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THEORETICAL AND EXPERIMENTAL INVESTIGATION OF SOME LATERAL CONTROL DERIVATIVES OF RECTANGULAR WING

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Abstract

In the field of Applied Aerodynamics, the evaluation of aerodynamic derivatives is essential for analysis of stability and control of flying bodies. In the present work some of important stability and control derivatives are calculated theoretically for a rectangular wing in an incompressible flow The theoretical solution is realized on the basis of two different methods; the Multhopp's method and the Vortex Lattice Method (VLM). An experimental evaluation of the calculated derivatives is also performed. The experimental study was carried out by the testing of a wing model in a general-purpose low speed wind tunnel. A good agreement was found between theoretical and experimental results.

1. Introduction

The rolling moment due to aileron deflection, and the associated hinge moment around the aileron axis of rotation are important terms in the aircraft lateral stability and control. The determination of these moments is based on the solution of both spanwise and chordwise aerodynamic load distributions on a wing with deflected control surfaces.

The present work includes theoretical and experimental determination of rolling and hinge moment derivatives. Such derivatives due to aileron and flap defections are evaluated for a rectangular wing with defected control surfaces in an incompressible flow. Among the available methods for determination of the aerodynamic load distribution on wings, Multhopp's solution of the lifting line theory together with the computational Vortex Lattice Method (VLM) were chosen to calculate the rolling moment due to their simplicity and easy applicability. As Multhopp's method is silent with respect to the chordwise load distribution, the hinge moment has been calculated using the VLM.

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The aerodynamic characteristics of a rectangular untwisted wing were measured on the Military Technical College (MTC) wind tunnel with closed test section of dimensions $1.5x1.15x3m^3$. The measurements were performed at low Reynolds number of about $5.2x10^5$, for different angles of attack and control surfaces deflections. Necessary wind tunnel boundary corrections were considered when evaluating experimental results.

2. Theoretical Solution

The development of the theory of wings of finite span, as mentioned in [1], is mainly the work of Lanchester and Prandtl . Prandtl's method was the first rational attempt for predicting loads on subsonic, three-dimensional wings. Prandtl and Tietjens [2] established the integro-differential equation representing the distribution of circulation on finite span wing. This is the fundamental equation of the lifting line theory which provides the relation between the wing parameters and the unknown circulation distribution. Among many others, Multhopp's method [3], solves the Prandtl's equation for the calculation of spanwise circulation. The lifting surface theory was introduced to overcome the shortcoming in the lifting line theory. The wing is replaced by a vortex sheet whose local intensity varies along both the span and chords of the wing. Ashely et al. [4] represented a survey on lifting surface theory. They emphasized purely numerical approaches to the problem and emerged new applications to nonplanar configurations. They carried out numerical solutions for linearized problems in both incompressible and compressible fluids.

A progress in numerical solutions of the lifting surface theory for planar and nonplanar configurations was presented by Landahl and Stark.[5]. They considered in details the problem of thin wing in unsteady flow. Also, they listed many ways of formulating the unsteady lifting surface problem.

For steady flow condition the technique that represents the loading with discrete vortices is more convenient than assuming continuous vortex distribution. Falkner [6] was the first who approximated the continuous distribution of bound vorticity on the wing surface by a finite number of discrete horseshoe vortices in both spanwise and chordwise positions. This method - Falkner's method - is known as the Vortex Lattice Method (VLM).

From the previously reviewed methods, Multhopp's and the vortex lattice method were chosen to solve the aerodynamic loading about the given wing. Both methods are characterized by simplicity and easy applicability.

2.1 Multhopp's Method

Multhopp's method, based on lifting line theory, solves Prandtl's equation for the calculation of the spanwise distribution of circulation. The method is extended to find the load distribution for cases of twisted wings, deflected flaps and deflected ailerons or any combination by linear superposition.

2.1.1 Dimensions form of Prandtl's equation

The equation of Prandtl is given by:

$$\Gamma = \frac{1}{2} b V_\infty a_\infty (\alpha_a - \alpha_i) \qquad (1)$$

Transforming the equation into dimensionless form by the substitution of the new variables;

$$\gamma = \frac{\Gamma}{l V_\infty}, \quad \bar{b} = \frac{b}{l/2}, \quad \bar{z} = \frac{z}{l/2} \qquad (2)$$

we obtain;

$$\gamma = \frac{1}{4} \bar{b} a_\infty (\alpha_a - \alpha_i) \qquad (3)$$

The induced angle of attack is expressed in terms of γ and $\bar{Z}$ as follows

$$\alpha_i(\bar{Z}) = -\frac{1}{2\Pi} \int_{-1}^{+1} \frac{d\gamma}{d\bar{Z}_1} \frac{d\bar{Z}_1}{(\bar{Z}_1 - \bar{Z})} \qquad (4)$$

By introducing the transformation $\bar{Z} = -\cos\theta$ and substituting for α_i in equation (1), then the Prandtl in dimensionless form becomes

$$\gamma = \frac{1}{4} a_\infty \bar{b} (\alpha_a - \frac{1}{2\Pi} \int_0^\Pi \frac{d\gamma}{d\theta_1} \frac{d\theta_1}{(\cos\theta_1 - \cos\theta)}) \qquad (5)$$

