DELAYED GUIDANCE FOR ROLLING MISSILES WITH BOOST-SUSTAIN PROPULSION MOTOR

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ABSTRACT

The results of a 6-Degrees-of-freedom (6DOF) simulation model of a short-range homing missile are presented. The missile considered is roll-rate stabilized and is guided by proportional navigation law. The missile motor is of boost-sustain type. In this paper, the effect of the boost acceleration is analyzed. During the boost phase, it is found that the presence of a relatively large incidence angle increases the demanded missile maneuver to correct for the input errors to the guidance loop. Mathematical analysis shows that this large incidence angles couples a considerable amount of the boost acceleration to the lateral plane. This can be regarded as an input acceleration bias that increases the effort done by the guidance loop. In the sustain phase; however, the sustainer acceleration is low and this effect can be neglected. To reduce the demanded effort from the guidance loop, a delayed guidance scheme is proposed. In this scheme, the guidance and control of the missile starts few moments after launch. The results obtained via numerical computations are in complete agreement with the theoretically obtained conclusions.

INTRODUCTION

Examination of the flight behaviour of a rolling missile involves solving a complicated problem with nonlinear aerodynamic functions and time varying parameters. This is because the dynamics of the airframe vary with changes in missile incidence and velocity. These types of problems can only be accurately solved through digital simulation. An altrnative approach is to linearize the problem and thus, enables a slightly less accurate estimate of the system performance to be obtained in much shorter time.

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Linearization technique is used for the nonrolling missile analysis and can be extended for missiles rolling with a constant roll rate and fixed values of aerodynamic derivatives. However, the effect of roll motion complicates the analysis since, instead of examining a single plane system, the complete two plane analysis must be considered. This is because one effect of the missile roll motion is to cross-couple the airframe pitch and yaw planes [1].

An analytical linearization technique had been used in the past for the investigation of rolling missiles. It involves combining the two planes together into a single complex plane [2]. The intention was to condense the two plane problem back down to the order of single plane situation. The two guidance command inputs to the control system are combined into one complex input variable with the real part corresponding to the input in one plane and the imaginary part corresponding to the input in the other plane. Similarly the output variable and the transfer function are complex. Linearized schemes, however, suffer from their limited applicability specially in the early stages of flight where the missile speed is no longer constant and the missile incidence angles are relatively large. Thus, the assumption that the aerodynamic coefficients are linear functions of the flight parameters is not a strong assumption. Hence, the results obtained via the linearized model during the early stages of flight will not be as accurate as usually required. For this reason accurate nonlinear models are often necessary for the proper invstigation of the flight dynamics just after launch and in the transition period between boost and sustain phases.

In this paper, the resúlts of a 6DOF nonlinear simulation model for a short range rolling missile are presented. A computer code written in Fortran 77 is developed to solve the model. Runge- Kutta 4 is used to numerically solve the differential equations pertinent to the system. The obtained results indicate that portion of the high booster acceleration is coupled to the lateral plane of the velocity vector. This induced component represents an acceleration bias injected in the guidance loop. This results in a larger maneuver demanded by the guidance system. In order to reduce this effect on the missile lateral control, the guidance loop is closed few moments after launch. The choice of the closing moment depends on the system static stability, instanteneous missile velocity, dispersion at launch, and gathering capability of the control system. The numerical analysis carried out shows that delaying the missile guidance few moments after launch considerably reduces the missile peak maneuver.

