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COMMAND GUIDANCE SYSTEMS SIMULATION : AIRFRAME ANALYSIS

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ABSTRACT

A complete six-degrees of freedom flight simulation model for anti-aircraft command guided missile system is developed. A computer code that solves the model is constructed with BORLANDC. Modular concept is considered in the code development. The non-linear differential equations that describe the model are solved by Runge-kutta 4 method. The integration step is chosen small enough that the numerical errors are negligible. The aerodynamic non-linear coefficients that describe the missile airframe are calculated by the aid of standard NASA curves. The missile is roll angle stabilized throughout the flight. It is controlled in the lateral planes via two pairs of rear control fins. The pitch and yaw control channels are identical except for constant gravity bias added to the pitch channel. The missile airframe is deeply investigated. The step response of the airframe to unit step fin deflection is obtained. The results indicate that the airframe can be accurately represented by a second order lag system. The weathercock natural frequency and the damping coefficient are obtained by transforming the time-domain data to the frequency domain by the Fast Fourier Transform. The obtained results show that the missile airframe is heavily underdamped. As well, the airframe bandwidth increases by increasing the missile speed. Doubling the missile speed fairly doubles the weathercock frequency at the expense of aerodynamic damping coefficient reduction.

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I. INTRODUCTION

Simulation is a practical tool for system evaluation, synthesis, optimization and modification to meet desired performance criteria. Generally, the designers of the guided weapons carry out accurate laboratory simulation for the system involved before any real experimentation. Past experience showed that accurate simulation would save large amount of the system budget. Hardware simulation, software simulation, and hybrid combinations of both are usually adopted in the system evaluation. Probably, the primary simulation model for the guided system is the mathematical model. A mathematical model, in its simplest, and most common form, is a set of differential equations whose inputs correspond to the physical stimuli given to the system and whose outputs describe the behavior of the system [1].

The missile system under consideration is guided from a ground (or ship) based guidance system linked with a guidance radar that provides the coordinates of the engaged target and the launched missile. The generated guidance commands are sent to the missile through wireless data link. The simplified block diagram of the guidance system is shown in Fig. 1. Although more than one guidance method can be used, only the line-of-sight (LOS) guidance law is adopted in the presented work and is given by

$$\varepsilon_m = \varepsilon_t ,$$

$$\text{and} \quad \beta_m = \beta_t , \tag{1}$$

where ε_m, β_m, ε_t, and β_t are shown in Fig. 2. Thus, the method of control is based on an estimation of the lateral displacement of the missile from the target LOS which is given by [2]

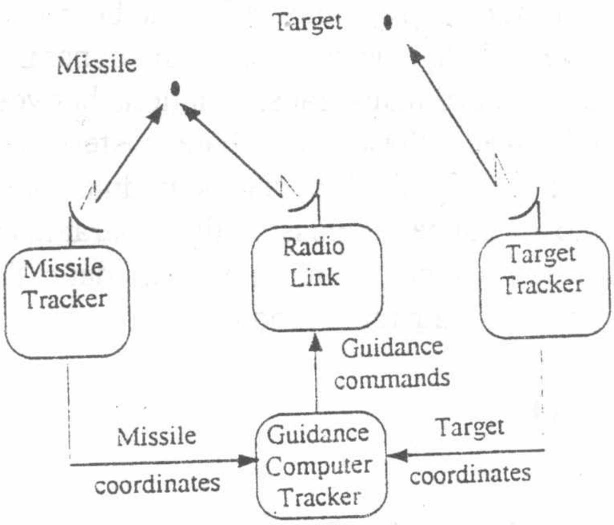
$$h_\varepsilon = r_m (\varepsilon_t - \varepsilon_m),$$

$$\text{and} \quad h_\beta = r_m (\beta_t - \beta_m) \cos\varepsilon_m , \tag{2}$$

where r_m and r_t are the missile and target ranges as shown in Fig. 2.

The missile motor is of boost-sustain type. The booster motor is contained in the first stage that is disposed at the end of the boost phase. The missile is aerodynamically controlled via two pairs of rear control fins. It has two identical control channels, each channel has lateral acceleration autopilot loop that controls the missile lateral acceleration to be very close to that issued by the guidance computer. The autopilot consists of a pneumatic fin servo, one accelerometer, one rate gyro, and the conditioning electronic circuits. In addition to that, a roll position control loop is utilized to keep the missile attitude fixed throughout the flight. At the end of the guidance phase, a radio proximity fuse is initiated to determine the optimum detonation instant of the warhead.

