MILITARY TECHNICAL COLLEGE
CAIRO - EGYPT

7th INTERNATIONAL CONF. ON
AEROSPACE SCIENCES &
AVIATION TECHNOLOGY

Analysis of a 16-ary Quadrature Amplitude Modulation for Land Mobile Satellite Data Communication

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ABSTRACT

The paper is concerned with evaluating the performance of a 16-ary quadrature amplitude modulation (QAM) signaling when transmitted through a land mobile satellite channel. The channel exhibits fading and shadowing as well as additive white Gaussian noise. Fading and shadowing cause both envelope and phase variations of the received signal. The paper derives upper bounds of the probability of error due to each one of these variations. An expression of the irreducible probability of error is also derived. The derived expressions are useful for the design of land mobile satellite data communication systems.

I. Introduction

In a recent paper [1], we have evaluated the probability of error of a quadrature-partial responce signaling when used in a shadowed rician fading land mobile satellite channel. The present paper extends the analysis given in [1] to the case of a 16-ary quadrature amplitude modulation (QAM) signaling. Fig.1 shows a signal constellation of a 16-ary QAM system obtained from four levels on each quadrature channel. Fig.2 shows a basic QAM modulator and demodulator structure along with a representative waveform for 16-ary QAM.

The performance of multilevel QAM operating in linear and nonlinear channels exhibiting Gaussian noise and other interference environments has been investigated by a number of researches[2-4]. However, much work is still required to investigate the performance of QAM in other important channels such as mobile satellite channel where fading and shadowing can affect the signaling performance. This paper provides an analysis of the probability of error of a 16-ary QAM signaling transmitted through a land mobile satellite channel. The paper considers the existence of both line-of sight (LOS) and multipath components of the received signal. Moreover, it considers shadowing by trees. The channel model used in the analysis is the one described in [5] and [6]. This model characterizes the combined effect of fading and shadowing of a land mobile satellite link in a rural environment. Due to shadowing , the LOS component follows a lognormal probability distribution. The multipath component is modeled as a narrow-band Gaussian random process whose envelope has a Rayleigh probability distribution. Analytical expressions for the probability of symbol error of a 16-ary QAM signaling due to envelope and phase variations caused by fading and shadowing will be given.

The paper is organized as follows: Section II is concerned with the analysis of error probability due to envelope variation. Aanalysis of error probability due to phase variations is given in section III. Section IV provides some numerical results. Section V concludes the paper.

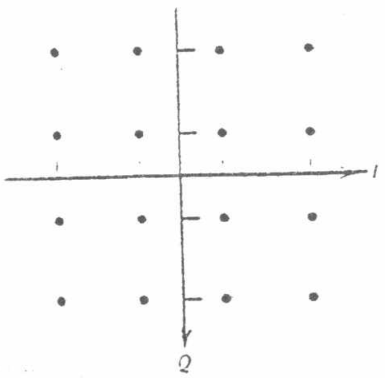

Fig. 1 Signal constellation of 16-ary QAM modulation

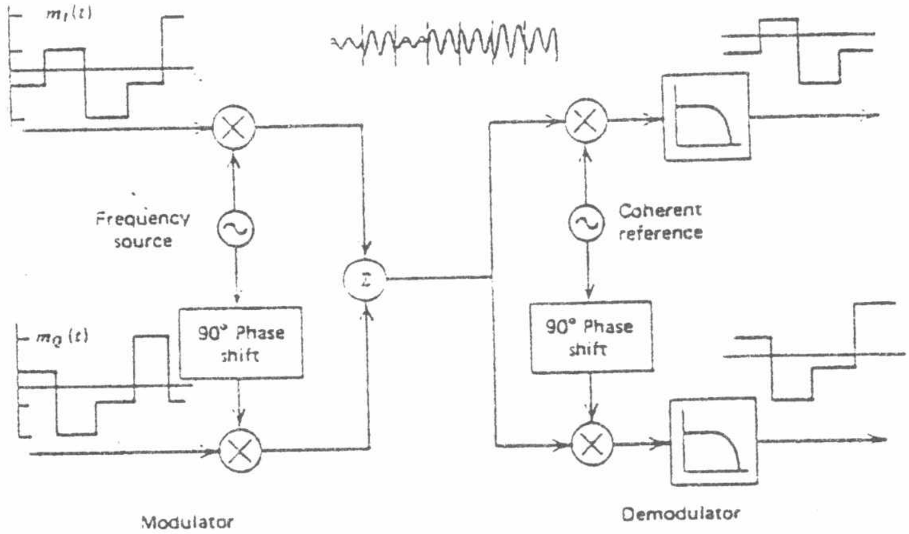

Fig.2 QAM modulator/demodulator

II. Error Probability Due to Envelope Variation

For an additive white Gaussian noise channel , the probability of error of a16-ary QAM signaling has the upper bound given by the following inequality[7]