EFFECT OF THRUST ON THE MISSILE LATERAL CONTROL

To investigate the effect of the booster acceleration on the missile lateral maneuver, the total forces that act on the missile airframe are resolved on the velocity coordinate system axes. The coordinate systems involved in the present model are shown in Fig. 1. The velocity and board coordinate systems are denoted by x, y, z and x_1, y_1, z_1 ; respectively. The reference coordinate system which is not shown in the figure is denoted by x_r, y_r, z_r. The angles between the velocity and board coordinate systems are α and β in the pitch and yaw planes; respectively. The projections of the total missile velocity $\vec{V}$ on the board system axes are denoted by U, v, and, w. p, q, and r represent the angular rotation rates of the missile board along the body system axes. The thrust vector $\vec{T}$ is assumed to act along the missile longitudinal axis given by x_1. The aerodynamic forces along the board axes are denoted by X, Y, and, Z. The translation motion of the missile C.G. is obtained from the rate of change of the linear momentum as:

$$\frac{d}{dt}(m\vec{V}) = \sum Forces, \tag{1}$$

where m is the missile mass. During the boost phase, the mass rate of change is relatively fast. Thus, Eq. 1 can be written as:

$$\vec{V}\frac{dm}{dt} + m\frac{d\vec{V}}{dt} = \vec{R} + \vec{T} + \vec{G}, \tag{2}$$

where $\vec{R}$ is the resultant aerodynamic force and $\vec{G}$ is the gravity force. Assuming that the weight force is small relative to the other forces, then Eq. 2 can be resolved along the board system axes as follows:

$$U\frac{dm}{dt} + m(\dot{U} + qw - rU) = X + T$$

$$v\frac{dm}{dt} + m(\dot{v} + rU - pw) = Y = ma_y \tag{3}$$

$$w\frac{dm}{dt} + m(\dot{w} - qU + pv) = Z = ma_z$$

Where a_y and a_z are the missile lateral accelerations. In the sustain phase, the first term of Eq. 2 is neglected and Eq. 3 reduces to the standard Euler's equations [1]. In conjunction with Eq. 3, there are 3 moment equations. However, since the radii of gyration usually change slowly in the boost phase, the standard Euler's moment equations are valid.

To obtain the effect of the boost acceleration on the missile lateral acceleration, Eq. 3 is rewritten in the velocity coordinate system as:

$$V\frac{dm}{dt} + m\frac{dV}{dt} = X' + T_x'$$

$$m(r'V) = Y' + T_y'$$

$$-m(q'V) = Z' + T_z' \qquad (4)$$

where V is the resultant missile velocity and is given by

$$V = \sqrt{U^2 + v^2 + w^2},$$

and the superscript $'$ means the projection of the corresponding quantity on the velocity coordinate axes. The angles of incidence α and β have small values, thus the following relations are valid:

$$\alpha = \frac{w}{U}$$

and

$$\beta = \frac{v}{U} \qquad (5)$$

The thrust components along the velocity system axes are approximated by

$$T_{x'} = T cos\alpha cos\beta = T$$

$$T_{y'} = -T sin\beta = -T\beta \qquad (6)$$

$$T_{z'} = -T cos\beta sin\alpha = -T\alpha$$

The body rates r' and q' are also approximated by

$$r' = rcos\alpha - pcos\beta sin\alpha = r - p\frac{w}{V}$$

$$q' = qcos\beta - psin\beta = q - p\frac{v}{V} \qquad (7)$$

Examination of Eq. 6, reveals that the thrust force components $T_{y'}$ and $T_{z'}$ act normal to the missile velocity direction. Mathematical manipulations of Eqs. 3 to 7 give the following equations:

$$m\dot{v} + v\frac{dm}{dt} = T\beta$$

and

$$m\dot{w} + w\frac{dm}{dt} = T\alpha \tag{8}$$

It is clear that $T\alpha$ and $T\beta$ components do affect the lateral velocities v and w. Thus, the total normal acceleration is given by that part produced by the aerodynamic control and the thrust components T_α and T_β. This effect can be considered as an acceleration bias or an extra source of error added to the acceleration demanded by the guidance system.

