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SIMULATING AND MAXIMIZING THE EFFECT OF JAMMING ON PROPORTIONAL NAVIGATION HOMING GUIDANCE

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ABSTRACT

In this paper the kinematics of a proportional navigation homing guidance trajectory have been simulated, first without any initial heading errors or target maneuvers and then with certain inserted angular errors to simulate a jamming effect that be maximized. We demonstrate in the same paper that the inserted angular error has to be time varying; such that additional false guidance commands are continuously generated which deviate the missile from its correct trajectory. Different methods can be adopted to induce such angular errors which are out of the scope of this paper.

The simulation results have been congruent with the theoretical discussion: Zero miss-distance has resulted without jamming. With constant inserted angle error the guidance system has succeeded to recover with a negligible miss-distance. Different laws of variation have been proposed for the inserted angular error and their parameters have been optimized for maximum possible miss-distance. Miss-distances in excess of 4 kilometers have been achieved.

KEY WORDS

proportional navigation, final miss-distance, impact point, target kill probability, self-protection jammer, jamming-to-signal ratio, missile seeker, speed gate stealing.

NOMENCLATURE

λ	=	*the missile-target line of sight angle*
$\Delta\lambda_j$	=	*the induced angular error due to jamming*
v_c	=	*the missile-target closing velocity*
J/S	=	*jamming-to-signal power ratio*
JSR	=	*10 log(J/S) [dB]*
R_{TM}	=	*Target - Missile range*

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L = *missile lead angle with respect to the instantaneous line of sight*
ECM = *Electronic Counter-Measures.*

INTRODUCTION

A proportional navigation guidance law issues a normal acceleration command proportional to the product of the line of sight rate and the closing velocity [1]. In a homing missile, those guidance commands are issued by the missile-borne guidance computer and implemented by the missile moving fins in two orthogonal planes in order to correct the missile deviations from its planned trajectory. The goal of the guidance system is to minimize the final miss-distance; in order to maximize the probability of target destruction by the guided missile. On the other hand; the goal of a target-borne self-protection jammer is to maximize the miss-distance and; consequently, minimize the target kill probability. The self-protection jammer starts by maximizing the jamming-to-signal power ratio (J/S ratio); in order to facilitate the application of an appropriate angular deception technique which creates false angle errors within the line of sight tracking system of the missile seeker. Such a maximization is usually achieved by stealing the basic coordinate tracking gate [2]; resulting in an infinite JSR.

In this paper a simulation study is introduced, where an angular error in the horizontal plane is induced by the jammer. The effects of such a jamming on the missile trajectory and; consequently, on the final miss-distance, are evaluated and maximized. As a result of the simulation, the best law of variation of the induced angular error is specified. System designers may impose certain limits to protect the missile against mechanical damage in cases of excessive normal acceleration. The effect of varying those limit has also been studied.

SIMULATION PROBLEM FORMULATION AND ASSUMPTIONS

The target is assumed to move along a straight line at a constant velocity v_T; which is not far from realistic situations. Moreover, if any target maneuvers or missile initial heading errors were assumed they would degrade the guidance performance and falsely enhance the jamming effectiveness. The missile is assumed to move along its proportional navigation trajectory at a constant velocity magnitude v_M; heading at a lead angle L, with an instantaneous normal acceleration given by :

$$a_n = N' \cdot \dot{\lambda} \cdot v_c \qquad (1)$$

where

$\dot{\lambda}$ = $\frac{d\lambda}{dt}$ = *the line-of-sight rate of change*

N' = *effective navigation ratio; a proportionality constant to be selected by the guidance system designer, between 3 and 5.*

The whole Kinematic model is solved in horizontal plane, referred to two stationary orthogonal coordinate axes as shown in **Fig.1**. The gravitational and drag forces are neglected for simplicity.