$$P_e \leq 4\,Q\left(\frac{2r}{\sqrt{5N_o}}\right) \tag{1}$$

Where r is the root mean square amplitude of the received signal, N_o is the noise power in bit-rate bandwidth, and the function Q(x) is defined as:

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp\left(-\frac{u^2}{2}\right)du.$$

It can be shown that the two above equations yield

$$P_e \leq 2\ \exp\left(-\frac{2r^2}{5N_o}\right) \tag{2}$$

In the case of a land - mobile satellite channel, r is random due to random variations of the channel. The probability density of r has been found in[5] and it is given by.

$$P(r) = \frac{r}{(b_o\sqrt{2\pi d_o})}\int_0^{\infty}\frac{1}{z}\exp\ [-(\ln z-\mu)^2/2d_o-(r^2+z^2)/2b_o].I_o(rz/b_o)dz \tag{3}$$

Where $I_o(\ )$ is the modified Bessel function of zeroth order.

b_o represents the average scattered power due to multipath,

μ is the mean value due to shadowing, and

$\sqrt{d_o}$ is the standard deviation due to shadowing.

Now the probability of error due to the envelope variation can be obtained by averaging (2) over P(r). Thus,

$$P_e \leq 2\int_0^{\infty} P(r).\ \exp\left(-\frac{2r^2}{5N_o}\right) dr \tag{4}$$

Substituting the expression of P(r) of (3) into (4) and interchanging the order of integration gives

$$P_e \leq \frac{2}{\sqrt{2\pi d_o}}\int_0^{\infty}\int_0^{\infty}\frac{1}{z}.\exp\left(-\frac{(\ln z-\mu)^2}{2d_o}\right).\exp\left(-\frac{z^2}{2b_o}\right).\frac{r}{b_o}.\exp\left(-\frac{r^2}{2}\left(\frac{1}{b_o}+\frac{4}{5N_o}\right)\right).I_o\left(\frac{rz}{b_o}\right)drdz \tag{5}$$

Let:

$$F_1 = \int_0^{\infty}\frac{r}{b_o}.\exp\left(-\frac{r^2}{2}a^2\right).I_o\left(\frac{rz}{b_o}\right)dr,$$

$$a^2 = \frac{1}{b_o}+\frac{4}{5N_o} \tag{6}$$

Using the substitution **u=ar**, then

$$F_1 = \frac{1}{a^2 b_o}\int_0^{\infty} u.\exp\left(-\frac{u^2}{2}\right).I_o\left(\frac{uz}{ab_o}\right)du \tag{7}$$

Let

$$\gamma = \frac{z^2}{a^2 b_o^{\ 2}} \tag{8}$$

Then, (7) implies that

$$F_1 = \frac{1}{a^2 b_o}.\exp\left(\frac{\gamma}{2}\right)\int_0^\infty u.\exp\left(-\frac{u^2+\gamma}{2}\right).I_o\left(u\sqrt{\gamma}\right)du \tag{9}$$

The integrand in (9) is in the form of a Rice pdf [8]. Consequently,

$$\int_0^\infty u.\exp\left(-\frac{u^2+\gamma}{2}\right).I_o\left(u\sqrt{\gamma}\right)du = 1 \tag{10}$$

Then, (9) and (10) yield

$$F_1 = \frac{1}{a^2 b_o}.\exp\left(\frac{\gamma}{2}\right) \tag{11}$$

After some algebraic manipulation, equations (5),(6) and (11) imply that, the average probability of error of a 16-ary QAM signaling due to the combined effect of shadowing and fading is bounded by

$$P_e \le \frac{1}{\sqrt{2\pi d_o}}\left(\frac{10N_o}{5\ N_o+4b_o}\right)\int_0^\infty \frac{1}{z}.\exp\left(-\frac{(\ln z-\mu)^2}{2d_o}\right).\exp\left(-\frac{2z^2}{(5N_o+4b_o)}\right).dz \tag{12}$$