SIMULATION MODEL DESCRIPTION

The simulation model involved is broken down into five major routines as follows: missile target geometry, guidance, autopilot, airframe, and kinematics. In the missile target geometry routine, the missile position relative to the target and the LOS turning rates are calculated. These guidance parameters are then fed to the guidance routine where the guidance command signals are generated. The guidance command signals are then supplied to the autopilot routine to steer the missile in the space. As well, signals from the on-board sensors are fed back at the input of the fin servo amplifier. The fin deflection control signals are thus generated and provided to the airframe routine. In the airframe routine, the forces and moments equations are solved. Solution of these equations involves the computation of the aerodynamic coefficients which depend on the aerodynamic pressure and missile incidence angles. Finally, the kinematics routine produces the missile position and velocity from the acceleration and the missile body turning rate data it receives from the airframe routine. Fig. 2 shows a simplified block diagram that describes this model.

A computer code written in Fortran 77 is developed to solve the model. Runge-Kutta 4 is used to numerically solve the differential equations pertinent to the system [3]. The input parameters to the model are numerous. For instance, the initial missile and target speeds and positions, the effective navigation ratio, and the initial heading error. For simplicity, the target is assumed to move in the vertical reference plane $x_r z_r$. This consideration is useful for two reasons; firstly, the missile-target trajectories are presented and depicted in the vertical plane which facilitates results analysis. Secondly, the missile trajectory projection in the horizontal plane $x_r y_r$ is interpreted as an image of the cross coupling between the horizontal and vertical guidance channels.

As mentioned earlier, three coordinate systems are involved in the present model. The transformations between these coordinate systems are achieved by the quaternion method [4]. The angular turning rates of the missile body; p, q, and r; are used to update the quaternion variables. The wind vector is assumed to be constant in the horizontal $x_r y_r$ plane. The atmospheric density is supposed to decay exponentially with height. The thrust vector is considered along the missile longitudinal axis. The center of gravity location, mass and moments of inertia are assumed linear functions of time with different slopes in the boost and sustain phases as shown in Fig. 2. The aerodynamic force and moment coefficients denoted by C_x, C_y, C_z and C_l, C_m, C_n; respectively are stored aperiori as functions of the missile speed. to reduce the required memory size, the curves that represent these coefficients are quasilinearized before storage. As a rolling missile, it has one control channel. Thus, only two control surfaces exist and their deflections are denoted by δ_1 and δ_3. The missile body products of inertia are neglected due its configuration.

The proportional navigation guidance law is adopted in the missile guidance[5]. The line-of -sight turninig rate signal derived from the missile-target geometry routine is multiplied by the navigation coefficient to yield the missile turning rate in the space. The navigation coefficient, N_{av} is assumed to vary with time as

$$N_{av} = N_o + N' t \tag{9}$$

Thus, at the beginning of flight, small navigation constant is employed to reduce the initially demanded acceleration in the presence of large heading errors. However, near the end of flight, the large navigation constant ensures better engagment accuracy.

SIMULATION RESULTS

The code of the prescribed model is run for different engagement scenarios. The only source of error is the initial heading error. Fig. 3 shows the lateral acceleration component a_{y_v} computed in a plane lateral to the velocity vector. As shown in the figure, a_{y_v} is sinusoidally oscillating function with varying amplitude. The frequency of the oscillations is given by the rolling frequency. This oscillatory nature is attributed to the rotation of the body and the velocity coordinate systems with respect to the reference frame. The missile resultant lateral acceleration is then computed as

$$a_n = \sqrt{{a_{y_v}}^2 + {a_{z_v}}^2} \tag{10}$$

Fig. 4 shows the resultant lateral acceleration as given by Eq. 10. It is clear that the initial missile normal accelation is zero due to its low initial speed and its finite inertia. During the boost phase, the missile speed increases, and hence; the guidance loop becomes more effective. Thus, the missile maneuver to correct for the initial heading errors increases and attains its peak at the end of the boost phase. In the sustain phase, however; the guidance loop continues in compensating the heading errors. As a result, the missile maneuver decreases gradually as the flight proceeds.

