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# MULTIVARIABLE SET POINT TRACKING USING POLE ZERO PLACEMENT CONTROLLERS

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## ABSTRACT

The paper describes the theory and application of multi-input/multi-output (MIMO) self tuning controllers where the control objective is the tracking of reference signals. The two schemes previously introduced by the authors for SISO case are extended for the case of MIMO. The two schemes require pole placement of the closed loop transfer function as well as zero placement of the error transfer function. The problem in the case of MIMO systems is more difficult than that for SISO, mainly because, the matrices describe the system are not commute. Simulation example is given to demonstrate that the second scheme is more suitable than the first one.

## 1. INTRODUCTION

Several papers have been recently appeared on multivariable self tuning control. The earlier papers extended the minimum variance controller to multivariable systems. To overcome the restriction of stably invertible (minimum phase) system the technique of Clarke and Gawthrop (8) has been extended to multivariable systems. More recent papers have, however, focused on pole assignment objectives. The attractiveness of pole assignment methods include their ability to handle nonminimum phase systems, and the fact that desired closed loop performance characteristics are easily specified via pole configuration. Furthermore new classical objectives, such as decoupling and the reduction of steady state errors, introduce slight modifications of the basic algorithms.

In this paper, we shall extend the concept of set point tracking to the case of multivariable systems. This concept is modified and studied well in the case of SISO systems. The paper extend two schemes previously proposed for SISO systems (9). The two schemes are compared with the original pole placement technique and proves good tracking. Also it is found that the second scheme is more suitable and give better results than the first one.

The paper proceeds as follows. Section 2 presents the OFF line design using the schemes for tracking. In section 3 the ON line algorithm is introduced. Section 4 presents a simulation example. Main results and conclusions are found in section 5.

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## 2. OFF LINE DESIGN

Consider the general form of feedback control system shown in Fig.(1), where the system is an m-input/m-output linear system described by the model

$$A(z^{-1})\ y(t) = z^{-K}\ B(z^{-1})\ u(t) \tag{1}$$

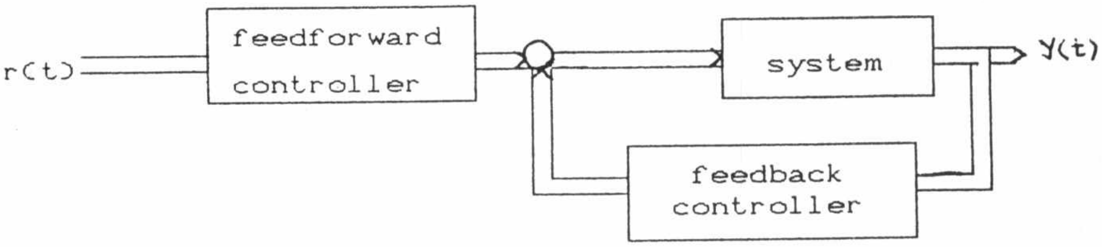


Fig.(1) General feedback control system

where $u(t)$ is $(m\times1)$ system input vector, $y(t)$ is $(m\times1)$ system output vector, $z^{-1}$ is the backward shift operator and $k$ is the time delay in the system which is a multiple integer of the sampling period. $A(z^{-1})$ and $B(z^{-1})$ are $(m\times m)$ matrices of order $na$ and $nb$ respectively and have the following structure, with $A_o = I$

$$X(z^{-1}) = X_o + X_1\ z^{-1} + \ldots + X_{nx}\ z^{-nx} \tag{2}$$

Let the control input vector given by

$$u(t) = T(z^{-1})\ R^{-1}(z^{-1})\ r(t) + S(z^{-1})\ R^{-1}(z^{-1})\ y(t) \tag{3}$$

where $r(t)$ is $(m\times1)$ reference input vector and $T(z^{-1})$, $S(z^{-1})$ and $R(z^{-1})$ are $(m\times m)$ matrices of order $nt$, $ns$ and $nr$ respectively, and have the structure given by (2), with $R_o=I$.

The output $y(t)$ can be written as

$$y(t) = R\ [\ A\ R + z^{-k}\ B\ S\ ]^{-1}\ z^{-k}\ B\ T\ R^{-1}\ r(t) \tag{3}$$

and the error $e(t)$ is given by

$$e(t) = r(t) - y(t)$$

$$e(t)=R[AR+z^{-k}BS]^{-1}[AR+z^{-k}BS-z^{-k}BT]^{-1}Rr(t) \tag{4}$$

equations (3) and (4) are the basic equations for pole placement and set point tracking.

