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GEOSTATISTICAL ANALYSIS OF TOPSOIL SODICITY USING COLLOCATED COKRIGING

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ABSTRACT

Geostatistics provides descriptive tools to characterize the spatial distribution of soil attributes. Kriging techniques rely on the spatial dependence between observations to predict attribute values at un-sampled locations. Cokriging on the other hand, utilizes the spatial correlation between two variables to map the primary one, which is under-sampled, using information content of the secondary variable. Collocated cokriging is used when the primary and the secondary variables are sampled at the same location. The present study aimed at applying collocated cokriging to map topsoil sodicity (primary variable) measured in 28 samples, using the information content of topsoil salinity (secondary variable) measured in 134 samples. Topsoil sodicity (SAR) ranged between 3.65 and 28.85, whereas topsoil salinity (EC) varied from 0.36 to 12.24 dS/m. The correlation coefficient, r, between the two variables is 0.98, which satisfies the most important criteria for carrying out cokriging. The fitted semivariograms for both variables are Gaussian, and the cross-semivariogram between the two variables is also Gaussian. The cokriged spatial distribution of topsoil sodicity was mapped and compared to kriged SAR. The cokriging results were cross-validated and the standard error of estimation was matched to that of kriging. The study showed the superiority of cokriging upon kriging as a spatial mapping method, especially if the primary variable is under-sampled.

Keywords: Geostatistical analysis, Collocated cokriging, Kriging, Salinity, Sodicity, Cross-semivariogram, Semivariogram,

INTRODUCTION

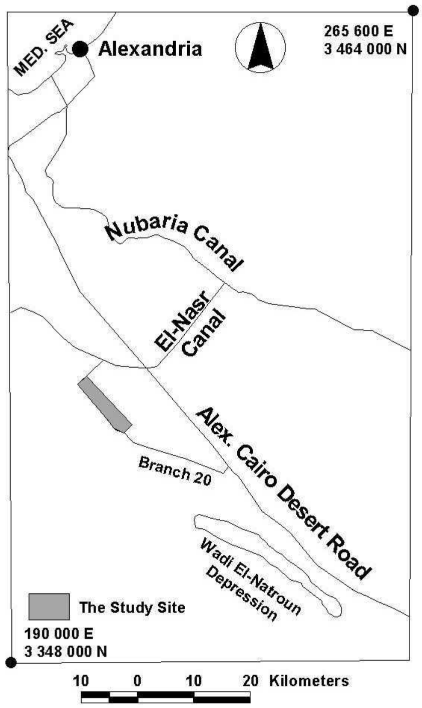
Geostatistics has been applied by many researchers to describe the spatial variability using the semivariogram and predict the values of soil attributes at un-sampled locations by different kriging (named after D.J. Krige) techniques (Trangmar et al., 1985; Warrick et al., 1986; Webster and Oliver, 1989; Burrough, 1989; Webster, 1991; Goovaerts, 1992, 1998b and 1999; Bahnassy et at. 1995; Bahnassy and Morsy, 1996 and El-Zahaby et al. 1999), ecological properties (Banerjee and Gelfand, 2002), and categorical variables (Bogaert, 2002). The term cokriging is used for spatial linear regression that uses data defined by different attributes. The data set will contain the primary variable of interest in addition to one or more secondary variables, which are spatially cross-correlated with the primary variable. Thus, the dataset will contain useful information about the primary variable. The cross-correlation between variables is utilized to improve these estimates, and to reduce the variance of the estimation error. The usefulness of the secondary variable is often enhanced by the fact that the primary variable of interest is under-sampled (Issacks and Srivastava, 1989). The spatial relationship between the values of the attribute is governed by the regionalized variable theory, which states that observations close to each other are more correlated than observations taken at a further distance (Journel and Huijbregts, 1978). This means that points spatially close to the estimation points should be given higher weights than those further away (Cressie, 1993). The coregionalized variable theory deals with the same situation as the regionalized variable theory, but the variables under consideration are correlated, and behave the same (McBratney and Webster, 1983 and 1986). Consequently, the cross-semivariogram can be modeled as a joint function between the two variables (Issacks and Srivastava, 1989). The linear coregionalization model allows for different ranges of spatial correlations for each variable (Wackernagel, 1994 and 1995). Due to computation and notation difficulties related to cokriging system (Journel and Huijbregts, 1978; Myers, 1982; and Deutsch and Journel, 1998), a limited number of researches have been carried out utilizing cokriging as a best linear unbiased estimator (B.L.U.E.). Danielsson et al, (1998) applied cokriging to estimate the total amounts and the spatial distribution for organic carbon, nitrogen and phosphorus in the Gulf of Riga surficial sediments, using loss on ignition as a covariable. Goovaerts (1998) used different methods of kriging and cokriging to model the spatial distribution of pH and electrical conductivity in two transects in forest and pasture soils. Ishida and Ando (1999) utilized disjunctive cokriging to estimate soil organic matter from Landsat Thematic Mapper image. Rivoirard (2001) indicated that the cokriging could be collocated or multi-collocated depending on the configuration of data and the location at which the value will be estimated.