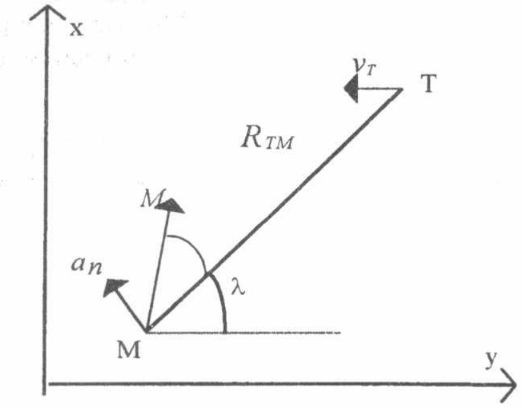


Fig.1. Homing Guidance Kinematic Model

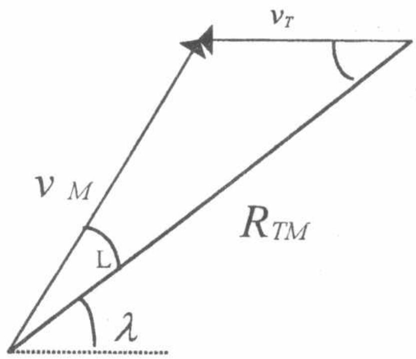


Fig.2. The collision triangle

From Fig.1. it is evident that :

$$\tan(\lambda) = \frac{y_T - y_M}{x_T - x_M} \tag{2}$$

From the collision triangle shown in Fig.2. it is evident that :

$$\frac{v_T}{\sin(L)} = \frac{v_M}{\sin(\lambda)} \tag{3}$$

A simulation model based on these relations and assumptions and applying Runge-Kutta numerical integration, similar to that described in reference [1], has been built using MATLAB software package. The jammer induced errors have been simulated by adding a controlled error component to the measured line-of-sight angle every time it is computed; such that :

$$\lambda_m = \lambda_r + \Delta\lambda_j \tag{4}$$

where

λ_m = *measured value of the line-of-sight angle*

λ_r = *real value of the line-of-sight angle*

$\Delta\lambda_j$ = *jammer-induced angular error*

An upper limit has been imposed on the missile normal acceleration magnitude. This limit has been given different values to study its effect on the results. Different time variables of the model have been monitored and plotted during the model execution; such as the jammer-induced angular error, the actual and measured values of the line-of-sight angle λ, its rate of change, the missile normal acceleration, the missile and target trajectories and their closing speed. At the end of every run the final miss-distance d_m has been computed.

The simulation model has been executed many times to study the effects of different parameters. Five basic cases have been considered :

1. No jamming
2. Constant induced angular error
3. Sinusoidally varying induced angular error
4. Linearly varying induced angular error
5. Saw-Tooth varying induced angular error

In all these cases, the following numerical values have been given to the model parameters; which are typical for medium range air-to-air guided missiles :

missile velocity magnitude = 1000 [m/s]
target velocity = 250 [m/s] (along the negative x axis)
initial missile launch range = 15 [km]

The effect of varying the missile launch range between 10 and 20 [km] has also been studied in the third case; where it does have a significant effect on the results.

FIRST CASE : NO JAMMING

In this case, the missile approaches the target in a straight line at a constant closing velocity. No variation occurs in the line-of-sight; resulting in a zero guidance command. As already expected; The simulation has resulted in a zero miss-distance (Fig.3.).

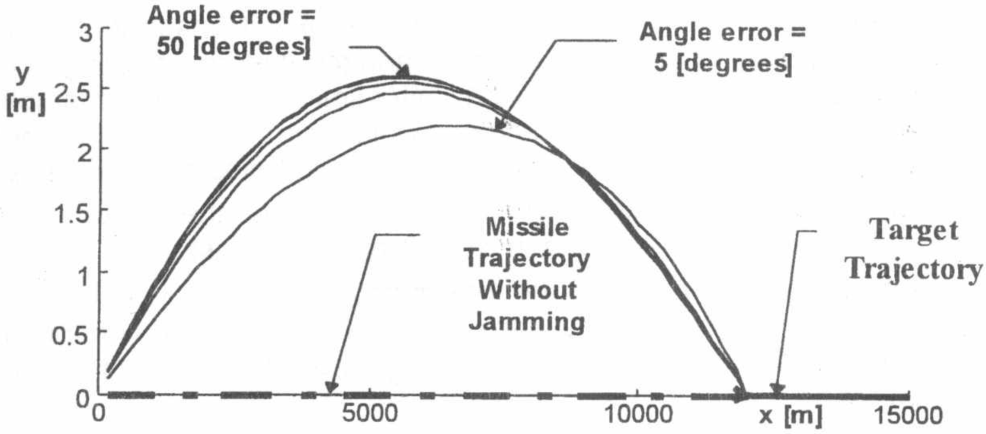


Fig.3. The first Two Cases

SECOND CASE : A CONSTANT INDUCED ANGULAR ERROR

Most ECM literature [3] discuss just inducing angle tracking errors without studying their time variations. In this case, although the induced angular error may take different constant values, its rate of change is zero and; consequently, the guidance command starts with a big value to correct the induced error and decreases as long as the error is being corrected. We have simulated this case to demonstrate this main concept.

