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The Quantum Entropy for Dilute Two Component Relativistic Plasma

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ABSTRACT
Entropy is one of the most important properties of thermodynamic functions to determine the nature of the changes that occur in the system. The aim of this paper is an attempt to identify the quantum entropy of dilute plasma model that contains two components, electrons, and ions. We calculated the quantum thermodynamic entropy in terms of the non-extensive parameter q which describes the degree of non-extensively in our model. The quantum entropy is directly proportional to increasing velocities and temperature in the velocity of particle interval (0,0.1c) for dilute relativistic plasma, where c is the speed of light. However, at sufficiently high speeds, this approximation is no longer valid. Also, it is having oscillatory behaviour with increasing velocities for dilute relativistic two component plasma model which evaluated by using the q-non extensive parameter.

1. Introduction

Entropy is considered one of the most important thermodynamic properties which is used to describe systems that contain a large number of particles. There are various methods for finding entropy such as Shannon Entropy [1] or Rényi entropy (RE) [2] in which the particle's states can be described by fair probabilities.

The Gibbs entropy of a classical macroscopic system can be defined as a function of the probability distribution over the phase space for an ensemble. Whereas Boltzmann entropy defined for an individual system also it is function on phase space. The essence of RE is clear in terms of different coding theorems since it has a firm operational meaning [3,4].The non-condensing entropy was proposed by Tsallis in 1988 and it was called q-entropy [5]. Also, the non-extensive statistics were based on the q-entropy and becoming a useful approach to the statistical property for many of various complex systems.

In fact, Kaniadakis entropy decreases with a projective measure. This characteristic is directly associated with the concavity of the entropic form [6-8].

In probability distributions, the calculating of entropy can be interpreted not only as a measure of uncertainty but also as a measure of information [4]. Also, the amount of information we get when we observe the outcome of an experiment (depending on chance) can be considered equal to the amount of uncertainty regarding the outcome of the experiment before it is performed [5].

In statistical mechanics the classical particle's states are described by the probability-distribution function f (r, v) on the phase space where r and v are, respectively, the particle's position and velocity [5].

Davis and Peebles (1977) applied the apparatus of the hierarchy equations (BBGKY) Born-Bogoliubov-Green-Kirkwood-Yvon to compute approximate similarity solutions for the evolution of two-point and three-point correlation functions in an expanding gravitating universe [9].

Lebowitz (2000) gave an overview for statistical mechanics and defined the entropy for nonequilibrium macroscopic system [10]. Boltzmann's great achievement is arriving at an understanding of what entropy means and why it almost never decreasing and must always tend to increase [11].

Homogeneous relativistic plasma is characterized by two important parameters: the density of particles ρ and the absolute temperature T (if there are several kinds of particles it is necessary to know their concentrations) [12]. There are three different energies associated with relativistic plasmas The first is called the rest mass per particle mc^2 and the second is known as Kinetic energy which is of order kT and the third is called Coulomb energy of order $e^2\rho^{1/3}$ per particle. The ratios of these energies give us the two important dimensionless parameters of the plasma; one of them is known as the dilution constant of the plasma ε_d and the other is the thermal coefficient of the plasma μ.

The present study is valid for relativistic dilute plasma in equilibrium which have ε_d<<1 and (μ>>1), i.e., the plasma can be slightly relativistic.

There are inequalities for entropies associated with probability distributions of the position and velocity for slightly relativistic plasma [13]. Recently, [14] new entropic uncertainty relations which based on properties of Reni [12] entropy was found. New inequalities for tomographic probability distributions obtained by using the entropic uncertainty relations for continuous variables [15].

Jiulin (2004) suggested that the Tsallis statistics could be statistics suitable for describing the nonequilibrium systems with inhomogeneous temperature and long-range interactions [16].

