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MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

PREDICTION OF PRESSURE DROP ACROSS AIR FILTER FIBROUS MEDIA

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ABSTRACT

In the domain of air filtration, the pressure drop across filtering medium represents, together with the retention efficiency the principal measures of filter performance. Theoretical prediction of the pressure drop is essential, especially in case of design of new filters. The Pich formula is widely used for prediction of the pressure drop across a fibrous medium in terms of medium structural parameters and fluid properties. This paper presents a comprehensive investigation of this formula on the basis of experimental measurements of the pressure drop across various types of fibrous media and comparison of the test results with the theoretical prediction by Pich for different values of air flow rate. Analysis of results and conclusions are included.

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THEORETICAL ANALYSIS

Of primary importance in the argument of air filtration is the pressure drop and retention efficiency. Both of these parameters combine to determine the overall performance characteristics of air filter for practical application. The pressure drop is of particular significance, since it is not difficult to prepare a filtering medium of practically absolute efficiency, but the higher associated pressure drop would limit the filter utilization. Generally, the pressure drop across an air filtering medium depends essentially on medium structure and size, velocity and temperature of air passing through medium and degree of medium contamination by dust. Numerous investigators have derived different formulae for theoretical prediction of pressure drop across the fibrous media applying the classical hydrodynamics laws with particular assumptions[2,3]. One of the widely applied formulae is that proposed by Pich, where the pressure drop δp is expressed as follow [1,2] :

$$\Delta p = F^{*} \mu \; u \; l_f^{'} \tag{1}$$

Where, F^{*} is the dimensionless fiber drag parameter, μ is the air dynamic viscosity ($\mu = 1.81 \times 10^{-5} N.s/m^2$), u is the air velocity through medium and l_f is the length of medium fiber per unit medium area.

According to Kuwabara theory for the flow around cylindrical fibers [2,3], F^{*} is defined as :

$$F^{*} = \frac{4 \pi}{K^{'}} \tag{2}$$

in which, $K^{'}$ is the hydrodynamic factor given by the following relationship :

$$K^{'} = -\frac{1}{2} \ln \beta + \beta - \frac{1}{4} \beta^2 - \frac{3}{4} \tag{3}$$

where β is the packing density of filtering medium obtained as :

$$\beta = 1 - \varepsilon \tag{4}$$

ε is the total porosity of filtering medium.

The air velocity u is determined as follows:

$$u = \frac{Q}{S} \tag{5}$$

where Q is the volume flow rate of air and S is the filtering medium unit area.

$l_f^{'}$ is given by the following relation :

$$l'_f = \frac{4\ m_f}{\pi\ d_f^2\ \rho_f} \tag{6}$$

where, m_f is the specific mass of medium fibers (fibers mass per medium unit area), d_f is the fiber diameter and ρ_f is the fiber density.

m_f is determined as follows:

$$m_f = \rho_f \beta\ L$$

where L is the thickness of filtering medium, obtained as:

$$L = \frac{V_B}{S} \tag{7}$$

V_B : bulk volume of medium. Substituting the above expression of m_f in equation (6), we get :

$$l'_f = \frac{4\ \beta\ L}{\pi\ d_f^2} \tag{8}$$

It is to be pointed out that Pich formula is valid in the regime of laminar flow, i.e. at low velocity of air or small values of Reynolds number. The formula assumes that the air passing through the filtering medium is absolutely clean.

EXPERIMENTAL WORK

Different types of filtering media were subjected to practical testing of pressure drop. On the other hand, for theoretical calculation of pressure drop across the tested media applying Pich formula, the porosity of media was determined experimentally.

Tested Filtering Media :

Two groups of fibrous filtering media used for automotive air filtration and available in the local market were adopted for conducting the pressure drop tests :

- paper filtering media of four types designated as Pa_1, Pa_2, Pa_3 and Pa_4.
- synthetic filtering media of three types : S_1, S_2 and S_3.