In every simulation run of this case an initial deviation occurred in the missile trajectory; but the proportional navigation guidance system succeeded to recover (Fig.3); resulting in final miss-distances of the order of a few meters; which are much smaller than the radius of the missile warhead destruction zone.

THIRD CASE : A SINUSOIDALLY VARYING ANGULAR ERROR

The induced angular error has been varied according to a sinusoidal law :-

$$\Delta\lambda_j = J_0 + A.\cos(\omega t) \qquad (5)$$

where

J_0	=	the average angular error	[degrees]
A	=	the amplitude of angular error variation	[degrees]
ω	=	the angular frequency of time variation [rad / sec]. It can be normalized with respect to 1 [rad/sec] to give $\omega_1 = \omega / 1$ [rad/sec].	

The line-of-sight rate will be proportional to $sin(\omega\, t)$. Consequently; the guidance commend will change sinusoidally with time and the missile trajectory is expected to take a sinusoidal shape. The acceleration limiter limits the maximum amplitude of the trajectory. Such a limitation effect decreases with decreasing the angular frequency. As this frequency decreases the crest of the missile trajectory increases. By further decrease of ω_1 the system gets more chance to recover the induced errors and the crest starts to decrease. Thus the missile trajectory crest has a maximum at a certain value of ω_1 depending on the maximum acceleration limit, the missile speed and the initial launch range. The final miss-distance depends on the phase of the sinusoidal missile trajectory at the impact point; and is therefore very sensitive to the initial launch range.

Fig.4 shows the simulated target and missile trajectories for different values of ω_1 with a maximum normal acceleration 20 G. The axes are scaled in meters. It is clear that the trajectory crest becomes maximum for some value of ω_1 near 3. A search for this value has been done and the results are shown in Fig.5 with unlimited normal acceleration and Fig.6 with limited normal acceleration.