2. Definition of Entropy and BBGKY Hierarchy in Statistical Mechanics

Defining the reduced s-particle distribution functions by of order s < N,

$$F^{(s)}(t_A, r_A, u_A) = \int F^N \prod_{R=s+1}^{N} d^3 r_R d^3 u_R, \qquad (1)$$

The relativistic BBGKY hierarchy deals with s-particle distribution functions in phase space. And it was defined by Lapiedra and Santos [١٠] as:

$$\begin{aligned} & u_A^\alpha \frac{\partial F^{(s)}}{\partial r_A^\alpha} + \sum_B \zeta_{AB}^\alpha \frac{\partial F^{(s)}}{\partial u_A^\alpha} \\ & + \sum_R \int \zeta_{AB}^\alpha \frac{\partial F^{(s+1)}}{\partial u_A^\alpha} \prod_{R=s+1}^{N} d^3 r_R d^3 u_R = 0, \quad A \neq B \end{aligned} \qquad (2)$$

Here ξ_{AB} is the acceleration of the particle A in the presence of the particle B .

We note that in (BBGKY) hierarchy, the n^{th} correlation function relates with the (n+1) correlation function and assumes a closure relation that connects them.

In quantum information theory, quantum relativistic entropy is a measure of distinguishing between two quantum states. It is the quantum mechanical analogue of relativistic entropy.

We propose a new quantum generalization of the Rényi entropy family containing the von Neumann entropy. In 1960, Rényi [13] defined a generalization of Shannon entropy which depends on a parameter. The entropy of a quantum state was defined by von Neumann for a state α as:

$$S_\alpha = -tr\alpha \log \alpha \tag{3}$$

If p_i is a probability distribution on a finite set, its Rényi entropy of order q is defined to be

$$S_q = \frac{1}{1-q} \ln \sum p_i^q \tag{4}$$

where $0 < q < \infty$ and $q \neq 1$, then for determining the quantum entropy alternatively to the distribution function of phase-space f (r, v) we can use:

$$S_q = \frac{1}{1-q} \ln \int (f(r,v))^q \, dr dv \tag{5}$$

In 1988, the Tsallis entropy was defined by Constantino Tsallis as a generalization of the standard Boltzmann–Gibbs entropy [17-18]. The q Tsallis entopy can be written by

$$S_{12}^T = \frac{k_B}{1-q} \int (f^q - f) d^3x d^3v \tag{6}$$

Where the q-nonextensive parameter or degree of non-extensivity is the relation between temperature gradient and potential energy of the system. Therefore, the deviation of q-parameter from unity indicates the degree of inhomogeneity of temperature or deviation from equilibrium.

Hashemzadeh (2015) defined the q non extensive velocity distribution function as the one particle distribution function is obtained by [19]

$$F_1^{qu}(r,v) = \frac{N}{V}\left[\frac{m}{2\pi k_B T}\right]^{\frac{3}{2}} v^2 \left[1-(1-q)\frac{mv^2}{2k_B T} - e\phi(r)\right]^{\frac{1}{1-q}} \qquad (7)$$

where v is the macroscopic entirety-moving velocity of the system, N is the number density of particles, T is the temperature and the potential field $\phi(r)$ can be drive from Poisson's equation.

For the relativistic quantum two component plasma system which consists of both electrons and ions the potential field $\phi(r_{12})$ can be drive from Poisson's equation:

$$\nabla^2\phi(r_{12}) = \frac{e}{\varepsilon_0}\left[\gamma_1 n_1 - \gamma_2 n_2\right] \qquad (8)$$

Where $\gamma_1 = \sqrt{1-\frac{v_e^2}{c^2}}$, $\gamma_2 = \sqrt{1-\frac{v_i^2}{c^2}}$, n_1 *and* n_2 *are* the electron and ion density field, respectively.