III. Error Probability Due to Phase Variation

The probability of error of a 16- ary QAM signaling with phase error is given by[7]

$$P_e \le 4Q\left(\frac{2r}{\sqrt{5N_o}}\cos\phi\right) \tag{13}$$

where ϕ is the received signal phase error. Using the relation obtained by Jain and Blachman[10]

$$Q\left(\frac{r}{\sqrt{N_o}}\cos\phi\right) = \frac{1}{4}[1-\frac{2}{\pi}\sum_0^\infty(-1)^n \frac{\Gamma\left(n+\frac{1}{2}\right)\left(-\frac{r}{\sqrt{2N_o}}\right)^{2n+1}}{(2n+1)!}{}_1F_1\left(n+\frac{1}{2},2n+2,-\frac{r^2}{2N_o}\right)\cos(2n+1)\phi\ ] \tag{14}$$

where ${}_1F_1(\ )$ is the confluent hypergeometric series defined in [11].

$\Gamma(\ )$ is the gamma function.

In the case of fading and shadowing, ϕ is random. The probability density function of ϕ has been found to be approximately Gaussian [9]. Thus,

$$P(\phi) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(\phi-m)^2}{2\sigma^2}\right) \tag{15}$$

Where m and σ are the mean and the standard deviation of the signal phase, respectively.

A performance estimate of a 16-ary QAM due to phase variation caused by fading and shadowing can be obtained by averaging (14) over $P(\phi)$. Hence, the probability of error will be

$$P_e \le 1-\left(\frac{2}{\pi}\sum_0^{\infty}(-1)^n \frac{\Gamma\left(n+\frac{1}{2}\right)\left(\frac{2r}{\sqrt{10N_o}}\right)^{2n+1}}{(2n+1)!}\, {}_1F_1\left(n+\frac{1}{2},2n+2,-\frac{1}{\sqrt{5}}\frac{r^2}{N_o}\right).\int_{-\infty}^{\infty}\cos(2n+1)\phi P(\phi)d\phi\right) \quad (16)$$

Rewrite the integration in (16) in complex form as:

$$\int_{-\infty}^{\infty}\cos(2n+1)\phi\; P_\phi(\phi)d\phi = \mathrm{Re}[\int_{-\infty}^{\infty}\exp(jv\phi)P(\phi)d\phi] \quad (17)$$

where **Re** denote the real part and $v = 2n+1$.

The term between square brackets on the RHS of (17) is the characteristic function of $P(\phi)$ [11]. Due to (15), one has

$$\mathrm{Re}[\int_{-\infty}^{\infty}\exp(jv\phi)\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(\phi-m)^2}{2\sigma^2}\right)d\phi] = \mathrm{Re}\left[\exp\left(jvm-\frac{v^2\sigma^2}{2}\right)\right] \quad (18)$$

Assuming that the phase has a zero mean, i.e. $m = 0$, then (16)-(18) yield

$$P_e \le 1-\left(\frac{2}{\pi}\sum_0^{\infty}(-1)^n \frac{\Gamma\left(n+\frac{1}{2}\right)\left(\frac{2r}{\sqrt{10N_o}}\right)^{2n+1}}{(2n+1)!}\, {}_1F_1\left(n+\frac{1}{2},2n+2,-\frac{1}{\sqrt{5}}\frac{r^2}{N_o}\right).\exp\left(-\frac{(2n+1)^2}{2}\sigma^2\right)\right) \quad (19)$$

Equation (19) represents the probability of error for a 16-ary QAM signaling due to phase variation caused by fading and shadowing. For large ,more than 15 dB, values of signal-to-noise, one can approximate the confluent hypergeometric series by the following expression [10].

$${}_1F_1\left(n+\frac{1}{2},2n+2,-\frac{1}{\sqrt{5}}\frac{r^2}{N_o}\right) \cong \frac{\Gamma(2n+2)}{\Gamma\left(n+\frac{3}{2}\right)}\left(\frac{1}{\sqrt{5}}\frac{r^2}{N_o}\right)^{-\left(n+\frac{1}{2}\right)} \quad (20)$$

