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COMPUTER AIDED OPTIMUM DESIGN OF A CAM ACTUATED SHOE BRAKE

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ABSTRACT

An optimization model for a cam actuated pivoted shoe brake used in Automobiles is presented. In this model the objective functions are: to minimize the mass of the brake drum, surface area of the brake lining and wear of the brake lining. The first and second objectives reflect the brake assembly weight; whereas the third one represents the service life of the brake. Eleven design variables are considered. Ten of them are related to the dimensions of the wheel brake assembly while the eleventh variable is the maximum air pressure in the brake system. The restrictions imposed on these variables are: the geometric constraints which reflect the size of the car wheel and behavior constraints which restrict the air pressure of the brake system, the lining pressure, the self-locking of shoes, the slip between the surface of lining and the surface of the drum, overheating of the brake assembly and the skidding of the vehicle wheels. A computer program is developed to solve this model. The program allows the designer to participate interactively in the process of seeking an optimum solution which satisfies the conflicting objectives. A numerical example is presented.

INTRODUCTION

The study of optimum design of vehicle brakes has been of continual interest since the later part of 20th century. From the review of available literature of the topics relating to brake optimum design for motor vehicles the following may be concluded [1-4] :

- The objective function may be the minimum stopping time, the minimum mass of brake assembly and/or minimum lining wear.
- The most significant objective function is the brake assembly mass.
- The design variables are the dimensions, the fluid pressure and the lining material.
- The constraints on the design are: geometry constraints due to tire size, constraint to avoid skidding of wheels, constraint to avoid over heating of lining, constraint due to lining material properties, constraint on the actuating force or oil pressure and constraint on the stopping time if it is not considered as an objective.

In the present work, a *Computer Aided Optimum Design* (CAOD) program is developed for a *cam actuated pivoted shoe brakes* used for vehicles. The design objectives are minimum drum mass, minimum lining surface area, minimum lining wear, or any combination of them(multi objectives).

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OPTIMIZATION MODEL SETUP

Generally speaking the optimum design problem can be formulated in the following way :

Find vector of design variables $X^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]^T$

such that :

$$f(X^*) = \text{optimum } f(X) \tag{1}$$

and such that:

$$g_i(X) \geq 0 \qquad i = 1,2,\ldots,m \tag{2}$$

$$h_j(X) = 0 \qquad j = 1,2,\ldots,p \tag{3}$$

where; $X = \left[x_1, x_2, \ldots, x_n\right]^T$ is the vector of design variables defined in the n-dimensional Euclidean space E^n, $f(X) = \left[f_1(X), f_2(X), \ldots, f_k(X)\right]^T$ is the vector function defined in k-dimensional Euclidean space E^k, each element of $f(X)$ represents an individual objective which has to be examined in the optimization model, and $g_i(X)$, $h_j(X)$ are functions of the design variables and represent the constraints imposed on the design variables.

For the optimization problem of a cam actuated pivoted shoe brake, shown in Fig.1, the following optimization model is assumed and for compactness; the final formulas are given while complete derivation of the given equations is presented in reference [5]:

