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An Investigation into the Performance of Control Fin Drives

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Abstract

The control system of a guided missile is composed of the missile itself equipped with an automatic pilot and of a guidance system which is either on the missile or outside of it. The guidance system measures the missile's coordinates and out of them in addition to the target coordinates creates the control signals for deflecting the fins in such a direction that should result in the minimum deflection of the missile from the required trajectory. To control the missile during its flight in space, two control signals are sufficient. However, in some cases it might be required to control also the rotation of the missile around its longitudinal axis in which case there should be three control signals. The direction of the missile's motion most often takes place in two mutually perpendicular planes. In order to ensure correct functioning of the control fins in flight, the missile must not rotate around its longitudinal axis using a gyroscopic stabilization channel within the missile. Therefore, the missile autopilot has three channels two of them serving for control and one for stabilization. The main requirement imposed on the autopilot is the transfer of the control signals into mechanical rotation of the control fins. Therefore, each autopilot channel always contains a control fin drive, mainly of the servo-amplifier and the servomotor (actuator). During missile flight it is necessary to keep the missile's dynamic properties approximately fixed or slowly varying. Thus, any undue changes in the missile's maneuverability can be avoided using feedbacks from the angular speed and from the missile's normal acceleration to stabilize and to improve the dynamic properties of the missile and of the whole control circuit. Therefore, this paper is devoted to investigate the performance of the control fin drives in a guided missile including the effects of hinge moments upon its dynamic properties. The dynamic pressure had changed between three values to investigate the fin performance during minimum, medium and maximum loading conditions. Then, necessary feedbacks and cascaded correction networks are designed and analyzed to improve the system performance.

Keywords: Guidance and Control, Aerodynamics, Mechanics of Flight

1- Introduction

The autopilot and the guidance system are the main elements/subsystems constituting the guided missile control system. The guidance system measures the

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missile's coordinates and out of them in addition to the target coordinates creates the control signals for deflecting the fins in such a direction that should result in the minimum deflection of the missile from the required trajectory. To control the missile during its flight in space, two control signals are sufficient. However, in some cases it might be required to control also the magnitude of the missile's speed in which case there should be three control signals. In order to ensure correct functioning of the control fins in flight, the missile must not rotate around its longitudinal axis. Therefore, the missile autopilot has three channels, as a rule, two of them serving for control and one for stabilization using a gyroscopic stabilization channel.

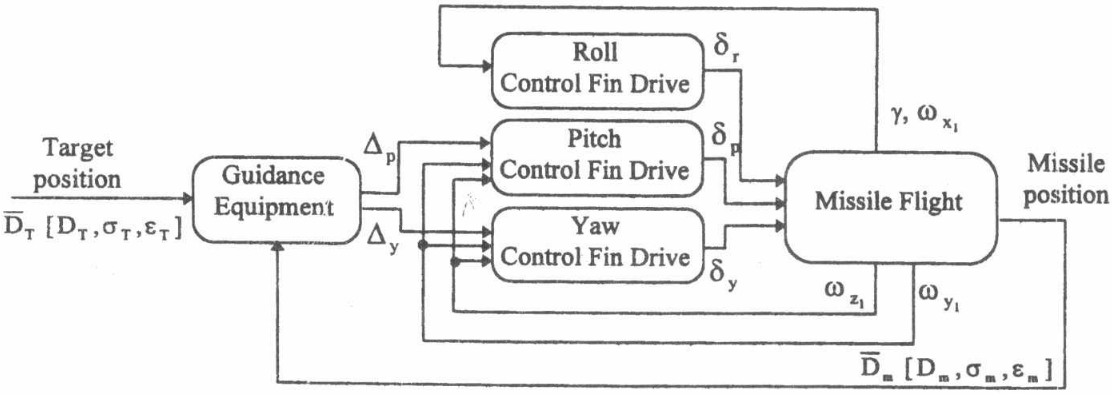


Fig. 1: Block diagram of missile control system

The main requirement imposed on the autopilot is the transfer of the control signals into mechanical rotation of the control fins. Therefore, each autopilot channel contains a control fin drive, mainly of the servo-amplifier and the servomotor (actuator). The missile properties usually has not got suitable dynamic properties, especially the damping of the missile oscillation around its center of gravity is low. The aerodynamic forces and moments change considerably with the dynamic pressure of the air, i.e. with the missile speed and with its altitude of flight. Therefore, the maneuverability of the missile changes with the increasing speed and altitude. Generally, the maneuverability of the missile is deteriorated with the increasing altitude. A change in the missile mass or a change in its center of gravity causes a change in the missile's maneuverability. From the control system stability point of view, any changes in the missile's dynamic properties are undesirable. Therefore, to avoid any undue changes in the missile's maneuverability, feedbacks from the angular speed and from the missile's cross linear (normal) acceleration are used to stabilize and to improve the performance (the dynamic properties) of the missile and of the whole control circuit. The control system of an antiaircraft guided missile is shown in Fig. 1, where the guidance device ensures the coordinates of both the missile and target continuously and by means of which it produces the control signals (commands) Δ_p and Δ_y for deflecting the elevators (for pitch) and rudders (for yaw).

2- Control Fin Drives

The control fin drive represents the main part of the missile autopilot and it is used to deflect the control surfaces in dependence on the strength and polarity of the control signals. Its structure is shown in Fig.2, where free gyroscopes are used for the stabilizing channel, the damping or rate gyro and accelerometer are used as damping elements used to damp the missile oscillations around its c.g. According to the shown structure in Fig.1, the control circuit should be internally stable. The feedback element may be rigid which means that it is of the P-type or elastic which means that it is of the PI-type. The P-type is the easier to stabilize the system and therefore it is either used alone or as initial guess in case of using the PI-type or controller. The increased order due to the PI controller is paid to diminish the aerodynamic disturbances. The control fin drives may be divided into pneumatic, hydraulic, electro-pneumatic, electro-hydraulic and electric. The choice between them depends upon many factors among them are the source of energy, weight of the drive and its dynamic properties.

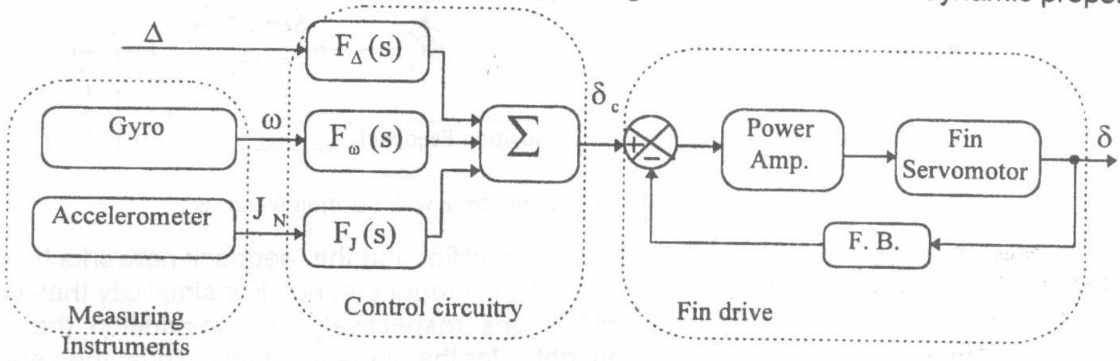


Fig. 2: Control fin drive with measuring instruments

It follows from the above discussions and Fig. 2 that the control fin equation of the control channel can be put in the following general form:

$$\delta = G_f(s)\delta_c = G_{f_c}\left[F_\Delta(s)\Delta + F_\omega(s)\omega + F_J(s)J_N\right] \tag{1}$$

where, δ is the control fin deflection, δ_c is the input signal to the control fin drive, Δ is the control signal or command, ω is the angular speed of the missile, J_N is the cross linear acceleration of the missile measured at the missile's center of gravity, $G_f(s)$ is an operator transfer of the fin control drive and the transfer $F_\Delta(s), F_\omega(s), F_J(s)$ correspond to equalizing networks of the individual loops.