Fig. 5 shows the longitudinal and normal acceleration of a missile launched with initial heading error. The oscillations that appeared in Fig. 4 are removed by taking the average over the rolling period. It is obvious from the figure that sudden shutoff of the booster results in a large drop in the longitudinal acceleration. This drop is accompanied by a small dip in the normal acceleration. This is attributed to the finite projection of the thrust force on the plane lateral to the velocity vector as given by Eq. 6.

To accurately evaluate the effect of the boost acceleration on the missile lateral maneuver, the normal acceleration is computed in $y_v z_v$ and yz planes as shown in Fig. 6. The difference between the two curves in the boost phase is obvious. The peak value of the acceleration in the velocity coordinate system exceeds that in the board coordinate system by one and half times. The difference is the component of the boost acceleration on $y_v z_v$ plane. In the sustain phase; however, since the longitudinal acceleration is small, there is no noticeable difference between the values of the normal accelerations in the velocity and board frames.

Thus, the presence of finite angles of incidence in the boost phase, drastically increases the effort done by the missile control system to correct for the initial heading errors. To reduce the effect of the boost acceleration on the missile lateral control, the incidence angles should be kept as small as possible in the boost phase. However, the values of these angles are indirectly controlled by the guidance loop. In the present system, an acceleration control autopilot is employed. The demanded acceleration issued by the guidance system is used to move the control fins. Accordingly, control moments followed by lateral forces are generated to steer the missile. Since the aerodynamic gain of the missile airframe during the early stages of flight is relatively low, large incidence angles are usually attained. This is obvious from Fig. 6 where the angle of incidence reaches its saturation value in the boost phase.

One way to reduce the values of the incidence angles is to disable the guidance few moments after launch. This keeps the values of the incidence angles as low as the values necessary for missile flight stability. Another way is to modify the guidance scheme such that the early demanded acceleration be as minimum as possible. However, disabling guidance seems to be more attractive since very minor modifications on the conventional PN guidance system are only required. It is obvious that the unguided period will not exceed the boost period. As well, target tracking by the seeker system should not be lost during this unguided period. This imposes an upper limit on the proposed guidance free time.

Fig. 7 shows the missile normal acceleration for two cases corresponding to conventional and delayed guidance schemes. A reduction of ten percent in the peak value of the normal acceleration is achieved by delaying the control over the missile half the boost duration. It is expected that more reduction can be achieved if longer delays are considered. However, the tracking capabilities of the guidance system and the instanteneous field of view of the missile seeker should be carefully taken into consideration.

CONCLUSIONS

The results of a short range rolling missile nonlinear model are presented. The effect of the thrust force on the missile guidance and control is discussed. It is found that the induced component of the thrust on the missile lateral plane resembles an aceleration bias error input to the guidance loop. This effect increases the effort done by the control system. To reduce this effect, a delayed guidance scheme is proposed. In this scheme, the guidance loop is disabled few moments after launch. The numerical results obtained shows a respectable improvement

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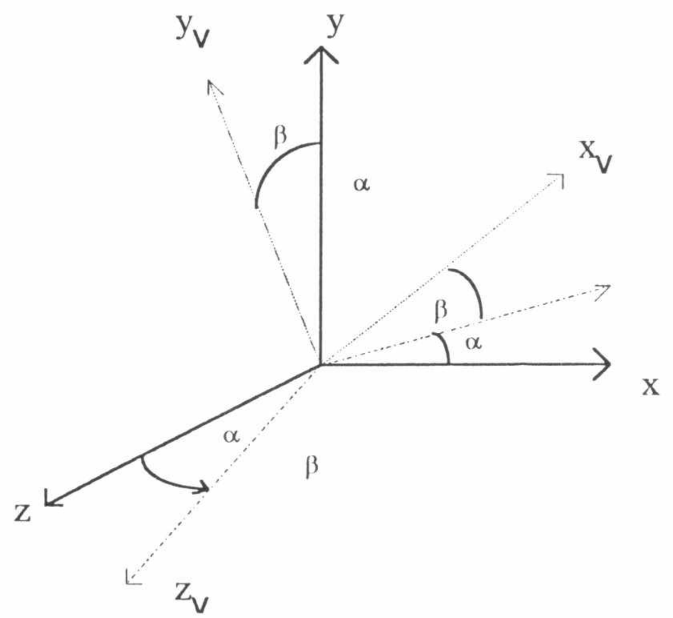