2.1.2 Approximate formula for the circulation distribution

To achieve zero circulation at wing tips, $\gamma(\theta)$ is expressed as Fourier sine series as follows;

$$\gamma(\theta) = 2 \sum_{k=1}^{m} A_k \sin k\theta \qquad (6)$$

The Fourier coefficient A_k is given by

$$A_k = \frac{2}{\Pi} \int_0^\Pi \frac{\gamma(\theta)}{2} \sin k\theta . d\theta \qquad (7)$$

writing $\gamma(\theta) \sin k\theta = f(\theta)$, then

$$A_k = \frac{1}{\Pi} \int_0^\Pi f(\theta) d\theta \qquad (8)$$

This integral can be approximated by means of M individual values of f(θ), distributed regularly within the interval (θ-π), as in Fig.1. From Fig. 1.

$$\theta_n = \frac{n\Pi}{M} \qquad 1 \le n \le M-1 \tag{9}$$

Substituting by expression (9) into (8) one obtains;

$$\gamma(\theta) = \frac{2}{M}\sum_{n=1}^{M-1}\gamma_n(\theta)\sum_{k=1}^{M-1}\sin k\,\theta_n \sin k\theta \tag{10}$$

Which is the Multhopp's approximate formula for the distribution of circulation. the solution was carried out numerically to obtain the individual values of γ_n satisfying the Prandtl equation.

2.1.3 Change of circulation distribution due to deflections of flaps and aileron

In the region of flaps and ailerons, the flap (or aileron) deflected by an angle δ leads to change in local lift coefficient by the value.

$$\Delta C_y = \frac{\partial C_y}{\partial \delta}\delta \tag{11}$$

which can be rewritten as

$$\Delta C_y = \frac{\partial C_y}{\partial \alpha}\frac{\partial \alpha}{\partial \delta}\delta = \frac{\partial C_y}{\partial \alpha}\alpha_{eq} \tag{12}$$

This enables to substitute the effect of flap deflection by the equivalent amount of the aerodynamic incidence, α_{eq}.

2.2 Solution by the Vortex Lattice Method

Although lifting line theory provides a reasonable estimate of lift, aerodynamic induced drag and rolling moment; however, an improved flow model is needed to calculate the lifting flow field about the wing of interest. Such method enables analysis of both chordwise and spanwise distribution of circulation for cases of deflected flaps and ailerons. The Vortex Lattice Method is adopted here to investigate the flow field about a wing with deflected flaps and ailerons with reasonable accuracy.

For wing which is subjected to a flow of low speed and operating at a small angles of attack, the resultant flow may be assumed to be steady, inviscid, irrotational and incompressible. For such kind of flow, the vortex lattice method approximates the continuous distribution of bound vorticity over the wing surface by a finite number of discrete horseshoe vortices, as shown in fig.2. The individual vortices are placed in rectangular panels, also called elements or lattices.

The bound vortex coincides with the quarter-chord line of the panel and all the panels are located on the mean camber surface of the wing. When the trailing vortices detach from the wing, they flow the curved path. However, for many engineering applications, suitable accuracy could be obtained using linearized theory in which straight-line trailing vortices extend downstream to infinity. In the linearized approach the trailing vortices are aligned either parallel to the free stream or the vehicle axis. Both orientations provide the same accuracy within the assumptions of linearized theory. In order to perform simple computation of influence of the various vortices, the trailing vortices have been assumed to be parallel to the vehicle axis. Furthermore, with the geometric coefficients don't change with the angle of attack.

The tangency boundary condition, of the flow to the wing surface, is sustained at the control point of every panel. The control point of each panel is centered spanwise on the three-quarter chord line midway between the trailing vortex legs. The induced velocity by all vortices is calculated at every control point. A system of algebraic equations in n-unknown vortex circulation strength is obtained. When solving this system of equations, and obtaining the strength of all vortices, the resultant required aerodynamic forces and moment are calculated. The solution is performed at different configurations and a complete aerodynamic characteristics are obtained.

The solution using VLM is completely explained in Ref. [7]. The results obtained by both Multhopp's and VLM are compared by the experimental ones. An analysis of results indicates a satisfactory coincidence between the theoretical and experimental results exists.

3. Experiment Set Up

In the experimental part of this work, the model is described; the test procedures are explained; and the measured cases are introduced.

3.1 Model Description

The test model is a rectangular untwisted wing of 1.2m span, 0.3m chord and airfoil type NACA 66_2-015. The wing has a plane flap (flap chord $b_{fl} = 0.09$m, flap span $l_{fl} = 0.62$m). The wing has a left and a right aileron every of span $l_{al} = 0.29$m and has the same flap chord, Fig.3. The hinge moment about the flap axis is measured by a mechanical moment balance designed and manufactured in the laboratory. The flap deflection angle (δ_{fl}) could be adjusted by wire and screw connected in the moment balance. The ailerons deflection angle (δ_{all}, δ_{alr}) are adjusted manually for different cases of measurements. The tested wing is connected with the hinge moment balance as shown in Fig.4. Location of the wing inside the test section of the wind tunnel is illustrated in Fig. 5.