The feedbacks of the angular speed signal and of the cross acceleration signal have positive influence upon the stabilization of the missile maneuverability. The missile dynamic properties (the oscillation of the missile's c.g. around the required trajectory can be damped) can be improved and the proper oscillation of the missile around its center of gravity can be damped effectively using these feedbacks. Thus, an improvement of the missile's maneuverability can be achieved by automatic regulation of the gain of the control fin drive. It is known that the missile's amplification or gain decreases with the dynamic pressure. In order to achieve a constant gain for the control circuit, the gain of the control fin drive can be regulated so that the total sum of the missile and the control fin drive amplification should be approximately constant. It can be achieved by automatic regulation of the drive gain in dependence on the dynamic pressure of the air. This regulation must be designed so that the amplification of the control fin drive should increase with the drop of the dynamic pressure of the air, i.e. with the decrease of the aerodynamic forces and moments. For the aerodynamic regulation of the control fin drive, the signal of the dynamic pressure's measuring element or that of the hinge moment of the control fin can be used. The control fin drives can be: pneumatic, hydraulic, electro-pneumatic, electro-hydraulic, and electric. The choice of the control fin drive depends first of all on the chosen sources of energy within the missile. The electro-pneumatic and electro-hydraulic systems are the most used. The electro-pneumatic ones are lighter than the electro-hydraulic ones but the later have better dynamic properties.

3- Dynamics of Linear Control Fin Drive

In a control fin drive with position feedback, the control fin deflection δ is proportional to the magnitude of the control signal i.e. to the required control surface deflection δ_{ref}, Fig. 3. The control fin drive is composed of a servomotor (actuator) and a power amplifier in addition to stabilizing feedback and prefiltering of the commanded input signal. The preamplifier can be considered as static element with gain k_{pr}. The prefilter is used to regulate, aerodynamically, the sensitivity (regulate the input signal level) of the control fin drive and it has the transfer k_{pf}. The position feedback is most often realized through a potentiometeric or induction reader of the control surface deflection and it has the gain k_{fb}.

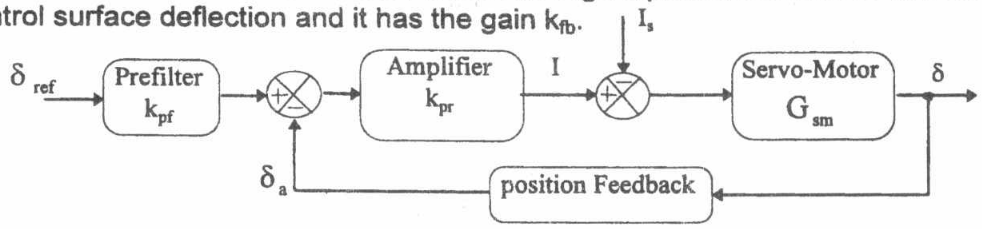


Fig. 3: Block diagram of the fin drive with position feedback

The reference shaping prefilter, the cascade amplifier and the feedback networks have to be designed for satisfying prespecified performance requirements. For simplicity they could be considered as static elements with amplifications, respectively, k_a. In addition, the negative position feedback is taken as k_a to quantify for the hinge moments. Thus, the servomotor transfer characteristics has the following form:

$$G_{sm} = \frac{\delta(s)}{I(s)} = \frac{k_m}{\tau_m^2 s^2 + 2\zeta_m \tau_m s + 1} = \frac{k_m}{(1+\tau_a s)(1+\tau_b s)} = \frac{k_m \omega_{ns}^2}{s^2 + 2\zeta_m \omega_{ns} s + \omega_{ns}^2} \qquad (2)$$

Assuming that the prefilter is unity and the amplifier and the feedback sensor have static transfer with gain k_a, the closed loop transfer function of the fin drive has the following form:

$$G_{cl} = \frac{k_a G_{sm}}{1 + k_a^2 G_{sm}} = \frac{k_a k_m / (1 + k_a^2 k_m)}{\tau_f^2 s^2 + 2\zeta_f \tau_f s + 1} \qquad (3)$$

Therefore, the characteristic polynomial of the closed loop fin drive has the following form:

$$\tau_f^2 s^2 + 2\zeta_f \tau_f s + 1 = 0 \qquad (4)$$

from which in addition to Eqn (3), it is clear that the fin drive has the following parameters:

the natural time constant $$\tau_f = 1/\omega_{n_f} = \tau_m / \sqrt{1 + k_a^2 k_m} \qquad (5)$$

the proportional damping $$\zeta_f = \zeta_m / \sqrt{1 + k_a^2 k_m} \qquad (6)$$

the compensated open loop gain $$k_{f_o} = k_a^2 k_m \qquad (7)$$

The compensated open loop gain k_{f_o} is obtained such that the conditions of optimum transition are satisfied i.e. $\zeta_f = 0.8$ and $\omega_{n_f} > 5\,\omega_{n_m}$ and consequently, the control fin servomotor should has the following damping:

$$\zeta_m \geq 4 \tau_m \omega_{n_m} \qquad (8)$$

If the fin drive servomotor or actuator has characteristics that not satisfy the above condition, then a stability correction network is necessary. The electric servomotors require such a type of correction while the pneumatic and hydraulic actuators did not. The phase margin/safety of an aerodynamically non-loaded control fin drive, during ground functional tests, is always lower than that with aerodynamic loading. This is due to the fact that the aerodynamic hinge moment increases favourably the motion damping. To increase the system proportional damping while keeping the natural frequency constant, a rate feedback can be used as shown in Fig. 4.

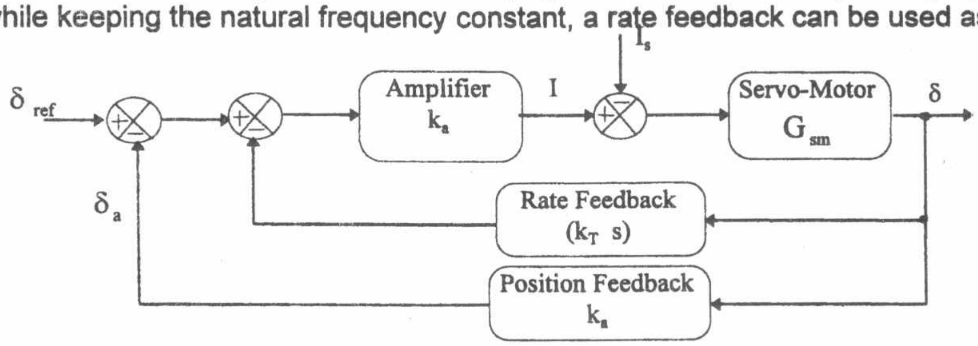


Fig. 4: Block diagram of the loaded fin drive with rate feedback

However, to increase the natural frequency of the system as well as its damping, a derivative or lead network has to be cascaded to the system within the forward loop as shown in Fig. 5.

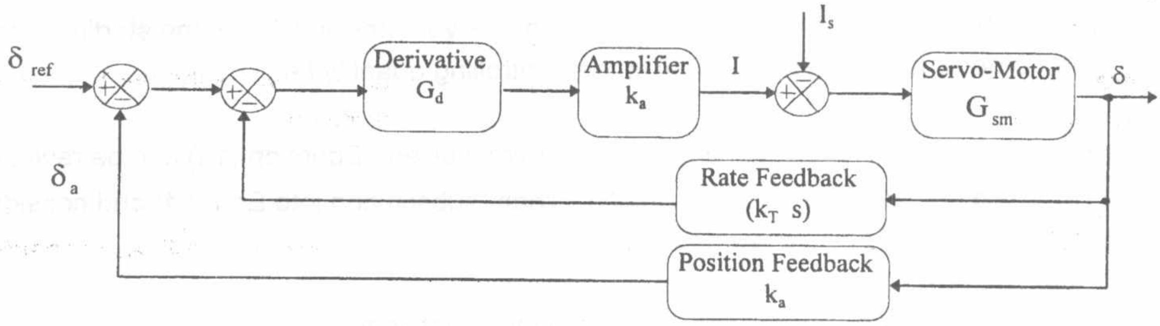


Fig. 5: Block diagram of the loaded fin drive with derivative network

The transfer characteristics of the derivative network has the following form:

$$G_d = \frac{1}{\alpha}\frac{(1+\tau_1\, s)}{(1+\tau_2\, s)} \tag{9}$$

where the two time constants are related to each other by the relation: $\tau_2 = \tau_1 / \alpha$ and the design factor α is selected such that the required increase of natural frequency and the proportional damping are achieved. That is, the new form of the natural frequency and the proportional damping are obtained as follows:

$$\omega_{n_f}^{d} = \omega_{n_f}\sqrt{\alpha} \tag{10}$$

$$\zeta_f^d = \zeta_f \, \frac{\alpha + 1 + (\alpha - 1)\sqrt{1 - 1/\zeta_m^2}}{2\sqrt{\alpha}} \tag{11}$$

The stability of the control fin drive can be investigated either algebraically, in case of low order systems, or using any of the known frequency stability analysis methods in conjunction with the computer aided design (CAD) packages such as MATLAB or CC.