Fig. 1. Velocity and board coordinate systems

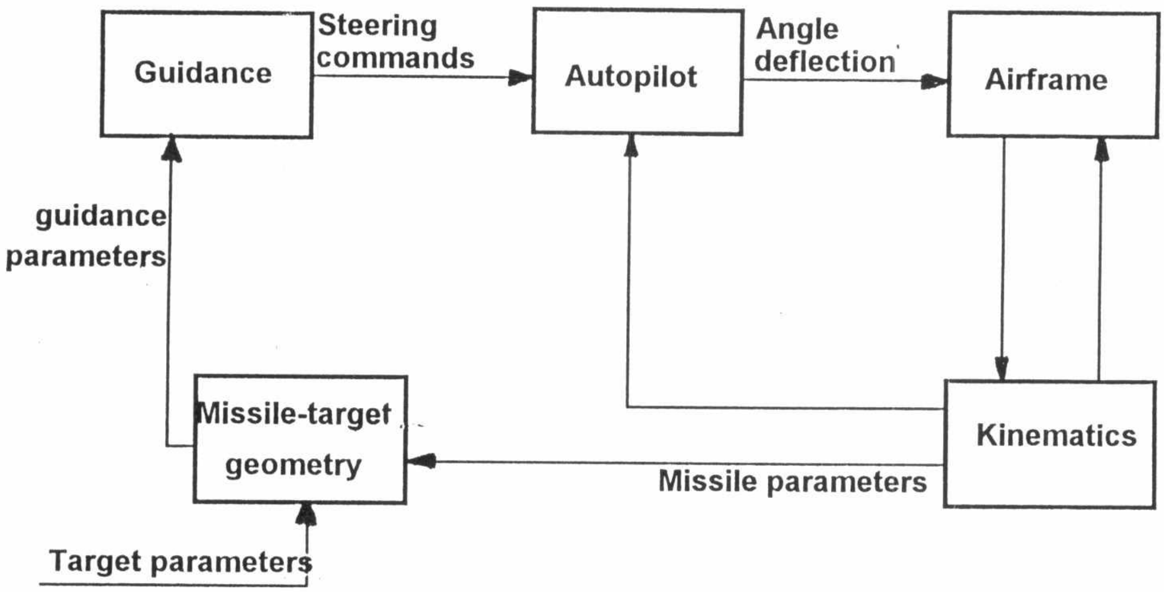

Fig. 2. Simplified block diagram of the simulation model

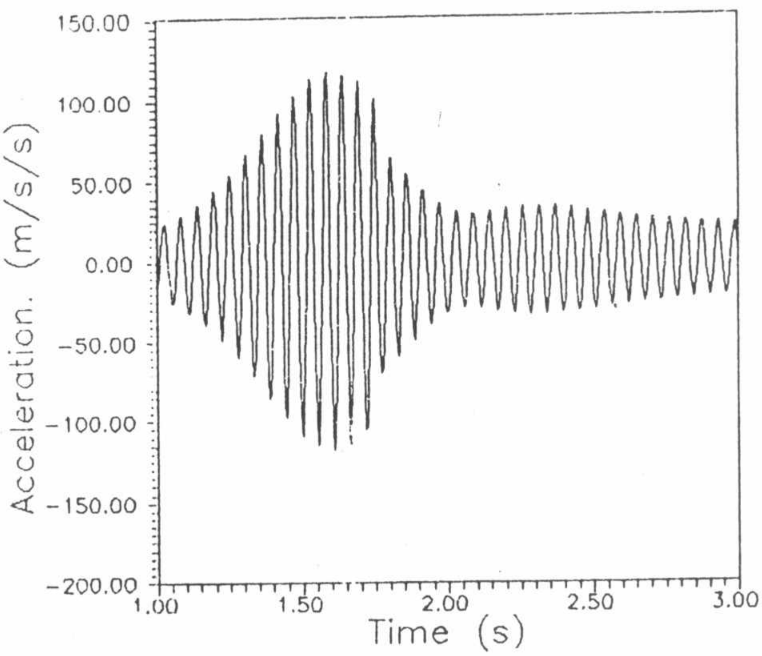