In this paper, the mathematical model that simulates the six-degrees-of-freedom motion of the prescribed system is presented. The operation of the roll loop and the proximity fuse system are not considered. A computer code is developed to solve the model. The code is broken down into modules as shown in Fig. 3. Each module simulates a physically existing subsystem in the guidance and control loop [3].



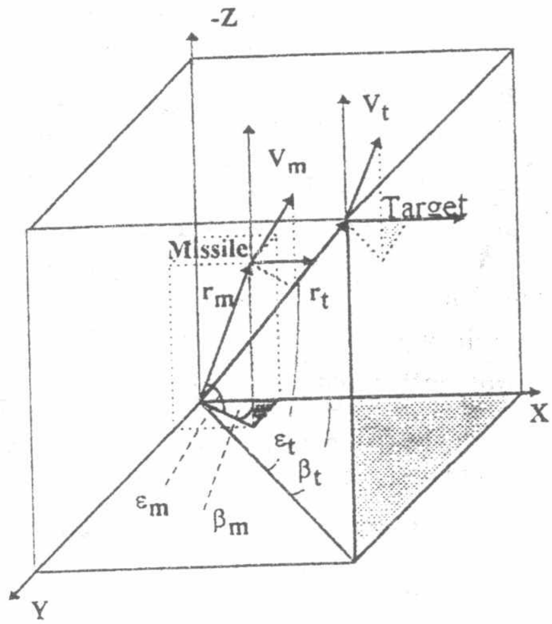


Fig. 1. Simplified block diagram of the guidance system. Fig. 2. Geometry of command guidance system.

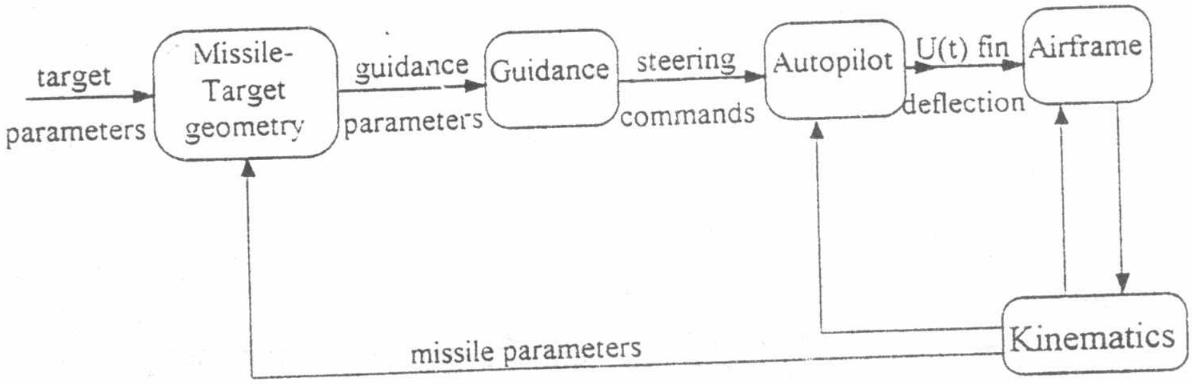


Fig. 3. Simplified block diagram of the main simulation model modules.

In the missile-target geometry module, the missile position relative to the target and the guidance parameters are calculated. The guidance parameters are the deviation errors between the ideal (theoretical) and actual position of the missile measured by guidance radar. The guidance module operates on the guidance parameters to generate the guidance steering command signals. The guidance module includes the compensators and limiters for the various error signals that contribute to guidance commands signals and the weighted summators that sums up these signals to produce missile commands. The guidance command signals are supplied to the autopilot to steer the missile in space.

In the airframe module various forces and moments are calculated. They involve aerodynamic, weight, thrust, and control forces and moments. The aerodynamics forces and moments are calculated in the velocity coordinate system. However the thrust and weight forces are computed in the board and reference coordinate systems; respectively. Thus, the solution of the dynamical problem necessitates a reliable means for coordinates transformations between these systems. The transformation between these coordinate systems are achieved by the Euler's angles method. Finally, the kinematics module solves the forces and moments equations to produce the missile flight parameters which are the instantaneous acceleration, velocity, and position data. The flight path variables are, then derived from the airframe module.