4- Effect of fin hinge moment on the servomotor performance

During the missile flight there is acting an aerodynamic hinge moment upon the control fin and consequently it influences the dynamic properties of the control fin drive (actuators) considerably. The hinge moment acting upon the elevators and rudders can be expressed, respectively, as follows:

$$M_{h_p} = M_{hp}^{\delta}\cdot\delta_p + M_{hp}^{\dot\delta}\cdot\dot\delta_p + M_{hp}^{\alpha}\cdot\alpha + M_{hp}^{o} \tag{12-a}$$

$$M_{h_y} = M_{hy}^{\delta}\cdot\delta_y + M_{hy}^{\dot\delta}\cdot\dot\delta_y + M_{hy}^{\beta}\cdot\beta + M_{hy}^{o} \tag{12-b}$$

where $\delta_{p,y}$ is the deflection of control fin, α is the missile angle of attack and β is the missile side slip angle. The magnitude of the hinge moment does not influenced considerably by the value of α or β. From Eqns (12) it is clear that the hinge moment has components proportional to the fin deflection and its derivative. That is, the hinge moment has a directive and viscous character and consequently it, not only increases the damping of the control fin's motion but it also limits the size of its deflection. The directional component of the hinge moment has a similar effect as the spring.

If there is a transfer N_t between the servomotor shaft and the control fin, then the rate of fin deflection is given by

$$\dot\delta = N_t \cdot \omega \tag{13}$$

where ω is the angular speed of the servomotor shaft. The motion of the servomotor can be described by the following differential equation:

$$I_{xx}\dot\omega = M_d - M_h N_t \tag{14}$$

where I_{xx} is the total moment of inertia reduced to the servomotor shaft and the driving moment M_d is given as follows:

$$M_d = M_s - d_v \omega \tag{15}$$

where d_v is the viscous damping coefficient of the servomotor and M_s is the starting moment of the servomotor which is proportional to the controlling quantity i.e.

$$M_s = k_m I \tag{16}$$

where k_m is the servomotor gain and I is the control current. Equation (13) can be rewritten in the form $\omega = \dot{\delta} / N_t$ which implies $\dot{\omega} = \ddot{\delta} / N_t$. Then, substituting into Eqn (14) and considering the gears transfer yield the following relation:

$$I_{xx}\ddot{\delta} = M_d N_t - M_h N_t^2 \tag{17}$$

From Eqn (15) and Eqn (16) the following equation is obtained

$$M_d N_t = k_m I N_t - d_v \dot{\delta} \tag{18}$$

Equations (12) can be put in the following short form, bearing in mind that the two channels are similar,:

$$M_h = M_h^{\delta} \cdot \delta + M_h^{\dot{\delta}} \cdot \dot{\delta} + M_h^{\alpha} \cdot \alpha + M_h^{o} \tag{19}$$

Now, substituting Eqns (18, 19) into Eqn (17) yields the following relation:

$$I_{xx}\ddot{\delta} = (k_m I N_t - d_v \dot{\delta}) - N_t^2 (M_h^{\delta} \cdot \delta + M_h^{\dot{\delta}} \cdot \dot{\delta} + M_h^{\alpha} \cdot \alpha + M_h^{o}) \tag{20}$$

Rearranging this equation yields

$$I_{xx}\ddot{\delta} + (d_v + N_t^2 M_h^{\dot{\delta}})\dot{\delta} + (N_t^2 M_h^{\delta})\delta = k_m I N_t - N_t^2 M_h^{\alpha} \cdot \alpha - N_t^2 M_h^{o} \tag{21}$$

For aerodynamically unloaded fin drive, $M_h = 0$ and consequently Eqn (21) reduces to the following form:

$$I_{xx}\ddot{\delta} + d_v \dot{\delta} = k_m I N_t \tag{22}$$

Comparing Eqn (21) and Eqn (18) it is clear how the hinge moment considerably influence the dynamic properties of the control fin drive. The starting current is given by the following relation:

$$I_s = N_t (M_h^{\alpha} \cdot \alpha + M_h^{o}) / k_m \tag{23}$$

During the missile flight the dynamic pressure changes with altitude and so do the parameters I_s , k_a , and τ_a. Therefore, it is necessary to investigate the performance of the control fin drive using different values for the dynamic pressure and consequently different values for the hinge moment. On gauging the control fin drive (servomotor) it is necessary to start with the basic conditions to enable the chosen maximum deviation δ_m i.e. $\max(\delta) \geq \delta_m$. This condition can be achieved if the maximum starting moment M_s is higher than the hinge moment of the control fin drive with maximum fin deflection and with maximum angle of incidence i.e.

$$M_{s_{max}} \geq N_t (M_h^{\delta} \cdot \delta_m + M_h^{\alpha} \cdot \alpha_m + M_h^{o}) \tag{24}$$

Therefore, the nominal output of the servomotor can be calculated as $\{(0.333 \text{ to } 0.5) M_{h_{max}} \cdot \dot{\delta}_n\}$, where $\dot{\delta}_n$ is the nominal speed of the control fin drive. From these discussions it is clear that the control fin servomotor has different properties during missile flight than those during ground functional tests. Therefore, it is necessary to have a robust control fin drive to reduce this difference or discrepancy due to the hinge moment. If $\max(\delta) = \delta_m$ then the control fin drive has proportional properties. However, with excessive servomotor output gauging, the maximum fin deflection grows very high (i.e. $\max(\delta) >> \delta_m$) and the drive will have integrational properties.

5- Dynamics of the Fin Actuator (Servo-motor)

A pneumatic servomotor can be used to control the fin drive of a guided missile, the construction of which is shown in Fig. 6-a. The control fin drive is composed of a servomotor (actuator) and a power amplifier (say magnetic amplifier) as shown in Fig. 6-b. Therefore, the objective is to, firstly, consider the hinge moment of zero value i.e. $M_h = 0$ and investigate the performance of the control fin drive then change its value to show how the control fin drive will behave, given the following data:

1. The control current $I = 0 \div 9.2[\mathrm{mA}]$, with the maximum current $I_{max} = 9.2[\mathrm{mA}]$, a maximum pressure of value $\Delta P = 11 \cdot 10^5[\mathrm{Nm}^{-2}]$ acts on piston area $P_A = 0.00241[\mathrm{m}^2]$
2. The moment of inertia is $I_{xx} = 0.41[\mathrm{kgm}^2]$ and viscous damping factor is $d_v = 20.7[\mathrm{Nm\,sec}]$
3. The force of the servomotor acts on arm of length $l_m = 0.047[\mathrm{m}]$

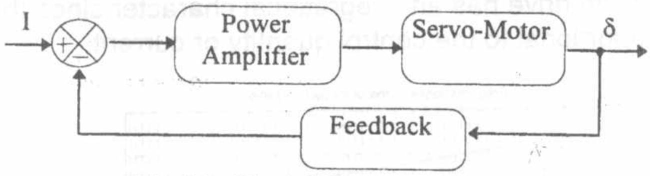


Fig. 6-b: Block diagram of fin drive

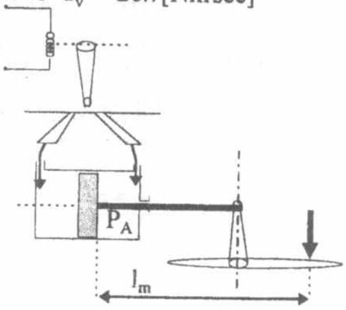


Fig. 6-a: Pneumatic servomotor

Considering this data in conjunction with zero hinge moment and Eqns (15, 16, 17), the maximum starting moment is obtained as follows:

$$M_{s_{max}} = P_{s_{max}} \cdot l_m = \Delta p_{max} \cdot P_A \cdot l_m = 124.55[\mathrm{Nm}] \quad (25)$$

Therefore, the motor gain is obtained from the maximum starting moment and maximum input current as follows:

$$k_m = M_{s_{max}} / I_{max} = 13.5\,[\mathrm{Nm/mA}] \quad (26)$$

The dependence of the driving moment on the speed of fin control can be obtained as follows:

$$M_d = 124.55 - 20.7\dot{\delta} = 124.55\,(1 - \dot{\delta}/6.017) \quad (27)$$

That is, the driving moment of the servo-motor equals the starting moment for zero fin speed and equals zero for fin speed of value $\dot{\delta} = 6.017\,[1/\mathrm{sec}]$, see Figs. 7-a,b.