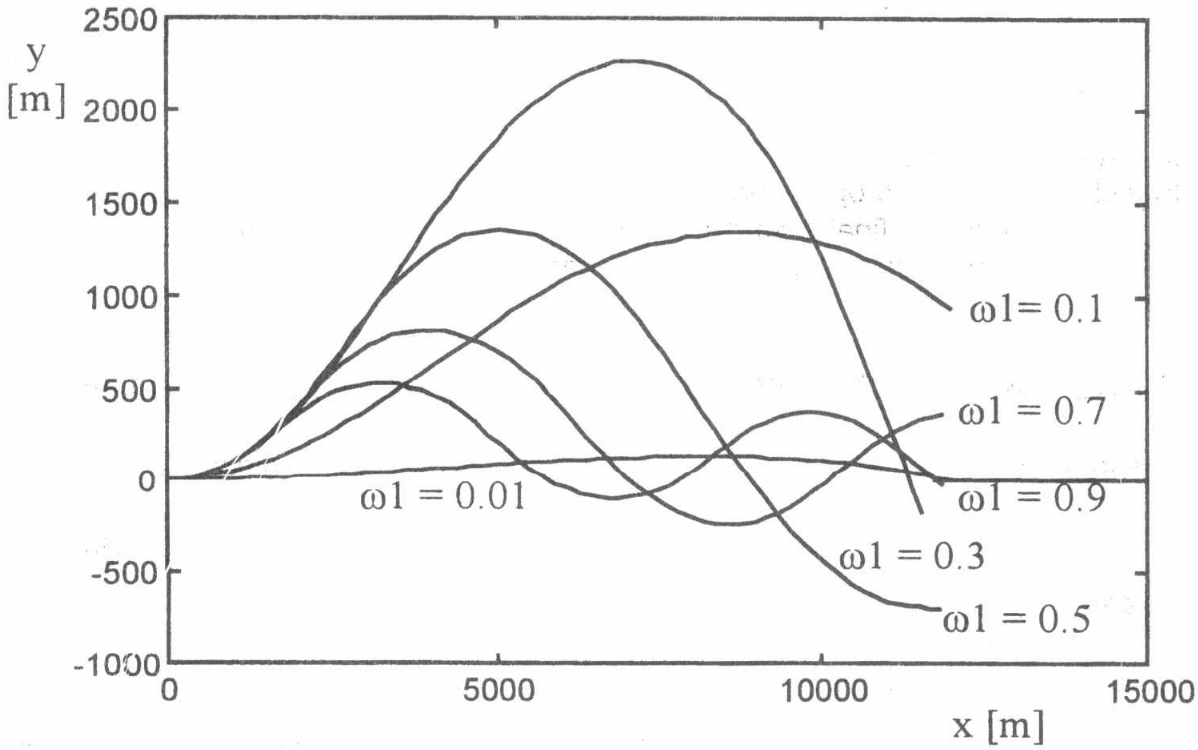
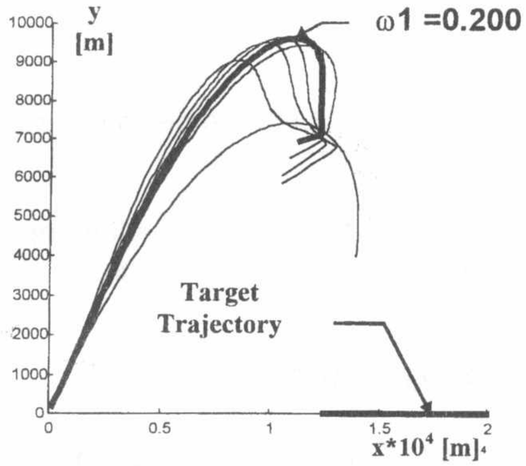
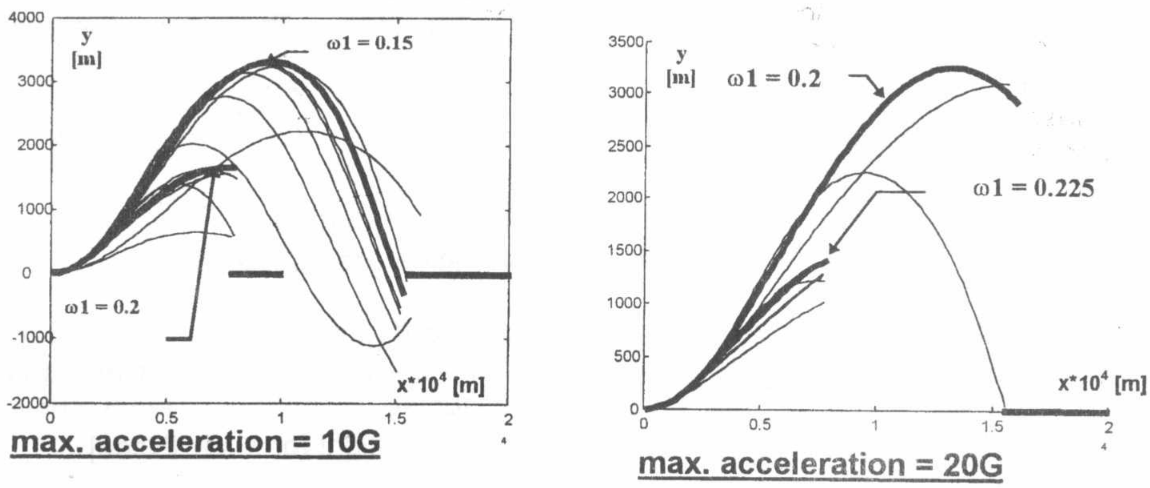


Fig.4. Trajectories for different values of ω1



Missile trajectories for different values of ω_1 between 0.1 and 0.3 with no limitation on the normal acceleration. The optimum value of ω_1 has been found 0.2; giving the highest trajectory crest and the largest miss-distance. Although they have been computed only for 20 [km] launch ranges; it is evident that shorter ranges would result in optimum values of the same order.