The purpose of this study is to apply cokriging to predict the values of the primary variable (topsoil sodicity), which is sparsely sampled and hard to measure (requires flame photometry for Na^+ and K^+, and titration for Ca^{++}), using the information content of topsoil salinity, which is densely sampled and easy to measure, taking into consideration the fact that these two variable are correlated. The cokriged sodicity (SAR) is compared to the kriged sodicity (SAR) and the standard error of estimation for both methods was matched.

MATERIALS AND METHODS

The Study Site

The study site is located about 90 km south of Alexandria, to the west of Alex-Cairo desert road on Branch 20 irrigation canal. It comprises part of the newly reclaimed sandy soils in West Nubaria region, which was distributed to the graduates in 1990. The total acreage of the study area is about 2500 hectares (map 1). Sampling Scheme

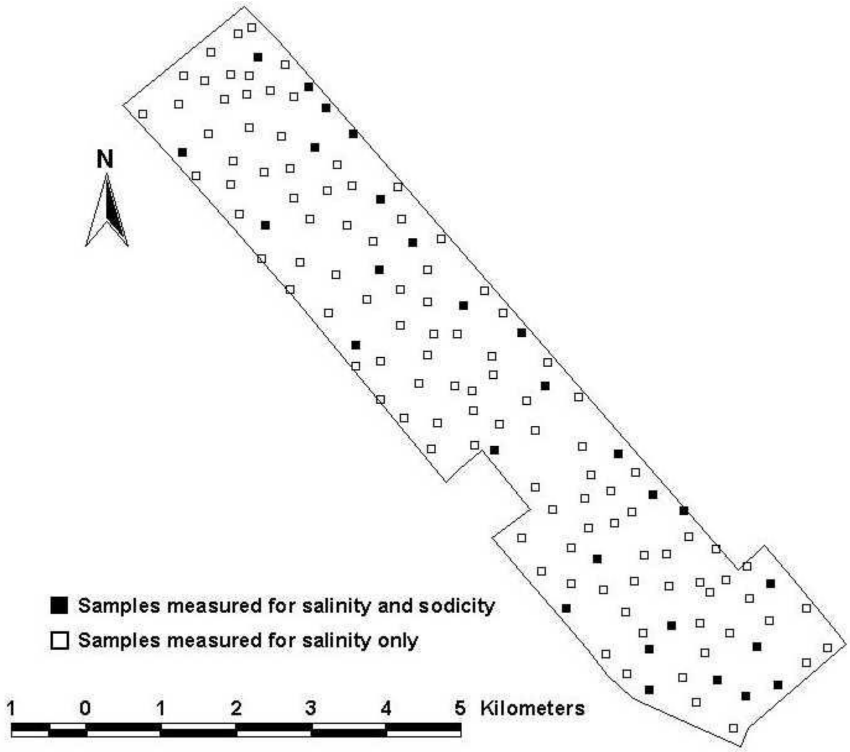


Map 1: Location of the study area.

One hundred thirty four soil observations were collected over the study area. The topsoil was analyzed for salinity (EC, dS/m). These soil observations were used as a secondary data for interpolating the sodicity. Twenty-eight soil observations were taken randomly as a subset of the original data and analyzed for sodicity (SAR), which is considered as the primary variable. The samples locations were georeferenced to the UTM coordinate system. The spatial configuration of the soil observations used for salinity and sodicity is shown in map 2.