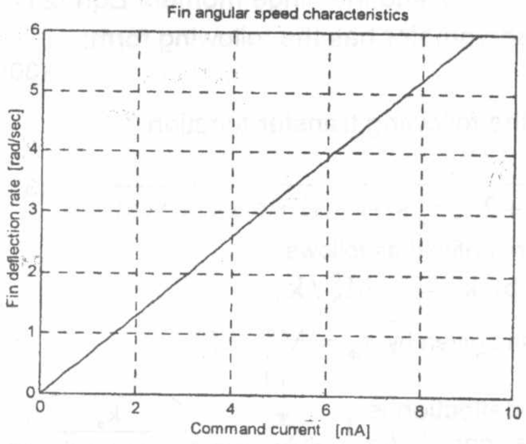


Fig. 7-a: Fin angular speed characteristics

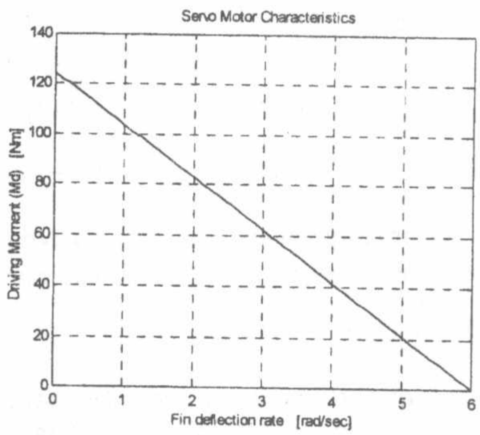


Fig. 7-b: Servomotor characteristics

5.1 Aerodynamically unloaded servo-motor

The Laplace transform applied to Eqn (22) and using $N_t = 1$ yield the transfer function of the unloaded servo-motor as follows:

$$\frac{\delta(s)}{I(s)} = \frac{N_t k_m}{s(I_{xx}s + d_v)} = \frac{k_o}{s(\tau_o s + 1)} \tag{28}$$

where the gain and time constant, respectively are: $k_o = N_t \cdot k_m / d_v = 0.652\,[\sec^{-1}\,\mathrm{mA}^{-1}]$ and $\tau_o = I_{xx} / d_v = 0.02\,[\sec]$. That is, the unloaded servomotor transfer function has the form:

$$\frac{\delta(s)}{I(s)} = \frac{0.652}{s(1 + 0.02 \cdot s)} \tag{29}$$

which has integrational properties. The control fin moves in the steady state at a free-running angular speed of $\dot{\delta}(0) = 0.652 \cdot I(s)$. That is, the control fin angular speed is proportional to the control current I, see Fig. 7. In addition, the transfer characteristics of the aerodynamically unloaded servomotor can be obtained through step response and frequency response, as shown in Fig. (8-a,b). From these figures it is clear how the system response is divergent with integrational character. The unloaded control fin drive has an integrational character since the steady angular speed of the control fin is proportional to the control quantity or current.

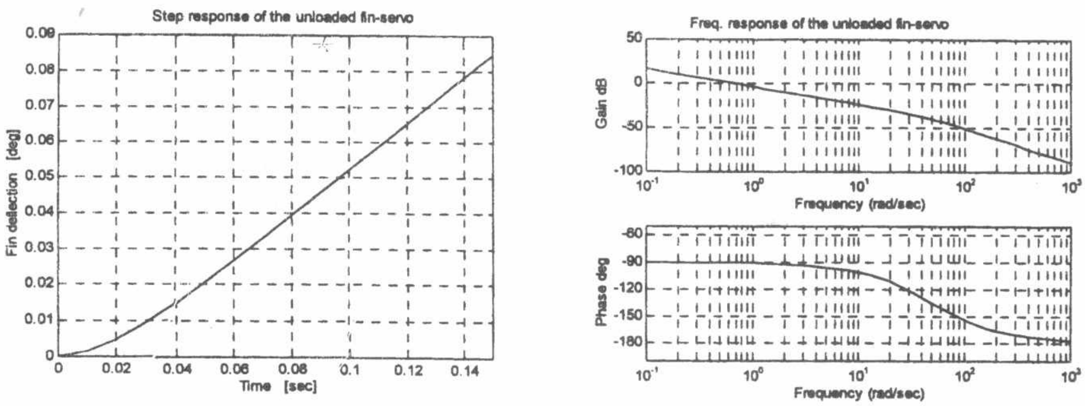


Fig. 8: Unloaded fin drive (a) Step response (b) Frequency response

5.2 Aerodynamically Loaded Servomotor

The dynamic properties of the control fin drive or servomotor are influenced by the aerodynamic loading or hinge moment and therefore they are investigated in this section. Considering zero values for the moments $M_h^o = M_h^\alpha = 0$ and the hinge moment Eqn (21), the equation of motion of the aerodynamically loaded servomotor has the following form:

$$I_{xx}\ddot{\delta} + (d_v + N_t^2 M_h^{\dot{\delta}})\dot{\delta} + N_t^2 M_h^{\delta}\delta = I k_m N_t \tag{30}$$

Applying the Laplace transform to Eqn (32) yields the following transfer function:

$$\frac{\delta(s)}{I(s)} = \frac{N_t k_m}{I_{xx}s^2 + (d_v + N_t^2 M_h^{\dot{\delta}})s + N_t^2 M_h^{\delta}} = \frac{1/k_a}{\tau_m^2 s^2 + 2\zeta_m \tau_m s + 1} = \frac{k}{(1+\tau_a s)(1+\tau_b s)} \tag{31}$$

where the system parameters describing its dynamics are defined as follows:

k_a is the gain of the aerodynamical feedback and given by $k_a = N_t M_h^{\delta} / k_m$

τ_a is the time constant of the aerodynamical feedback and given by $\tau_a = M_h^{\dot{\delta}} / M_h^{\delta}$

It is clear that during missile flight, the control fin deflection is proportional to the control signal I. That is, the control fin servomotor lost its integrational character due to the influence of the directional component of the hinge moment. In other words, the dynamic order of the servomotor has decreased by one degree and keep only its static properties.

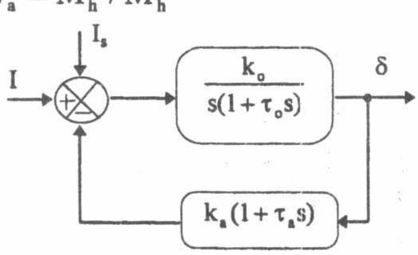


Fig. 9: Block diagram of loaded servomotor

The block diagram of the aerodynamically loaded servomotor is shown in Fig. 9, with the forward and feedback transfer functions given in each block. The starting current is given by the following relation:

$$I_s = N_t (M_h^{\alpha} \cdot \alpha + M_h^{o}) / k_m \tag{32}$$

5.2.1 Minmum control fin hinge moment action

Considering the dynamic pressure has the value $q = 12 \cdot 10^4 [Nm^{-2}]$, the following hinge moment acts on the fin controls $M_{h_{max}}^{\delta} = 74.5\,[Nm/rad]$ and $M_h^{\dot{\delta}} = 9.43\,[Nm\,sec/rad]$. Therefore, the transfer function of the aerodynamically loaded servomotor has the form:

$$\frac{\delta(s)}{I(s)} = \frac{0.181}{1 + 0.4011 \cdot s + 0.0055 \cdot s^2} \tag{33}$$

Consequently, the transfer characteristics of the aerodynamically loaded servomotor in time and frequency are shown in Fig. (10). The value of the **control signal to yield a maximum steady state fin deflection** under the action of the minmum value of control fin hinge moment is given by the relation: $I_{max} = \delta M_{h_{min}}^{\delta} / k_m = 2.21 [mA]$.