Fig.5. Missile trajectories for different values of ω1 with unlimited normal acceleration



With the missile normal acceleration upper limit is = 20 G; the missile trajectory gets its highest crest for ω_1 = 0.2 and 0.225 for a launch range 20 and 10 [km] respectively. Since a smaller oscillation frequency of the induced angular error results in a longer oscillation path of the missile trajectory and does not allow the trajectory to complete half a cycle before the impact point; slower error variation rates may lead to higher values of miss-distance. When the maximum missile acceleration is 10 [G]; the optimum value for ω_1 is 0.15 and 0.2 for a launch range of 20 [km] and 10 [km] respectively. The graphs show an evident return of the missile trajectory towards the target path for higher excitation frequencies; leading to smaller values of miss-distance. This is not allowed when ω_1=0.15. Thus we can consider ω_1=0.15 a good compromise for different launch ranges between 10 and 20 [km] for the given velocities and the assumed acceleration limits.

Fig.6. Missile trajectories for different values of ω1 with limited normal acceleration

From the simulation runs of this case it has turned out that :-
i. Appreciable values of final miss-distance can be achieved.
ii. The higher the error variation frequency ω; the lower the missile deviation from its ideal trajectory. For $\omega > 10$ the miss-distance does not exceed 7 [m].
iii. The final miss-distance is very sensitive to the initial launch range.
iv. At a certain value of ω_1 the trajectory gets a maximum crest. This value depends on the maximum allowable normal acceleration and the initial launch range.

FOURTH CASE : LINEARLY VARYING ANGLE ERROR

Since an oscillating induced error creates an oscillating normal acceleration; the trajectory itself oscillates around its correct path; giving rise to relatively small deviations. If we create a contiguously increasing angular error in one direction; we can get a continuous creation of guidance commands in the same direction which causes large deviations from the correct trajectory. This technique can be characterized by :

$$\Delta\lambda_j = J_o + K.\,.t \qquad (6)$$

where

J_o	=	the average angular error	[degrees]
K	=	the rate of angular error variation	[degrees/sec]

It is evident that increasing the angular error rate K increases the deviation and the final miss-distance. The simulation results are displayed in Fig.7 for different values of K. A maximum normal acceleration is assumed to be 10G. Fig.8. shows the variation of the normal acceleration during the missile flight for K =2 [°/sec].

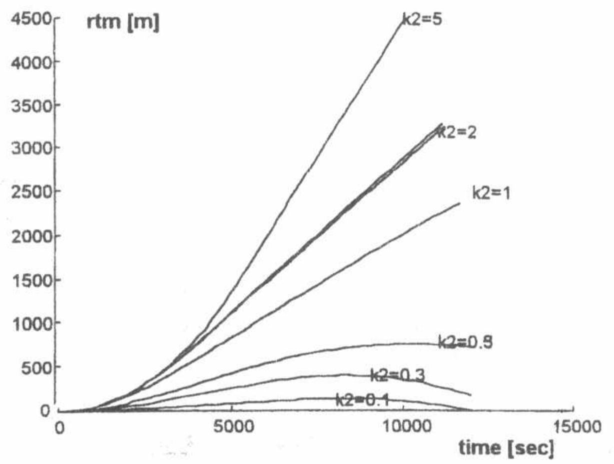


Fig.7. Trajectories for the fourth and fifth cases

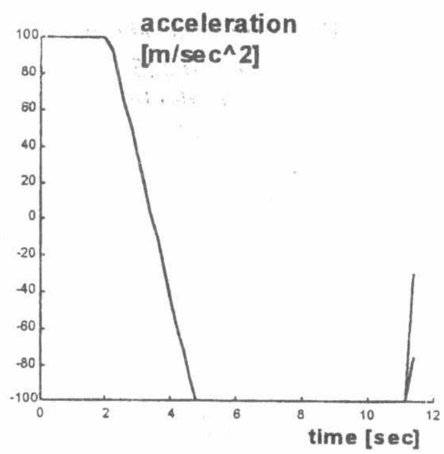


Fig.8. Normal Acceleration for both cases (k = 2)

FIFTH CASE : SAW-TOOTH VARYING ANGLE ERROR

Although a linearly varying induced angular error may result in the best possible deviation from the correct trajectory; it is difficult to realize by known jamming techniques. The nearest realizable waveform that gives a similar effect is the saw-tooth (ST) law of variation. We have tried to adjust the slope of the ST to that of the goal linear variation; resulting in the following law of change:

$$t2 = w.\ t - integer\ (w.\ t)$$
$$\Delta\lambda_j = j_{o2} + k_2 .\ (t2 / w) \qquad (7)$$

where w, k_2 and j_{o2} are jamming parameters to be adjusted for maximum effectiveness. Fig.9 demonstrates how a linear variation law is approximated sawtooth laws of variation with different frequencies w_i.