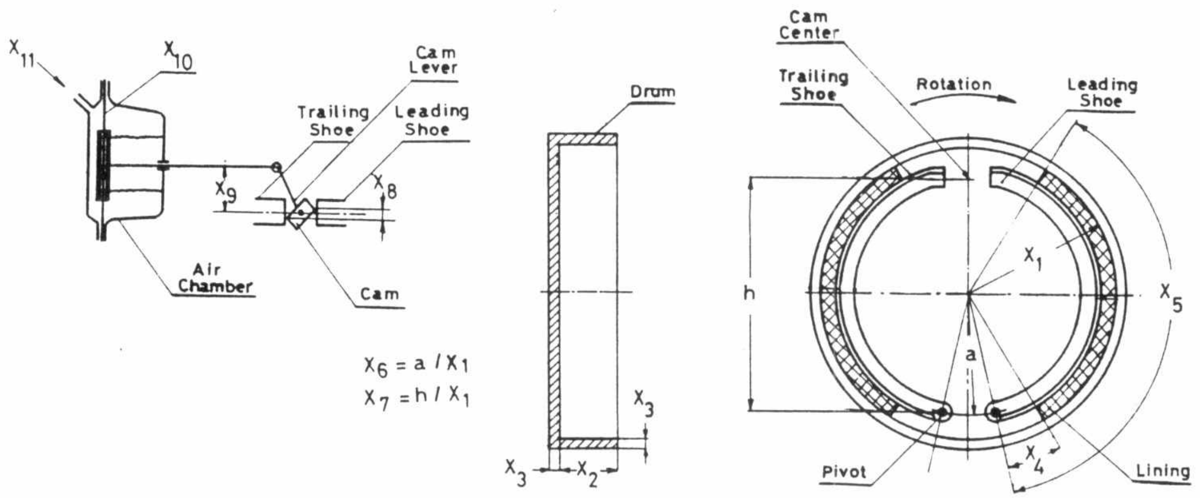

Fig.1 Schematic of cam actuated pivoted shoe brake [6]

A) The vector of design variables considered in this problem is of dimension n = 11, and can be represented as follows:

$$X = \left[X_1, X_2, \ldots, X_{11}\right]^T \tag{4}$$

where;

X_1 is the drum inner radius (m),
X_2 is the drum or lining width (m),
X_3 is the drum thickness (m),
X_4 is the lining start angle (Rad.),
X_5 is the lining end angle (Rad.),
X_6 is the pivot distance factor,
X_7 is the cam distance factor,
X_8 is the distance between the shoes actuating forces (m),
X_9 is the cam lever arm length (m),
X_{10} is the air chamber area (cm^2), and
X_{11} is the air pressure of the brake system (Pa).

B) To represent the design objectives, the vector function f(x) is formulated as follows :

$$f(x) = \left[f_1(x), \; f_2(x), \; f_3(x)\right]^T \tag{5}$$

where;

$f_1(X)$ is the minimum drum mass objective (Kg),
$f_2(X)$ is the minimum lining surface area objective (m^2),and
$f_3(X)$ is the minimum lining wear objective (m).

A formulation of the first and the second objective can be derived in a simple way using Fig.1.

$$f_1(X) = \pi \, G_d \left\{ \left[\left(X_1 + X_3\right)^2 - X_1^2 \right] X_2 + \left(X_1 + X_3\right)^2 X_3 \right\} \tag{6}$$

$$f_2(X) = 2 \, X_2 \, X_1 \left(X_5 - X_4\right) \tag{7}$$

The third objective may be given by:

$$f_3(x) = \frac{P_m}{(9 \; Sy/K)} \left(\frac{S_b}{r_d}\right) X_1 \tag{8}$$

where;

$$P_m = \frac{F \; X_7}{X_2(A_f - B_f)} \quad , \tag{9}$$

$$F = \frac{X_9 \; X_{10} \; X_{11}}{X_8} \left(1 - \frac{B_f}{A_f}\right) \quad , \tag{10}$$

$$A_f = \frac{X_6 \; X_1}{4} \left[2\left(X_5 - X_4\right) + \sin 2X_4 - \sin 2X_5\right] \quad , \tag{11}$$

$$B_f = \mu \; X_1 \left[\cos X_4 - \cos X_5 + \frac{X_6}{2}\left(\sin^2 X_4 - \sin^2 X_5\right)\right] \quad , \tag{12}$$

$$S_b = 0.5 \; t_{in} \; V_o + \frac{V_o^2}{2a_{max}} - \frac{t_{in}^2 \; a_{max}}{8} \quad , \tag{13}$$

$$a_{max} = \eta_b \; \mu_t \; g \quad , \tag{14}$$

G_d is the specific mass of the drum material (Kg/m^3),
P_m is the maximum pressure on the lining (Pa),
Sy is the lining yield strength (Pa),
K is the wear coefficient, its value is chosen according to the materials of both drum and lining [1],
S_b is the braking distance (m),
r_d is the wheel dynamic radius (m),
A_f, B_f is the lining factors (m),
F is the leading shoe actuating force (N),
μ is the lining coefficient of friction,
a_{max} is the maximum deceleration (m/S^2),
t_{in} is the time during which deceleration increased to its maximum value (0.2 S - 0.45 S),
V_o is the initial vehicle speed (m/S).
η_b is the braking efficiency,
g is the gravity acceleration (9.8 m/S^2), and
μ_t is the road coefficient of adhesion.