3. Evaluation of Quantum Entropy for dilute relativistic plasma Model

To calculate the binary relativistic distribution function substituting from equations (7) and (8) into equation (2) we get:

$$F_{12}^{qu}(r_1,v_1,r_2,v_2)=\frac{N^2}{V^2}\left(m_1m_2\right)^{\frac{3}{2}}\left[\frac{1}{2\pi k_BT}\right]^3 v_1^2v_2^2[1-(1-q)\frac{mv_1^2}{2k_BT} -(1-q)\frac{mv_2^2}{2k_BT}+(1-q)^2\frac{m_1v_1^2m_2v_2^2}{4k_B^2T^2}-e_1\phi(r_1)-e_2\phi(r_2)+...]^{\frac{1}{1-q}} \quad (9)$$

By substituting from equations (9) and into equation (5) we get the quantum Rényi entropy as:

$$S_{12}^{R}=\frac{1}{1-q}\ln\int \frac{N^2}{V^2}\left(m_1m_2\right)^{\frac{3}{2}}\left[\frac{1}{2\pi k_BT}\right]^3 v_1^2v_2^2[1-(1-q)\frac{mv_1^2}{2k_BT} -(1-q)\frac{mv_2^2}{2k_BT}+(1-q)^2\frac{m_1v_1^2m_2v_2^2}{4k_B^2T^2}-e_1\phi(r_1)-e_2\phi(r_2)+...]^{\frac{q}{1-q}}dr_1dr_2dv_1dv_2 \quad (10)$$

By substituting from equations (8) into equation (6) we get the quantum Tsallis entropy as:

$$S_{12}^{T}=\frac{k_B}{1-q}\int[\frac{N^2}{V^2}\left(m_1m_2\right)^{\frac{3}{2}}\left[\frac{1}{2\pi k_BT}\right]^3 v_1^2v_2^2[1-(1-q)\frac{mv_1^2}{2k_BT} -(1-q)\frac{mv_2^2}{2k_BT}+(1-q)^2\frac{m_1v_1^2m_2v_2^2}{4k_B^2T^2}-e_1\phi(r_1)-e_2\phi(r_2)+...]^{\frac{q}{1-q}} -\frac{N^2}{V^2}\left[\frac{m}{2\pi k_BT}\right]^3 v_1^2v_2^2\exp\left[-\frac{m(v_1^2+v_2^2)}{2k_BT}-e\phi(r_{12})\right]]dr_1dr_2dv_1dv_2 \quad (11)$$

4. Results and Discussion

This study evaluates the quantum entropy by using the q-non extensive parameter for dilute relativistic two component plasma model. Figure (1) show the quantum distribution function for one component plasma in the velocity of particle interval (0,0.5c) for dilute relativistic Plasma in different temperatures; We notice that as the temperature increases, the value of the distribution function decreases. Also, the values of the distribution functions reach zero as the speed of particles reaches the speed of light and this occurs faster in high temperatures than low one. The quantum distribution function for two component plasma in the velocity of particle interval (-0.2c,0.2c) for dilute relativistic Plasma is shown in figure (2).As the temperature increases, the distribution function becomes more concentrated. At lower temperatures, the particles have less energy therefore, the speeds of the particles are lower, and the distribution has a smaller range value. Moreover, when the temperature of the particles increases, the distribution becomes more concentrated in two component plasma model. Because the particles have greater energy at higher temperature, the particles have high velocity and high distribution function value.

Figures (3) and (4) show the direct proportionality between entropy and temperature at low speeds, while the entropy behavior changes in inverse proportion with higher temperatures for higher speeds. The quantum Tsallis Entropy is shifted to higher speeds and expanded at higher temperatures. Also, it increases with increasing the velocity, and it reaches a maximum and then it decreases with enhancing the velocity.

We note from figure (5) that the quantum Rényi entropy is directly proportional to increasing velocities and temperature in the velocity of particle interval (0,0.1c) for dilute relativistic Plasma. It is seen that the quantum Rényi Entropy increases with enhancing the velocity for all temperatures. It is observed that quantum Rényi behavior is different at low and high temperatures. However, at sufficiently high speeds, this approximation is no longer valid.

The quantum Tsallia Entropy in the velocity of particle interval (-c,c) have oscillatory behaviour with increasing velocities for dilute relativistic two component plasma model in different temperatures as seen in figure (6).

Figures (7) and (8) show the direct proportionality between Rényi and Tsallia entropy with the q-non extensive parameter at v= 0.5c for dilute relativistic plasma model. With setting q parameter, we can obtain the best results in comparison with experimental data.