5.2.2 Medium control fin hinge moment action

Considering the dynamic pressure to has the value $q = 20 \cdot 10^4 [Nm^{-2}]$, the following hinge moment acts on the fin controls $M_h^{\delta} = 124\,[Nm/rad]$ and $M_h^{\dot{\delta}} = 15.7\,[Nm\,sec/rad]$; where the system parameters describing its dynamics, considering $N_t = 1$, are calculated and therefore, the transfer function of the aerodynamically loaded servomotor has the following form:

$$\frac{\delta(s)}{I(s)} = \frac{0.109}{1 + 0.2896 \cdot s + 0.003249 \cdot s^2} \tag{34}$$

Consequently, the frequency characteristics of the aerodynamically loaded servomotor and its transient characteristics for a step or jump change in the control current I are obtained using the MATLAB as shown in Fig. 10.

5.2.3 Maximum control fin hinge moment action

Considering the dynamic pressure has the value $q = 50 \cdot 10^4 [Nm^{-2}]$, the following hinge moment acts on the fin controls $M_{h_{max}}^{\delta} = 310\,[Nm/rad]$ and $M_h^{\dot{\delta}} = 39.3\,[Nm\,sec/rad]$. Therefore, the transfer function of the aerodynamically loaded servomotor has the form

$$\frac{\delta(s)}{I(s)} = \frac{0.0435}{1 + 0.1958 \cdot s + 0.0014 \cdot s^2} \tag{35}$$

Consequently, the transfer characteristics of the aerodynamically loaded servomotor in time and frequency are shown in Fig. (10). The **steady state fin deflection** or inclination due to the action of the maximum value of control fin hinge moment is given by the relation $\delta = k_m I_{max} / M_{h_{max}}^{\delta} = 0.4 [rad] \cong 23°$.

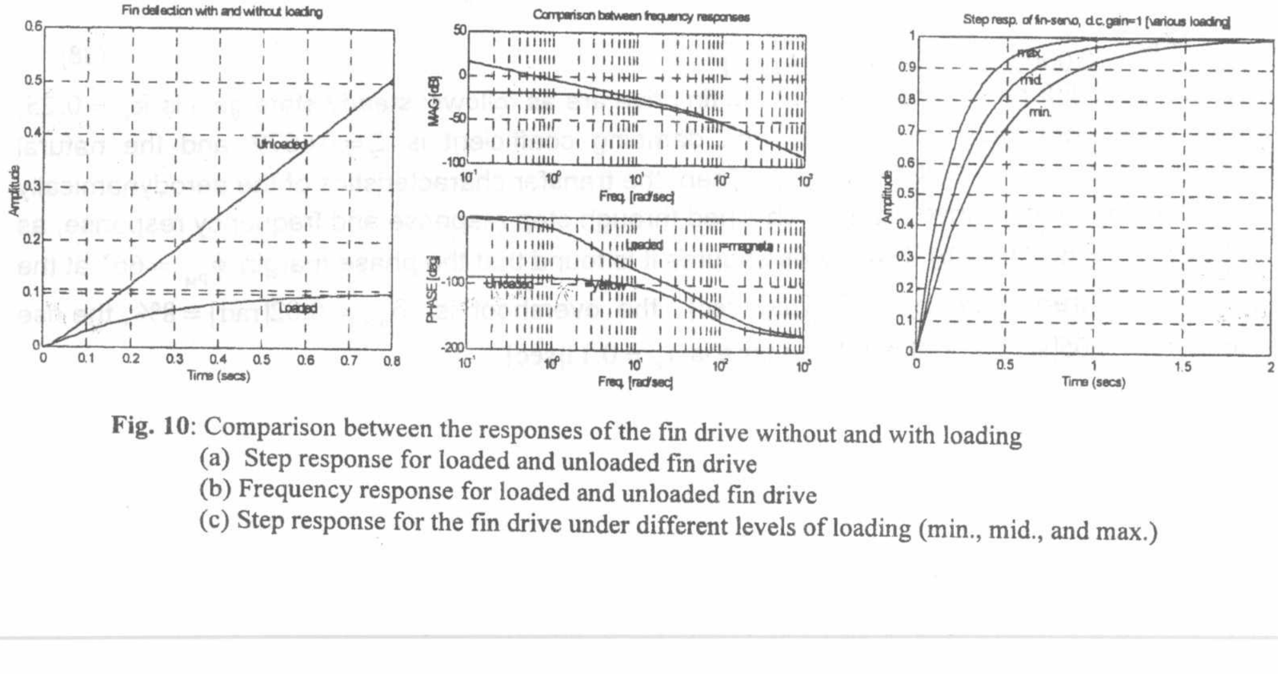


Fig. 10: Comparison between the responses of the fin drive without and with loading
(a) Step response for loaded and unloaded fin drive
(b) Frequency response for loaded and unloaded fin drive
(c) Step response for the fin drive under different levels of loading (min., mid., and max.)

From Figures 8 and 10, it is clear that the aerodynamically loaded servomotor has lost, substantially, its integrational character and therefore its performance differ from the unloaded case. That is, the steady state error becomes very small in this case. In addition, the speed of response is increased with increasing the level of aerodynamic loading as clear from Fig.10-c.

6- Performance Analysis of a linear control fin drive

The control fin drive is composed of a servomotor (actuator) and a power amplifier in addition to stabilizing feedback and prefiltering of the commanded input signal, as shown in Fig. 3. The transfer function of the closed loop fin drive system is obtained as follows:

$$G_{fin} = \frac{k_{pf} k_{pr} k}{\tau_m^2 s^2 + 2\zeta_m \tau_m s + (1+k_1)} = \frac{k_f}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{36}$$

where $k_1 = k_{fb} k_{pr} k$, the steady state gain is $k_f = k_{pf} k_{pr} k/(1+k_{fb} k_{pr} k)$, the damping coefficient is $\zeta = \zeta_m / \sqrt{1+k_1}$, the time constant is $\tau = \tau_m / \sqrt{1+k_1}$ and the natural frequency is $\omega_{nf} = 1/\tau = \sqrt{1+k_1}/\tau_m$. The phase margin ϕ_{PM} of the control fin drive is calculated for the value of the natural frequency ω_{n_m} of the missile oscillatory motion.

6.1 Unloaded control fin drive

Considering the preamplifier as static element with gain k_{pr} = 9.1, the prefilter, used to regulate the sensitivity of the control fin drive, with transfer k_{pf} = 2.5, and the feedback potentiometer with gain k_{fb} = 10 in addition to the transfer of unloaded servomotor in Eqn (28); the transfer function of the unloaded control fin drive can be obtained as follows:

$$G_{fin} = \frac{0.25}{0.000337 s^2 + 0.0168 s + 1} \tag{37}$$

where; u represents the input reference fin deflection, the gain $k_1 = 59.4$, the steady state gain of the fin drive is $k_f = 0.24586$, the time constant of the fin drive is $\tau = 0.0183$, the damping coefficient of the fin drive is $\zeta = 0.46$ and the natural frequency of the fin drive is $\omega_{nf} = 54.6[\text{rad/sec}]$. The transfer characteristics of the aerodynamically unloaded fin control drive can be obtained through step response and frequency response, as shown in Fig. (11). From these figures it is found that the phase margin $\phi_{PM} = 48^\circ$ at the cross-over frequency $\omega_c = 45[\text{rad/sec}]$, the overshoot is $\delta_{ovs} = 0.2[\text{rad}] = 20\%$, the rise time $t_r = 0.04[\text{sec}]$ and the settling time is $t_s = 0.17[\text{sec}]$.