C) A total of 27 constraints are considered to be imposed on the design variables. These constraints are:

Design variable limits constraints :

The design Variable limits constraints consist of a group of inequality constraints which reflect the size available for the brake and the air pressure limits. These constraints are expressed by:

$$g_j(x) = X_{mxj} - X_j \geq 0 \quad , j = 1,11 \tag{15}$$

$$g_k(x) = X_j - X_{mnj} \geq 0 \quad , k = 12,22 \tag{16}$$

where;

X_{mxj} is the maximum limit of the design variable X_j,

X_{mnj} is the minimum limit of the design variable X_j, and

X_j is the design variable as defined by equation 4

Lining pressure constraint :

The maximum pressure between brake shoe lining and the drum must not exceed the allowable value of the lining pressure. This is to guard against crushing.

$$g_{23}(X) = P_{m_{max}} - P_m \geq 0 \tag{17}$$

where;

$P_{m_{max}}$ is the Lining pressure allowable value of lining material (Pa).

Self locking constraint :

This constraint is to prevent the self locking of the leading shoe and is expressed by [5] :

$$g_{24}(x) = 0.7 - \frac{B_f}{A_f} \geq 0 \tag{18}$$

Slip constraint :

The slip speed between the lining and the drum surface must not exceed its allowable value and accordingly;

$$g_{25}(x) = V_{s_{max}} - V_s \geq 0 \tag{19}$$

where;

$V_{s_{max}}$ is the permissible value of slip speed of the lining material (m/s), and

V_s is the slip speed (m/s), and can be given by:

$$V_s = V_o \frac{X_1}{r_d} \tag{20}$$

Temperature constraint :

The overheating of lining is limited as follows:

$$g_{26}(x) = T_{max} - T_d \geq 0 \tag{21}$$

where;

T_{max} is the permissible temperature of lining material (^{o}C), and

T_d is the surface temperature of the drum during the braking (^{o}C), and can be given by:

$$T_d = T_o + \frac{N_d\, X_3}{\lambda_d\, \Omega_d^2} \left[0.5 + 0.0778\, \Omega_d^4 \right] \tag{22}$$

$$N_d = \frac{E_d}{t_b} , \tag{23}$$

$$E_d = \frac{\sigma_d\, y\, W(1 + Z) V_o^2}{2\, g\, A_d} , \quad \text{(for a wheel of the front axle)} \tag{24}$$

$$E_d = \frac{\sigma_d\, y'\, W(1 + Z) V_o^2}{2\, g\, A_d} , \quad \text{(for a wheel of the rear axle)} \tag{25}$$

$$\sigma_d = \frac{1}{1 + 0.09\, \dfrac{(X_5 - X_4)}{\pi}} , \tag{26}$$

$$\Omega_d = \frac{X_3}{\left(\alpha_d\, t_b\right)^{0.5}} , \tag{27}$$

$$\alpha_d = \frac{\lambda_d}{G_d\, C_d} , \tag{28}$$

$$y = \frac{W_{fs}}{W} + \left(\frac{H}{L}\right)\left(\frac{a_{max}}{g}\right) , \tag{29}$$

$$y' = \frac{W_{rs}}{W} - \frac{H}{L}\left(\frac{a_{max}}{g}\right) , \tag{30}$$

where;

T_o is the ambient temperature (^{o}C),

σ_d is the proportion of heat dissipated that enters the drum through rubbing path area ,

Z is the coefficient of mass increase due to rotating masses,

α_d is the thermal diffusivity of drum or rotor material,

λ_d, G_d, C_d are the thermal conductivity ($J/m\ S^{o}C$), specific mass (Kg/m^3)density, and specific heat($J/Kg\ ^{o}C$)of drum; respectively,

y, y' is the axle dynamic load coefficient of the front and rear axles; respectively,

W_{fs}, W_{rs} is the static load on the front and rear axles; respectively,

W is the vehicle weight (N),
H is the C.G. height of the vehicle (m), and
L is the wheel base of the vehicle (m).