II. MISSILE EQUATIONS OF MOTION

The Euler's force and moment equations can be written as [4], [5] :-

$$\begin{aligned}
&T_x - F_x - mg\sin\theta = m\left(\dot{U} + qw - rv\right),\\
&F_y + mg\cos\theta\sin\varphi = m\left(\dot{v} - pw + rU\right),\\
&F_z + mg\cos\theta\cos\varphi = m\left(\dot{w} - qU + pv\right),\\
&L = A\dot{p} + qr(C - B),\\
&M = B\dot{q} + pr(A - C), \quad \text{and}\\
&N = C\dot{r} + pq(B - A).
\end{aligned} \tag{3}$$

Where T_x is main thrust component. A, B, and C represent the missile moments of inertia. p, q, and r are the angular rates. U, v, and w are the velocity components along the missile axes as shown in Fig. 4. θ and φ are the

Euler's angles. F_x, F_y, F_z, L, M, and N are the aerodynamic forces and moments and are given by :-

$$F_{\begin{cases}x\\y\\z\end{cases}} = c_{\begin{cases}x\\y\\z\end{cases}} sQ, \text{ and } (L, M, N) = c_{\begin{cases}l\\m\\n\end{cases}} sQ\ell . \quad (4)$$

Where s is the reference cross-sectional area and is given by $\pi d^2 / 4$, with d being the missile reference diameter. Q is the dynamic pressure of atmosphere and is given by $\frac{1}{2}\rho v_m^2$, where ρ is the air density and v_m is the total missile velocity. ℓ is the missile length. c_x, c_y, c_z, and c_l, c_m, c_n are coefficients that describe the missile airframe. They are given by

$$\begin{aligned}
c_x &= c_{xo} + c_x^{\alpha}\alpha^2 , \\
c_y &= c_y^{\beta}\beta + c_y^{\delta}\delta_r , \\
c_z &= c_z^{\alpha}\alpha + c_z^{\delta}\delta_e , \\
c_l &= c_{lp} p , \\
c_m &= c_{m\alpha}\alpha + c_{m\delta}\delta + c_{mq} q , \text{ and} \\
c_n &= c_{n\beta}\beta + c_{n\delta}\delta + c_{mr} r .
\end{aligned} \quad (5)$$

These aerodynamic force and moment coefficients are calculated and stored aperiori as functions of the missile velocity [6].

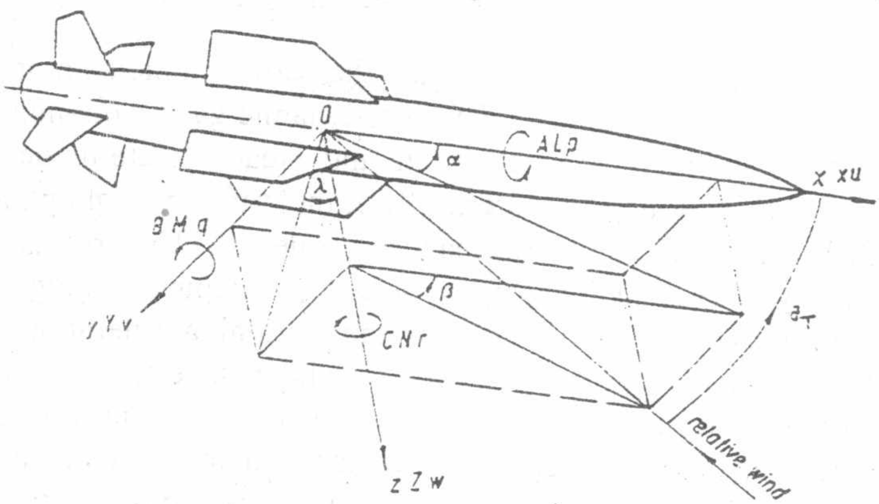


Fig. 4. Force, Moment, etc, Conventions.

III. ANALYSIS OF MISSILE AIRFRAME

The previously described model is constructed with BORLANDC and Runge-kutta 4 method [7] is used to solve the system of differential equations. The numerical integration step is chosen small enough that the numerical errors are negligible. Although the present simulation model is used for the missile airframe analysis only, it can be used for evaluating the missile system performance in the desired threats. It should be noted that the results of this analysis are the corner stone in the design of the proper autopilot for particular mission fulfillment. Thus, the airframe of the missile is excited by forcing its elevons to have 1° deflection at a specified instant t_0 as

$$\delta_e(t) = 1.U(t - t_o). \ [°] \tag{6}$$

The instant t_0 is chosen to be at the beginning of the guidance phase. Since the intenation is to evaluate the aerodynamic transfer function of the missile airframe, the velocity of sound and air density are kept constants, and the ground and thrust forces are omitted. The various flight parameters are recorded versus time for different longitudinal velocity values. It has to be noted that in the present analysis, the autopilot and guidance loops are no longer closed.