5. Conclusions

We have investigated the effects of velocity and temperature on quantum entropy and quantum distribution function using Tsallis and Rényi formalism. We first solve the BBGKY hierarchy equation and obtain the distribution function of the system of two component relativistic plasma. Then, we have calculated the variations of the quantum Tsallis and Rényi entropy as a function of velocity and temperature. At high velocity, the entropy obtained by Tsallis entropy is inversely proportional to increasing temperature, while it is directly proportional to it in lower velocities. And we noticed that the Rényi approximation showed the stability in the value of entropy with increasing speeds for different temperatures with the inverse proportion between both entropy and temperatures.

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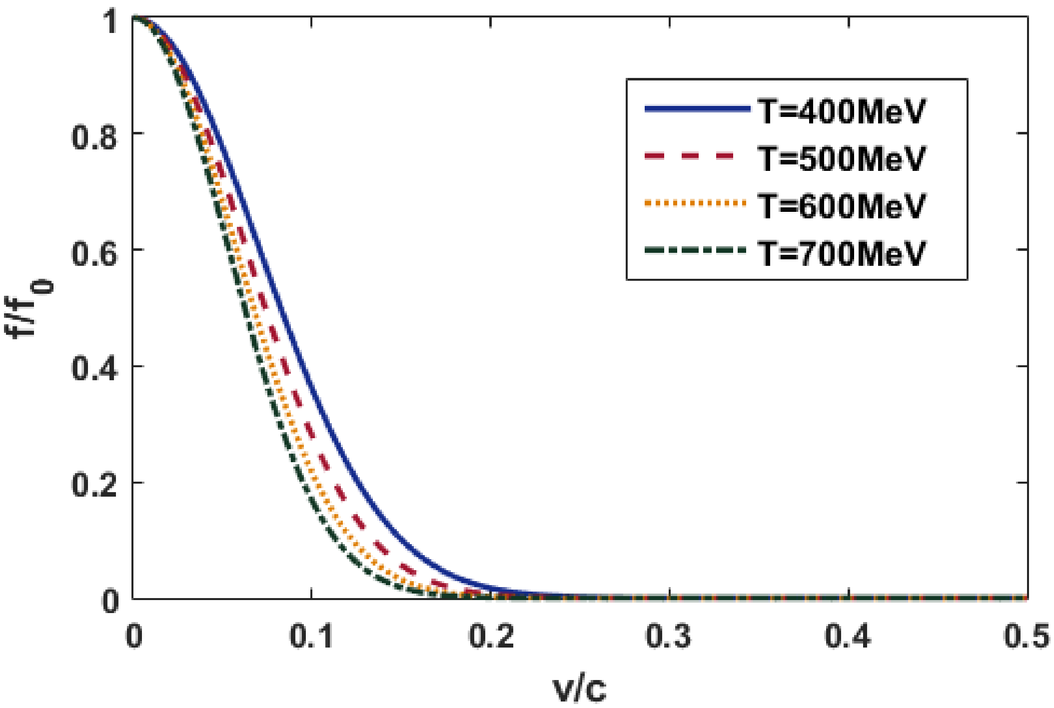

Figure 1. The quantum distribution function for one component plasma in the velocity of particle interval (0,0.5c) for dilute relativistic Plasma in different temperatures