3.2 Wind Tunnel Description

The model is tested in a low speed wind tunnel which has the following specifications [8]
-Type Dingler of a maximum speed of rectangular shape (1.15x1.5x3.0 m^3) with tapered corners.

- Low turbulence factor.
- Six - components mechanical balance.

3.2.1 Velocity profile across the test section

The maximum velocity deviation at each point is 0.45% from that velocity at the center of the test section, so the tunnel has acceptable uniform velocity distribution across the test section.

3.2.2 Wind tunnel constant ($\wp$)

The ratio between the dynamic pressure at the test section and that at outlet of the nozzle is called wind tunnel constant. It was determined along the closed test section; and was found that the change in $\wp$ lies in the acceptable limits. Wind tunnel constant for the present experiment was equal to 0.948 and considered during the calculation of the experimental results.

3.2.3 Static pressure variation along the test-section.

The static pressure variation along the test section was measured and a pressure gradient was obtained. This pressure gradient; which results in a horizontal buoyancy, lies in the practical ranges. It usually affects the drag which is of less importance in the case study.

3.2.4 Turbulence factor τ_f

The turbulence factor for the closed test-section was determined by two different methods, sphere drag method, and pressure sphere. By the first method, the turbulence factor was determined to be equal 1.132, and 1.2 by the second one. Both obtained results are in the normal limits which ranges from acceptable turbulence in the closed test-section configuration.

3.2.5 Accuracy of measured air loads

The six-components balance is highly accurate one. The errors in weighing have been determined, and they are considered during evaluation of the experimental results.

3.3 Test Procedures

The model is tested in the mentioned wind tunnel, with closed- test section configuration, at constant dynamic pressure of about 640 N/m^2. The test is operated at different angles of attack from -2.5^o up to 12.5^o with step of 2.5^o. The flap defection angles are from 0 up to 40^o with step of 10^o. The aileron deflection angles are 0, 5, 10, and 15 degrees. Aerodynamic forces, and moments, including the flap hinge moment, are measured for different angles of attack, flap and aileron deflections. All experiments were carried out at Reynolds number of $5.2x10^3$. When computing the experimental results, the wind tunnel boundary corrections were considered. Both the theoretical and experimental results are presented.

4. Results and Analysis

The theoretical results obtained by Multhopp's method and VLM together with the experimental evaluation are presented. For comparing the theoretical results with the experimental ones, both are plotted together in figures.

Fig.6 represents the lift curve obtained theoretically and experimentally for the wing at zero flap deflection. The theoretical results coincide with the experimental ones at small angles of attack between -3° to 3°. The deviation between the theoretical and experimental results, shown at high α, is due to boundary layer separation on the wing upper surface. This occurred early due to the separation of the laminar boundary layer that having weak stability at such low Reynolds number, for testing.

Multhopp's results coincide better with the experimental ones than the results obtained by VLM. The results obtained by VLM show a linear variation of C_y within the whole considered range. This is usual as the effect of viscosity is completely ignored in the solution by that method. The dependence of the rolling moment coefficient on the aileron deflection angle for different wing angles of attack and this is shown in Fig.7. The plotted curves show a good coincidence between the theoretical results obtained by Multhopp's solution and the experimental ones in range of aileron deflection till $\delta_{al} = 10°$. Results obtained by VLM are of higher values. This is due to the consideration of purely potential flow besides the Kutta condition at the control surface trailing edge is not fulfilled. The discrete distribution of vortices results in the non zero value at the trailing edge. This shortcoming can be improved by increasing the number of lattices in the region of control surfaces to allow the gradual decrease of circulation near the rear edge.

The dependence of rolling moment derivative, with respect to aileron deflection angle ($m_x^{\delta al}$) on the angle of attack is shown in Fig.8 . This derivative is obtained for the linear part of coarse of variation in the range of considered angles of attack. The curve shows good coincidence between Multhopp's method and experimental results. The value of the derivative ($m_x^{\delta al}$) decreases with increasing α due to the growing effect of boundary layer separation. The solution by VLM shows that ($m_x^{\delta al}$) is independent on α, that is due to the same reasons given in the analysis of the previous Fig.7 .

The dependence of hinge moment coefficient against flap deflection δ_{fl} in the range of the considered angles of attack is shown in Fig.9 . The presented curves show some difference between theoretically and experimentally obtained hinge moments. The difference increases with increasing the flap deflection . The theoretical hinge moment tends to be double of the measured one. This may refer to that the flow the upper surface of the flap is seperated at high flap angles, which is not considered in the theoretical solution. This reason is supported by the experimental results which show no remarkable change in hinge moment at the same flap deflections for different angles of attack.

The rate of variatin of hinge moment with flap deflection angle ($m_{he}^{\delta fl}$) is represented in Fig.10. The curve shows a slight decrease in the measured derivative ($m_{he}^{\delta fl}$) with increasing angle of attack α . The theoretically obtained $m_{he}^{\delta fl}$ is approximately constant and this is due to the assumption of potential flow only.

5. Conclusions

The determination of airplane lateral control derivatives is very essential in the field of applied aerodynamics. In the present work, theoretical determination of air-loads on a rectangular wing of moderate aspect ratio and with deflected control surfaces, is carried out by application of both Multhopp's method and VLM. From the obtained results, the following conclusions could be introduced.