The normal acceleration, the incidence angle, and the body rate in the pitch plane are displayed versus time for different velocity values given by 2M, 3M, 4M, and 5M; respectively. The results are shown in Fig. 5. a, b, c.

Inspection of the figures reveals that the aerodynamic gain of the airframe is proportional to the velocity. The aerodynamic gain is defined by the ratio of the steady state normal acceleration and steady state fin deflection [4]. As expected, the fast.fourier transform [8] of the different flight parameters shows that they all have same resonance frequency. This resonance frequency increases by increasing the airframe velocity as depicted in Figs. 6. a, and 6. b, where the magnitude and phase of the normal acceleration and body rate spectra are shown. Thus, the ability of the missile to fulfill missions against highly maneuvering targets is greatly enhanced by increasing its velocity. Comparison of the phase spectra of the normal acceleration and the body rate as shown in Fig. 7 shows that the body rate phase advances that of the normal acceleration. This observation is consistent with the physical nature of the guidance process where the fin deflection generates an unbalanced moment that forces the missile to rotate around its C.G.. In turn, large incidence angle is developed and the normal force is generated.

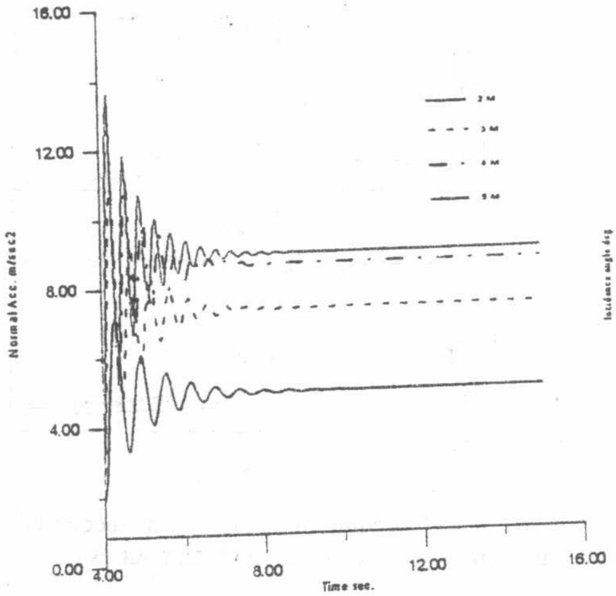


Fig. 5. a. The normal acceleration response due to unit step fin deflection for various mach numbers as given in the figure inset.

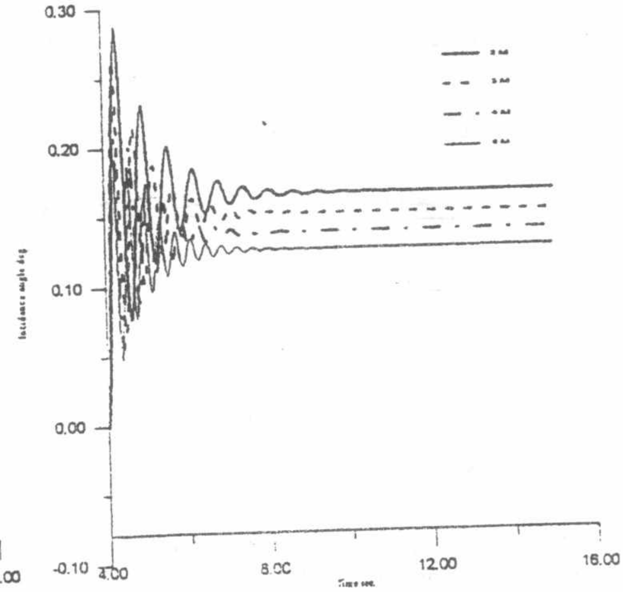


Fig. 5. b. Body rate response due to unit step fin deflection for various mach numbers as given in the figure inset.