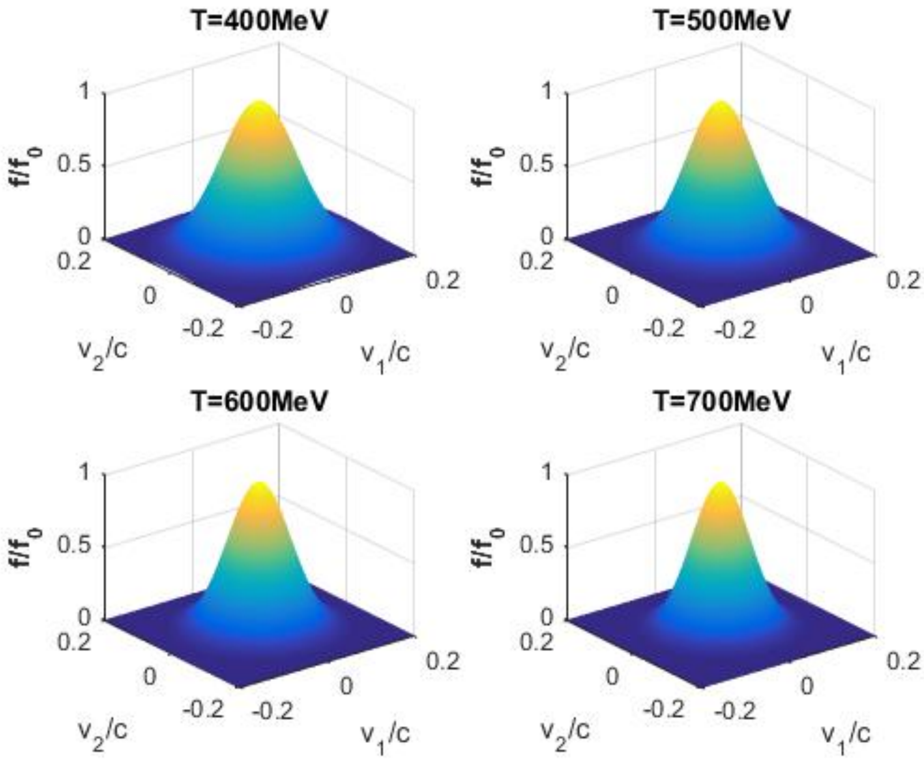

Figure 2. The quantum distribution function for two component plasma in the velocity of particle interval (-0.2c,0.2c) for dilute relativistic Plasma in different temperatures.

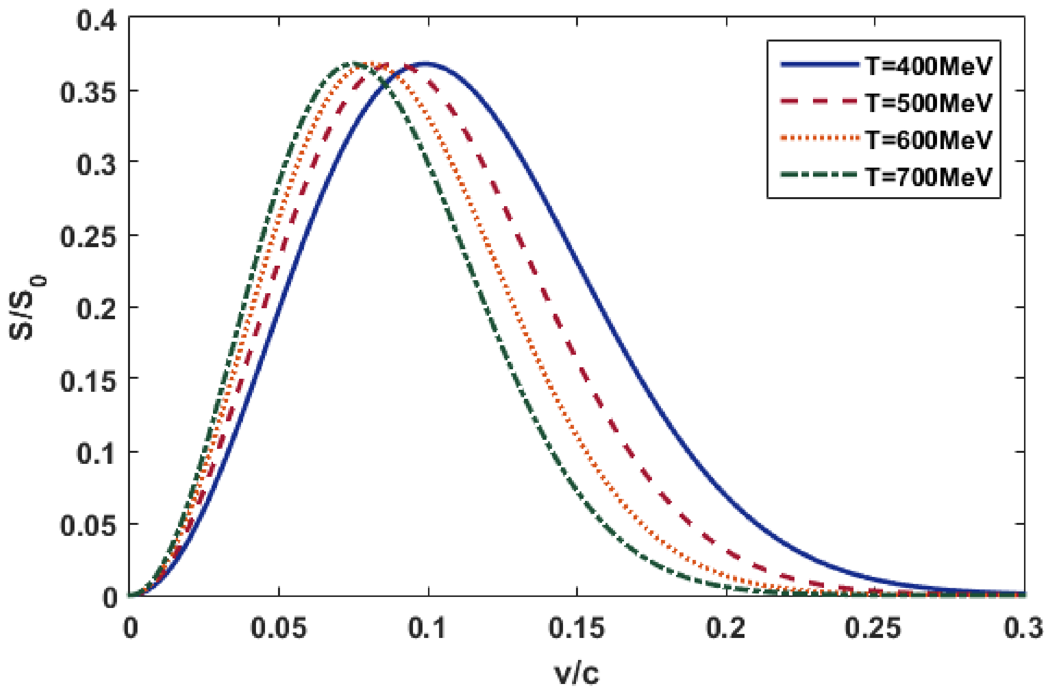

Figure 3. The quantum Tsallis Entropy in the velocity of particle interval (0,0.3c) equation (11) for dilute relativistic Plasma for different temperatures.