After some algebraic manipulation, (19) and (20) imply that

$$P_e \le 1-\frac{4}{\pi}\left(\sum_0^{\infty}\frac{(-1)^n}{(2n+1)}\exp\left(-\frac{(2n+1)^2}{2}\sigma^2\right)\right) \quad (21)$$

Immediately, one notices that (21) is independent of signal-to-noise ratio. This independence is due to the fact that for large signal-to-noise ratio, the effect of phase error dominates the effect of noise. Inquality (21) represents the irreducible probability of error of a 16-ary QAM signal due to phase variation caused by fading and shadowing. This is the bottom effect of phase variation which cannot be down-crossed whatever large is the available SNR. Finally, it is worth mentioning that the effects of envelope variation and phase variation have been evaluated individually in this paper. The probability of symbol error due to the combined effect of envelope and phase variations is bounded from above by the sum of the individual error probabilities provided by (12) and (19).

IV. Numerical Results

The results are obtained by numerically evaluating the probability of error expressions derived in the previous sections. Table I shows the channel model parameter examples used in the numerical calculation. Results for the error probability due to envelope variation are obtained using equation (12). The integral in (12) has been evaluated numerically using Gaussian quadrature integration.

Table I. Channel Model Parameters

	Light shadowing and fading	Average shadowing and fading	Heavy shadowing and fading
b_0	0.158	0.126	0.0631
μ	0.115	-0.115	-3.91
$\sqrt{d_0}$	0.115	0.161	0.806
σ	0.36	0.45	0.52

Fig.3 shows the probability of error versus SNR for the cases of light, average, and heavy shadowing and fading. The results show that for the case of heavy shadowing and fading, a very high SNR is required to compensate for the effect of fading and shadowing. For example, at probability of 10^{-3}, a margin of 34 dB is required. Also, for the same probability of error, margins of 14 and 19 dB are required to compensate for light and average shadowing and fading, respectively.

A 50-term approximation of the series in (19) was used to obtain numerical values of the probability of error due to phase variation. Also, a 1500-term approximation of the series in (21) was used to obtain results of the irreducible probability of error. The confluent hypergeometric series, ${}_1F_1(.)$, was evaluated using a recurrence relation [10] and in terms of the modified Bessel functions of zero and first orders. As described by Jain and Blachman [9], in their evaluation of the confluent hypergeometric series for low probability of error or high SNR, terms not only need to be more accurate , but many terms are necessary for high SNR.

Fig.4 shows the probability of error versus SNR of a 16-ary QAM signal due to phase error caused by fading and shadowing. It was found that a 50-term approximation of (19) is adequate for the case of heavy shadowing and is reasonably accurate for the cases of light and average shadowing. This conclusion was arrived at by observing the differences in results between the irreducible probability of error calculation which is considered more accurate (with a 1500-term approximation) and the calculation of a 50-term approximation.

Fig.5 shows the probability of error of a 16-ary QAM signal at σ ranging from σ = 0.1 to 0.9 radian where σ is the rms phase error. The results show, for practical systems, that the irreducible probability of error is very small when the rms phase error σ is less than 0.3 radian. For larger values of σ, the irreducible probability of error increases rapidly as shown in Fig. 5.

V. Conclusions

The symbol error probability of a 16-ary QAM signaling has been evaluated for a land mobile satellite link in a rural environment. The presence of fading, shadowing, and additive noise has been considered. The analysis includes both envelope and phase variations caused by fading and shadowing. The irreducible error probability due to channel phase error has also been obtained. The results show that at a 10^{-3} probability of error, very high signal-to-noise ratio margins of 14, 19, and 34 dB are required to compensate for light, average, and heavy shadowing respectively when considering envelope variation only. Also, the results show that the irreducible probability of error is less than 10^{-3} when the rms phase error is less than 0.3 radian. For larger phase errors, the probability of symbol error increases rapidly and dominates the performance of the a 16-ary QAM.