1. In cases of non-deflected control surfaces, all aerodynamic coefficients resulted from both the theoretical solutions are in good agreement with the experimentally obtained ones.
2. Multopp's method is very suitable for the determination of the change in rolling moment due to aileron deflection angles at small values regime.
3. Extended Multhopp's solution, with changed lift curve slope of airfoil and with changing $d\alpha/d\delta$, is more suited to the experimental results.
4. The results obtained by application of the VLM are higher than those obtained by Multhopp's method and the experimentally calculated ones. This is referred to the neglected effect of viscosity in the solution by this method.
5. The solution by the VLM is affected by the distribution and the number of lattices on the wing surface.
6. The hinge moment obtained theoretically by the VLM is highly deviated from the experimental results, this is due to the boundary layer seperation in flap region. The theoretical hinge moment is approximately twice the experimental one because the lower surface of the flap is actually the only effective one.
7. In future more accurate numerical method be applied to consider the wing thickness and viscosity of the fluid.

5. Nomenclature

a	lift curve slope
b	chord length
C_x	drag coefficient
C_y	lift coefficient
l	wing span
M	number of Multhopp's sections
m_{he}	hinge moment coefficient
m_x	rolling moment coefficient
V	velocity
x, y, z	Cartisian coordinate axes
α	angle of attack
Γ	circulation
γ	dimensionless circulation
δ	control surface deflection angle
$\wp$	wind tunnel constant
ρ	mass density
τ	trailing edge angle
τ_f	turbulence factor

Subscripts

a	aerodynamic
all	left aileron
alr	right aileron
al	aileron
eff	effective
eq	equivalent
fl	flap
i	induced
o	incompressible
∞	infinite span win or undisturbed flow

superscripts

$\_$	dimensionless value

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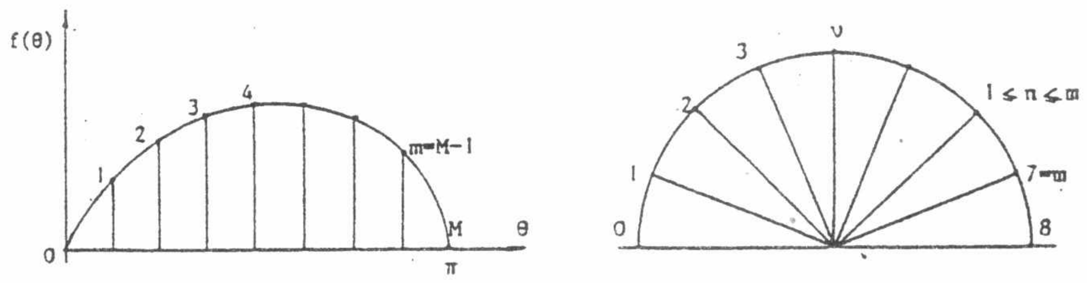

