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MODELLING OF VIBRATION DAMPING IN COMPOSITE STRUCTURES

By

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ABSTRACT

An improved dynamical model for vibration damping in composite structures is introduced to incorporate the code number and the degree of isotropism against the modal parameters.

A substantial development has been carried out from the fitting of modal measurements with lowest residual errors to permit an establishment of quasi uniform mass damping models in terms of normal coordinates system.

The analysis of the obtained results proves not only the efficiency of the developed model but also its applicability in any wide range of frequency spectrum of composites.

1. INTRODUCTION

Composite damping or energy dissipation property of vibrating composite structures, is a name for complex physical dynamic nature that is amenable to rheological modal analysis. In a broad class of composite structures, the distinguishing characteristic of the damping mechanism is its strong- dependence on the eigenfrequencies such that it exhibits little damping at high frequency level [1,3].

In contrast to the dynamic nature of isotropic domain, a further complication arises in composite domain due to the mutual effects for various parameters, such as code number, degree of isotropism (volume fraction), type of fixation as well as the vibrating mode number on the damping and

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stiffness distributions. As an example, the decreasing of volume fraction of fiber enhances energy dissipation by increasing the loss associated with matrix composite[2,3,4]. It might be expected that the natural frequencies of vibrating composite structures and in sequence the damping capacity, can be altered by changing layer's orientations and stacking sequence[5,6], so that the damping nature as a function of frequencies of composites should be further studied.

At the present time, it is still difficult to determine accurately the modal characteristics of composite structures (particularly damping parameters) throughout analytical approach. The experimental confirmation prediction is therefore at very least desirable and can be used to form analytically the mathematical model. In turn it can be used to more clearly understand the configuration of parameters controlling the dynamic of composite states.

Recently, a mathematical model representing the damping capacity of composite was established[1]. Based on the student distribution approximation of the measured values of damping in the fundamental mode, the modal relationships between the fundamental frequencies and the damping factors were developed in equivalent to an uniform mass damping of isotropic structures.

In the present work, an attempt has been made to improve the convergence characteristics of the model within a wide range of frequencies for different code numbers at two levels of volume fraction.

Basically, a weight factor (α) has been introduced for correlating and updating the mathematical model to the experimental data throughout the utilization of the curve fitting response function. This has resulted generalized quasi rectangular hyperbolic relationships between the loss factors and the natural frequencies with the confidence level 99.5%. These results permit the uncoupling of simultaneous equations of motion of composite structures with the lowest residual errors.

In the experimental work, Fig. (2), cantilever composite beams made from fiber reinforced plastic FRP are considered as the object of the study for their simplicity and for wide national applications. Various specimens made from three plies, Fig. (1), are utilized for two levels of volume fraction (a) a weakly composite 15% and (b) an average composite 45%.

In order to evaluate accurately the influnces of code number on the damping capacities and natural frequencies, twelve specimens of unidirectional cross - ply and angle-ply laminate have been fabricated in the laboratory by using hand lay up technique.

Numerically the first four natural frequencies at the two levels of volume fraction are computed by the use of modified formula M.F.M [7] and listed in the second column of the tables (1) and (2).

For the sake of verification, the experimental results of the natural frequencies and the loss factors for the first four natural modes are listed in the third and fourth columns in these tables respectively.

To highlight the idiosyncratic nature of the damping parameters, various curves representing mutual relationships of modal parameters were plotted in Fig. (3) and (4).

The close agreement between the mathematical and experimental results leads to more easly understand and enrich properly the modelling of the dynamic nature of vibrating damping composite structures.

2. PROBLEM STATEMENT

The governing equations of free damped vibration of laminated structural model of n degrees of freedom may be expressed as :

$$\underset{\sim}{m}\ddot{\underset{\sim}{q}} + \underset{\sim}{c}\dot{\underset{\sim}{q}} + \underset{\sim}{k}\underset{\sim}{q} = 0 , \qquad (1)$$

where, $\underset{\sim}{m}$, $\underset{\sim}{c}$, $\underset{\sim}{k}$ are the n x n mass damping and stiffness matrices, respectively and $\underset{\sim}{q}$ is the corresponding n x 1 displacement vector.