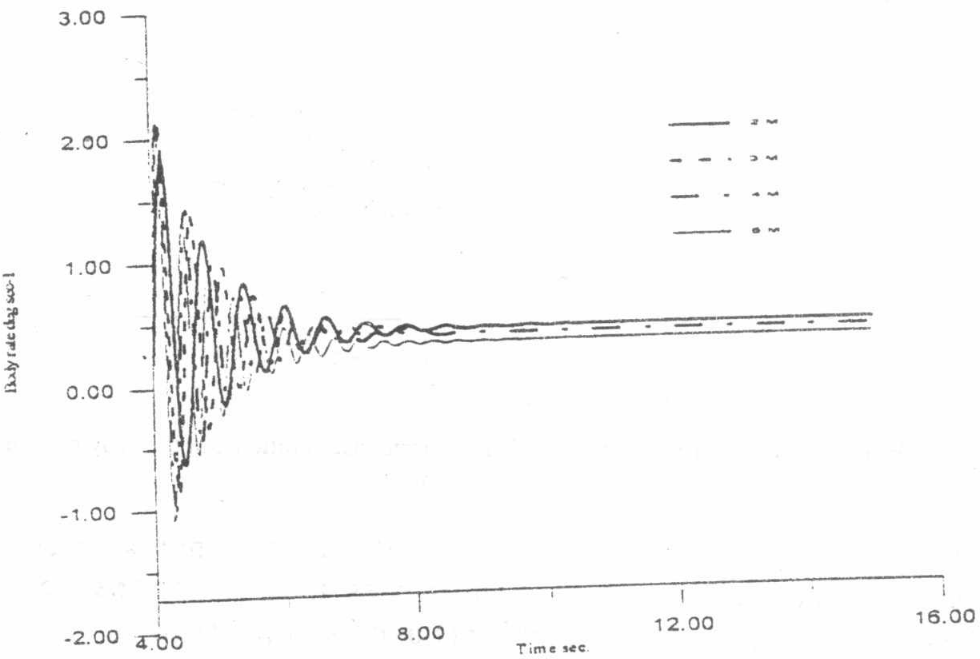


Fig. 5. c. Incidence angle response due to unit step fin deflection for various mach numbers as given in the figure inset.

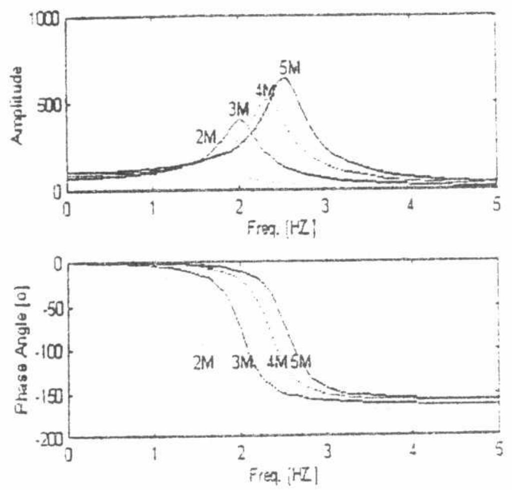


Fig. 6. a. The amplitude and phase spectra of the airframe normal acceleration for various mach number.

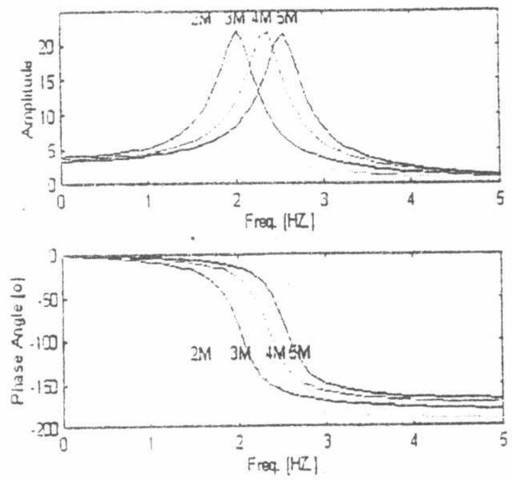


Fig. 6. b. The amplitude and phase spectra of the airframe body rate for various mach number.