Fig.1 Distribution of f(0) along wing span

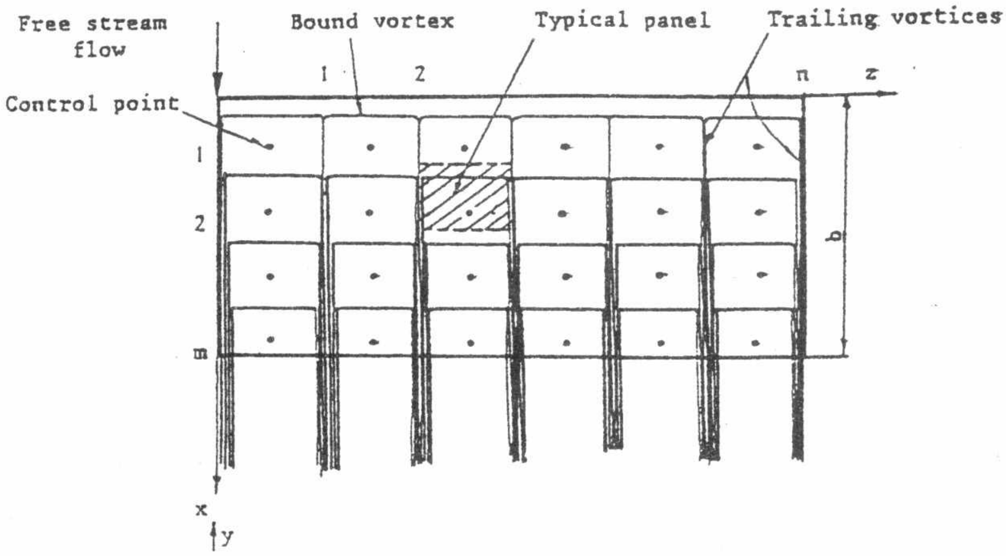

Fig.2 System of horse-shoe vortices

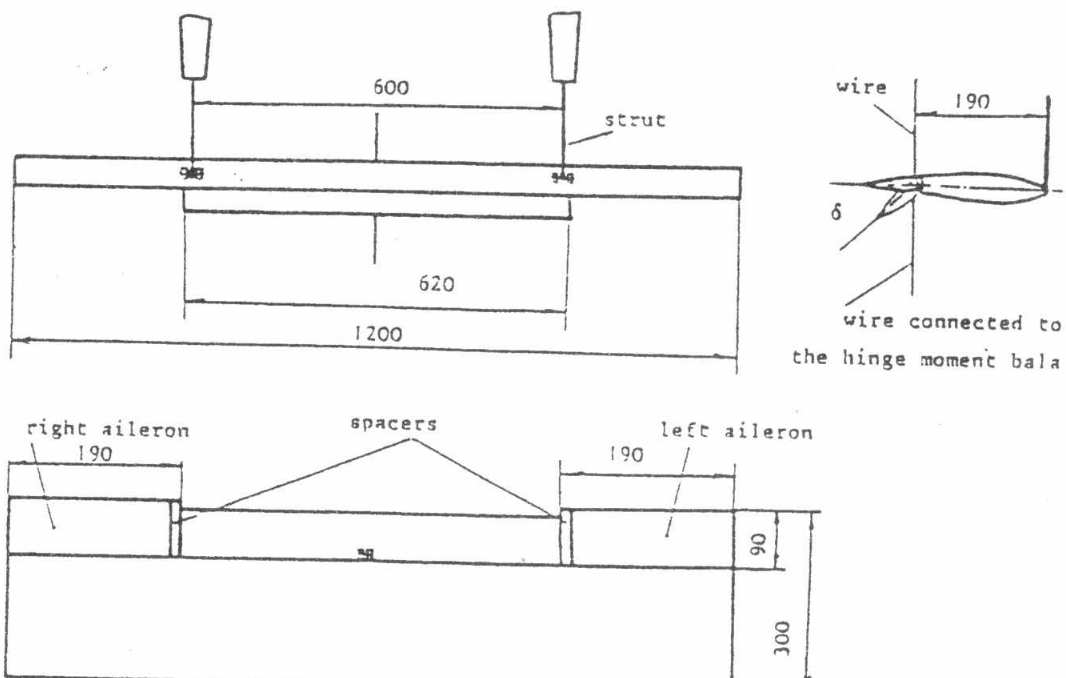

Fig.3 Three-view drawing of the tested wing