Fig. 3. Lateral acceleration along the y_V axis

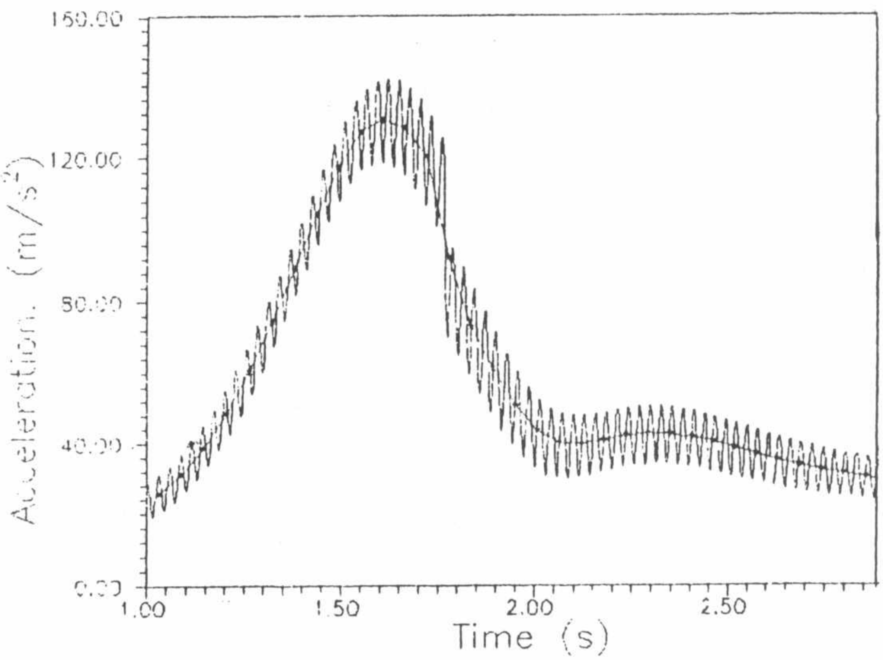

Fig. 4. Resultant normal acceleration

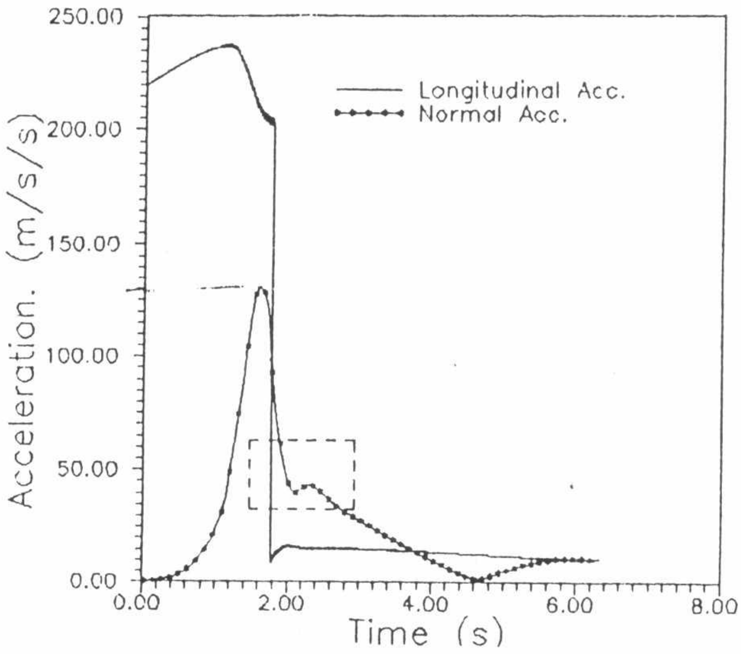

Fig. 5. Longitudinal and normal acceleration