Skidding constraint :

To avoid skidding of the vehicle wheels during braking, the braking torque must not exceed the adhesion torque. This constraint is an equality constraint to ensure minimum braking time and distance, and is expressed by:

$$h_1(x) = M'_{bw} - M_{bw} = 0 \tag{31}$$

where;

$$M'_{bw} = 0.5\, y\, \mu_t\, W\, r_d \quad , \text{(for a wheel of the front axle)} \tag{32}$$

$$M'_{bw} = 0.5\, y'\, \mu_t\, W\, r_d \quad , \text{(for a wheel of the rear axle)} \tag{33}$$

$$M_{bw} = 2\mu\, P_m\, X_2\, X_1^2 \left(\cos X_4 - \cos X_5\right) \quad , \tag{34}$$

M'_{bw} is the adhesion torque of one wheel (N.m), and
M_{bw} is the total shoe torque for one wheel (N.m).

METHOD OF SOLUTION

To solve the optimization model formulated above, a Computer Aided Optimum Design (CAOD) program has been developed. This CAOD program utilizes the International Math Scientific Library (IMSL) already existing on the VAX-11 computer available at the Military Technical College Computer Center. The modular programing technique is followed in order to easy add to or modify the program. Referring to Fig.2, the main modules of the developed CAOD program are:

a) Main title module
b) Input data module
c) Optimization module
d) Problem formulation module
e) Output data module

The main title module prints a main title showing the name of the program, the target of the program, the needed data for the program, and the restrictions on the program. The input data module receives the needed data for the program.

The optimization module, Fig.3, calculates the optimum design of the model by calling the subroutine, NCONF from IMSL. This subroutine solves this model using the successive quadratic programming algorithm and the finite difference gradient.

The problem formulation module, Fig.4, calculates the needed values at a given point such as A_f, B_f, F, P_m,.... etc. It also calculates the values of the objectives and the constraints at that point.