6.2 Loaded control fin drive

6.2.1 Minimum loading

Considering that the dynamic pressure has the value $q = 12 \cdot 10^4 [\text{Nm}^{-2}]$ and the hinge moment is $M_h^\delta = 74.5[\text{Nm/rad}]$ and $M_h^{\dot\delta} = 9.43[\text{Nm sec/rad}]$, in addition to Eqn (33); the transfer function of the closed loop system has the following form:

$$G_{fin} = \frac{0.25}{0.00033246 s^2 + 0.024351 s + 1} \tag{38}$$

where the different parameters of the fin drive are as follows: steady state gain is $k_f = 0.25$, the time constant $\tau = 0.01823$, the damping coefficient is $\zeta = 0.6679$ and the natural frequency is $\omega_{nf} = 54.85[\text{rad/sec}]$. Then, the transfer characteristics of the aerodynamically loaded fin control drive can be obtained through step response and frequency response, as shown in Fig. (12-a,b). From these figures it is found that the phase margin $\phi_{PM} = 66^\circ$ at the cross-over frequency $\omega_c = 37.5[\text{rad/sec}]$, the overshoot is $\delta_{ovs} = 0.02[\text{rad}] = 8\%$, the rise time $t_r = 0.05[\text{sec}]$ and the settling time is $t_s = 0.11[\text{sec}]$.

6.2.2 Medium loading

Considering that the dynamic pressure has the value $q = 20 \cdot 10^4 [\mathrm{Nm}^{-2}]$ and the hinge moment is $M_h^{\delta} = 124[\mathrm{Nm/rad}]$ and $M_h^{\dot{\delta}} = 15.7[\mathrm{Nm sec/rad}]$, in addition to Eqn (34); the transfer function of the closed loop system has the following form, taking into consideration that the gain of the prefilter has a value = 1.54,

$$G_{fin} = \frac{0.154}{0.00032755s^2 + 0.0291965s + 1} \tag{39}$$

where the different parameters of the closed loop fin drive are as follows: the steady state gain is $k_f = 0.154$, the time constant is $\tau = 0.018098$, the damping coefficient is $\zeta = 0.8066$ and the natural frequency is $\omega_{nf} = 55.25[\mathrm{rad/sec}]$. The transfer characteristics of the aerodynamically loaded fin control drive can be obtained through step response and frequency response, as shown in Fig. (13-a,b). From these figures it is found that the phase margin $\phi_{PM} = 75°$ at the cross-over frequency $\omega_c = 33[\mathrm{rad/sec}]$, the gain margin is $k_{gM} = \infty[\mathrm{dB}]$, the overshoot is $\delta_{ovs} = 0.003[\mathrm{rad}] = 2\%$, the rise time $t_r = 0.06[\mathrm{sec}]$ and the settling time is $t_s = 0.08[\mathrm{sec}]$.

6.2.3 Maximum loading

Considering that the dynamic pressure has the value $q = 50 \cdot 10^4 [\mathrm{Nm}^{-2}]$ and the hinge moment is $M_h^{\delta} = 310[\mathrm{Nm/rad}]$ and $M_h^{\dot{\delta}} = 39.3[\mathrm{Nm sec/rad}]$, in addition to Eqn (35); the transfer function of the closed loop system has the following form, taking into consideration that the gain of the prefilter has a value = 0.64,

$$G_{fin} = \frac{0.064}{0.00035367s^2 + 0.04946s + 1} \tag{40}$$

where the different parameters of the closed loop fin drive are as follows: the steady state gain is $k_f = 0.064$, the time constant is $\tau = 0.0188$, the damping coefficient is $\zeta = 1.3154$ and the natural frequency is $\omega_{nf} = 53.19[\mathrm{rad/sec}]$. The transfer characteristics of the aerodynamically loaded fin control drive can be obtained through step response and frequency response, as shown in Fig. (14-a,b). From these figures it is found that the phase margin $\phi_{PM} = 96.3°$ at the cross-over frequency $\omega_c = 20[\mathrm{rad/sec}]$, the gain margin is $k_{gM} = \infty[\mathrm{dB}]$, the overshoot is $\delta_{ovs} = 0\%$, the rise time $t_r = 0.15[\mathrm{sec}]$ and the settling time is $t_s = 0.2[\mathrm{sec}]$.

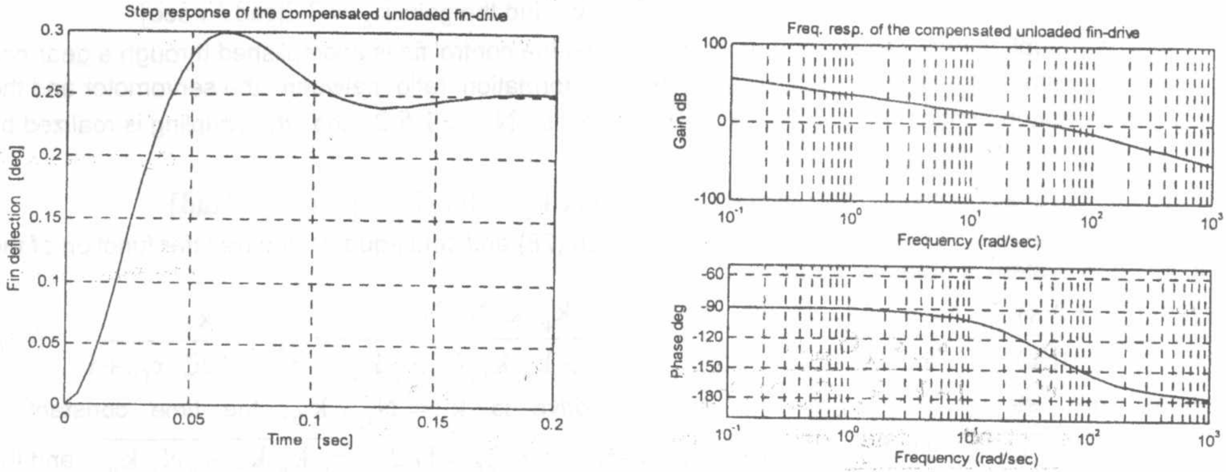


Fig. 11: Compensated unloaded fin drive (a) Step response (b) Frequency response

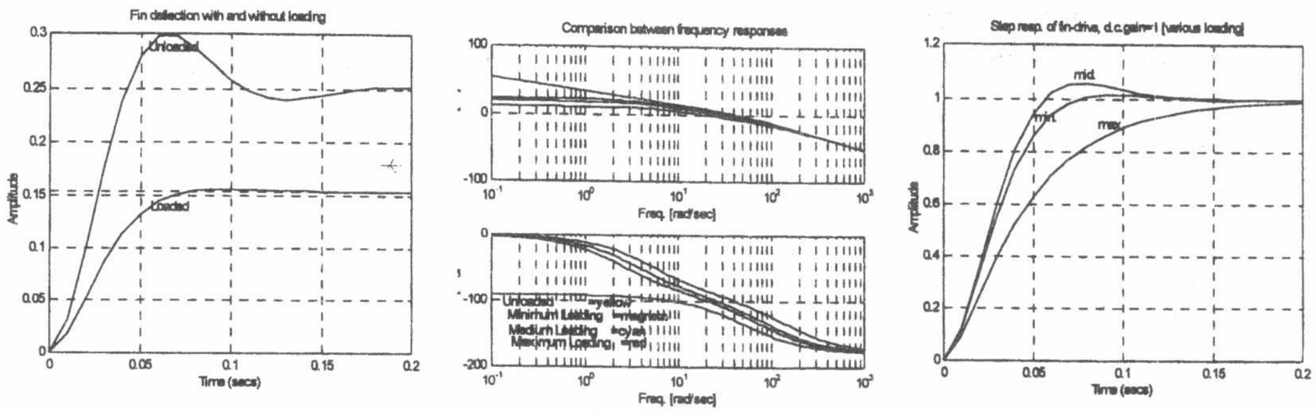


Fig. 12: Comparison between the responses of the fin drive without and with loading
(a) Step response for loaded and unloaded fin drive
(b) Frequency response for loaded and unloaded fin drive
(c) Step response for the fin drive under different levels of loading (min., mid., and max.)

From Fig. 12, it is clear that the aerodynamically loaded fin drive has lost, substantially, its integrational character and therefore its performance differ from the unloaded case. In addition, the speed of fin response and the overshoot are decreased with increasing the level of aerodynamic loading as clear from Fig. 12-c.