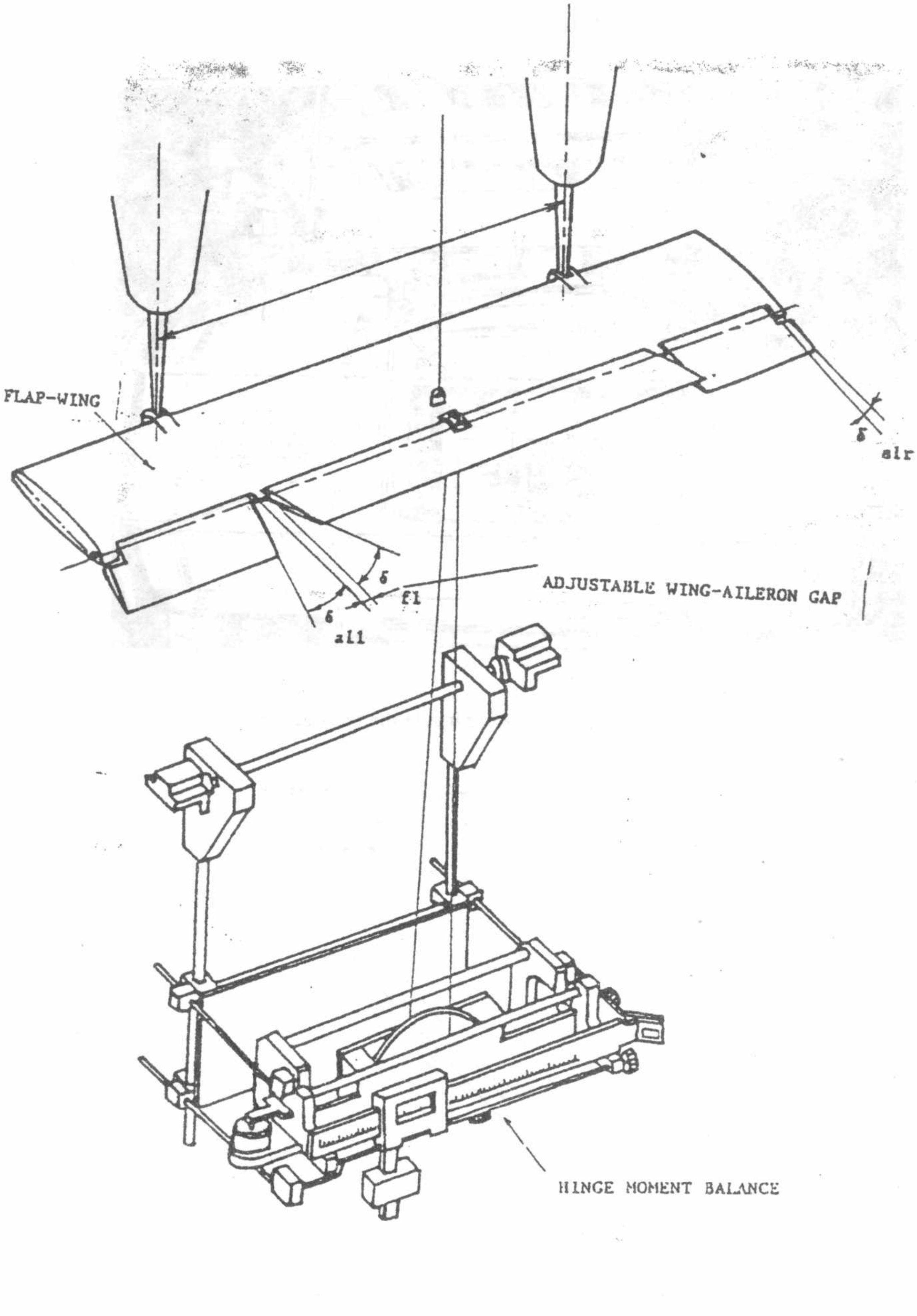

Fig.4 The tested wing connected to the hing moment balance

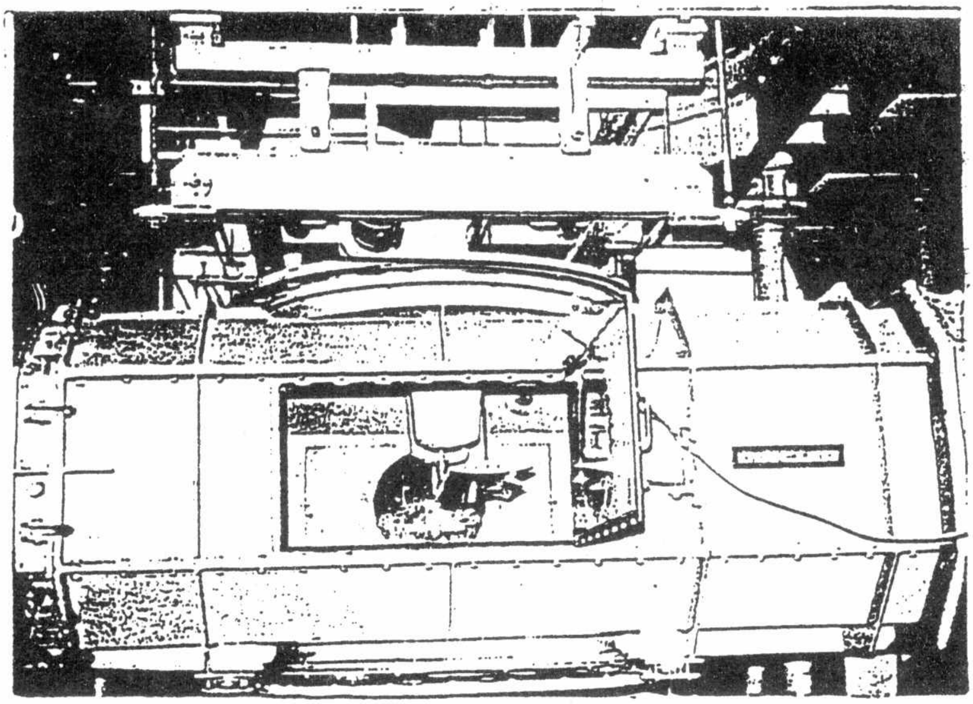

Fig.5 The tested wing inside the test-section

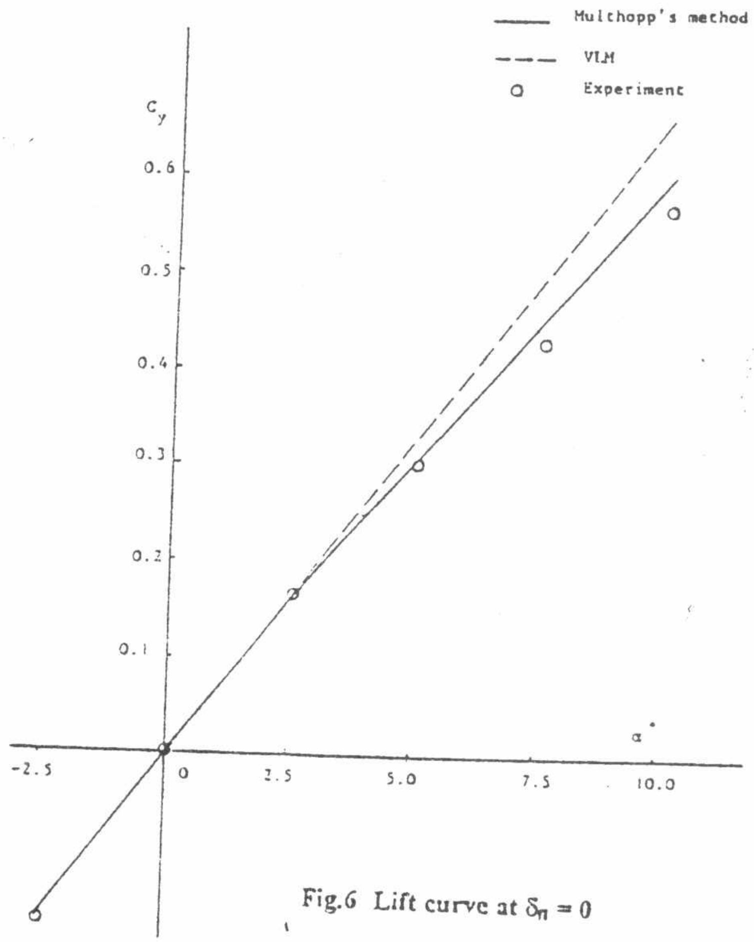