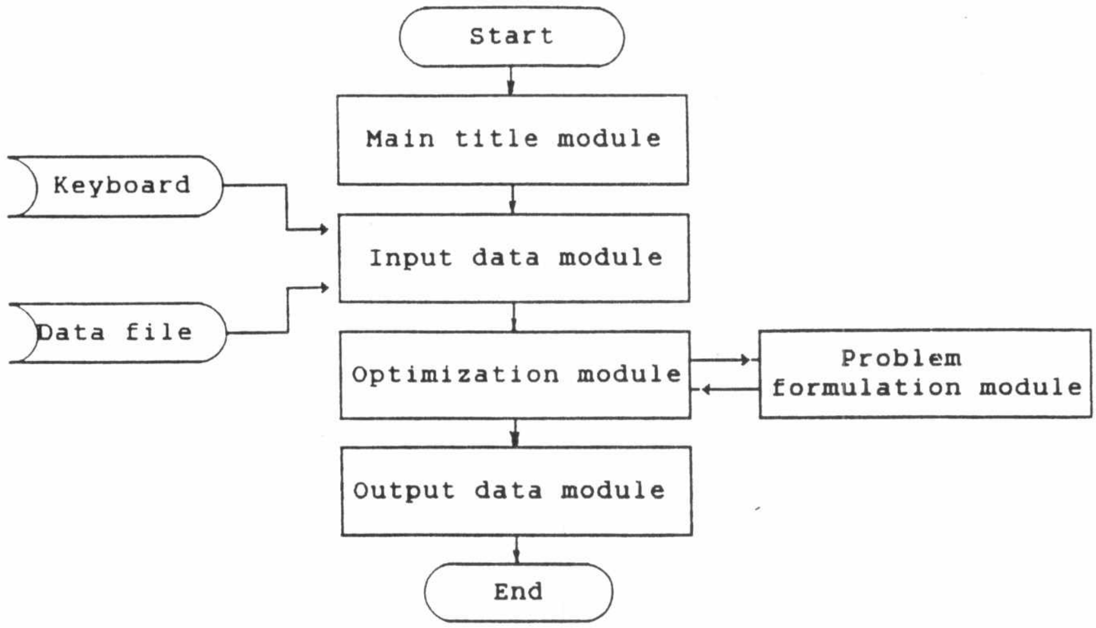

Fig.2 Layout of the CAOD program

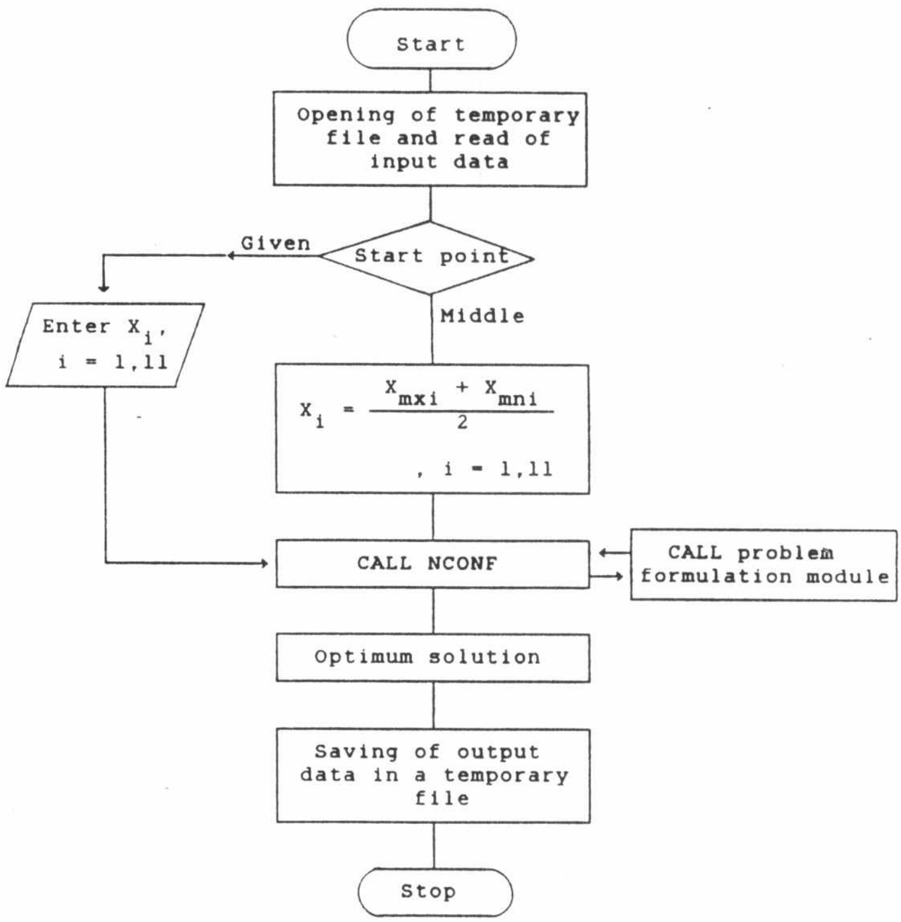

Fig.3 Flow chart of optimization module

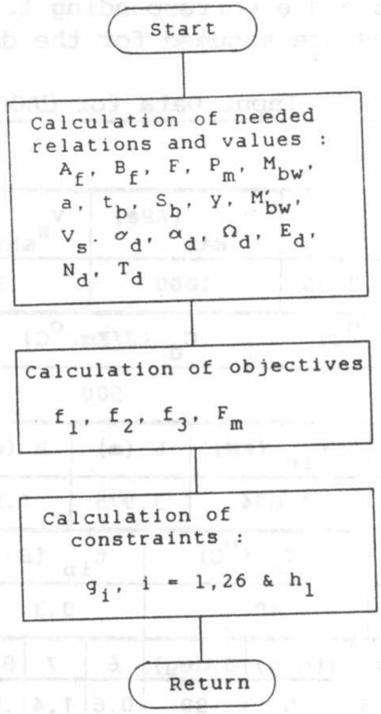

Fig.4 Flow chart of problem formulation module

It should be noted that in the objectives definition part, a weighted sum of the different objectives in the normalized form is considered to calculate the multi-objectives function as follows:

$$F_m = \sum_{i=1}^{3} w_i \frac{f_i - f_{mni}}{f_{mxi} - f_{mni}} \tag{35}$$

where;

F_m is the multi-objectives function,

w_1, w_2, w_3 are the weighting factors corresponding to f_1, f_2, f_3; respectively such that : $w_i \geq 0$, $w_1 + w_2 + w_3 = 1$,

f_i is the i^{th} objective, and

f_{mxi}, f_{mni} are the maximum and minimum values calculated for objective no. i; respectively.

The output data module shows the values of the objectives f_1, f_2, f_3, F_m, optimum value of the design variables, values of the constraints, dimensions of the brake assembly, and braking performance.

EXAMPLE OF THE OPTIMIZATION RESULTS

The optimization program illustrated in the previous section is used to determine the optimum design parameters of a cam actuated pivoted shoe brake with respect to minimum drum mass, minimum lining surface area, minimum lining wear and a multi-objectives of them. As an example, the brake of the military truck ZIL-131 is considered with the input data shown in Table 1. Most of these data are corresponding to the data available of this truck. Reasonable values are assumed for the data which are not given.

Table 1 Input Data for CAOD

Lining material properties	Sy (MPa)	μ	$P_{m\,max}$ (KPa)	$V_{s\,max}$ (m/s)	T_{max} (^oC)	K
	10	0.35	1000	35	250	5×10^{-6}

Drum material properties	λ_d (J/ms ^oC)	C_d (J/Kg ^oC)	G_d (Kg/m^3)
	50	500	7800

Vehicle configuration	W (KN)	W_{fs} (KN)	L (m)	H (m)	r_d (m)	Z	η_b
	99.813	31.654	3.975	1.5	0.53	0.05	1

Braking condition	μ_t	T_o (^oC)	t_{in} (S)	V_o(Km/hr)
	0.4	40	0.3	80

Limits for design variables	i	1(m)	2(m)	3(m)	4(deg)	5(deg)	6	7	8(m)	9(m)	10(cm^2)	11(MPa)
	X_{mni}	0.18	0.08	.005	25	90	0.6	1.4	.044	.110	70	0.5
	X_{mxi}	0.22	0.12	0.02	90	145	0.8	1.6	.066	.116	89.3	0.65

Entering these data to the CAOD program developed for the optimization model, the optimum values are obtained. Tables 2 & 3 show these results for the single objective (drum mass ,lining surface area or lining wear) and multi-objectives of them, respectively.

From these results, it is noticed that:

1- In case of minimum drum mass objective and minimum lining wear objective or any combination of them, the lining start angle is at its lower limit and lining end angle is at its upper limit. So in this case, these two design variables can be eliminated and accordingly the dimension of the design vector can be reduced by two i.e it will be nine only. However; in case of minimum lining surface area objective or a multi objectives in which the weighting factor of the lining surface area is greater than zero, these two design variables should be considered as they are very effective in the results.

2- In the example studied, the drum width is always at its upper limit. However; more study is needed before eliminating this variable from the design vector.

3- In case of minimum drum mass objective,the temperature constraint is at its critical value, and in case of minimum lining surface area objective the lining pressure constraint is at its critical value. This agrees with the logical expectation that the temperature rises with the drum mass decrease and the lining pressure increases with the decrease of the lining surface area.