7- Design of Compensation Networks for a control fin drive

The feedback elements are assumed to be mechanically coupled to the motor shaft through a gearbox with transformation ratio N_{t1} while the control surface itself is mechanically coupled to it through a gearbox with transformation ratio N_{t2}. For the loaded control fin drive consider that the hinge moment $M_h^{\delta} = 2.2[\mathrm{Nm/rad}]$, $M_h^{\dot{\delta}} = 0.1254[\mathrm{Nm\,sec/rad}]$, the maximum deflection of the control fin $\delta_{max} = \pm 15^{\circ}$, the angular rate of the control fin is $\dot{\delta} = 2[\mathrm{rad/sec}]$ and the natural frequency of the missile is $\omega_{n_m} = 7[\mathrm{rad/sec}]$. For carrying out this design and analysis, the different subsystems are assumed to have the following characteristics:

1. The preamplifier has a gain $k_{p_r} = 4\,[\mathrm{mA/v}]$
2. The power amplifier has a gain $k_{p_A} = 26.2\,[\mathrm{v/mA}]$ and neglected time constant
3. The servo motor time constant is $\tau_o = 0.05[\mathrm{sec}]$ and the gain $k_o = 7.2[\mathrm{rad/v\,sec}]$.
4. The coupling between the servomotor shaft and the control fin is acomplished through a gear box with transfer ratio $N_t = 1{:}520$ and the transformation ratio between the servomotor and the feedback potentiometer is $N_{t_1} = 1{:}100$. That is, $N_{t_2} = 1{:}5.2$ and the coupling is realized by gears with efficiency $\eta = 0.7$.
5. The feedback potentiometer is realized such that $k_{fb} = 0.25[\mathrm{v}/\circ] = 14.3[\mathrm{v/rad}]$

The transfer function of the servomotor is given in Eqn (28) and consequently the transfer function of the closed loop system is obtained as follows:

$$G_{fin} = \frac{\delta(s)}{u(s)} = \frac{k_{p_r} k_{p_A} G_{sm} N_{t_1} N_{t_2}}{1 + k_{p_r} k_{p_A} G_{sm} N_{t_1} k_{fb}} = \frac{k_{p_r} k_{p_A} k_o N_{t_1} N_{t_2}}{\tau s^2 + s + k_{p_r} k_{p_A} k_o N_{t_1} k_{fb}} = \frac{k_f}{\tau_f^2 s^2 + 2\zeta_f \tau_f s + 1} \qquad (41)$$

where, the steady state gain of the fin drive is $k_f = N_{t_2} / k_{fb}$, the time constant is $\tau_f = \sqrt{\tau_o / (k_{p_r} k_{p_A} k_o N_{t_1} k_{fb})}$, the damping coefficient is $\zeta_f = 1/2/\sqrt{\tau_o\, k_{p_r} k_{p_A} k_o N_{t_1} k_{fb}}$ and the natural frequency $\omega_{nf} = 1/\tau_f$. The phase safety (margin) ϕ_{PM} of the control fin drive is calculated for the value of the natural frequency ω_{n_m} of the missile oscillatory motion.

7.1 Unloaded control fin drive

The transfer function of the unloaded (i.e. $N_{t_2} = 1:1$) control fin drive can be obtained according to Eqn (41) as follows:

$$G_{fin} = \frac{0.06993}{0.0004634s^2 + 0.009268s + 1} \tag{42}$$

where the steady state gain $k_f = 0.06993$, the time constant $\tau_f = 0.0215$, the damping coefficient $\zeta_f = 0.215$ and the natural frequency of the fin drive is $\omega_{nf} = 46.454[\text{rad/sec}]$. Then, the transfer characteristics of the aerodynamically unloaded fin control drive can be obtained through step response and frequency response, as shown in Fig. (13-a,b). From these figures the phase margin $\phi_{p_M} = 24.27°$ at the cross-over frequency $\omega_c = 44.35\,[\text{rad/sec}]$, the overshoot $\delta_{ovs} = 0.035[\text{rad}] = 50\%$, the rise time $t_r = 0.05[\text{sec}]$ and the settling time $t_s = 0.4[\text{sec}]$.

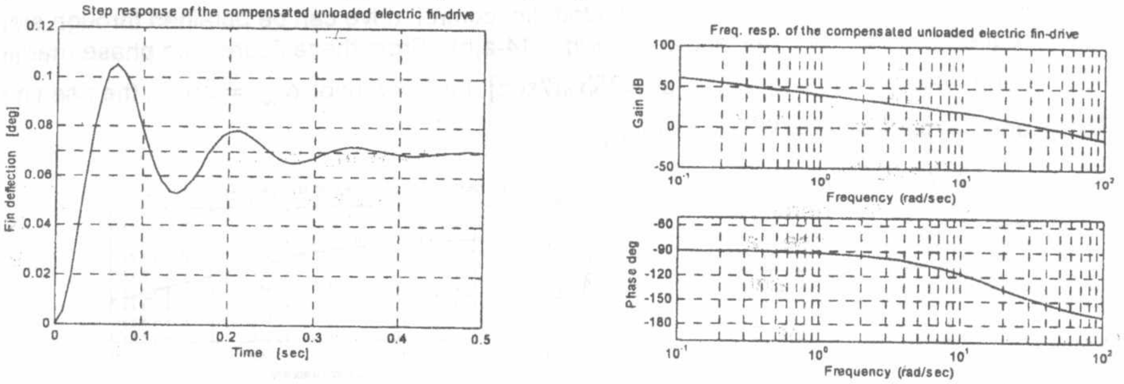


Fig. 13: Step and Frequency response for unloaded fin drive

7.2 Loaded control fin drive

The starting torque of the control fin servomotor can be expressed by the following relationship $M_s = k_m \cdot u_s$, where k_m is the servomotor constant or gain and u_s is the command or control voltage. For a maximum control signal $u_{s_{max}} = 40[v]$ and starting torque $M_s = 4.3 \cdot 10^{-2}[\text{Nm}]$, the constant of the servomotor can be obtained as follows: $k_m = M_s / u_{s_{max}} = 1.075 \cdot 10^{-3}[\text{Nm/v}]$. The influence of the aerodynamic hinge moment upon the control fin servomotor is evident as an effect of the rigid feedback round the servomotor as shown in Fig. 9.

The feedback due to loading has the transfer function $G_{f_L} = k_l(1 + \tau_l s)$, where the aerodynamic rigid feedback gain k_l and the time constant τ_l are given by $k_l = M_h^\delta \cdot N_t / k_m = 3.94[\text{v/rad}]$ and $\tau_l = M_h^{\dot{\delta}} / M_h^\delta = 0.057[\text{sec}]$. Thus, the transfer function of the aerodynamically loaded servomotor can be obtained as follows:

$$G_{sm} = \frac{k_{sm1}}{\tau_m^2 s^2 + 2\zeta_m \tau_m s + 1} \tag{43}$$

where, the steady state gain $k_{sm_l} = 0.254$, the time constant $\tau_m = 0.042[\text{sec}]$, the damping coefficient $\zeta_m = 1.1$ and the natural frequency of the loaded servomotor is $\omega_{n_{sm}} = 23.81[\text{rad/sec}]$. Since the damping coefficient is greater than unity ($\zeta_m > 1$), the transfer function (43) of the aerodynamically loaded servomotor can be written as a product of two aperiodic elements as follows:

$$G_{sm} = \frac{k_{sm}}{(1 + \tau_a s)(1 + \tau_b s)} \tag{44}$$

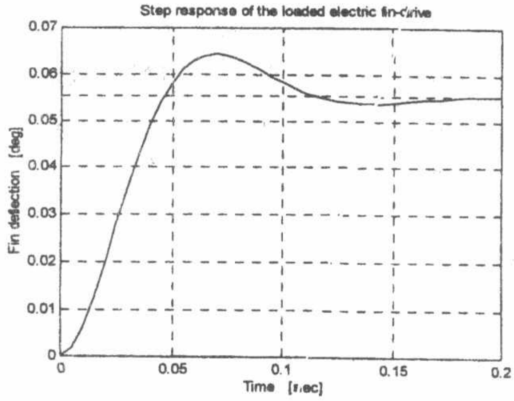
where the two time constants are given by: $\tau_a = \tau_m(\zeta_m + \sqrt{\zeta_m^2 - 1}) = 0.065[\text{sec}]$ and $\tau_b = \tau_m(\zeta_m - \sqrt{\zeta_m^2 - 1}) = 0.027[\text{sec}]$. Consequently, the transfer function of the servomotor has the following form:

$$G_{sm} = \frac{0.254}{0.001764s^2 + 0.0924s + 1} = \frac{0.254}{(1 + 0.065s)(1 + 0.027s)} \tag{45}$$

Then, the closed loop transfer function of the fin drive is obtained as follows:

$$G_{fin} = \frac{0.055381}{0.000365127s^2 + 0.019141s + 1} \tag{46}$$

where the time constant of the loaded fin drive is $\tau = 0.01911[\text{sec}]$, the damping coefficient of the loaded fin drive is $\zeta = 0.5008$ and the natural frequency of the loaded fin drive is $\omega_{n_f} = 52.3286[\text{rad/sec}]$. The transfer characteristics of the aerodynamically loaded fin control drive can be obtained through step response and frequency response, as shown in Fig. (14-a,b). From these figures the phase margin $\phi_{PM} = 66.4°$ at the cross-over frequency $\omega_c = 38[\text{rad/sec}]$, the overshoot $\delta_{ovs} = 20\%$, the rise time $t_r = 0.04[\text{sec}]$ and the settling time $t_s = 0.15[\text{sec}]$.