Fig.6 Lift curve at $\delta_n = 0$

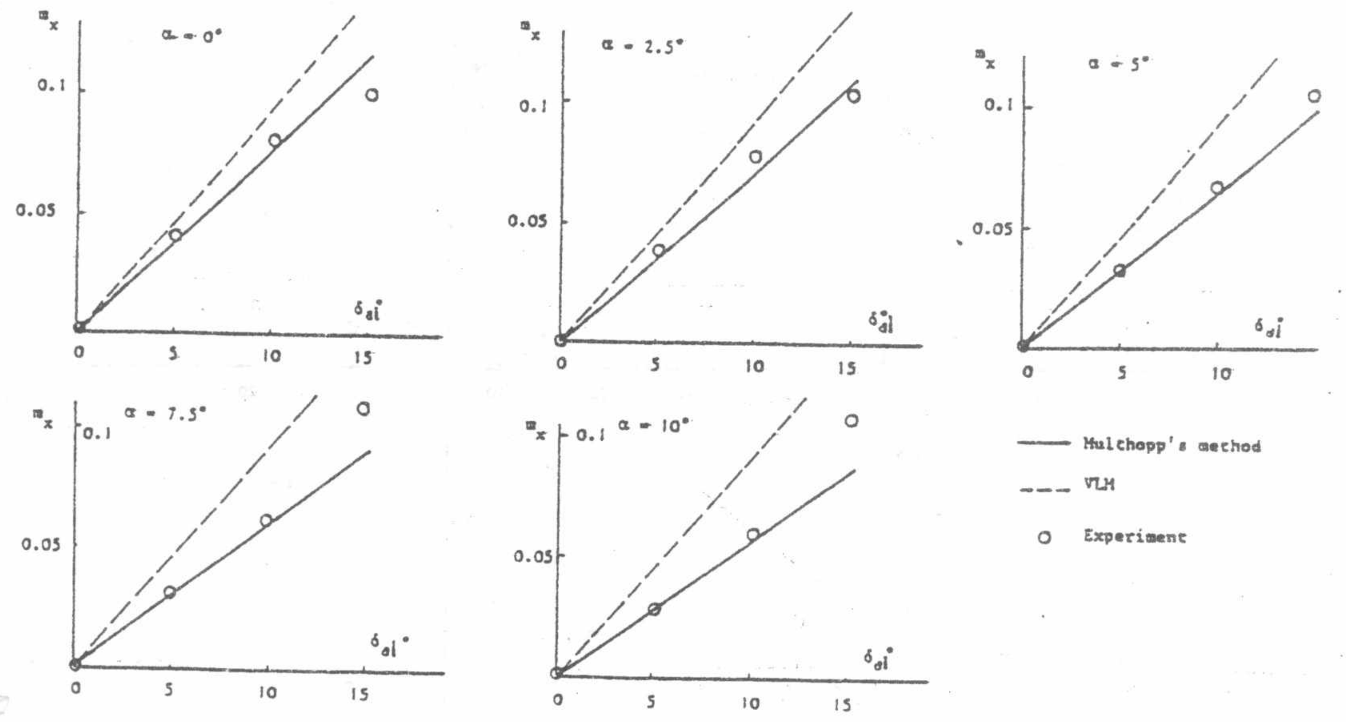

Fig.7 Variation of m_x with δ_{al} for different α

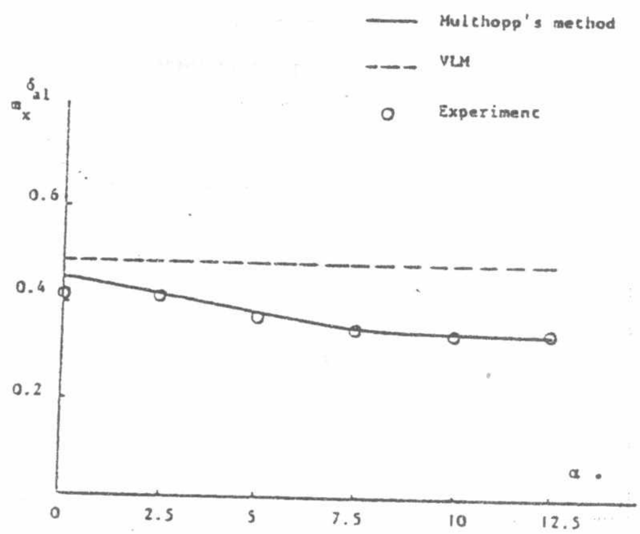

Fig.8 Dependence of $m_x^{\delta_{al}}$ on angle of attack (α)

