

where m_1 is constant.

The deceleration parameter is given by:

$$q = -\frac{R\ddot{R}}{\dot{R}^2}, \quad (30)$$

where R is the average scale factor and read as:

$$R^3 = \sqrt{-g} = \frac{r^2 \sin[\theta]}{\left(1+\frac{kr^2}{4}\right)^3}[M(t)]^3 \quad (31)$$

From (30) we get:

$$R = K\left[\frac{k_1 t}{2m-k_1 t}\right]^{\frac{1}{m}}, \quad (32)$$

Then,

$$M(t) = \left[\frac{k_1 t}{2m-k_1 t}\right]^{\frac{1}{m}}, \quad (33)$$

where $K(r,\theta) = \frac{\sqrt[3]{r^2 \sin[\theta]}}{\left(1+\frac{kr^2}{4}\right)}$, k_1 and m are constants.

From (5) and (6), we obtain:

$$p = -\frac{4k\left(\frac{tk_1}{2m-tk_1}\right)^{-2/m} + \frac{16(3-2m+2tk_1)}{t^2(-2m+tk_1)^2} + 3\left(\frac{tk_1}{2m-tk_1}\right)^{-6/m} m_1^2}{4\chi}, \quad (34)$$

$$\rho = \frac{3\left(4k\left(\frac{tk_1}{2m-tk_1}\right)^{-2/m} + \frac{16}{t^2(-2m+tk_1)^2} - \left(\frac{tk_1}{2m-tk_1}\right)^{-6/m} m_1^2\right)}{4\chi}, \quad (35)$$

Equation (29)

$$\beta = \left(\frac{tk_1}{2m-tk_1}\right)^{-3/m} m_1. \quad (36)$$

For the model (1), the volume element $V = \sqrt{\frac{r^4 \mathrm{Sin}[\theta]^2\left(\frac{tk_1}{2m-tk_1}\right)^{6/m}}{\left(1+\frac{kr^2}{4}\right)^6}}$, the Hubble parameter $H = \frac{2}{2mt-t^2k_1}$, the expansion scalar $\theta = \frac{6}{2mt-t^2k_1}$, the non zero components of the shear tensor (σ_i^j) given by $\sigma_1^1 = \sigma_2^2 = \sigma_3^3 = \frac{2}{t(-2m+tk_1)}$ and the shear $\sigma = \sqrt{\frac{6}{t^2(-2m+tk_1)^2}}$.

The thermodynamics function of the universe $\boldsymbol{S} = C_1$.

The enthalpy (**H**), the Helmholtz free energy (**F**) and the Gibbs free energy (**G**) read as:

$$\boldsymbol{H} = \frac{32\sqrt{\frac{r^4 \sin[\theta]^2\left(\frac{tk_1}{2m-tk_1}\right)^{6/m}}{(4+kr^2)^6}}\left(4k\left(\frac{tk_1}{2m-tk_1}\right)^{-2/m} + \frac{16(m-tk_1)}{t^2(-2m+tk_1)^2} - 3\left(\frac{tk_1}{2m-tk_1}\right)^{-6/m} m_1^2\right)}{\chi}, \quad (37)$$

$$\boldsymbol{F} = -\frac{C_1}{2m\pi t - \pi t^2 k_1} + \frac{48\sqrt{\frac{r^4 \sin[\theta]^2\left(\frac{tk_1}{2m-tk_1}\right)^{6/m}}{(4+kr^2)^6}}\left(4k\left(\frac{tk_1}{2m-tk_1}\right)^{-2/m} + \frac{16}{t^2(-2m+tk_1)^2} - \left(\frac{tk_1}{2m-tk_1}\right)^{-6/m} m_1^2\right)}{\chi}, \quad (38)$$

$$\boldsymbol{G} = -\frac{C_1}{2m\pi t - \pi t^2 k_1} + \frac{32\sqrt{\frac{r^4 \sin[\theta]^2\left(\frac{tk_1}{2m-tk_1}\right)^{6/m}}{(4+kr^2)^6}}\left(4k\left(\frac{tk_1}{2m-tk_1}\right)^{-\frac{2}{m}} + \frac{16(m-tk_1)}{t^2(-2m+tk_1)^2} - 3\left(\frac{tk_1}{2m-tk_1}\right)^{-6/m} m_1^2\right)}{\chi}. \quad (39)$$

The behavior of the thermodynamic functions of the universe with the time can be shown as follow. The values of the constants are taken as: $c_1 = 1$, $C_1 = 2$, $\theta = \frac{\pi}{2}$, $r = 3$, $k_1 = 0.113$, $m_1 = 1$, $m = 2$ and $\chi = 8\pi$.