The fiber diameter d_f of the tested media, as prescribed in manufacturer data is indicated in table 1, below.

Table 1 : Fiber diameter d_f of tested filtering media

Medium	Paper media				Synthetic media		
	Pa_1	Pa_2	Pa_3	Pa_4	S_4	S_2	S_3
d_f (μm)	6.5	15	17	18	50	64	76

Determination of Porosity :

The total porosity ε of a filtering medium is defined as follows :

$$\varepsilon = \frac{V_p}{V_B} \times 100 = \frac{V_B - V_m}{V_B} \times 100 = 1 - \frac{V_m}{V_B} \times 100 \quad (\%)$$

where, V_p is the pore volume in filtering medium, V_B is the medium bulk volume and V_m is the volume of medium matrix.

The bulk volume V_B is measured using the porosimeter (Fig.1). A square sample of the filtering medium of area 7 × 7 cm^2 is immersed in the porosimeter mercury chamber (of known volume) and the resulting mercury displacement is measured. Such displacement is the sample bulk volume V_B. This is based on the fact that due to mercury surface tension and its non-wetting properties, the mercury does not penetrate into the pores of immersed sample and consequently its displacement expresses the value of V_B.

The volume of filtering medium matrix V_m is determined, through measuring the weight of medium sample when dry and when immersed in water, and then applying the Archimedes Principle. The sample dry weight W and sample weight during immersion in water W are measured by means of a special balance (Fig.2).
The balance upper pan is used for measuring the dry weight W while the lower pan measures the wet weight W . From the values of W and W , V_m is determined according to the Archimedes Principle as follows:

$$W - W' = V_m \rho_m \qquad \rho_m: \text{water density} = 1 \ (g/cm^3)$$

Therefore, $$V_m = \frac{W - W'}{\rho_m}$$

The results obtained for the measurement of V_B, W and W' and the resulting porosity ε are given in table 2 below.

Table 2 : Determination of porosity ε of filtering media.

Filter medium	V_B [cm^3]	W [g]	W' [g]	ΔW = W-W' [g]	$V_m = \frac{\Delta W}{\rho_m}$ [cm^3]	ε [%]
Pa_1	1.62	0.6341	0.3741	0.260	0.260	83.951
Pa_2	4.21	1.0832	0.4972	0.586	0.586	86.081
Pa_3	3.10	0.7551	0.3541	0.401	0.401	87.064
Pa_4	3.29	0.7261	0.3661	0.360	0.360	89.058
Pa_5	41.3	3.6261	1.1761	2.450	2.450	94.068
Pa_6	41.3	6.1173	1.4813	1.636	1.636	95.579
Pa_7	26.4	1.5791	0.7951	0.784	0.784	97.030

Measurement of pressure Drop:

The pressure drop across the tested filtering media was measured using the air cleaner test rig available in the M.T.C., Tanks Department (Fig.3). Testing was conducted on rectangular specimens of the filtering media of dimensions 41 × 15 cm each. A special holder for the tested specimens with the necessary interface components with rig were manufactured.

The pressure drop across the tested media was measured at five values of volume flow rate of air : 0.062, 0.076, 0.09, 0.101, and 0.11 (m^3/s). This corresponds to values of air velocity through the tested specimens of 1.008, 1.463, 1.642, and 1.805 (m/s) respectively.

RESULTS AND DISCUSSION

The theoritical pressure drop ΔP_{th} is calculated according to Pitch formula expressed by the equation (1). In this equation, F^* is determined by equation (2) starting with, the predetermined values of ε (table 2), then calculating β and K according to equations (4) and (3) respectively. The air velocity u is calculated by equation (5) for the flow rate values indicated previously. l_f is determined by equation (8) using the given values of d_f (table 1) and values of L calculated by equation (7), with V_B taken from table (2).

The results of calculation of β, K', F^*, L and l_f' are illustrated in the following table.