Descriptive Statistical Analysis

The data for salinity and sodicity were analyzed for basic statistics including mean, variance, standard deviation, minimum, maximum, skewness, and kurtosis. The histogram for both variables was obtained, and the correlation between the two variables was calculated.



Map 2: Location of soil observations for sodicity and salinity

Semivariogram and Cross-semivariogram Analysis

The semivariogram is defined as half of the average squared difference between two attribute values separated by vector **h**, for one variable (Burrough and McDonnell, 1998):

$$\gamma(\mathrm{h}) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \{Z(x_i) - Z(x_i + h)\}^2$$

where *N*(*h*) is the number of pairs at lag **h**, Z(x_i) is the value of the attribute at location (x_i) and Z(x_i + h) is the value of the attribute at location (x_i + h) separated by distance **h**. The separation vector **h** is specified with some direction and distance (lag) tolerance. This semivariogram is used to model both salinity and sodicity, and then fitting them to one of the known semivariogram functions (Gaussian, Exponential, Spherical). In case of using two variables (cokriging) the cross-semivariogram is calculated as follows:

$$\gamma_{UV}(\mathrm{h}) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \{Z_U(x_i) - Z_U(x_i + h)\}\{Z_V(x_i) - Z_V(x_i + h)\}$$

where Zu (sodicity) and Zv (salinity) are the two variables. This equation is used to model sodicity using the information content of salinity, then fitting the obtained model to one of the known cross-semivariograms represented by Gaussian, Spherical, and Exponential functions.

Cokriging

A co-kriged estimate is a weighted average in which the value of U at location x_o is estimated as a linear weighted sum of co-variables V_k. If there are k variables k = 1, 2, 3,. . . V, and each variable is measured at n_v places, x_{ik} = 1, 2, 3.... N_k, then the value of one variable U at x_o is predicted by (Burrough and McDonnell, 1998):

$$Z_U^{'}(x_o) = \sum_{k=1}^{V}\sum_{i=1}^{n_v} \lambda_{ik} Z(x_{ik}) \qquad \text{for all } V_k$$

where λ_{ik} is the weight assigned to variable k and $Z(x_{ik})$ is the value of the variable at location i.

To avoid bias, i.e. to ensure that

$E[z_u(x_o) - z'_u(x_o)]=0$ and
the sum of weights λ_{ik} = 1 for U = V and
the sum of weights λ_{ik} = 0 for $V_k \neq U$

The first condition (sum of weights λ_{ik} = 1) implies that there must be at least one observation of U for cokriging to be possible. The interpolation weights are chosen to minimize the variance:

$$\sigma^2_u (x_o) = E[\{z_u(x_o) - z'_u(x_o)\}^2]$$

There is one equation for each combination of sampling site and attribute, so for estimating the value of variable j at site x_o, the equation for the g-th observation site of the k-th variable is:

$$\sum_{j=1}^{V}\sum_{i=1}^{n_v} \lambda_{ij}\gamma_{ij}(x_{ij}, x_{gk}) + \Phi_k = \gamma_{uv}(x_o, x_{gk})$$

for all g=1 to n_v and all k=1 to V, where Φ_k is the Lagrange's multiplier. These equations together make-up the cokriging system.

Cross Validation

Cross validation is a technique which is used to compare estimated and true values using the information available in the data set. In cross validation, the estimation method is tested at the locations of existing samples. The sample value at a particular location is temporarily discarded from the sample data set; the value at the same location is then estimated using the remaining samples. Once the estimate is calculated, it is compared to the true sample value that was initially removed from the sample data set.

This procedure is repeated for all samples. This could be expressed as (Issaks and Srivastava, 1989):

$$Error = r = v' - v$$

Where *v'* is the estimated value and *v* is the true value. Mean square error (MSE) is calculated from the formula:

$$MSE = \frac{1}{n}\sum_{i=1}^{n} r^2$$

Linking Geostatistics to Geographic Information Systems (GIS)

The estimates from cokriging and kriging, and the associated error (Gamma Design, 2001) were exported to Arc View GIS software (ESRI, 1997) for better visualization, mapping and printout.