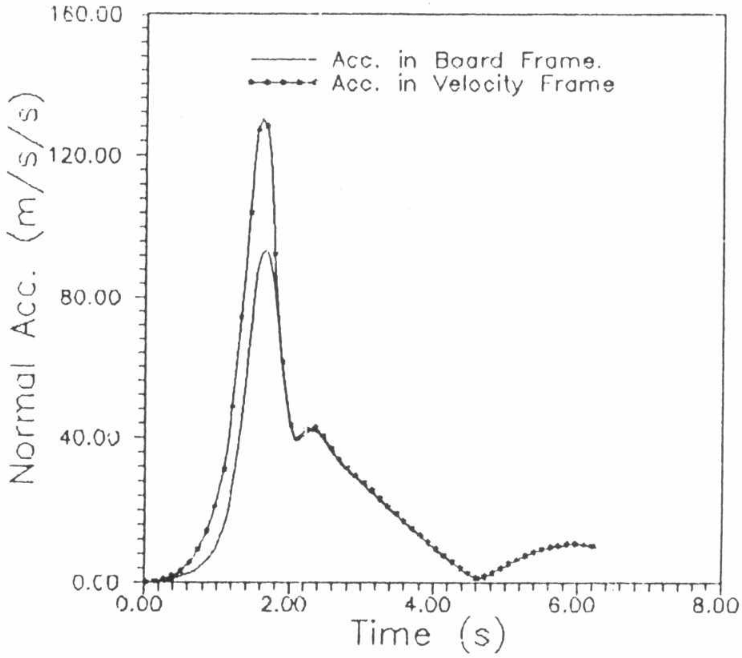

Fig. 6. Normal acceleration in the board and velocity coordinate systems

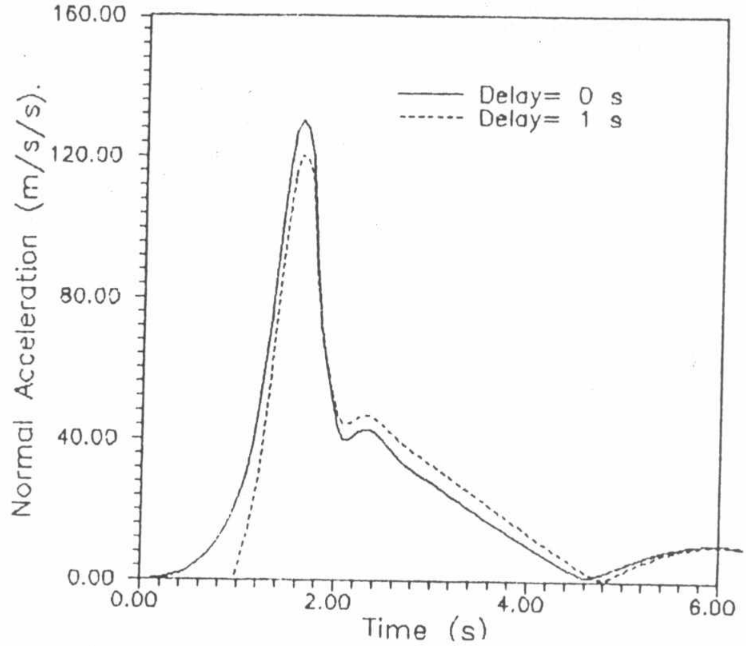

Fig. 7. Normal acceleration with and without delaying the guidance process