### 2.1. First Scheme :

In the first scheme it is assumed that $T(z^{-1}) = S(z^{-1})$ and therefore equations (3) and (4) becomes

$$y(t) = R\ [\ A\ R + z^{-k}B\ S\ ]^{-1}z^{-k}B\ S\ R^{-1}r(t) \tag{5}$$

$$e(t) = R\,[\,A\,R + z^{-k} B\,S\,]^{-1}\,A\,R\,r(t) \tag{6}$$

in equation (6) R is chosen such that the modes of the input are cancelled by R, so let us assume that

$$r(t) = \{\frac{N_1(z^{-1})}{M_1(z^{-1})}, \ldots\ldots, \ldots\ldots, \frac{N_m(z^{-1})}{M_m(z^{-1})}\}^T \tag{7}$$

In order to cancel $M_i(z^{-1})$, $i=1, \ldots, m$, let

$$R = R_D \,.\, R1 \tag{8}$$

$$\text{where}\quad R_D = \prod_{i=1}^{m} M_i(z^{-1})\, I_{mxm} \tag{9}$$

The pole placement problem is to find R and S such that the closed loop transfer function has certain poles, which lead to the Diophantine equation

$$A\,R + z^{-k}\,B\,S = A_m\,A_o \tag{10}$$

Both $A_m$ and $A_o$ are (mxm) matrices, where $A_m$ is the desired characteristic polynomial of the closed loop system and $A_o$ is another polynomial which can be considered as the observer polynomial. Substitute from (9) into (10), let $A_m A_o = A_{m1}$ and $AR_D = A_1$, we get

$$A_1\,R_1 + z^{-k}\,B\,S = A_{m1} \tag{11}$$

Thus equation (11) can be solved for $R_1$ and S. Then R is determined from equation (8).

2.2. Second Scheme :

In the second scheme, T is chosen different from S, and thus equations (3) and (4) are used. In equation (3) S and R are chosen such that

$$A\,R + z^{-k}\,B\,S = A_m\,A_o = A_{m1} \tag{12}$$

Substitute from (12) in (4), we get

$$e(t) = R[A\,R + z^{-k}\,B\,S]^{-1}[A_{m1} - z^{-k}\,B\,T]R^{-1}\,r(t) \tag{13}$$

In equation (13) T is chosen such that it cancels the modes of r(t), let

$$A_{m1} - z^{-k}\,B\,T = R_D\,R_2 \tag{14}$$

where $R_D$ is given by equation (9), and $R_2$ is another matrix to satisfy the compatibility degree of equation (14). Rearranging equation (14)

$$R_D\,R_2 + z^{-k}\,B\,T = A_{m1} \tag{15}$$

Equation (15) is solved for T and $R_2$. Note $R_2$ is not of interest.

### 2.3. Solution of the Diophantine Equation

The solution of the Diophantine equation of the form

$$A\,R + z^{-k}\,B\,S = A_{m1} \qquad (16)$$

has a unique solution, if and only if, the following conditions are satisfied [ 2 ]

$$\begin{aligned} nr &= nb + k - 1 \\ ns &= na - 1 \qquad (17) \\ nam1 &<= na + nb + k - 1 \end{aligned}$$

The solution is obtained by equating equal powers of $z^{-1}$ in both sides of equation (16), which is transformed to a set of simultaneous equations.

It is seen that solution of Diophantine equation requires inversion of matrix of order (na+nb+k-1)*m, subtraction, and matrix multiplication.

### 2.4. Control Law Application

The control law of equation (3) can be implemented by introducing a further assumption of commutivity, it is assumed that, there exist $\bar{S}$ and $\bar{R}$ such that

$$S\,R^{-1} = \bar{R}^{-1}\,\bar{S} \qquad (18)$$

where $\bar{S}$ and $\bar{R}$ are (mxm) matrices of order ns and nr respectively. The solution of equation (18) leads to a system of linear equations.

The solution of equation (18) requires inverse of matrix of order m*(nr+ns), unless nr=0, i.e. nb+k=1, where $\bar{R}$ = I and $\bar{S}$ = S. Or when m=1, where the transformation is unnecessary. From equation (18) the control law is given by

$$u(t) = \bar{R}_t^{-1}\,\bar{T}\,r(t) - \bar{R}_s^{-1}\,\bar{S}\,y(t) \qquad (19)$$

## 3. ON LINE DESIGN

When the parameters of the system are unknown, then, they must be estimated using recursive least square, and the controller is redesigned in each step using the estimated paameters instead of the true ones.