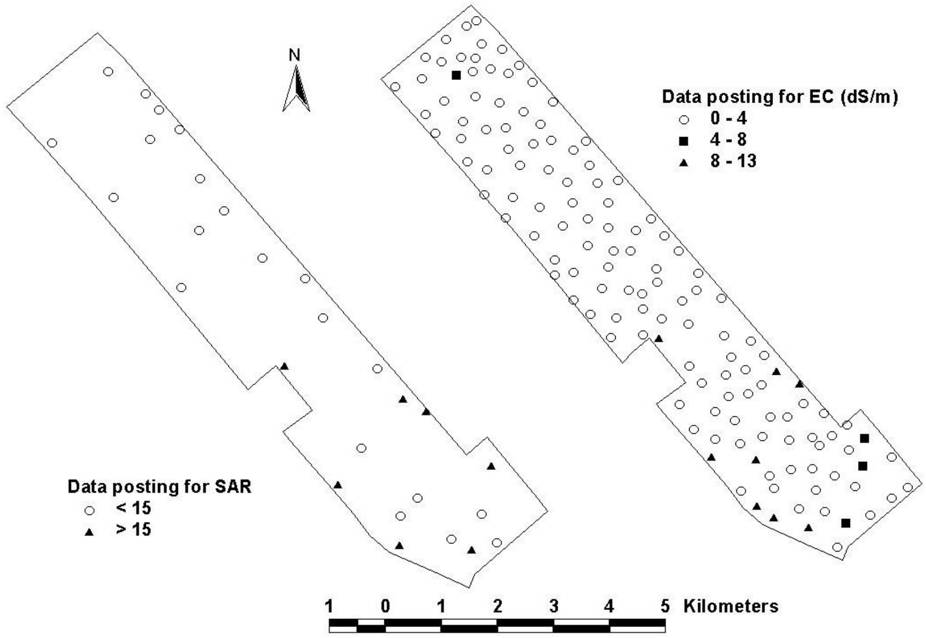
RESULTS AND DISCUSSIONS

Description of Spatial Patterns

The analysis of spatial data starts with posting the data values. Map (3) shows the spatial distribution of sodicity (SAR), and salinity (EC, dS/m), sampled at 28 and 134 locations, respectively. The spatial distribution of the variables is not random, but follows the regionalized theory, i.e., observations close to each other on the ground tend to be more alike than those further apart (Journel and Huijbregts, 1978). The presence of such spatial structure is prerequisite to the application of Geostatistics, and represent the first step towards spatial prediction (Burrough and McDonnell, 1998).

Descriptive Statistical Analysis

The statistical analysis of the salinity and sodicity is shown in table (1). It is clear that salinity has more variability than sodicity as the CV% is almost doubled. This is attributed to the greater number of soil samples (134) used in the analysis compared to the number of samples (28) used for sodicity analysis. Moreover, there is a greater number of soil samples with low salinity values (figure 1), which lowered the mean compared to the standard deviation. The histogram for both salinity and sodicity is shown in figure (1). The distribution of both variables is positively skewed, indicating the dominance of low values, with the presence of a very little high values that might have an impact of the final estimates (Isskas and Srivastiava, 1989). On the other hand, variance indicates that SAR has spread on a wide range contrary to EC, which is distributed around a high number of samples with low values (Figure 1).



Map 3: Data posting for salinity (EC, dS/m) and sodicity (SAR).

Regression analysis of both salinity and sodicity indicated a positively highly correlated two variables, which satisfies the need to carry out cokriging analysis of sodicity using the information content of salinity. The correlation coefficient for this analysis is 0.98. Yates and Warrick (1987) showed that if the correlation coefficient between a primary variable and the covariable exceeds 0.5, then the inclusion of the covariable is favorable, and cokriging performs better than kriging. The following equation represents the regression analysis of salinity and sodicity:

$$SAR = 4.08 + 2.1\ EC \qquad r = 0.98$$

Table 1: Descriptive statistical analysis for salinity (EC, dS/m) and sodicity (SAR)

Statistical Parameter	Sodicity(SAR)	Salinity (EC, dS/m)
Mean	11.60	1.71
Standard Deviation	7.43	2.25
CV% (coefficient of variation)	64.05	131.58
Variance	55.29	5.06
Minimum	3.65	0.36
Maximum	28.85	12.24
Skewness	0.92	2.91
Kurtosis	-0.29	8.20
N (number of samples)	28	134