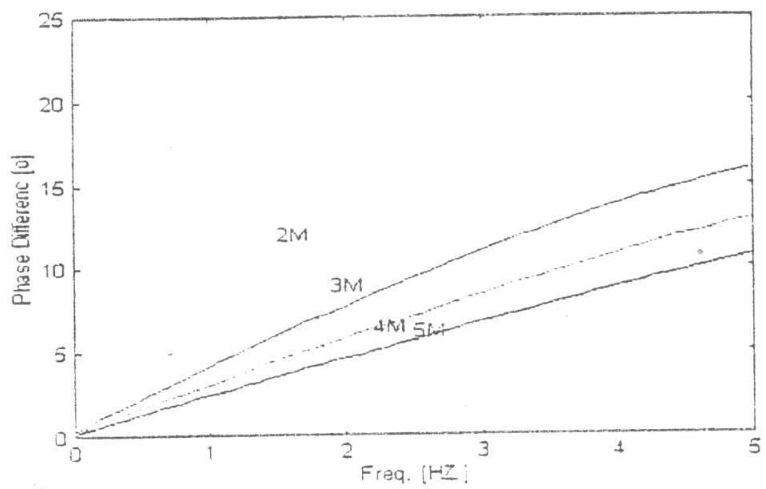


Fig. 7. The phase difference between the airframe normal acceleration and the body rate for various mach number.

Thus, the body rate phase should exceed that of the normal acceleration. The block diagram shown in Fig. 8 illustrates the relation between the normal acceleration developed on the airframe and its turn rate [4].

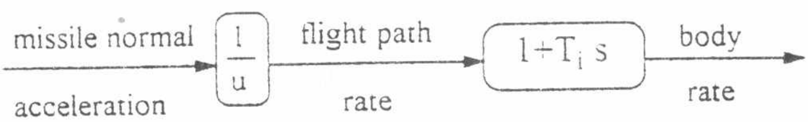


Fig. 8. Relation between body rate and normal acceleration.

The time constant that appears in Fig. 8 represents the time delay between the development of the body rate and the appearance of missile velocity turn rate. It can be written as :

$$T_i = \frac{\tan(\Delta\Phi)}{2\pi f}, \tag{7}$$

where $\Delta\Phi$ is the phase difference between the phase angle of the normal acceleration and that of the body rate at a specific operating frequency f.

In view of Fig. 8 and equation (7), T_i is calculated at different frequencies for various mach numbers as shown in Table 1. It is clear that $\Delta\Phi$ is approximately linear proportional to f which is consistent with equation (7).

It is clear that the time constant decreases as the missile speed increases. Hence, the missile response speed to maneuver is further enhanced as the mach number increases.

In the light of the results shown in Fig. 6. b, and by invoking the transient analysis of the second order system [9], the variation of the airframe natural frequency and the damping coefficients versus speed are shown in Table 2.

The increasing of the airframe maneuverability as velocity increases is apparent. However, the damping coefficient of the airframe decreases by increasing its velocity. This reduces the static stability of the missile and necessitate the presence of an autopilot to insure dynamic stability throughout the entire flight time.

Table 1. The time constant calculations for different mach numbers and different operating frequencies

Missile speed	$T_i * 10^{-4}$		
(Mach)	f =2 HZ.	f = 3 HZ.	f = 4 HZ.
2	2.74267	2.46782	2.18168
3	1.88385	1.75457	1.64838
4	1.41288	1.36672	1.30207
5	1.10814	1.07121	1.08044

Table 2. The undamped weathercock frequency and damping coefficient calculations for different mach numbers

Missile speed (Mach)	Open loop	
	Undamped weathercock frequency rad/sec	Damping ratio
2	9.95	0.16170
3	12.61	0.07787
4	14.95	0.06809
5	16.00	0.05359

CONCLUSION AND FUTURE WORK

Six degrees of freedom simulation model of command guidance system is developed. The missile airframe is analyzed. The weathercock undamped frequency and damping coefficient of the airframe are obtained from the airframe step response in the time domain. Also, the airframe time constant which corresponds to the delay between the swinging of the airframe around its C.G. and the start of its turn in space is computed. The calculations are made at different mach numbers given by 2, 3, 4, and 5 Mach.

It is found that the increase in the mach number of the airframe increases the weathercock frequency and decreases the damping coefficient. The calculated airframe incidence lag time shows that it decreases by the increase of the mach number. The obtained results indicates that the airframe bandwidth can be fairly doubled by doubling the mach number. This finding means that the tactical capabilities of the missile can by enhanced significantly by increasing its speed. Thus, the kinds of missions of this type of missiles can be extended to cover new harder tactical scenarios. The autopilot which is not discussed in this paper should be perfectly suited for each scenario. One of the interesting tactical situations that needs more investigation is engagement of hypersonic ballistic missiles. The method of increasing the missile velocity and the performance of the missile tracking system which is part of the parent guidance radar at these higher velocities are beyond the paper scope. These points represent future research areas that need deep investigation.

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