In the absence of damping, the natural frequencies ω and the corresponding mode shapes $\underset{\sim}{V}$ are obtained by solving the eigenvalue problem:

$$\underset{\sim}{E}\underset{\sim}{V} = \underset{\sim}{V}\underset{\sim}{P} \qquad (2)$$

where, $\underset{\sim}{E} = \underset{\sim}{m}^{-1}\underset{\sim}{k}$ is the n x n inverse dynamic matrix

$\underset{\sim}{V}$ is the n x n orthogonal classical modal matrix and $\underset{\sim}{P}$ is the n x n diagonal frequency matrix containing the n squared frequencies.

Having obtained the complete set of n eigenpairs, the orthogonality properties of mass and stiffness matrices are then expressed as :

$$\underset{\sim}{M} = \underset{\sim}{V}^T \underset{\sim}{m} \underset{\sim}{V} = [M_r], \qquad (3)$$

$$\underset{\sim}{K} = \underset{\sim}{V}^T \underset{\sim}{k} \underset{\sim}{V} = [K_r], \qquad (4)$$

where, $\underset{\sim}{M}$ and $\underset{\sim}{K}$ are the n x n diagonal mass and stiffness matrices.

Considering the invariance of the modal matrix under raising the matrices $\underset{\sim}{E}$ and $\underset{\sim}{P}$ to any rational positive number α, the eigenvalue problem(2) may be recast as :

$$\underset{\sim}{D}\underset{\sim}{V} = \underset{\sim}{V}\underset{\sim}{\Lambda} \qquad (5)$$

where $\underset{\sim}{D} = \underset{\sim}{E}^{\alpha}$ and $\underset{\sim}{\Lambda} = \underset{\sim}{P}^{\alpha}$.

Here α is the control weight factor to be choosen for minimizing the weighted residual. A special case, at which α (max) = 1, was considered for the uniform mass damping matrix of composite in equivalence to the damping of isotropic structure presented in Reference [1]

3. MATHEMATICAL MODEL OF VIBRATION DAMPING OF COMPOSITE STRUCTURE :

It was mentioned in Ref. [1] that the hyperbolic relations between loss factors (η) and natural frequencies (ω) of composite plate, vibrating at the first mode for different boundary conditions, provide a more reliable prediction throughout the utilization of an uniform mass damping scheme. These relations were established by utilizing the student distribution approximation with confindence level 95%.

The demand for more accurate modelling of composite structures for various code numbers within a wide range of frequency sperctrum requires a modification of hyperbolic relations by introducing the proper weight factor (α_i) and the damping constant (a_i) for i th code number.

The complication for selecting the proper current values of the damping parameters is a result of the mutual effects of the code number, volume fraction, aspect ratio, types of fixation and natural mode number.

For establishing a proper equivalent mathematical model let us start by successive premultiplication of equation (5) by a positive integer j followed by premultiplication of both sides of the result equation by $\underset{\sim}{V}^T\underset{\sim}{m}$ and we have:

$$\underset{\sim}{V}^T \underset{\sim}{m} \underset{\sim}{D}^j \underset{\sim}{V} = \underset{\sim}{M} \underset{\sim}{\Lambda}^j \qquad (6)$$

From definition, the right hand side of the previous equation is a diagonal matrix. It follows that $\underset{\sim}{m}\underset{\sim}{D}^j$ satisfies the orthogonality conditions.