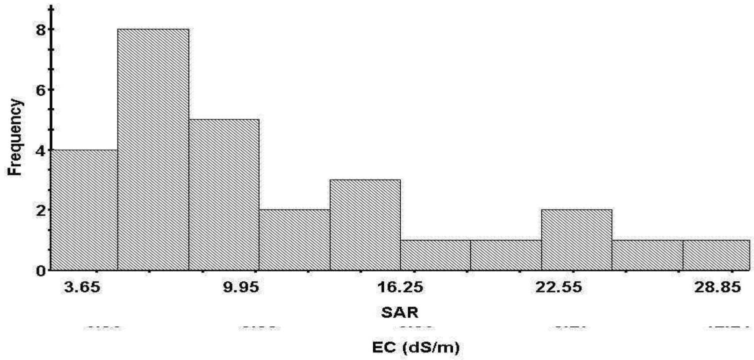


Figure 1: Histograms for salinity (above) and sodicity (below)

Salinity and Sodicity Semivariograms

The semivariograms for both salinity and sodicity were fitted to the Gaussian model as shown in the following equation:

$$\gamma(h) = Co + C1\{1 - \exp(-\frac{3h^2}{a^2})\}$$

Where C_o is the nugget, C_1 is the sill, **h** is the separation distance (lag) in meters, and ***a*** is the range. The parameters for the fitted semivariograms for both salinity and sodicity are shown in table (2), and the semivariograms are shown in figure (2). The formulated equations for these two variables are as follows:

$$\gamma_{SAR}(h) = 33.3 + 54.46\{1 - \exp(-\frac{3h^2}{(4670)^2})\}$$

$$\gamma_{EC}(h) = 3.73 + 8.74\{1 - \exp(-\frac{3h^2}{(11300)^2})\}$$

Table 2: Semivariogram types and parameters for salinity and sodicity

Variable	Model	Nugget (Co)	Sill (C1)	Range (a)	R^2	Lag (m)
SAR	Gaussian	33.3	54.46	4670	0.95	1500
EC	Gaussian	3.73	8.74	11300	0.91	500

It is clear that the coefficient of determination R^2 for both models exceeds 0.90, which indicates the goodness of the estimation. Moreover, The fitted Gaussian semivariogram indicates a smoothly varying pattern for both variables (Burrough and McDonnell, 1998).

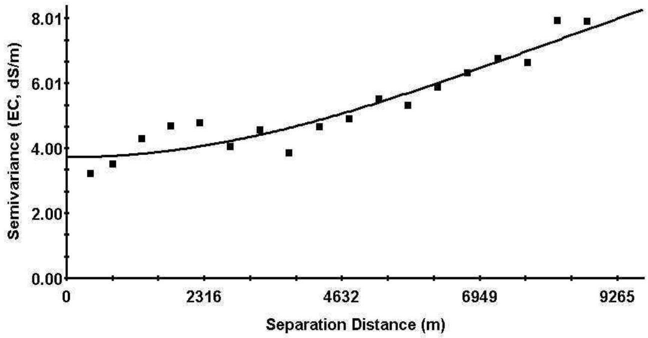
The Cross-semivariogram (Collocated semivariogram)

The cross-semivariogram of sodicity and salinity is of the collocated type, which means that the estimation was performed using variables measured at the same location. Table (3) and figure (3) indicate the parameters of the fitted Gaussian cross-semivariogram between sodicity and salinity. The Gaussian joint semi-semivariogram is as follows:

$$\gamma_{SAR-EC}(h) = 16.13 + 23.36\{1 - \exp(-\frac{3h^2}{(4750)^2})\}$$

Table 3: Cross-semivariogram parameters between sodicity (SAR) and salinity (EC, dS/m).

Variable	Model	Nugget (Co)	Sill (C1)	Range (a)	R^2	Lag (m)
SAR and EC	Gaussian	16.13	23.36	4750	0.95	1500



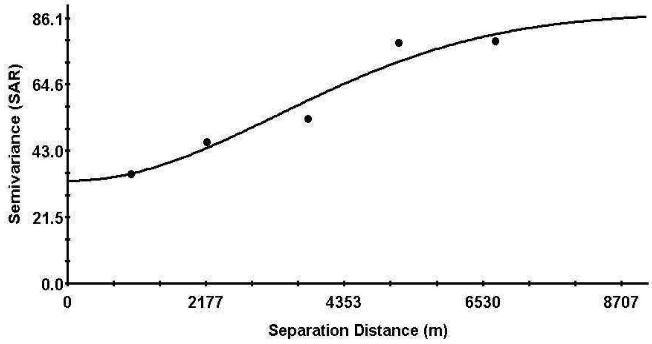


Figure 2: The semivariograms for salinity (above) and sodicity (below)