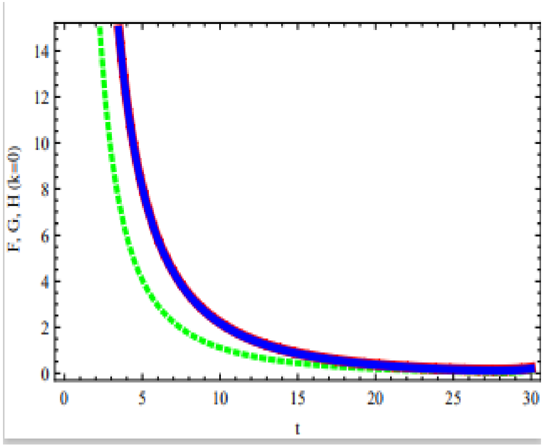

Fig. 5 The Helmholtz F (Green line), Gibbs free energy G (Red line) and the Enthalpy H (Blue line) vs. time t, $0 < t < 30$.

F, G and H begin with large values at $t = 0$ and decrease to reach zero values at the end evolution.

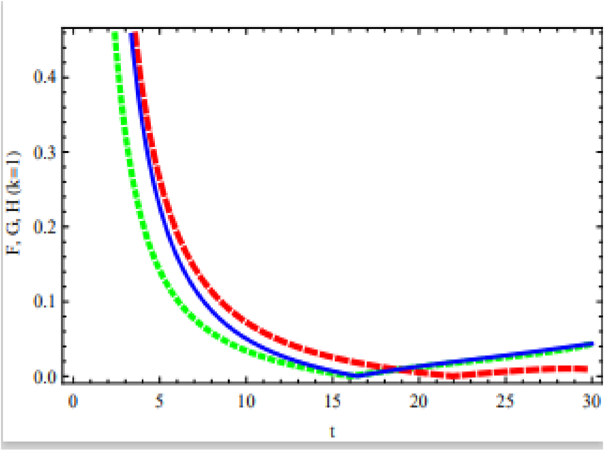

Fig. 6 The Helmholtz F (Green line), Gibbs free energy G (Red line) and the Enthalpy H (Blue line) vs. time t, $0 < t < 30$.

For $0 < t < 2$, $F, G, H = 0$, as $t > 2$, F decreases to reach zero at $t \simeq 16$ and increases again to reach large value at the end of evolution. as $t > 5$, H decreases to reach zero at $t \simeq 16$ and increases again to reach large value at the end of evolution. As $t > 5$, H decreases to reach zero at $t \simeq 16$ and increases again to reach large value at the end of evolution. As $t > 5$, G decreases to reach zero value at the end of evolution.

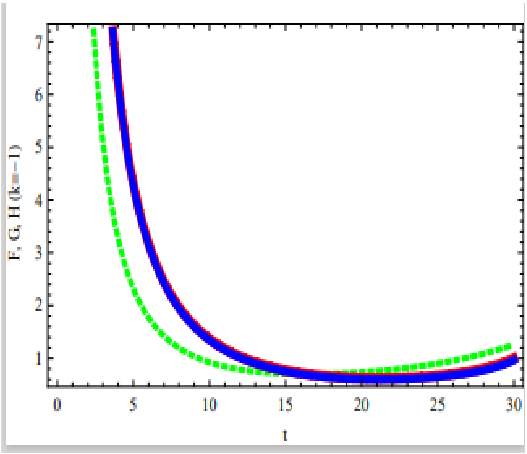

Fig. 7 The Helmholtz F (Green line), Gibbs free energy G (Red line) and the Enthalpy H (Blue line) vs. time t, $0 < t < 30$.