## 4. SIMULATION EXAMPLE

The schemes proposed for tracking as well as pole placement technique are compared using a simulation example. Consider the 2-input/2-output system described by equation (1), where

$$A(z^{-1}) = I + A_1 z^{-1} + A_2 z^{-2}$$

$$B(z^{-1}) = B_o + B_1 z^{-1}, \quad K=1$$

given $A_1 = \begin{bmatrix} -1.4 & -0.2 \\ -0.1 & -0.9 \end{bmatrix}$ $A_2 = \begin{bmatrix} 0.48 & 0.1 \\ 0 & 0.2 \end{bmatrix}$

$B_o = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $B_1 = \begin{bmatrix} 1.5 & 1 \\ 0 & 1 \end{bmatrix}$

The model is chosen to be

$$A_m(z^{-1}) = I + A_{m1} z^{-1} + A_{m2} z^{-2}$$

given $A_{m1} = \begin{bmatrix} -1.0 & 0 \\ -0.1 & -0.7 \end{bmatrix}$ $A_{m2} = \begin{bmatrix} 0.25 & 0 \\ -0.05 & 0.12 \end{bmatrix}$

while the observer polynomial is taken as $A_o(z^{-1}) = I$. The case of periodic step input is considered, where $M(z^{-1}) = 1 - z^{-1}$.

Simulation for the 3 cases are shown in figures 2,3,4 for the Off line design and figures 5,6,7 for the On line design. It is very clear that both schemes give good tracking than design using pole placement only. It is also clear by comparing figures 3 and 4 as well as 6 and 7 that the second scheme give better tracking specially at sudden changes of reference signal.

## 5. CONCLUSIONS

Two schemes for better set point tracking which are previously adopted for SISO systems are generalized here for the case of MIMO. The two schemes still prove good tracking. The second scheme is advantageous than the first one, both in the results obtained as well as computations required for the controller design.

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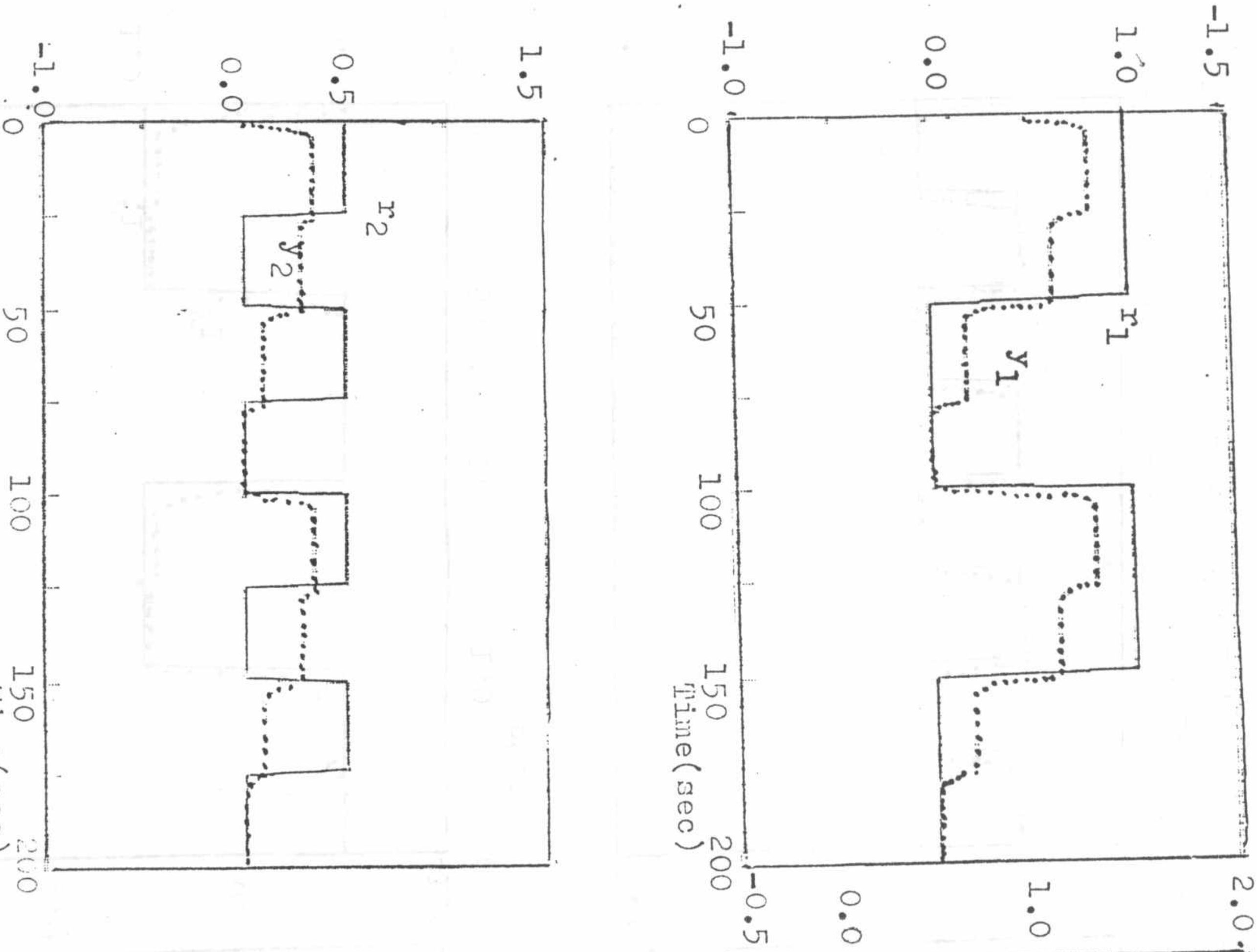


Fig.(2) Off line design using pole placement.