Let a damping matrix is represented as a linear combinations of the compound matrix given by :

$$\underset{\sim}{c} = \sum_{j=o} a_j \underset{\sim}{m} \underset{\sim}{D}^j \qquad (7)$$

The proposed form (7) satisfies the orthogonality conditions such that :

$$\underset{\sim}{c}\, \underset{\sim}{m}^{-1} \underset{\sim}{k} = \underset{\sim}{k}\, \underset{\sim}{m}^{-1} \underset{\sim}{c}.$$

With regard to equations (6) and (7) this diagonalized damping matrix is then given by :

$$\underset{\sim}{C} = [\,C_r\,] = \underset{\sim}{V}^T \underset{\sim}{c}\, \underset{\sim}{V} = \Sigma\, a_j \underset{\sim}{M}\, \underset{\sim}{\Lambda}^j \qquad (8)$$

Here the damping coefficient of the r<u>th</u> mode is expressed as :

$$C_r = \underset{\sim}{v}_r^T \underset{\sim}{c}\, \underset{\sim}{v}_r = \sum_{j=0} a_j \,.\, M_r\, \lambda_r^j \qquad (9)$$

where, $\lambda_r = \omega_r^{2\alpha}$

In view of the hyperbolic relationship between the loss factor (η) and the natural frequency derived in Ref. (1), The r<u>th</u> loss factor can be modified to be

$$\eta_r = \sum_{j=o} a_j\, \omega^{\alpha(2j-1)} \qquad (10)$$

In composite domain the computaion of damping constants requires high computational effort compared with isotropic one. For simplicity, the series given by equations (7), (9) and (10) have been truncated respectively to the forms;

$$\underset{\sim}{c} = a_o\, \underset{\sim}{m} \qquad (11)$$

$$C_r = a_o\, M_r \qquad (12)$$

$$\eta_r = a_o\, \omega_r^{-\alpha} \qquad (13)$$

In view of equation (13), the quasi hyperbolic relation can be expressed as:

$$\eta\, \omega^{\alpha} = a_o \qquad (14)$$

Here, the damping constant a_o increases as the volume fraction decreases and as the code number leading to a low stiff composite structure.

For the sake of graphical linearization, the previous equation can be transformed into the following logarithmic form :

$$\ln \eta + \alpha \ln \omega = \ln a_o \tag{15}$$

The isotropic state can be considered as a limiting case at which the weight factor reaches its maximum value here as :

$$\alpha^* = \tan \delta = 1$$

consequently equation (15) will be recast as :

$$\ln \eta + \ln \omega = \ln a^* \tag{16}$$

In equation (16) the two limiting isotropic states arise here as :

1. $a^* = a_1$ represents a fully fiber domain at which $v_f = 100\%$
2. $a^* = a_2$ represents a fully matrix domain at which $v_f = 0.0\%$

A set of family curves representing composite domains of various degrees of isotropism in the physical state can be then bounded as follow :

$$o < \alpha < \alpha^*$$
$$a_1 < a_o < a_2$$
$$o < \eta < 1$$

It is obvious that the rate of change (slope) depends on the degree of isotropism at which $\alpha \leq \tan^{-1} 45°$. Also the damping constants for each specimen increase as the volume of fraction decreases.

The validity of orthogonality condition permits the decoupling of natural modes. Consequently, the equation of motion in the r th modes is expressed as :

$$M_r \ddot{Y}_r + C_r \dot{Y}_r + K_r Y_r = F_r \tag{17}$$

The steady state response in the r th mode is then expressed as :

$$Y_r = \left(\frac{F_o}{K_r}\right) . \delta_r . \sin(\Omega t + \alpha) \tag{18}$$

Here the magnification factor in the r th mode is given by :

$$\delta_r = \left[(1 - Z_r^2)^2 + (\eta_r . Z_r)^2\right]^{\frac{1}{2}} \tag{19}$$

where, $Z_r = \Omega / \omega_r$

$\delta_r = 1/\eta_r$ for $\alpha < 1$,

$\delta_r = 1/\eta_r^*$ for $\alpha = 1$,

where, $\eta_r = \eta_r^* \; \omega_r^{1-\alpha}$

4. EXPERIMENTAL MODEL OF VIBRATION DAMPING OF COMPOSITE STRUCTURAL BEAM, FRP

A verifications of the equivalent mathematical model of the damping distribution in relation to the frequency spectrum of the composite structure in the light of the tested data is represented. The frequency response tests were performed on cantilever composite beams made from fiber reinforced plastic FRP by utilizing fast fourier transform dual channel analyzer in conjunction with the computer as shown in Fig. (2).