For $0 < t < 2$, $F, G, H = 0$, as $t > 2$, F decreases to reach zero at $t \simeq 15$ and increases again to reach large value at the end of evolution. As $t > 5$, G, H are decreasing to reach zero at $t \simeq 20$ and increase again to reach large value at the end of evolution.

From (22)-(24) we find the distance modulus μ in the form :

$$\mu = 5\ln(2.46[\frac{2}{kk_1}(\frac{2.46}{k(1+z)}+1)^{-1} - \frac{2}{kk_1}ln(2 - 2(\frac{2.46}{k(1+z)}+1)^{-1}) + 1](1+z)) + 25. \quad (40)$$

As a pervious case, we makes a comparison between the values of the distance modulus obtained for supernovae la and the values obtained for our model.

Redshift(z)	Supernovae la (μ)	Our model (μ')
0.0132	30.09 : 30.61	30.30
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0.0295	34.65 : 34.99	34.8
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0.0409	35.68 : 36	35.8
0.0399	35.74 : 36.08	35.9
0.0519	35.93 : 36.25	36.1
0.0721	36.16 : 36.5	36.3
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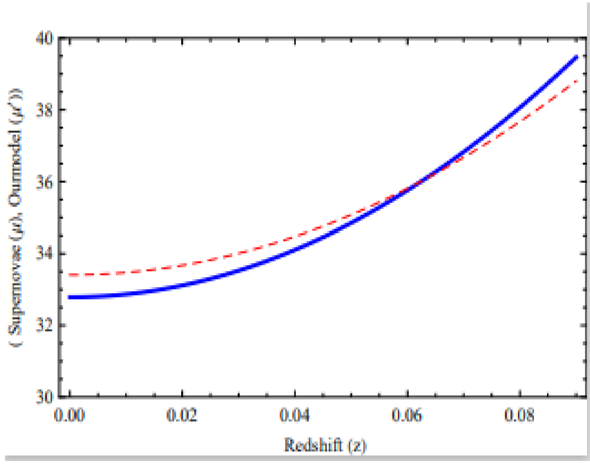

Fig. 8 The distance modulus μ' of our universe (dashed line) and the distance modulus μ of Supernova la μ (thick line).

For $0 < z < 0.6, \mu' > \mu$.At $t \simeq 0.06$, we obtain $\mu = \mu'$. As $t > 0.06, \mu' < \mu$. (Fig. 8).

7 Conclusion

In the present paper, we studied (FRW) cosmological model in Lyra geometry in different forms of the time component of the displacement vector β and the deceleration parameter q. In the case of $\beta = \beta_\circ$ (constant) we obtained a static cosmological model with $M(t) = constant$. This result is consistence with the result obtained by Halford [39]. The pressure p and the density ρ are constants. The entropy $\boldsymbol{S}$ of the universe is constant that means we deal with an adiabatic process. The thermodynamics function of the universe are constants. In the case of $\beta(t)$ is a function of t and q is a constant, we deal also with an a diabetic process as $\boldsymbol{S} = constant$. The functions $\boldsymbol{F}, \boldsymbol{G}$ and $\boldsymbol{H}$ for the cases $(k = 0,1,-1)$ begin with small values at the beginning of evolution and increase uniformly to reach large values at the end of evolution. In the case of β and q are functions of time t, we obtain a different behavior of the thermodynamic functions of the universe comparing with the second case $q = constant$. The functions $\boldsymbol{F}, \boldsymbol{G}$ and $\boldsymbol{H}$ begin with large values at the beginning of evolution and reduce to reach small values at the end of evolution. In three cases the additional term introduced by Lyra has an effect on the behaviores of the pressure and density but has no effect on the entropy of the universe because it is not a part of the energy momentum tensor. This result in agreement with Hegazy and Farook [36] and Hegazy [37]. The curve of the distance modulus μ' of our universe obtained in the case of time dependent deceleration parameter is closer to the distance modulus μ of Supernova la than the curve obtained in the case of constant deceleration parameter.

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