Table3:Calculated values of β, K', F^*, L and I_f' for the tested filtering media.

Filtering medium	β	K'	F^*	L [cm]	$I_f' \times 10^{-2}$ [m/m^2]
Pa_1	0.16049	0.3188	39.4177	0.033	15960.457
Pa_2	0.13919	0.3700	33.9770	0.086	4704.046
Pa_3	0.12936	0.3980	31.5860	0.063	3590.483
Pa_4	0.10942	0.4630	27.1520	0.067	2880.956
S_1	0.05932	0.7210	17.4360	0.843	2546.823
S_2	0.04421	0.8530	14.7380	0.843	1158.506
S_3	0.02970	1.0380	12.1110	0.539	362.354

The results of the theoretically calculated values of pressure drop Δp_{th} and the experimentally measured values Δp_{ex} are given in table 4 below for the test values of air flow rate Q.

Table 4 : Theoritical and experimental pressure drop at different flow rates.

Filtering Medium	Q = 0.062 [m³/s]		Q = 0.076 [m³/s]		Q = 0.090 [m³/s]		Q = 0.101 [m³/s]		Q = 0.111 [m³/s]	
	Δp_{th} [Pa]	Δp_{ex} [Pa]	Δp_{th} [Pa]	Δp_{ex} [Pa]	Δp_{th} [Pa]	Δp_{ex} [Pa]	Δp_{th} [Pa]	Δp_{ex} [Pa]	Δp_{th} [Pa]	Δp_{ex} [Pa]
Pa_1	1149.345	1491.12	1409.316	1726.56	1668.146	2099.34	1872.246	2452.50	2058.103	2707.56
Pa_2	419.242	627.84	514.070	784.80	608.483	1000.62	682.932	1167.39	750.726	1334.16
Pa_3	206.582	372.78	253.309	461.07	299.831	549.36	336.515	637.65	369.921	735.06
Pa_4	142.490	255.06	174.720	343.35	206.808	292.40	232.112	441.45	255.153	510.12
S_1	80.889	98.10	99.185	117.72	117.401	156.96	131.766	196.20	144.846	215.82
S_2	31.101	39.24	38.136	58.86	45.140	68.67	50.662	78.48	55.692	98.10
S_3	7.994	9.81	9.802	19.62	11.602	29.43	13.022	39.24	14.315	49.05

For the purpose of comparison, the results obtained for Δp_{th} and Δp_{ex} are interpreted graphically in figures 4, 5, 6, and 7 for the paper filtering media and figures 8, 9, and 10 for the synthetic media. The figures indicate that generally, the experimentally measured values of pressure drop Δp_{ex} are always higher than the theoretical values Δp_{th} given by Pich formula, but with different rates depending on the type of filtering medium. The discrepancy between Δp_{th} and Δp_{ex} could be referred to the fact that Pich formula was developed on the basis of approximating the fibrous filtering medium by a system of parallel cylinders which is not absolutely true. On the other hand, the air passed through the filtering media during experimentation is not absolutely pure but actually contains environmental fine particles which cause an additional rise in pressure drop. For quantitative evaluation of the argumentation of Δp_{ex} over Δp_{th} the ratio $\frac{\Delta p_{ex}}{\Delta p_{th}}$ was calculated for the different media at the different values of flow rate Q. The results obtained are illustrated in figures 11 and 12 for the paper and synthetic media respectively. The figures indicate a quasi-linear increase of this ratio with Q, of the form $\frac{\Delta p_{ex}}{\Delta p_{th}} = a + b\ Q$. The coefficients a, b for the different filtering media obtained through the least square approximation are given in the following table.

Table 5 : Coefficients of linear dependence of $\frac{\Delta p_{ex}}{\Delta p_{th}}$ on Q for the different filtering media.