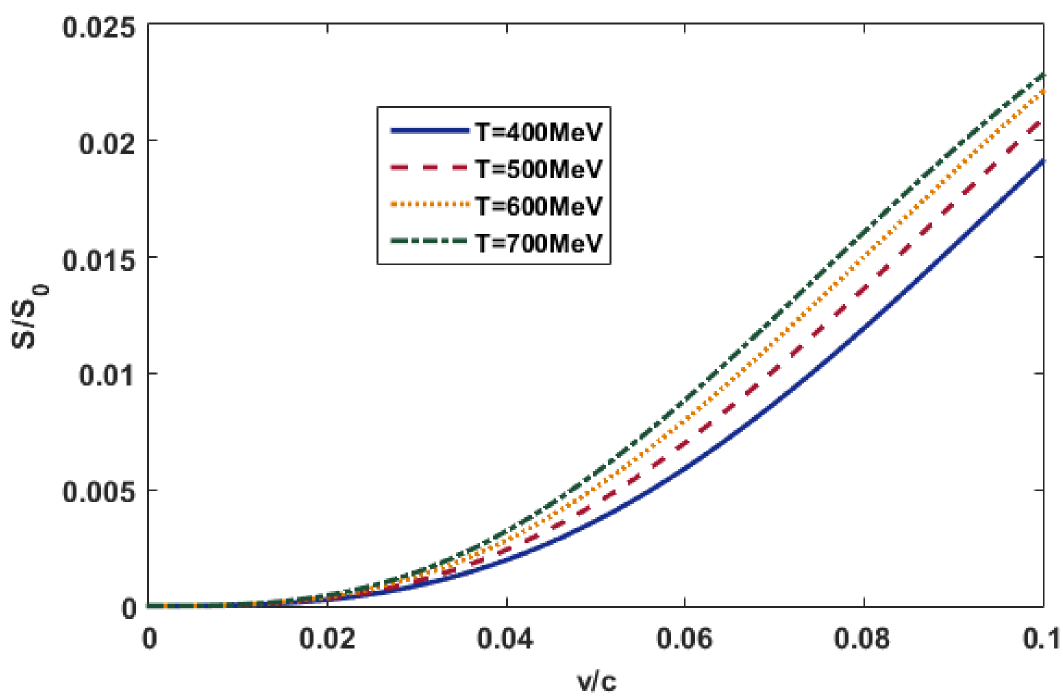

Figure 4. The quantum Tsallis Entropy in the velocity of particle interval (0,0.1c) for dilute relativistic Plasma for different temperatures.