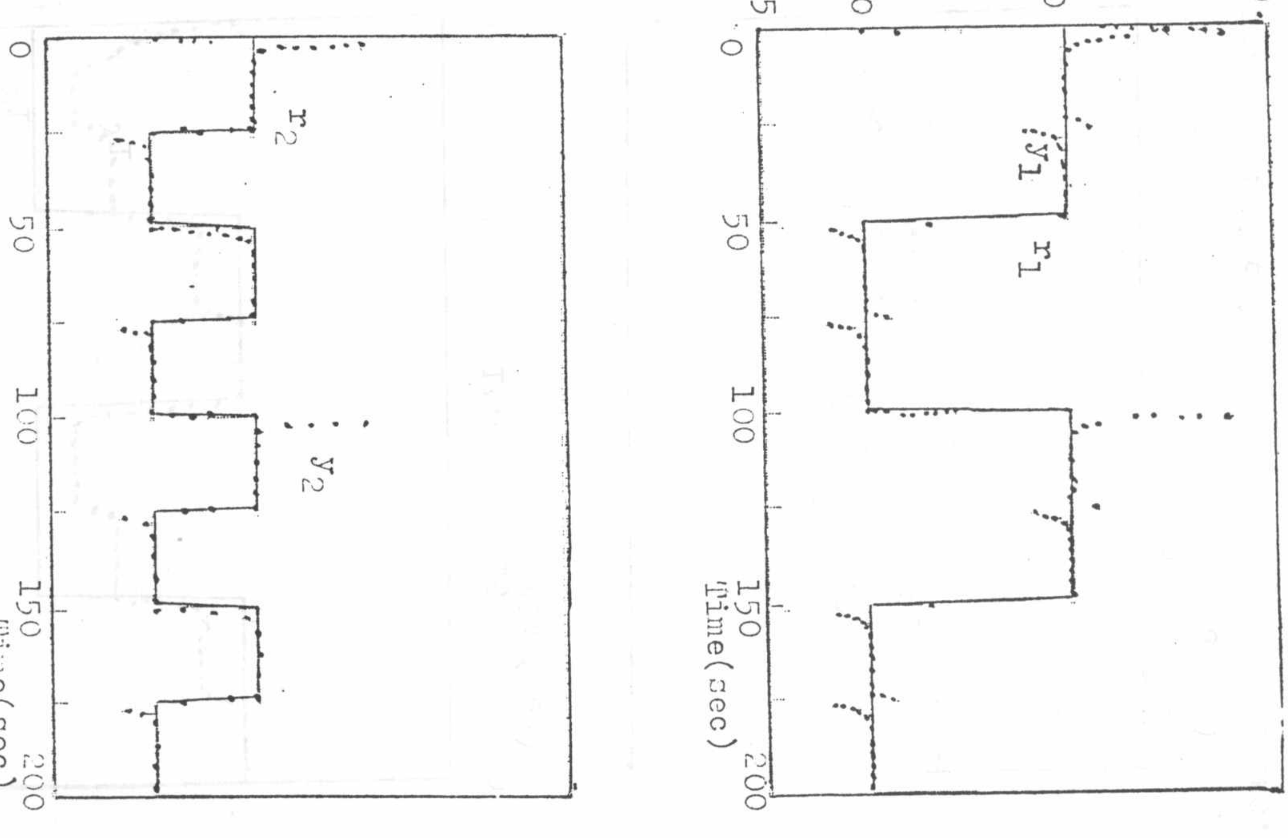


Fig.(3) Off line design using scheme 1.