A typical specimens FRP composite beam of dimension (210 x 20 x 3 mm) made of three plies with 1 mm thickness for each ply is shown in Fig. (1). Two composite level were selected for each code number. These are weakly composite specimen of volume fraction V_f = 15% and the average composite specimen of V_f = 45%.

To study the effect of lamina orientation and stacking sequence on the modal parameters, six code numbers of specimen were fabricated and stated as (0/0/0), (0/30/0), (0/45/0), (0/90/0), (45 / - 45 / 0) and (45 / 0 / 45) for each volume fraction.

Within the frequency range (8000) Hz, the frequency response and half power tests were performed for the measurements of the first four loss factors and the corresponding eigenfrequencies and listed in the third and fourth columns of tables (1) and (2). For the sake of verification, the first four natural frequencies listed in the second column of the tables were computed by modifying the developed formula [7] and the form;

$$\omega_i = \frac{\lambda_i^2}{2\pi L^2}\left(\frac{D^*}{\rho_c \cdot t}\right)^{\frac{1}{2}}$$

where, D^* = Condensed bending stiffness modulus.

With the utilization of the least square technique on the measured values of the loss factors and natural frequencies at the first four natural modes, the quasi rectangular hyperbolic curve fitting are plotted with the confindence level 99.5% as shown in Fig. (3, 4).

To study the effect of the ply orientation on the damping capacity, we have firstly consider the effect of changes of inner ply orientations and secondly the effects of changes of outer ply orientation compared with the changes of inner ones. A glance at Fig. (3) and (4) indicates that the damping capacity increases monotonically, (natural frequencies decrease), with the increase of angle of orientation of the inner layer.

With regard to the experimental results listed in tables (1) and (2), it is obvious that the changes of outer orientations have significant effects on the damping, (and stiffness), of the specimens compared with the changes of the inner orientation. As example the experimental results of the loss factors for (0/0/0), (0/45/0) and (45/0/45) show that the loss factor increases by 16% due to the increase of the inner layer (by 45°), while it increases by 87% and 105% due to the increase of the two outer layers (by 45°) for v_f = 45% and 15% respectively

In Fig. (5, 6) the logarithmic forms of quasi hyperbolic relations for all cases are plotted in linear forms. It is noticed that the slopes assigned by the weights factor are depending mainly on the degree of the isotropical state while the damping constant depends mainly on the flexibility of the specimes.

The interrelations of the weight parameters (α, a_o) and the code numbers for each level of volume fraction are listed in monotonic manner in table (3) and plotted in Fig. (7) and (8). It is found that the weight parameters increase linearly with the increase of ply-orientation of either inner or outer layers.

CONCLUSION

In the present work, the modelling of damping distribution in vibrating composite structures is established with the lowest residual errors. The fitted results of the measured values indicate that :

1. There is an existence of generalized quasi rectangular hyperbolic

relationships between the loss factors and natural frequencies of composite structures and can be found at any selected ranges of frequency spectrum.

2. The developed quasi uniform mass damping matrix by utilizing the proper weight factors permits the utilization of normal coordinate systems for uncoupling equations of motion of composite structure with the high confidence level 99.5%

3. The angle orientations of the outer laminate have significant effects on the modal parameters of the composite compared with the inner laminate.

4. The angle of orientation 45°, at which the shearing parameter reach highest values has the highest influence on the modal parameters compared with the other angle orientations.

5. The obtained values of the weight factors may be considered as indicators of the degree of isotropity of the composite structures.

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Table 1: The numerical and experimental modal parameters of the first four modes of the fixed-free GRP beam of volume fraction 15%

[0/0/0]

Mode no., i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	211.109	188.496	13.334
2	1322.981	1294.336	4.370
3	3704.384	3694.513	2.552
4	7259.115	7250.796	2.080

$\eta_i = 2.248738345\,(\omega_i)^{-0.53724390}$
confidence level = 99.4803%

[0/90/0]

Mode no., i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	208.482	175.929	16.072
2	1306.526	1231.504	5.612
3	3658.309	3568.849	2.728
4	7168.825	6572.212	2.486