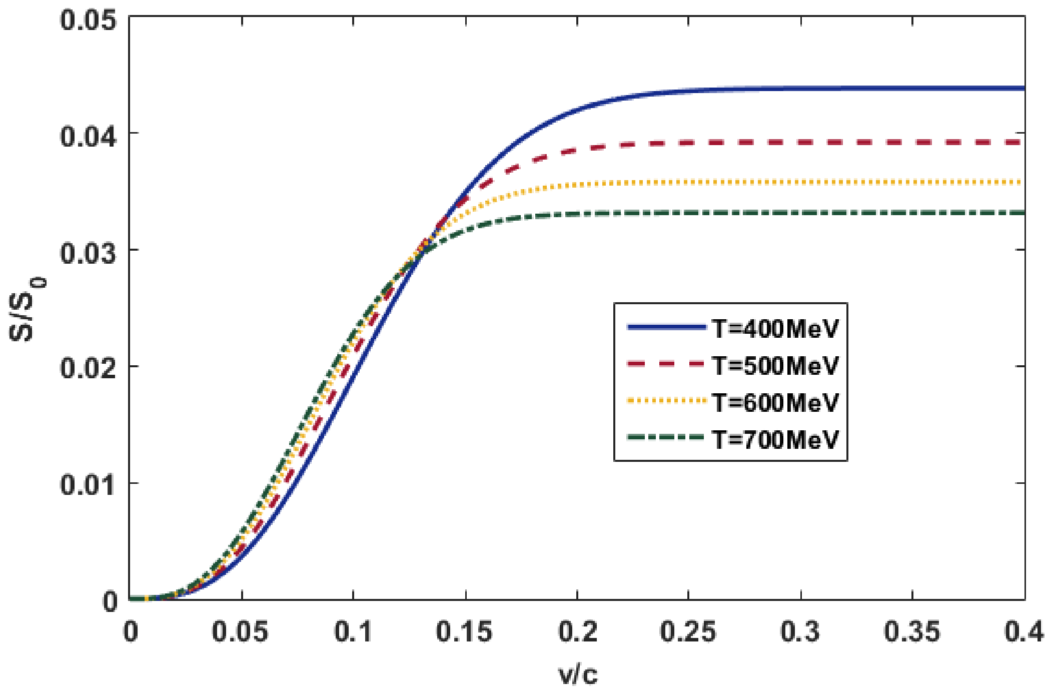

Figure 5. The quantum Rényi Entropy in the velocity of particle interval (0,0.3c) equation (10) for dilute relativistic Plasma for different temperatures.

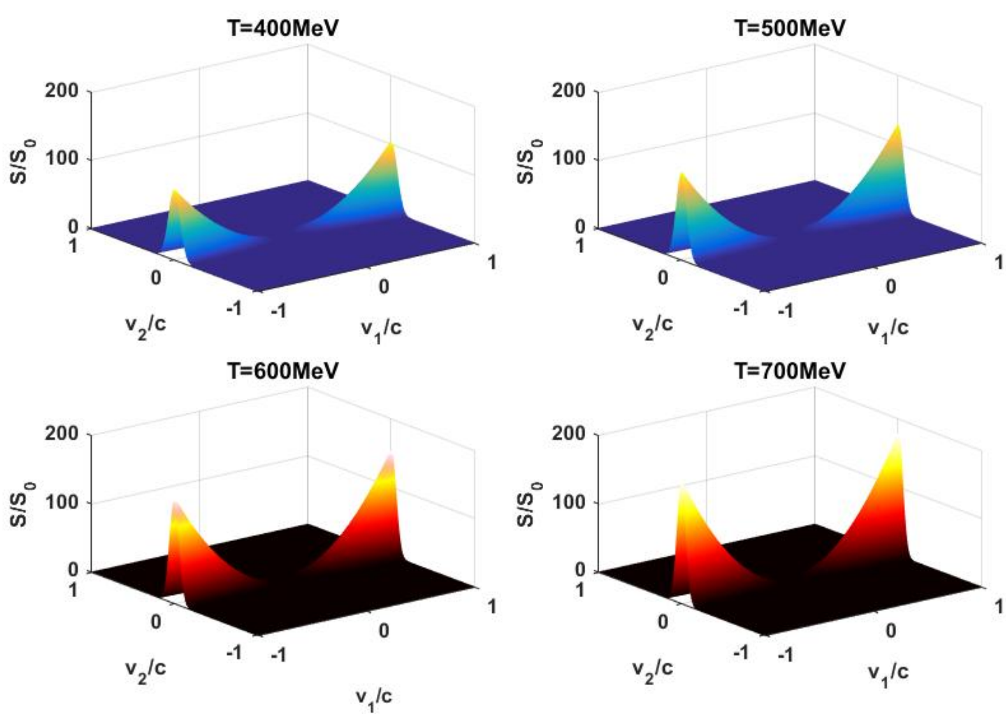

Figure 6. The quantum Tsallia Entropy in the velocity of particle interval (-c,c) for two component dilute relativistic Plasma in different temperatures**.**