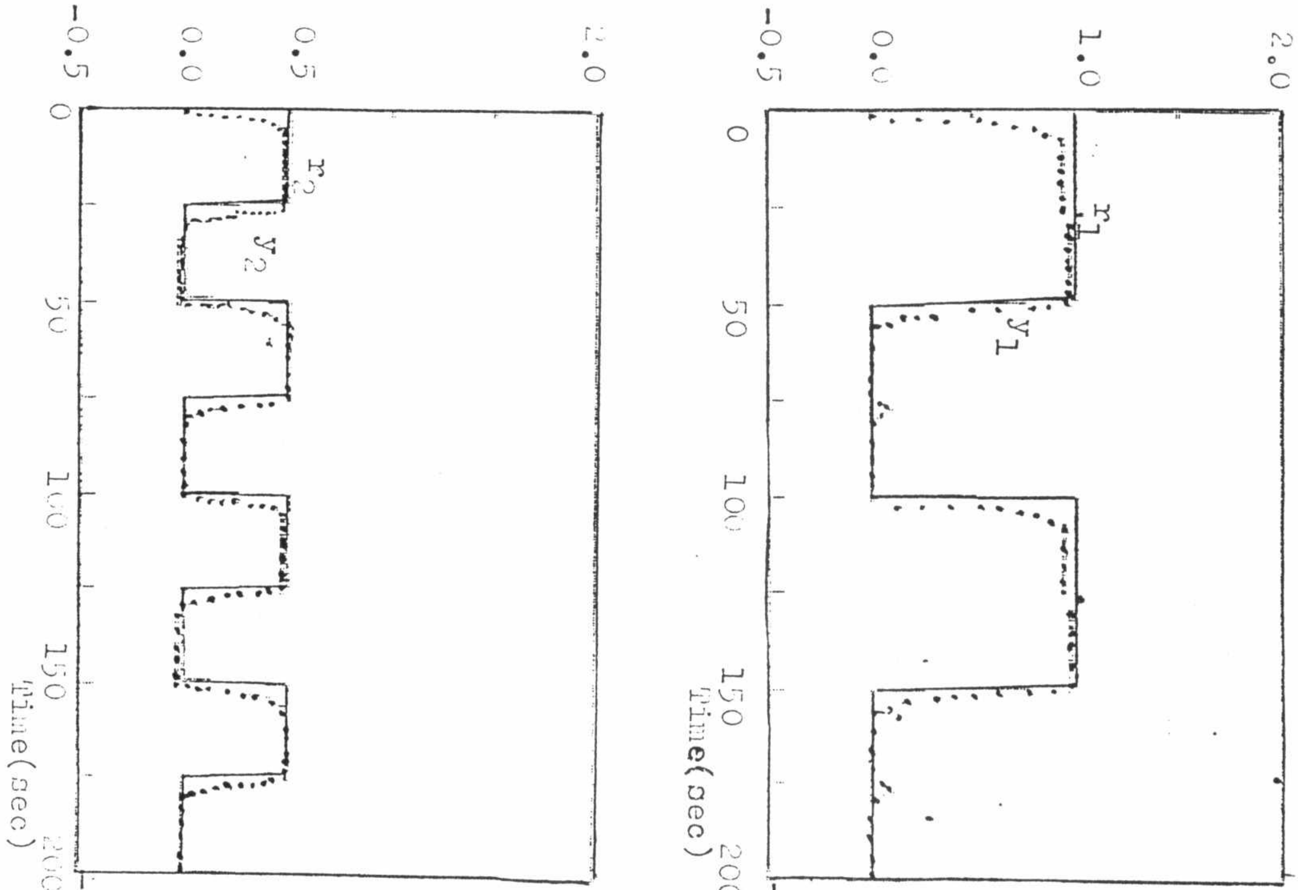


Fig.(4) Off line design using scheme 2.