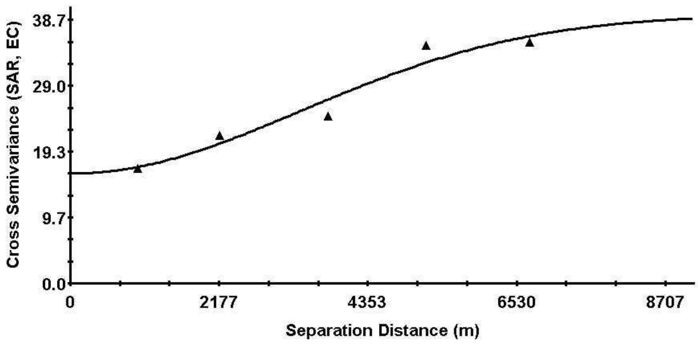
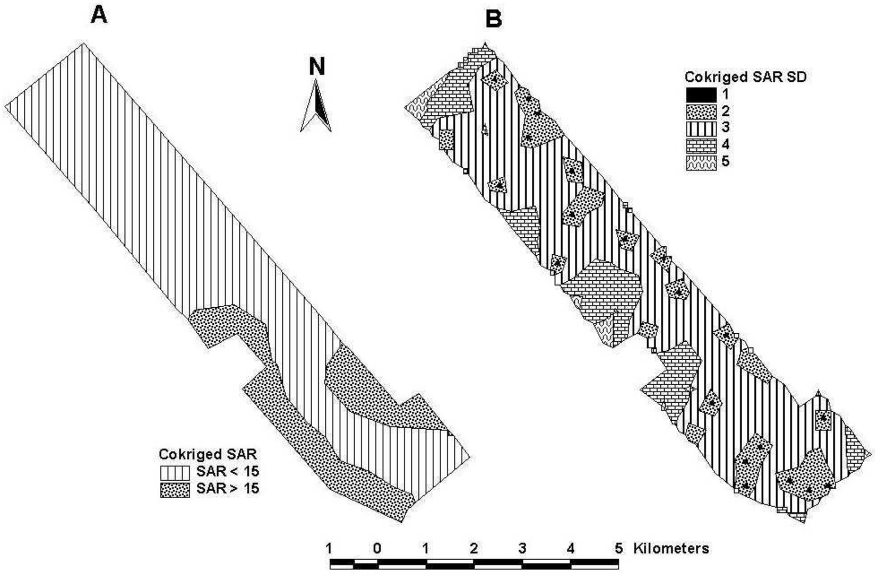


Figure 3: The cross-semivariogram between sodicity and salinity

The most important parameter in this estimation is the high R^2 (0.95) obtained from the fitting process. This high estimation regression coefficient comes very close to that of the simple linear regression (0.98) between sodicity and salinity. The advantage of cokriging over linear regression is that it takes into consideration the spatial variability of the surrounding points, rather than performing blindly the linear regression, which lacks this improvement.