Medium	Pa_1	Pa_2	Pa_3	Pa_4	S_4	S_2	S_3
a	1.23	1.11	1.57	1.65	0.73	0.8	-1.42
b	0.65	5.97	3.35	2.92	6.95	8.1	43.86

From figure 11 of paper media, it is noticed that the ratio $\frac{\Delta p_{ex}}{\Delta p_{th}}$ remains nearly constant for the medium Pa_1 with a value for about 1.28 and slightly increases in medial Pa_3 and Pa_4 ranging from 1.8 to 1.94. For the paper medium Pa_2 , the increase of ($\frac{\Delta p_{ex}}{\Delta p_{th}}$) with Q is more remarkable, varying from 1.46 to 1.76 in magnitude.
For the synthetic media, figure 12 shows more intensive increase of the ratio $\frac{\Delta p_{ex}}{\Delta p_{th}}$ with the air flow rate Q. This ratio reaches 1.47 for medium S_1, 1.56 for S_2 and 3.46 for S_3.

Modification of Pich Formula :

Due to the remarkable increase of the actual pressure drop found by experimentation over the theoretical prediction by Pich formula proved throughout this work, a correction factor γ is proposed to be introduced in this formula, which would take the form :

$$\Delta p = \gamma\ F^{*}\ \mu\ u\ I_f'$$

The correction factor γ is always greater than unity and is generally a function of the flow rate Q. The value and variation of γ with Q depend essentially on the type of filtering medium. This correction factor would be determined through implementing a series of pilot tests on specimens of the assigned filtering media and comparing the experimental results with the theoretical values attained by the original formula of Pich, as proceeded in this work. After determination of this factor, the modified formula of Pich could be applied for the design of the complete air filter in view of the pressure drop.

CONCLUSIONS

1. An analysis of the Pich formula for prediction of the pressure drop across a fibrous filtering medium used in air filters was introduced.
2. An experimental work has been developed for identification of the structural properties of seven selected types of paper and synthetic filtering media in order to calculate the pressure drop across media applying the Pich formula.
3. The pressure drop across the selected media was calculated by the original formula of Pich and measured experimentally at different values of air flow rate.
4. For investigating the effectiveness and precision of Pich formula, the experimental and theoritical results have been compared together. The comparison proved a significant discrepancy towards a remarkable increase of experimental results over the theoritical prediction by Pich. This necessitates the modification of the original Pich formula, which has been developed through introducing a correction factor into the formula. The methodology of determination of the correction factor was presented.

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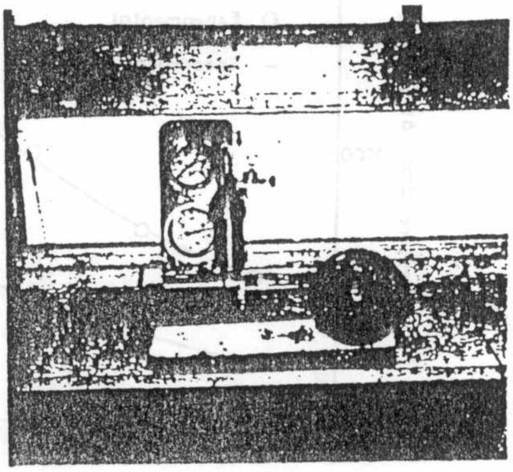