$\eta_i = 3.011008077\,(\omega_i)^{-0.5545636963}$
confidence level = 99.0864%

[0/30/0]

Mode no., i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	209.544	185.354	13.898
2	1313.186	1244.071	4.546
3	3676.958	3606.548	2.614
4	7205.362	7200.530	2.094

$\eta_i = 2.463872500\,(\omega_i)^{-0.5463537063}$
confidence level = 99.5838%

[45/-45/0]

Mode no., i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	171.160	163.363	20.000
2	1072.640	1043.009	6.024
3	3003.425	2978.230	3.798
4	5885.510	5811.946	2.648

$\eta_i = 3.56531542\,(\omega_i)^{-0.5694874495}$
confidence level = 99.6619%

[0/45/0]

Mode no., i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	208.936	179.071	14.912
2	1307.506	1237.788	5.076
3	3661.049	3581.416	2.632
4	7174.191	6754.424	2.280

$\eta_i = 2.743609311\,(\omega_i)^{-0.5531980819}$
confidence level = 99.3578%

[45/0/45]

Mode no., i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	123.615	119.381	23.684
2	774.666	766.549	8.196
3	2169.094	2161.416	4.534
4	4250.556	4234.867	3.086

$\eta_i = 3.797399516\,(\omega_i)^{-0.5762650855}$
confidence level = 99.9998%

Table 2: The numerical and experimental modal parameters of the first four modes of the fixed-free GRP beam of volume fraction 45%

[0/0/0]

Mode no.,i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	299.004	295.310	10.106
2	1873.816	1859.823	3.632
3	5246.742	5152.212	2.256
4	10281.522	10040.530	1.564

$\eta_i = 1.973894157(\omega_i)^{-0.5243924613}$
confidence level = 99.9324%

[0/90/0]

Mode no.,i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	294.631	276.460	11.932
2	1846.433	1822.124	4.396
3	5170.069	4913.451	2.430
4	10131.272	9952.566	1.704

$\eta_i = 2.765085107(\omega_i)^{-0.5522194691}$
confidence level = 99.9926%

[0/30/0]

Mode no.,i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	296.372	289.027	10.870
2	1857.310	1847.256	3.996
3	5196.766	4988.849	2.330
4	10190.975	10027.964	1.630

$\eta_i = 2.288586882(\omega_i)^{-0.5362827918}$
confidence level = 99.9956%

[45/-45/0]

Mode no.,i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	230.643	213.628	15.294
2	1445.403	1432.566	5.702
3	4047.176	4021.239	2.812
4	7930.844	7929.380	2.298

$\eta_i = 3.097594465(\omega_i)^{-0.5536083162}$
confidence level = 99.6279%

[0/45/0]

Mode no., i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	294.926	282.743	11.112
2	1848.281	1859.823	4.222
3	5175.234	4951.150	2.348
4	10141.394	10015.397	1.662

$\eta_i = 2.400855191(\omega_i)^{-0.5394284987}$
confidence level = 99.9872%

[45/0/45]

Mode no.,i	Frequency, ω (rad/sec)	Frequency, ω_d (rad/sec)	Damping, η (%)
1	141.529	138.230	20.910
2	886.960	841.947	7.462
3	2483.511	2475.575	3.808
4	4866.691	4863.185	2.842