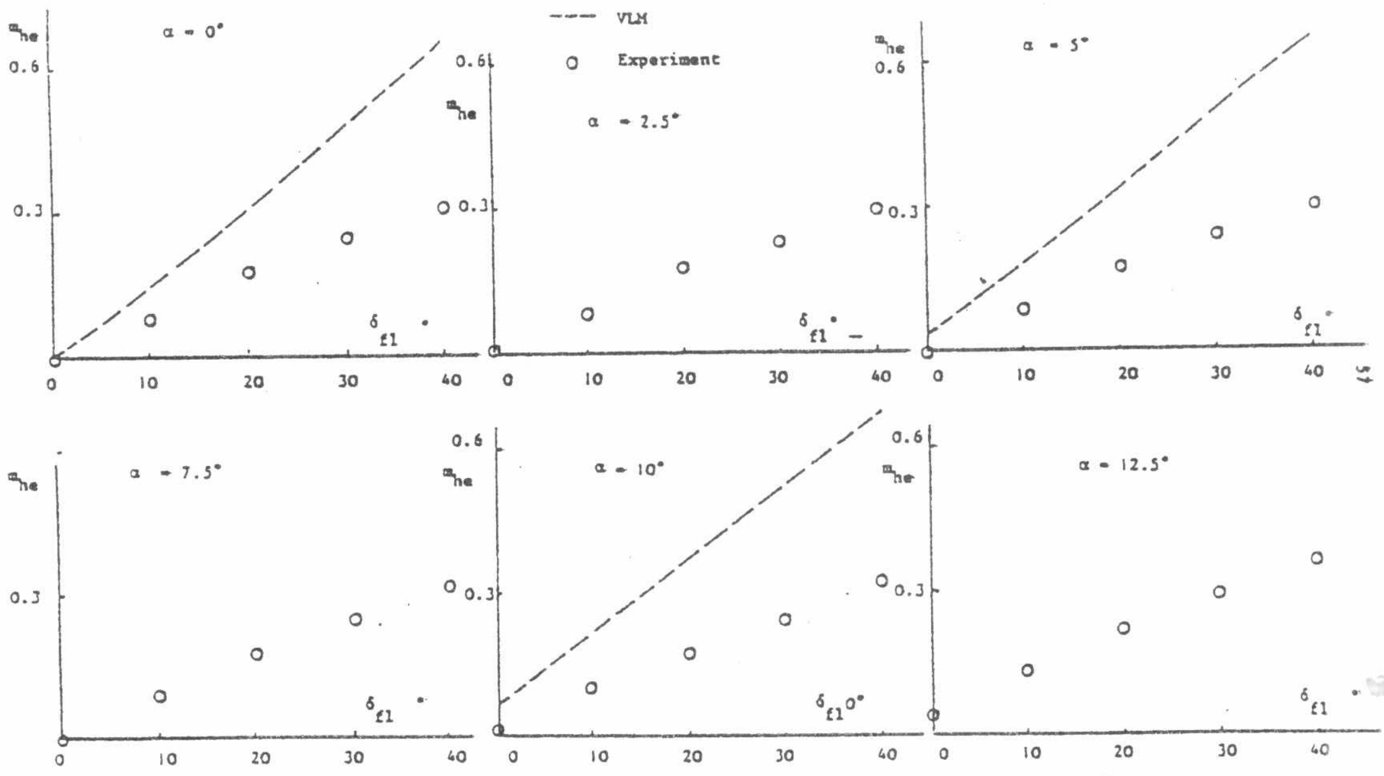

Fig.9 Dependence of m_{he} on δ_{fl} for different angles of attack (α)

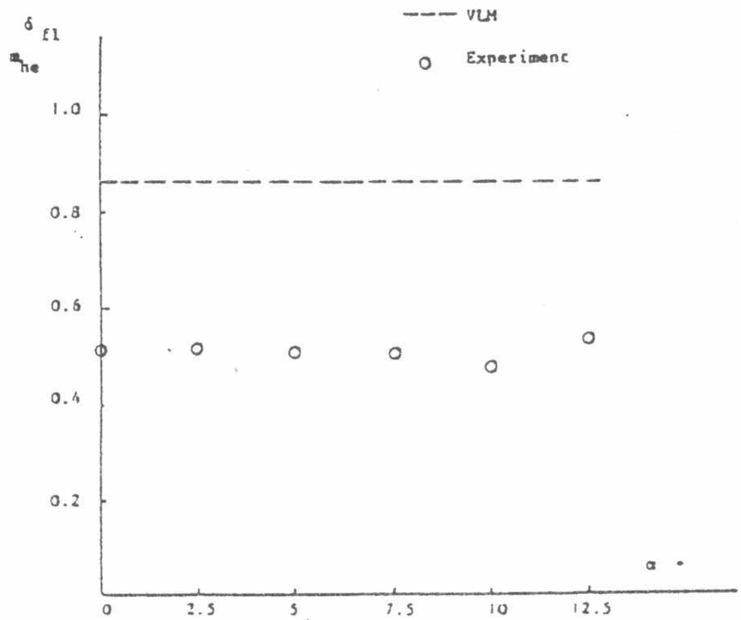

Fig.10 Dependence of $m_{he}^{\delta fl}$ on angle of attack (α)