Cokriging Compared to Kriging

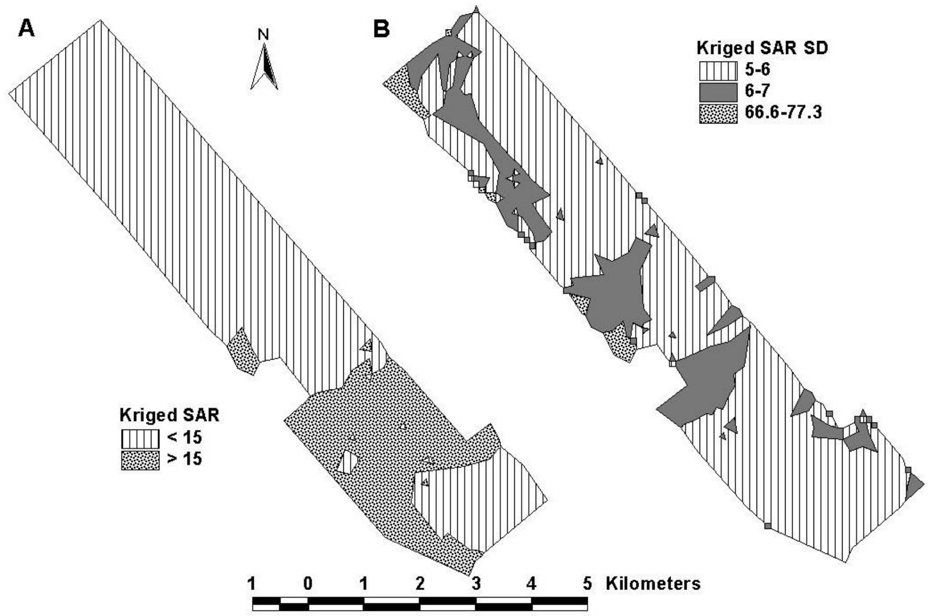
The output from cokriging process is a map of the spatial distribution of sodicity based on the information content and the high correlation with salinity. Map 4A shows the cokriged sodicity and the associated standard error of the estimates (Map 4B) for the study area.



Map 4: Cokriged SAR (A) and the associated standard deviation (B).

It is clear that the cokriged SAR is smoothed out, because estimated values are less variable than actual values. This is expressed by an overestimation of small values while large values are underestimated; however the smoothing depends on the local data configuration (Goovaerts, 1999). The error (map 4B) is small in areas close to data locations and increases as the location being estimated gets further away from sampled locations, as compared with the map (3), which shows data posting. Another reason for the smoothing is that the studied soil is mainly sandy, in which salinity and sodicity is quite not a problem due to the dominance of the coarse sand fraction, which hinder the upward movement of saline water table by capillary rise.

Topsoil sodicity (SAR) was kriged in order to compare both the cokriging results, and the standard error. Map (5) indicates the results of kriging SAR and the associated standard deviation (SD).



Map 5: Kriged SAR (A) and the associated standard deviation (B).

It is clear that kriging aggregated the high sodicity values in one contiguous group due to the lack of information in the area between the topsoil sodicity samples. On the other hand, cokriging utilized the information content of soil salinity to predict the values of topsoil sodicity at un-sampled locations. Moreover, the kriging standard deviation (standard error) shown in map (5B) have much higher values especially at the boundary of the study area, and behaved very erratically due to the lack of surrounding points. For these reasons, cokriging is much preferred over kriging, especially if the primary variable is under-sampled, as in the case of topsoil sodicity (28 samples spread over an area of 2500 ha).

Cross Validation of Cokriging and Kriging

The process of cross validation between the estimated and the true value permits the evaluation of cokriging performance. Figure (4) shows the linear regression between the cokriged and actual values of sodicity (SAR). The standard error (SE) of prediction is high (4.69) due to the above-mentioned reasons related to smoothing effect of cokriging, and the configuration of the data. The regression equation resulted from the cokriging cross validation is as follows:

Cokriged SAR (predicted) = 3.92 + 0.76 SAR (measured)
r = 0.78 SE prediction = 4.69

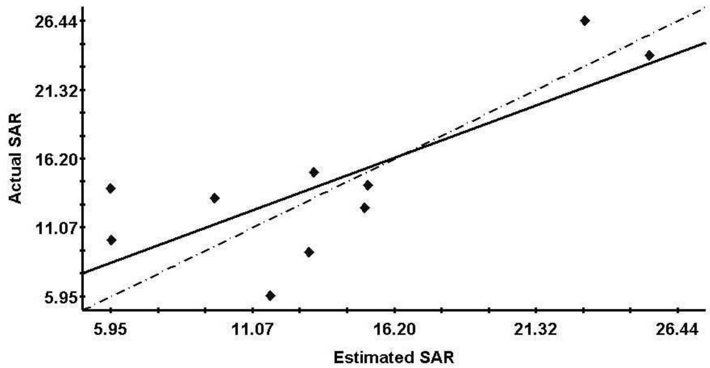


Figure 4: Cross validation between cokriged and actual values of SAR. (The solid line is the regression line, the dot-dash line is for r = 1)

For comparison sake, kriged SAR was cross validated to see how the standard error (SE) of prediction behaves (Figure 5) and check the results with cokriging estimates.