Fig. 1: The porosimeter for measuring of bulk volume of filtering medium

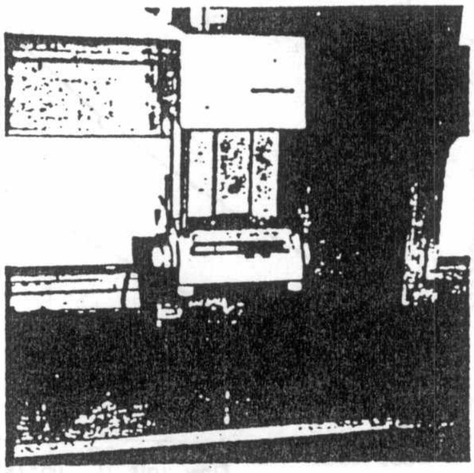
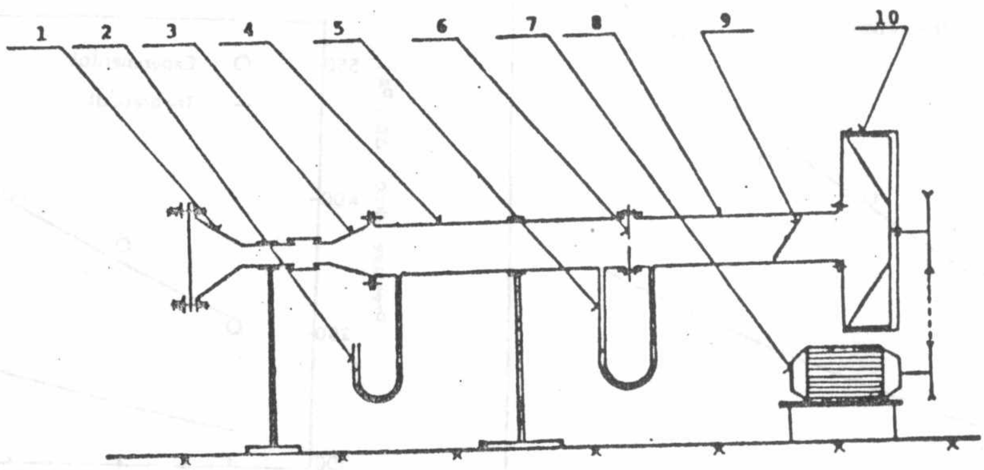


Fig. 2: The balance used for measuring the dry and wet weights of medium samples



1. Holder with tested sample.
2. Ordinary manometer.
3. Connecting cone.
4. Front pipe.
5. Differential manometer.
6. Orifice plate.
7. Electric motor.
8. Rear pipe.
9. Control shutter.
10. Centrifugal fan.

Fig. 3: Air cleaner test rig

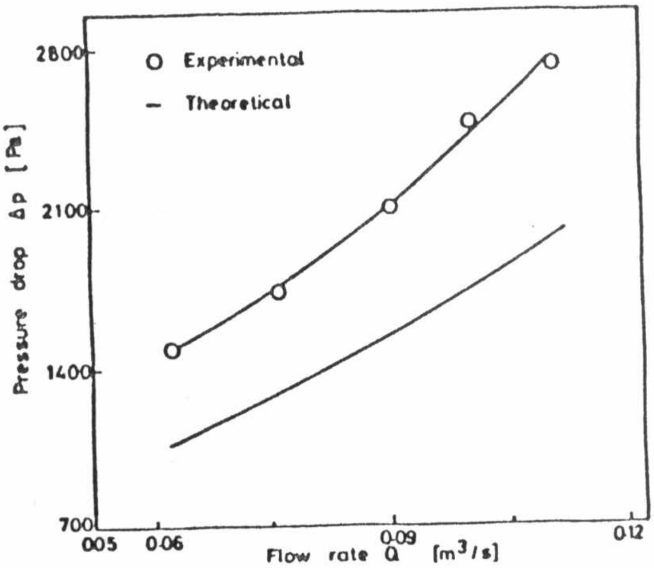


Fig. 4: Comparison of theoretical and experimental pressure drop-paper medium Pa_1