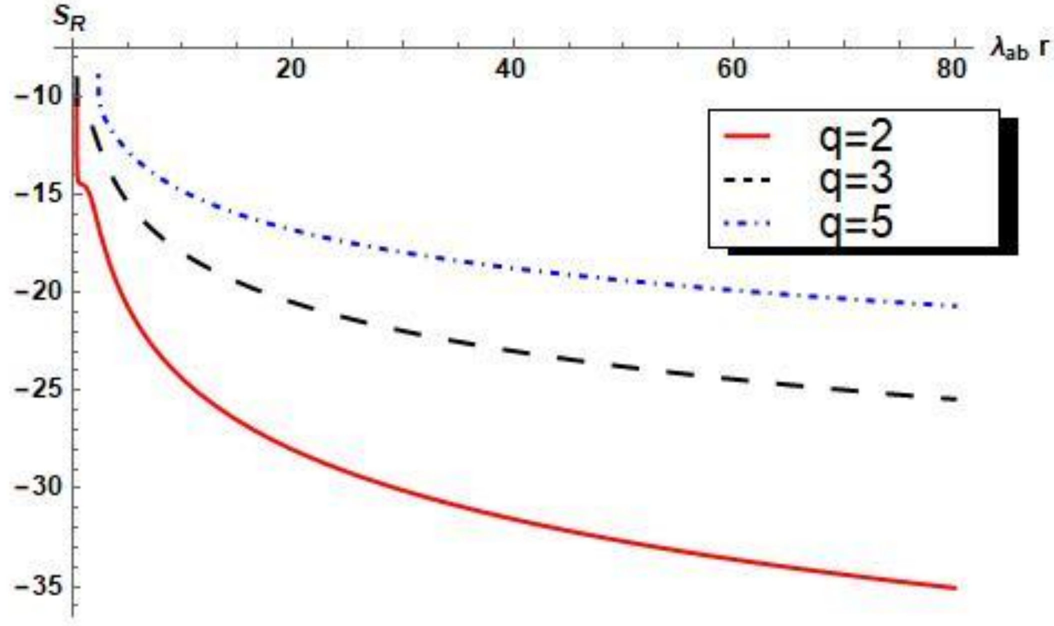

Figure 7. The quantum Rényi Entropy for dilute relativistic Plasma at q= 1,2 and 3 at v=0.5c.

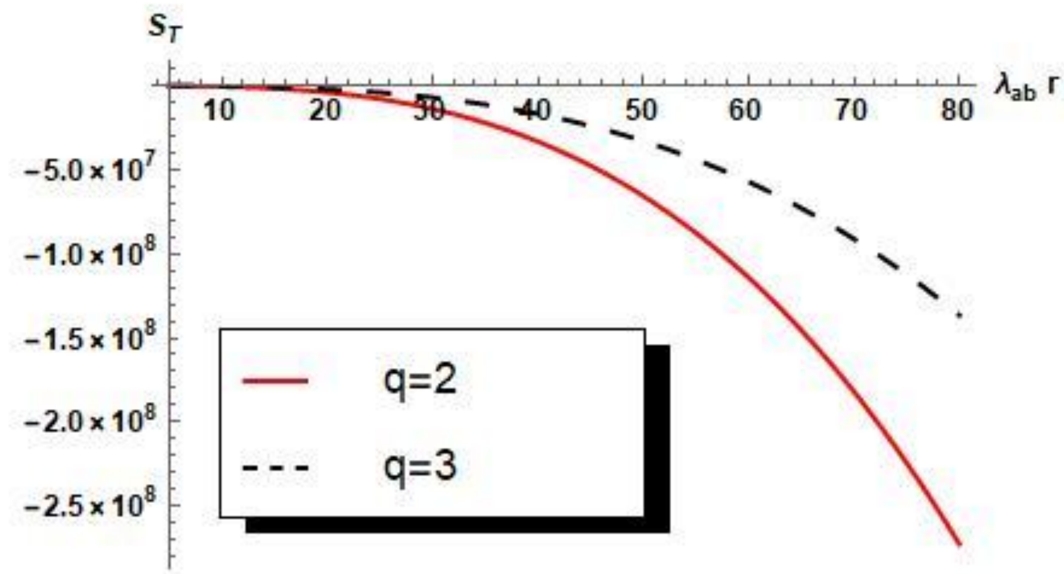

Figure 8. The quantum Tsallia for dilute relativistic Plasma at q=2 and 3 at v=0.5.