$\eta_i = 3.591001961(\omega_i)^{-0.5741157503}$
confidence level = 99.8897%

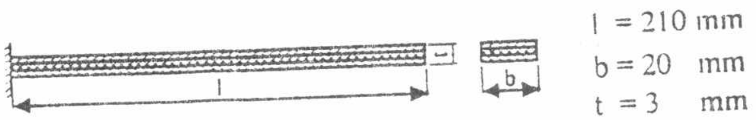
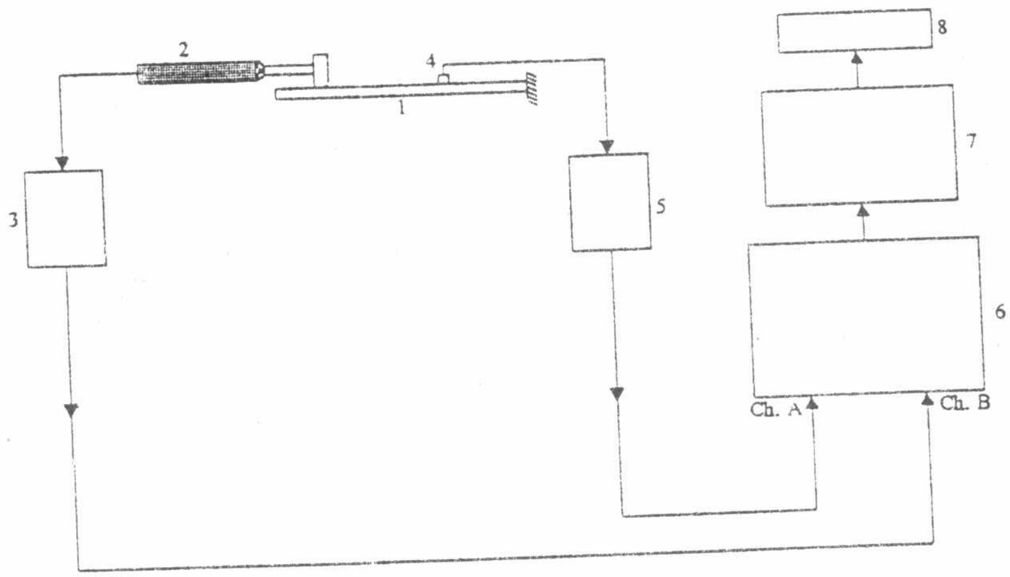


Fig. (1) : 3-layered beam model



1: Beam model
2: Impact hammer with built-in force transducer
3: Conditioning amplifier
4: Piezoelectric accelerometer
5 : Charge amplifier
6 : Dual-channel signal analyzer
7 : Computer
8 : Printer

Fig. (2): Schematic block diagram of the measuring circuit

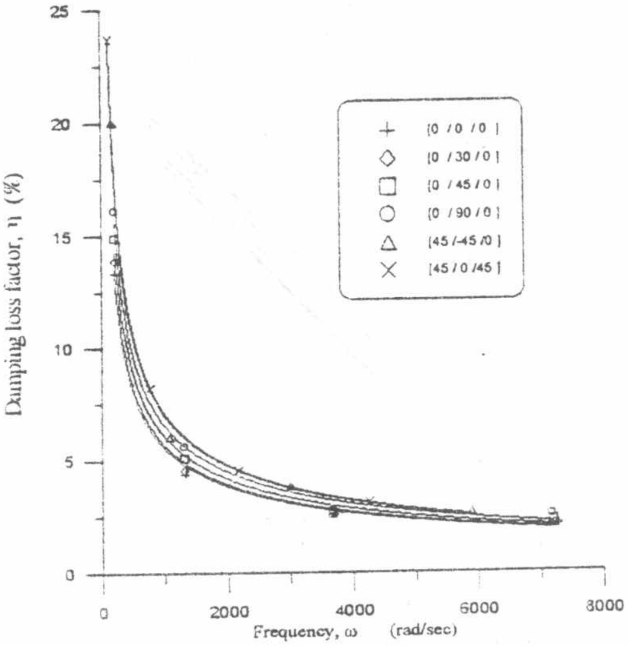


Fig.(3): Quasihyperbolic relation between the damping loss factor, η and the natural frequency, ω for the fixed-free GRP beam of volume fraction 15%