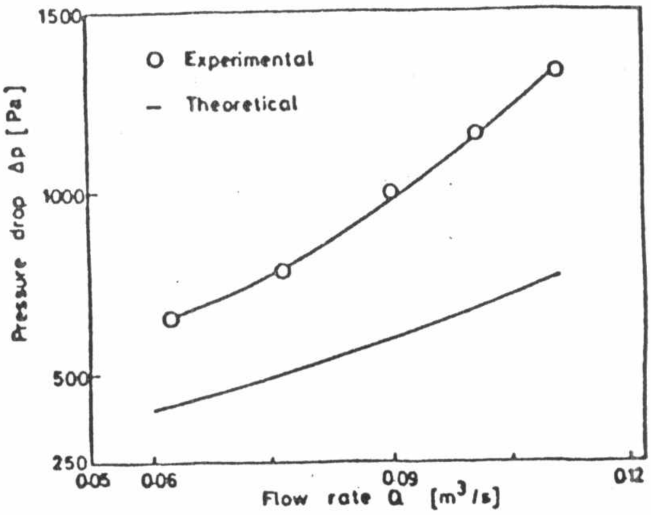


Fig. 5: Comparison of theoretical and experimental pressure drop-paper medium Pa_2

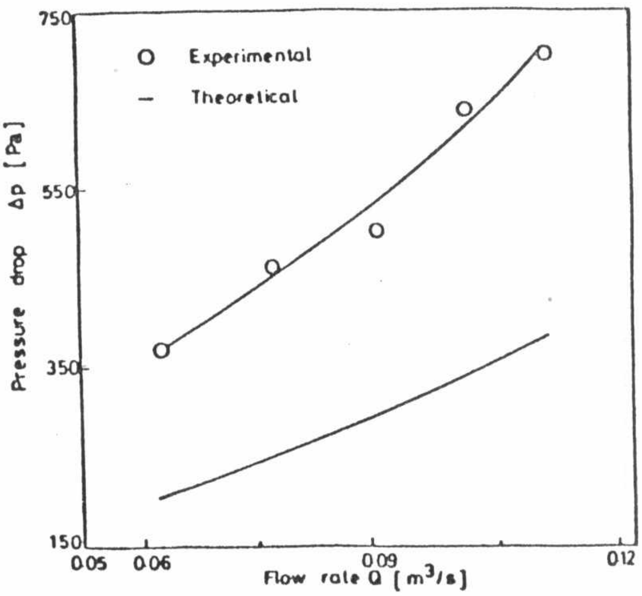


Fig. 6: Comparison of theoretical and experimental pressure drop-paper medium Pa_3

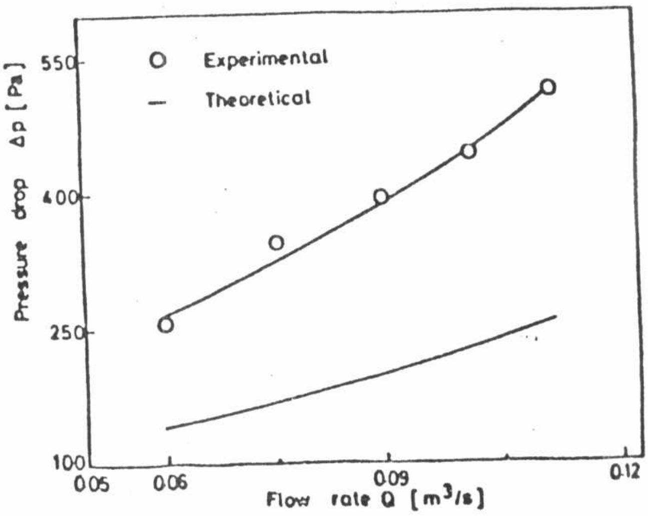


Fig. 7: Comparison of theoretical and experimental pressure drop-paper medium Pa_4