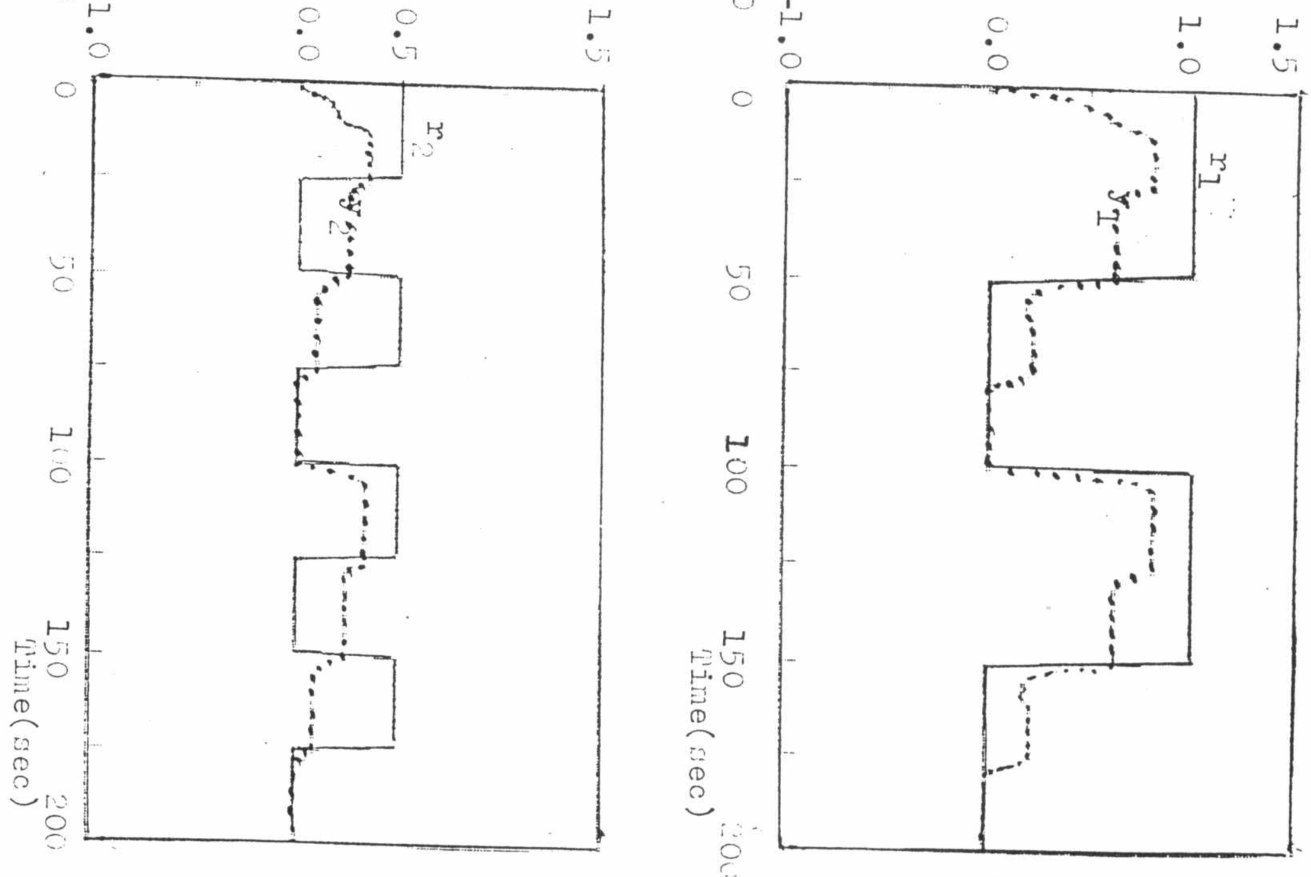


Fig.(5) On line design using pole placement.