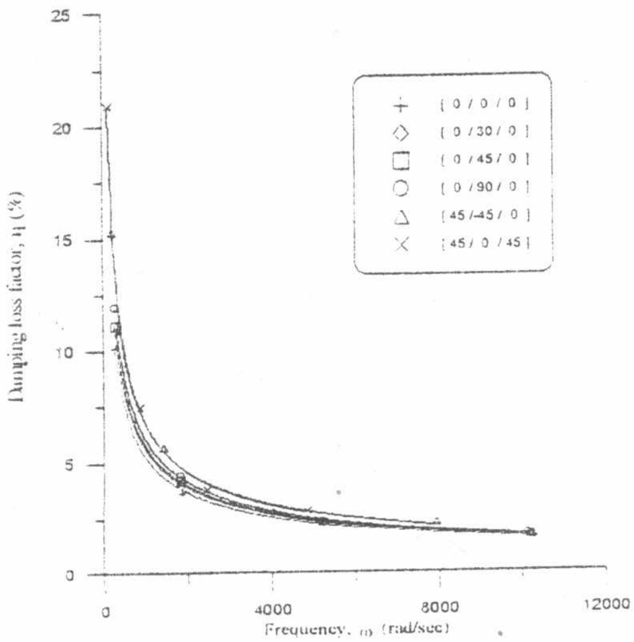


Fig.(4): Quasihyperbolic relation between the damping loss factor, η and the natural frequency, ω for the fixed-free GRP beam of volume fraction 45%.

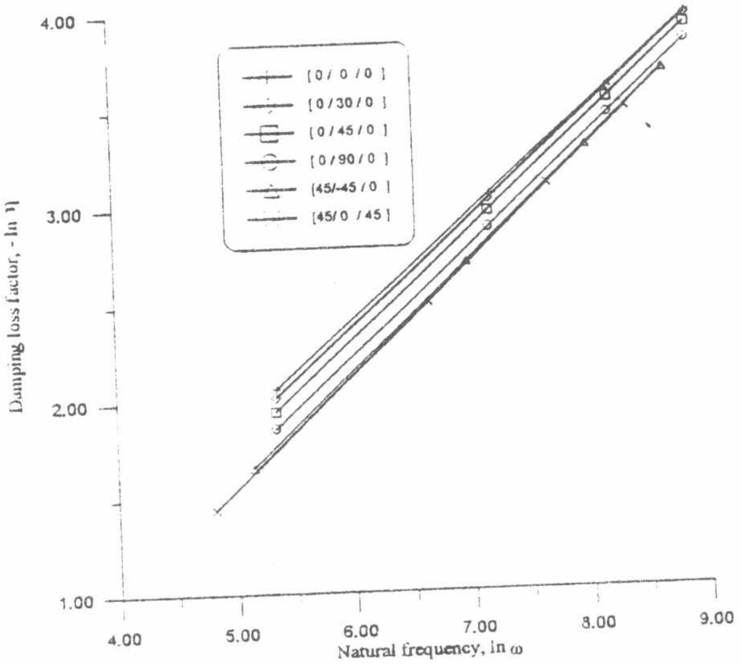


Fig. (5) Logarithmic relation between damping loss factor, η and natural frequency, ω of fixed-free GRP beam of volume fraction 15%.

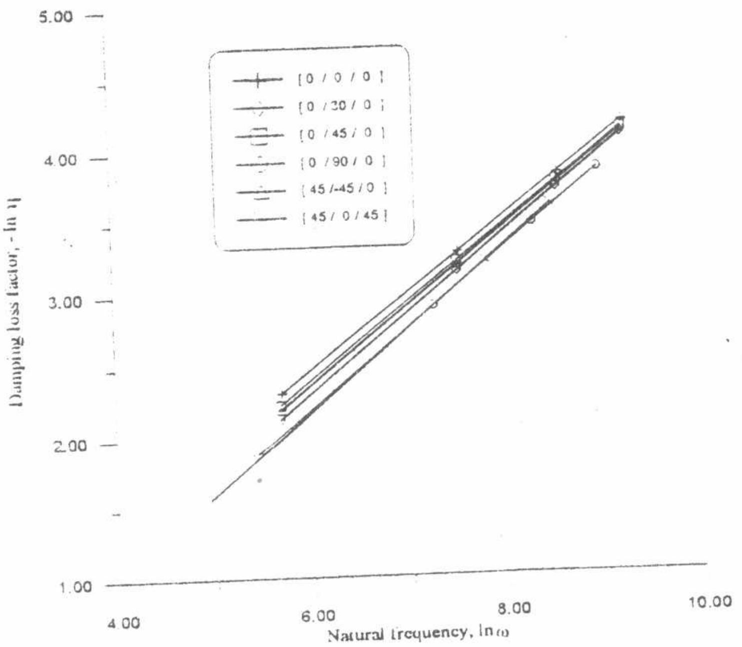


Fig. (6) Logarithmic relation between damping loss factor, η and natural frequency, ω of fixed-free GRP beam of volume fraction 45%.