Table 2 Summary of Single Objective Optimization Results

Optimum output data	Considered objective	Minimum drum mass	Minimum lining surface area	Minimum lining wear
Objectives	Drum mass (Kg)	11.00	33.2	33.2
	Lining area (Cm^2)	928	692	1106
	Lining wear (μm)	1.284	1.527	1.078
Design Variables	Drum radius (m)	0.185	0.22	0.22
	Drum width (m)	0.12	0.12	0.12
	Drum thickness (m)	0.0055	0.0125	0.0125
	Lining start angle (deg)	25	50	25
	Lining end angle (deg)	145	125	145
	Pivot distance factor	0.6	0.65	0.74
	Cam distance factor	1.6	1.54	1.51
	Distance between shoes actuating forces (m)	0.045	0.047	0.044
	Cam lever length (m)	0.116	0.116	0.116
	Air chamber area (Cm^2)	83.8	89.3	89.2
	Air pressure (MPa)	0.575	0.575	0.575
Constraints 0 or +ve value is OK	Lining pressure (KPa)	0.49	0.002	294
	Self locking	0.04	0.13	0.16
	Mean slip (m/s)	27	26	26
	Temperature (^oC)	0	82	82
	Skidding (N.m)	0	0	0

Table 3 Summary of Multi-objectives Optimization Results

Multi-Objective weighting factors								
	W_1	0.70	0.50	0.333	0.25	0.25	0.15	0.15
	W_2	0.15	0.25	0.333	0.50	0.25	0.15	0.70
Optimum output data	W_3	0.15	0.25	0.333	0.25	0.50	0.70	0.15
Objectives	Drum mass (Kg)	11.20	11.30	12.80	12.80	12.80	12.8	12.8
	Lining area (Cm^2)	972	859	695	695	1005	1028	695
	Lining wear (μm)	1.220	1.308	1.520	1.520	1.141	1.125	1.527
	Multi-Objective	0.1537	0.2343	0.3553	0.269	0.2773	0.2035	0.1620
Design Variables	Drum radius (m)	0.196	0.202	0.22	0.22	0.22	0.22	0.22
	Drum width (m)	0.12	0.12	0.12	0.12	0.12	0.12	0.12
	Drum thickness (m)	0.0052	0.005	0.005	0.005	0.005	0.005	0.005
	Lining start angle(deg)	26	41	50	50	36	34	50
	Lining end angle (deg)	145	143	125	125	145	145	125
	Pivot dis. factor	0.6	0.65	0.65	0.65	0.71	0.71	0.64
	Cam dis. factor	1.58	1.5	1.54	1.56	1.47	1.47	1.54
	Distance between the shoes actuating forces	0.044	0.044	0.047	0.046	0.044	0.044	0.046
	Cam lever length (m)	0.116	0.116	0.116	0.116	0.116	0.116	0.116
	Air chamber A. (Cm^2)	77.5	89.3	89.3	86.2	89.3	89.3	86.6
	Air pressure (MPa)	0.575	0.575	0.575	0.575	0.575	0.575	0.575
Constraints 0 or +ve value is OK	Lining pressure (KPa)	102.1	68.8	4.3	4.3	522.6	263.2	0.001
	Self locking	0.04	0.09	0.13	0.13	0.13	0.13	0.13
	Mean slip (m/s)	27	27	26	26	26	26	26
	Temperature (^oC)	0	0	16	17	17	17	16
	Skidding (N.m)	0	0	0	0	0	0	0

4- The lining wear objective will get value toward the single objective value if the weighting factor of drum weight objective is higher than 0.333. This is because there is a mutual effect between these objectives.

CONCLUSIONS

The optimization model presented for designing the cam actuated pivoted shoe brakes and the method of solution of this model allow the designer to find the optimum solution in a short computation time. The computer program for this purpose has been prepared in a general form and thus only simple data given by the designer are needed to introduce his problem to the optimization program. The program is run interactively and thus the designer can easily find the solution that meets his requirements.

For the case studied, the minimum values of drum mass, lining wear or any combination of them can be obtained by choosing the lining angle as maximum as possible and searching for the optimum values of the other design variables. However; in case of minimum lining surface area objective or a multi objectives in which the weighting factor of the lining surface area is greater than zero ; the lining angle is a very effective design variable in the results.

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