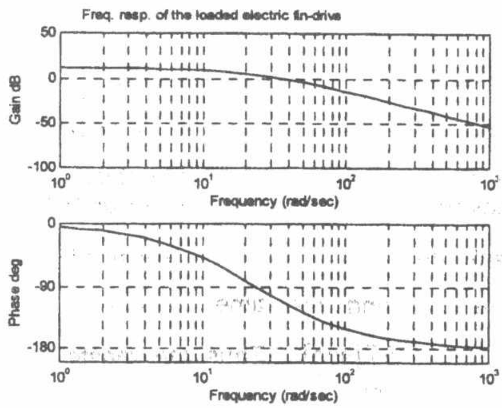


Fig. 14: Step and Frequency response for loaded electric fin drive

7.3 Loaded control fin drive with rate feedback

The designed system is required to have a damping coefficient $\zeta_m = 0.8$ i.e. the following inequality has to be hold $\zeta_m \geq 4\tau_m\omega_n$. According to the obtained results $\zeta_m = 1.1 \geq 4\tau_m\omega_n = 1.226$. That is, from the obtained results it is clear that the response is not satisfactory, for which reason a rate feedback is utilized as shown in Fig. 5 . The rate feedback has the transfer $G_{r_{fb}} = k_T s$, where the objective is to determine the gain k_T such that the damping coefficient is $\zeta_f = 0.8$. The closed loop transfer function is obtained as follows:

$$G_{fin} = \frac{k_{p_r} k_{p_A} k_{sm} N_{t_1}}{\tau_m^2 s^2 + 2\zeta_{m_r}\tau_m s + 1} \tag{47}$$

where; $\zeta_{m_r} = \zeta_m + (k_{p_r} k_{p_A} k_{sm} N_{t_1} k_T)/2\tau_m$. Thus, the damping coefficient of the fin drive is given by

$$\zeta_f = \frac{\zeta_{m_r}}{\sqrt{1 + k_{f_{ol}}}} = \zeta + \frac{k_{p_r} k_{p_A} k_{sm} N_{t_1} k_T}{2\tau_m\sqrt{1 + k_{f_{ol}}}} \tag{48}$$

From Eqn(48), the gain of the rate feedback k_T can be obtained such that the damping coefficient is $\zeta_f = 0.8$ as $k_T = 0.20755[\text{v}/(\text{rad/sec})]$. The rate feedback can be realized using a tachogenerator coupled with the servomotor shaft after the gear box with transfer N_t in addition to a convenient voltage divider. With this type of feedback, the open loop transfer of the fin drive is obtained as $\zeta_{f_r} = 1.754$. Since, the damping coefficient is greater than unity, the system is divided into two aperiodic elements with the time constants: $\tau_{a_r} = 0.1342[\text{sec}]$ and $\tau_{b_r} = 0.01315[\text{sec}]$. Then, the closed loop transfer function of the loaded electric fin drive is given by

$$G_{fin} = \frac{0.0553812}{0.000367\,s^2 + 0.030718\,s + 1} \tag{49}$$

where; the time constant of the loaded fin drive is $\tau = 0.01916[\text{sec}]$, the damping coefficient of the loaded fin drive is $\zeta = 0.802$ and the natural frequency of the loaded fin drive is $\omega_{n_r} = 52.2[\text{rad/sec}]$. Then, the transfer characteristics of the aerodynamically loaded fin control drive with rate feedback can be obtained through step response and frequency response, as shown in Fig. (15-a,b). From these figures the phase margin $\phi_{PM} = 92.65^\circ$ at the cross-over frequency $\omega_c = 4232[\text{rad/sec}$, the overshoot $\delta_{ovs} = 0\%$, the rise time $t_r = 0.05[\text{sec}]$ and the settling time $t_s = 0.1[\text{sec}]$.

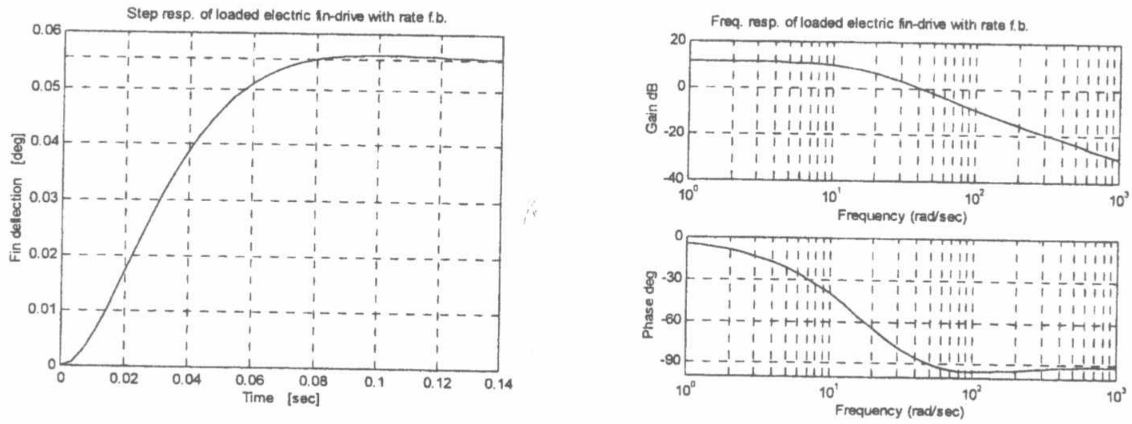


Fig. 15: Step and Frequency response for loaded electric fin drive with rate F.B.

7.4 Loaded control fin drive with cascaded derivative correction

Determine the magnitude of the rate feedback gain so that the proportional damping of the whole circuit/system might achieve the value of $\zeta = 0.8$. The series derivative network can be realized using an RC-circuit whose elements are chosen such that the transfer characteristics, Eqn (9), has the following form:

$$G_d = \frac{(1 + 0025\,s)}{4\,(1 + 0.00625\,s)} \tag{50}$$

Due to this type of correction, the natural frequency and the damping coefficient can be increased utilizing $\alpha = 4$ to achieve the required damping $\zeta_d = 0.8$ as follows: $\omega_{n_d} = 104.66[\text{rad/sec}]$ and $\zeta_d = 0.7812$. Then, the closed loop transfer function of the loaded fin drive with rate feedback and derivative correction is given as follows:

$$G_{fin} = \frac{0.000852464 + 0.0340986\,s}{0.0000056491\,s^3 + 0.00137669\,s^2 + 0.0698147\,s + 1} \tag{51}$$

where; the time constant of the loaded fin drive is $\tau = 0.01916[\text{sec}]$, the damping coefficient of the loaded fin drive is $\zeta = 0.802$ and the natural frequency of the loaded fin drive is $\omega_{n_r} = 52.2[\text{rad/sec}]$. Then, the transfer characteristics of the aerodynamically loaded fin control drive with rate feedback can be obtained through step response and frequency response, as shown in Fig. (16-a,b). It is clear that the series derivative or the rate feedback can be used for the correction of or improving the control fin drive properties. However, the rate feedback is used to increase the damping of the open loop without affecting its natural frequency while the series correction can increase both the damping and the natural frequency of the open loop.