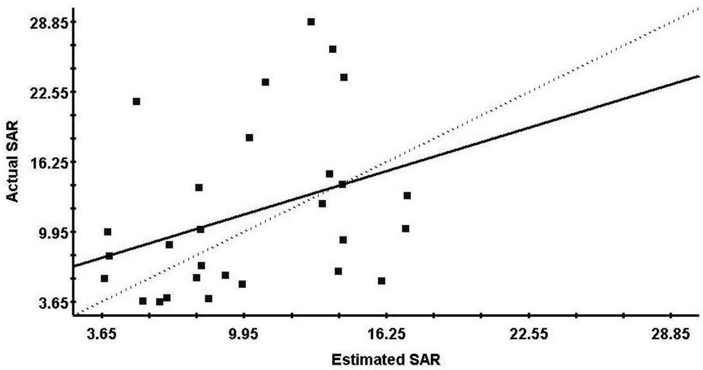


Figure 5: Cross validation between kriged and actual values of SAR. (The solid line is the regression line, the doted line is for r = 1)

Figure (5) shows the linear regression between the kriged and actual values of sodicity. The regression equation resulted from the cokriging cross validation is as follows:

Kriged SAR (predicted) = 5.31 + 0.62 SAR (measured)
r = 0.35 SE prediction = 6.97

The standard error of kriging prediction is much higher (6.97) than that of cokriging (4.69). The kriging correlation coefficient is very poor (0.35), as compared to the cokriging one (0.78).

For these reasons, cokriging is much preferred over kriging, especially in the case of under-sampling the variable of interest. Moreover, there must be an intensely sampled covaraible, which is correlated with the variable of interest.

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تحليل جيواحصائي لصودية الطبقة السطحية باستخدام طريقة حساب التواجد المشترك

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تقدم الإحصاء الجيولوجية أدوات وصفية لتمييز التوزيع الفراغى لخواص الأرض. تعتمد طرق kriging على الارتباط الفراغى بين العينات للتنبؤ بقيم الخاصية تحت الدراسة فى مواقع لم يؤخذ منها عينات. من جانب آخر, فان cokriging يستفيد من الارتباط الفراغى بين خاصيتين لرسم خريطة توزيع الصفة الأساسية تحت الدراسة (والتى يؤخذ منها عينات قليلة) باستخدام المحتوى المعلوماتى للمتغير الثانوى (والذى يؤخذ منه عينات كثيرة). تستخدم طريقة collocated cokriging عندما يتم جمع عينات المتغير الأساسي و المتغير الثانوى من نفس المكان. تهدف الدراسة الحالية إلى استخدام طريقة cokriging لرسم خريطة توزيع نسبة ادمصاص الصوديوم SAR (المتغير الاساسى) للطبقة السطحية و المقاسة فى ٢٨ عينة باستخدام معلومات ملوحة الطبقة السطحية (المتغير الثانوى) والمقاسة فى ١٣٤ عينة. وقد تراوحت قيم SAR فى الطبقة السطحية بين ٣,٦٥ و ٢٨,٨٥ , فى حين اختلفت قيم ملوحة الطبقة السطحية بين ٠,٣٦ و ١٢,٢٤ dS/m . وقد كان معامل الارتباط r بين الملوحة و الصودية ٠,٩٨ مما يوفى أهم شرط لاستخدام cokriging وهو وجود معامل ارتباط عالى بين المتغيرين. وقد تم عمل fitting لشكل توزيع الاختلافات semivariogram لكل من الخاصيتين (الملوحة و الصودية) وقد كان يتبع نموذج Gaussian وكذلك التصاحب بين الخاصيتين cross-semivariogram كان يتبع نموذج Gaussain. وقد تم رسم خريطة توزيع الصودية باستخدام cokriging و مقارنتها بخريطة توزيع الصودية باستخدام kriging. بالإضافة إلى أن القيم المقدرة بكل من الطريقتين قد تم عمل cross-validation لها ورسم خريطة توزيع الخطأ القياسى لمقارنته بالقيم المحسوبة. وقد أوضح هذا البحث أفضلية استخدام الـ cokriging عن استخدام kriging كأحد طرق رسم الخرائط, خاصة إذا تم تجميع عينات قليلة للمتغير تحت الدراسة, مع وجود متغير آخر له علاقة ارتباط قوية مع المتغير الأساسى .