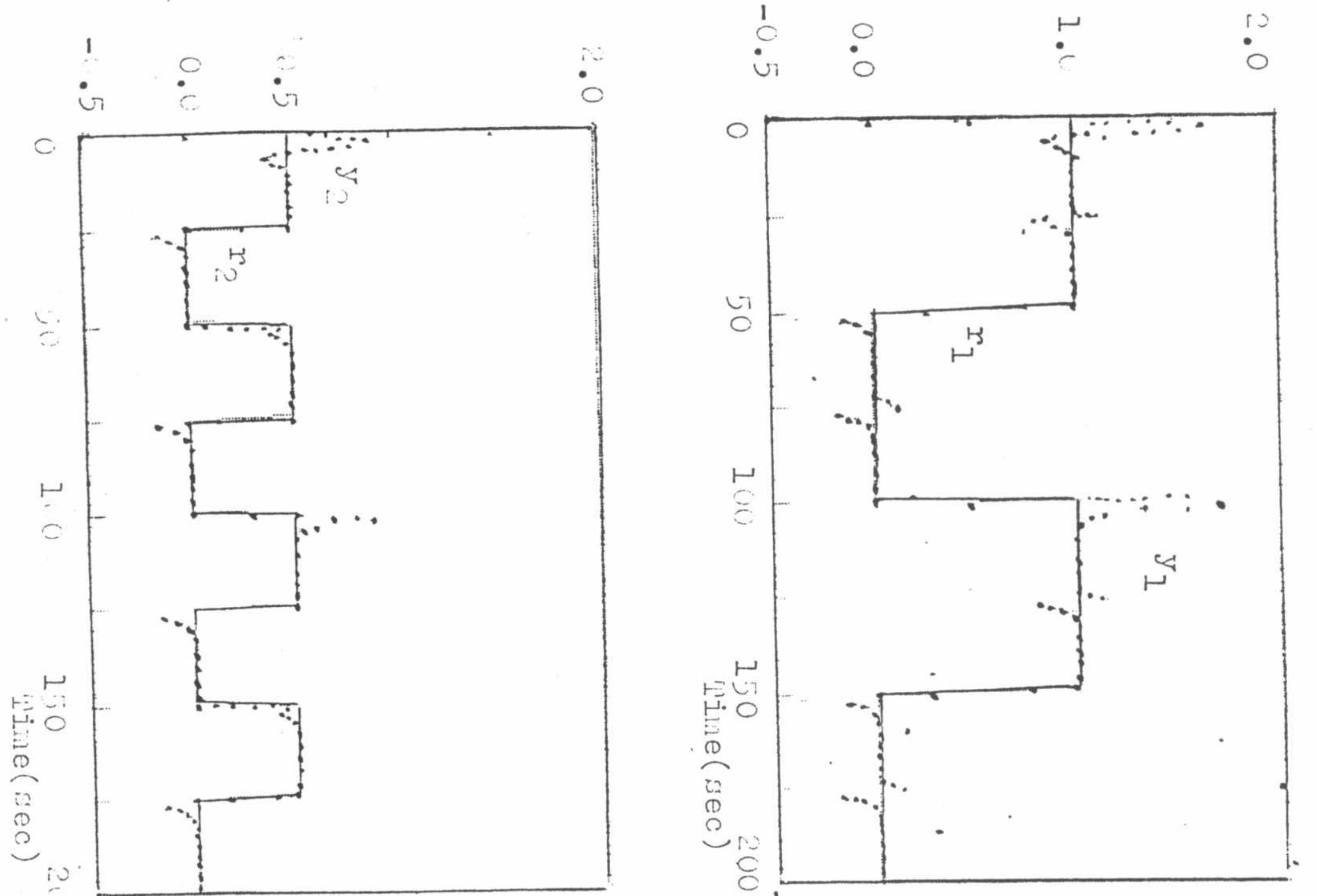


Fig.(6) on line design using scheme 1.

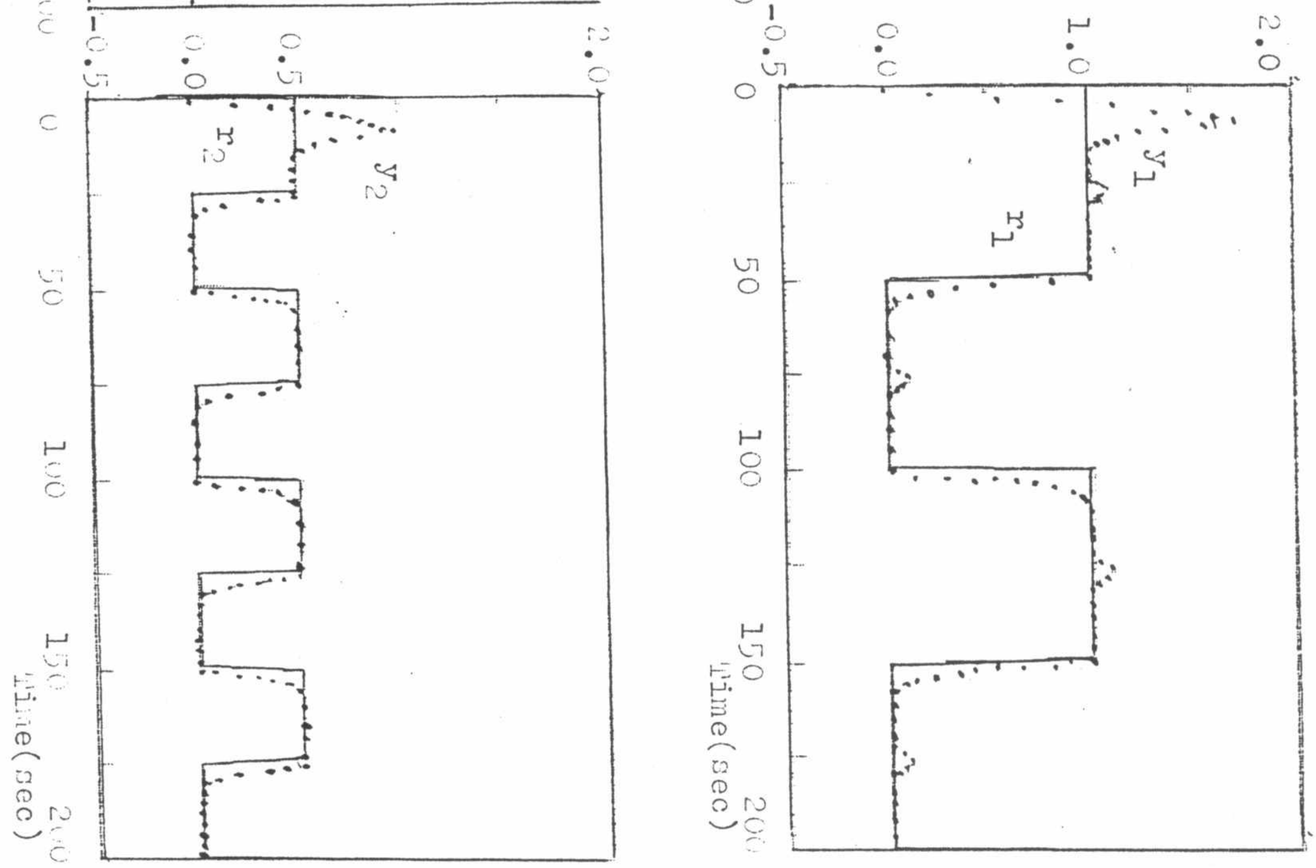


Fig.(7) on line design using scheme 2.