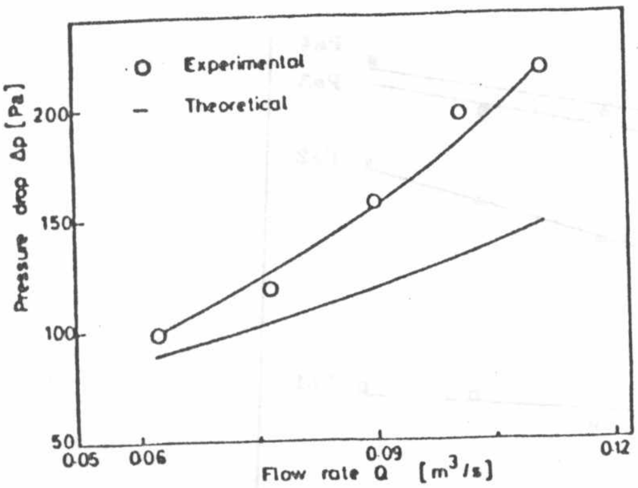


Fig. 8: Comparison of theoretical and experimental pressure drop-paper medium S_1

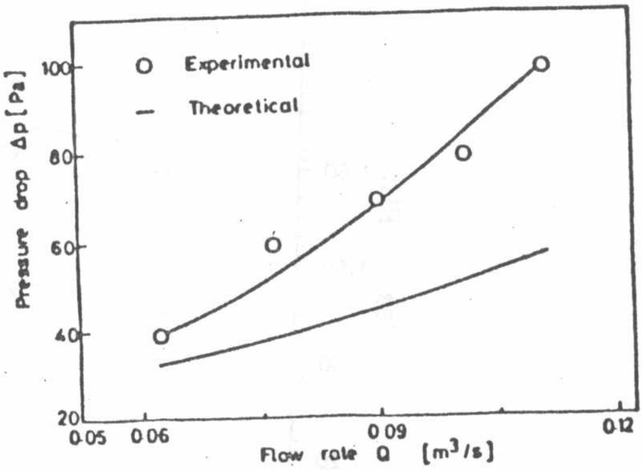


Fig. 9: Comparison of theoretical and experimental pressure drop-paper medium S_2

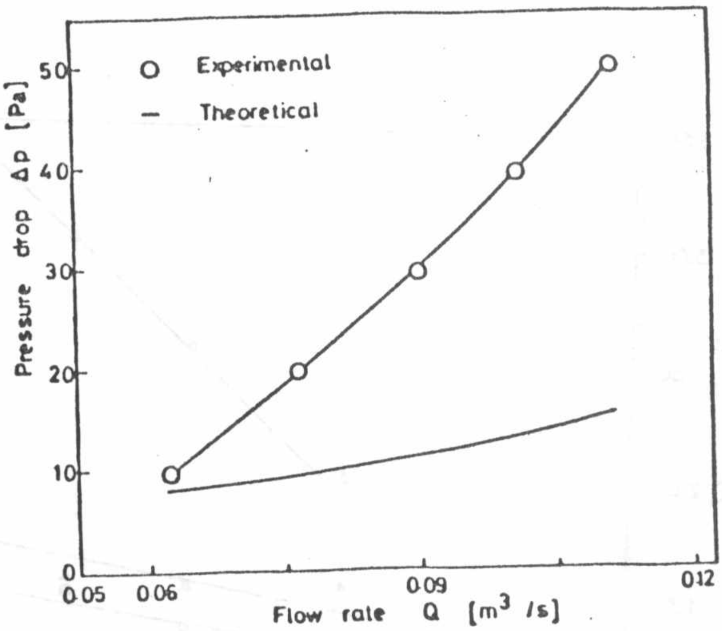


Fig. 10: Comparison of theoretical and experimental pressure drop-paper medium S_3