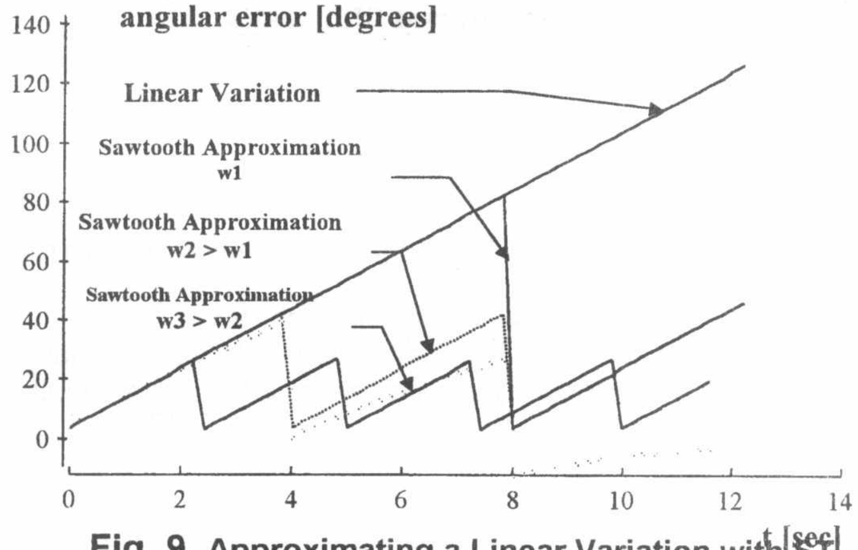


Fig. 9. Approximating a Linear Variation with ST.

The flyback time of the sawtooth must be as short as possible to minimize the total time during which the acceleration is negative. The value of w has to be as small as possible; in order to minimize the number of negative acceleration periods along the trajectory. The resulting trajectories for k2 = 2 and w = 0.5, are depicted in Fig.7 and Fig.8 where the coincidence of the results for linear and sawtooth laws of variation are evident. Fig.10 shows the time variation of the real values of the angle of sight λ and those values apparent to the missile tracking system; being deceived by jamming. In the same figure is shown the induced angular error which is the difference between the real and apparent values.

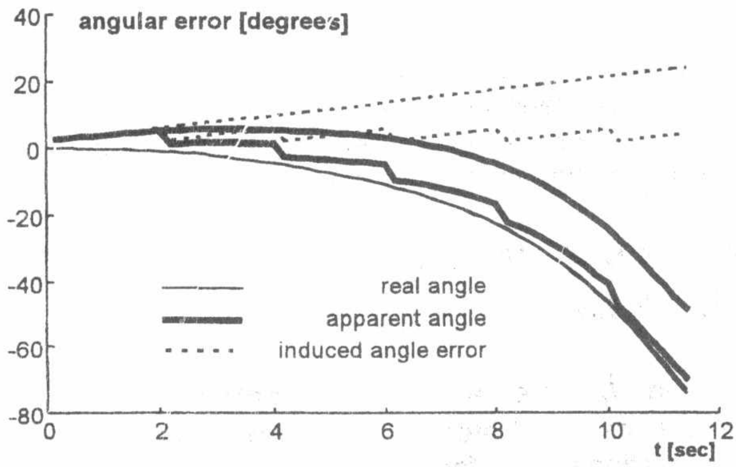


Fig. 10. Time variation of the real and measured values of the line-of-sight angle for the fourth and fifth cases.

CONCLUSION

1. Only time-varying induced angular errors affect a proportional navigation homing guidance process.
2. Sinusoidal variation effect is very sensitive to initial launch range.
3. The most effective variation law is the linear one. Sawtooth variation law is more easily obtainable by known jamming techniques and can give the same results.

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