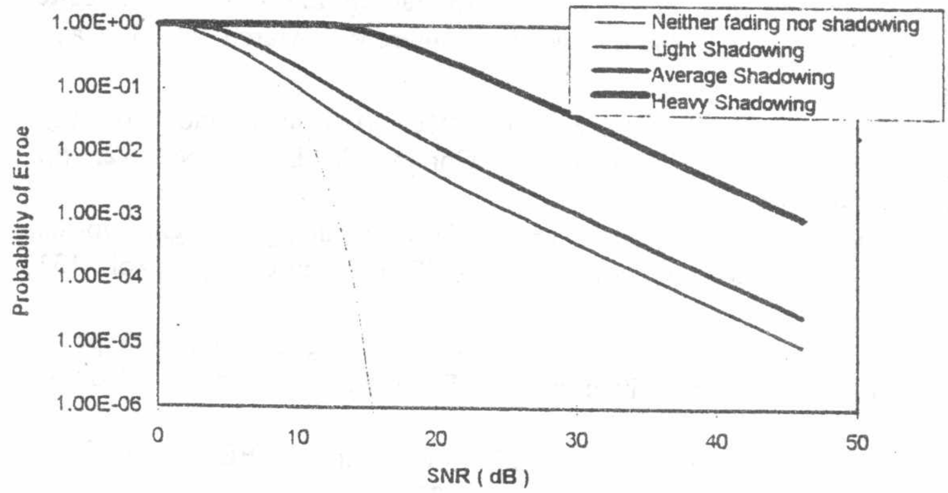

Fig.3 Probability of Error of a 16-ary QAM due to envelope variation.

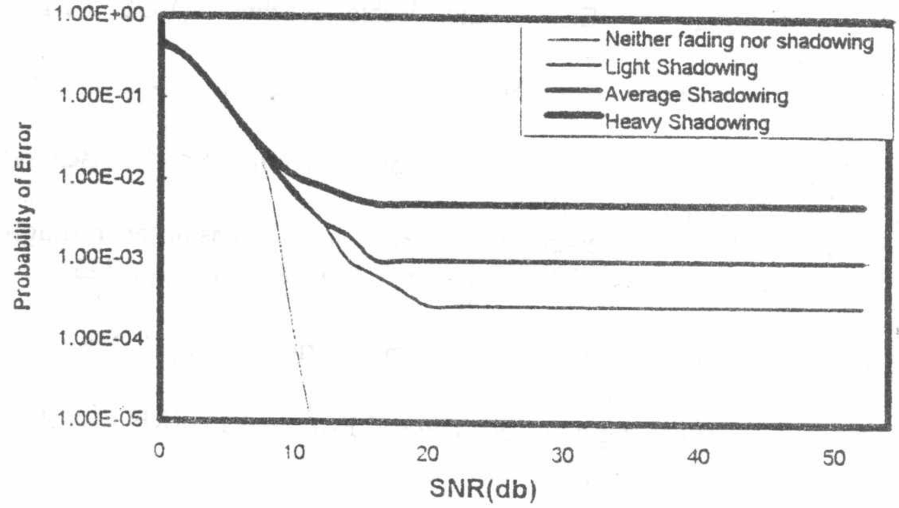

Fig.4 Probability of Error of a 16-ary QAM due to phase error caused by Light, Average, and Heavy Shadowing

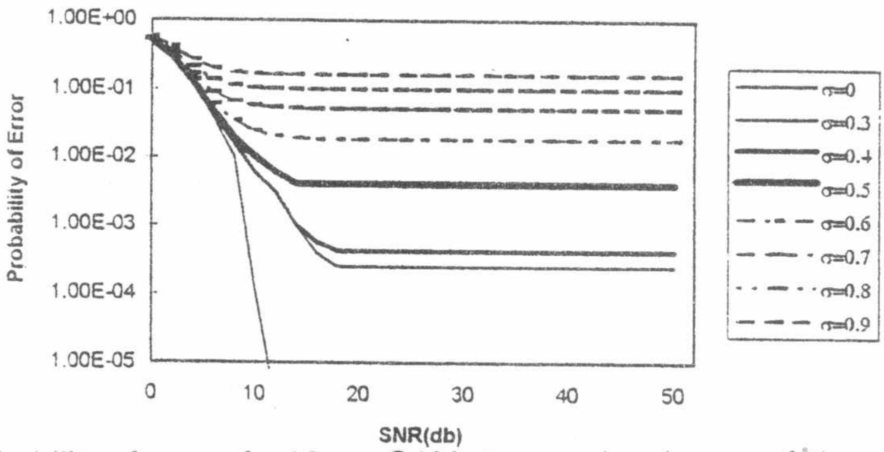

Fig.5 Probability of error of a 16-ary QAM at several variances of the phase error

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