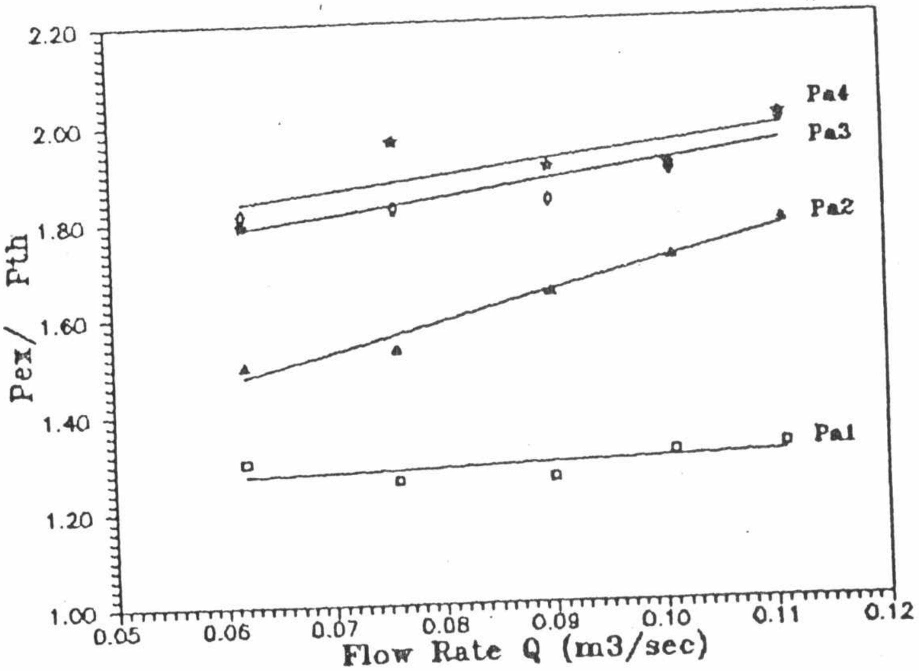


Fig. 11: Evolution of $\Delta P_{ex}/\Delta P_{th}$ with flow rate Q - paper media

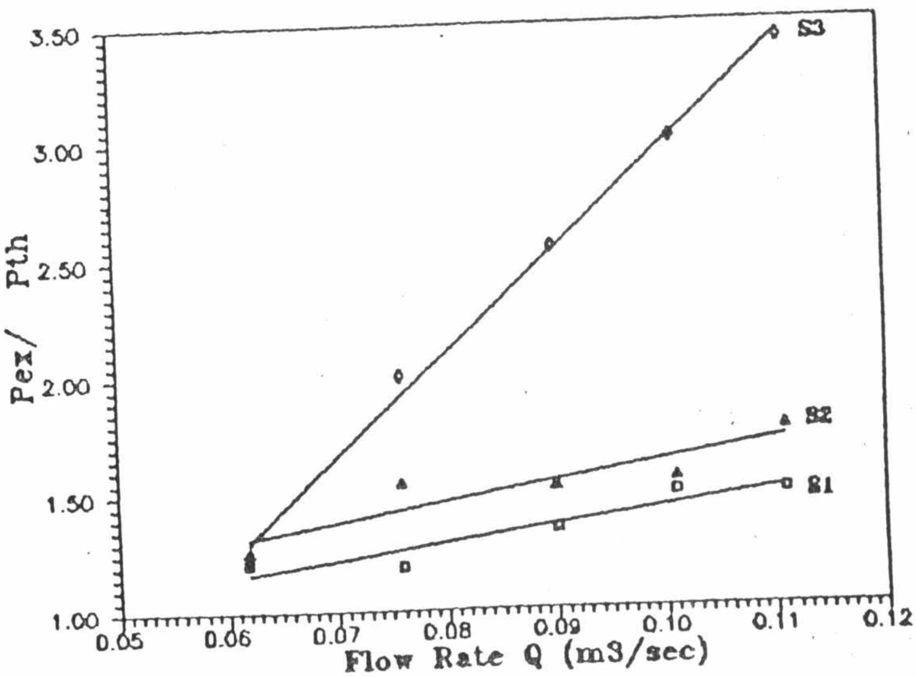


Fig. 12: Evolution of $\Delta P_{ex}/\Delta P_{th}$ with flow rate Q - synthetic media