Table 3 : Damping parameters of the six code numbers of fixed-free GRP beam for the two volume fractions, (V_f)

Code no.		[0/0/0]	[0/30/0]	[0/45/0]	[0/90/0]	[45/-45/0]	[45/0/45]
Vf= 15%	a	2.248738345	2.4638725	2.743609311	3.011008077	3.56531542	3.797399516
	α	0.53724390	0.5463537063	0.55319680819	0.5545636963	0.5694874495	0.5762650855
Vf= 45%	a	1.973894157	2.288586882	2.400855191	2.765085107	3.097594465	3.591001961
	α	0.5243924613	0.5362827918	0.5394284987	0.5522194691	0.5536083162	0.5741157503

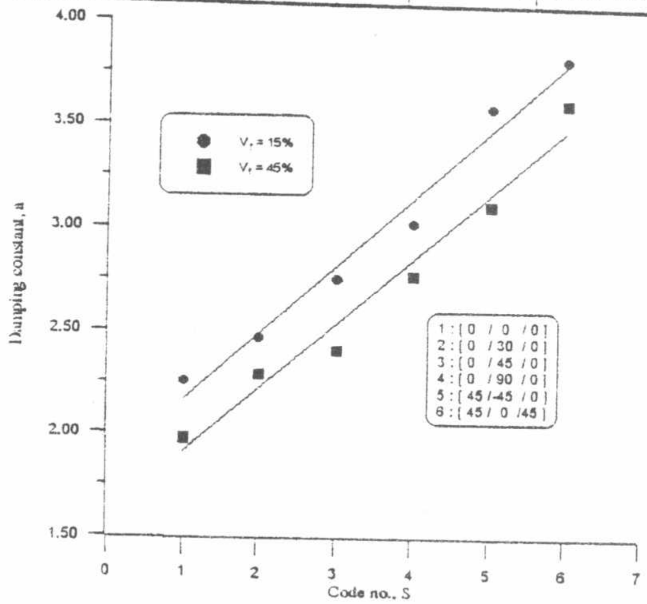


Fig. (7) : Quasilinear relation between the damping constant, a and the code no., S for the fixed-free GRP beam of volume fraction 15% and 45%

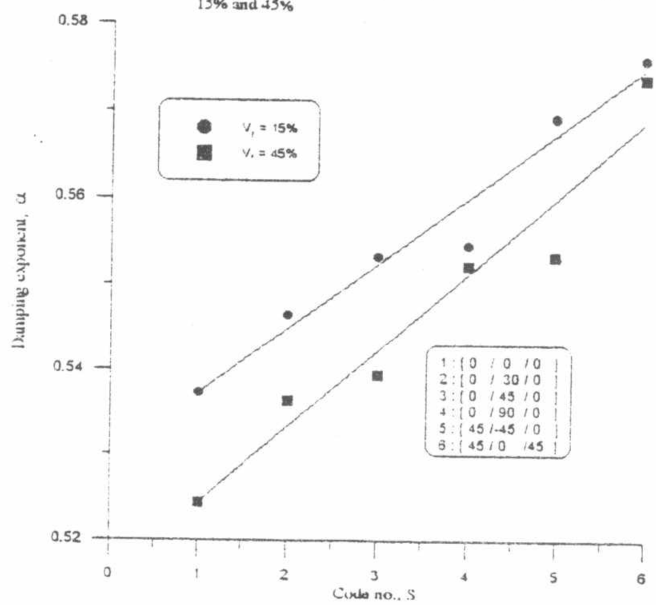


Fig.(8) : Quasilinear relation between the damping exponent, α and the code no., S for the fixed-free GRP beam of volume fraction 15% and 45%.