# A MODEL REFERENCE ADAPTIVE CONTROLLER (MRAC) WITH PERIODIC RESETTING OF ADAPTATION MECHANISM (PRAM)

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## ABSTRACT

It has been assured that standard adaptive control algorithms, designed in the so-called ideal case, would likely become unstable when one takes disturbances and neglected dynamics into account.

For the stability problems to be tackled; we propose a modified model reference adaptive controller that introduces the concept of periodic resetting of adaptation mechanism. In order to attain the best feasible performance; an optimum adaptation-time is computed on-line,via minimization of certain quadratic criterion.

In this paper, a modified algorithm of MRAC with (PRAM) has been developed. Such a controller can provide stability with better performance to a large family of real plants, where sinusoidal disturbances and unmodeled dynamics are most probable.

The proposed scheme prevents the dangerous problems due to unbounded parameter drift present in available MRAC schemes, in such practical conditions. The analytical arguments and improvements provided by the proposed algorithm, has been verified by simulation results.

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## 1. INTRODUCTION

An intensive investigation study of the stability and robustness properties of a wide class of conventional adaptive control algorithms in the presence of unmodeled high-frequency dynamics and persistent output disturbances, has been carried out(Rohrs and others, [14]). Their main conclusion was that:

1) Sinusoidal reference inputs at specific frequencies and / or,
2) Sinusoidal output disturbances at any frequency (including dc), can cause the loop gain to increase without bound leading to instability.

Since then, ROBUST MODEL REFERENCE ADAPTIVE CONTROL(MRAC) for real systems, under practical conditions, have been extensively studied in the literature during the last decade. For linear single-input-single-output (SISO) systems, several robust modifications have been proposed that can be turned into the following main classes:

(i) Modifying the adaptive law, such as, dead zone : ([10],[15]), $\sigma$ modification:[2], $e_1$ modification:[8], dead zone using an upper bound of the plant parameters [5], and with a variable width: ([9],[6]) , controller with bounded parameters using appropriate gain limiters [17].

(ii) Increasing the richness of the reference input:(Narendra and Annaswamy [7]) derived the sufficient conditions on the persistent excitation of the reference input, given the maximum amplitude of disturbance, for the signals in the adaptive system to be globally bounded.

Concerning nonlinear systems, several researches dealing with robust adaptive control are now available in the literature, however, few results have been obtained(see e.g.[11],[16],[3],[4], and [19]).

In this paper, we propose the PRAM-modification as a solution for stability problems of a plant subject to disturbances and unmodeled dynamics.

The proposed modification makes a conventional adaptive controller globally stable, and robust to uncertainties; in the sense that all the signals in the loop remain bounded. For reviewing the general structure of a standard MRAC algorithm and the associated infinite gain operators in the presence of such uncertainties (unmodeled dynamics and disturbances); the reader may refer to [13] , [14] , and [17].

## 2. PRAM - MODIFICATION

The essential idea of PRAM is to reset the adaptive gains to their optimum nominal values periodically. The time-period after which the adaptation mechanism is reset periodically, is named the adaptation period ( $T_a$). By defining a permissible limit for the ratio of adaptive gain drift to system mismatch error ( $\| \tilde{K}(t) \| / \| e(t) \|$ ), that would not be exeeded, we can derive the upper bound for the adaptation-period ($T_{a_{max}}$). Also a lower bound ($T_{a_{min}}$) can be determined according to the transient reponse of the control system. Depending on the selected value of the adaptation period, the resetting mechanism can be designed. Thus, the proposed modified algorithm is capable to prevent the un-definite drift of controller parameters $\tilde{\underline{k}}(t)$ ; by assuming a finite time of adaptation-period, after which the drift of parameters will be reset to zero periodically. The limiting bounds of adaptation period ( $T_{a_{min}}$ , $T_{a_{max}}$ ) , will be properly evaluated in order to contain the drift of adaptive parameters $\tilde{\underline{k}}(t)$ within the assumed permissible limits. PRAM-modification can be introduced in the adaptive mechanism of the available standard MRAC algorithm, as shown with CA1 in Fig.1.

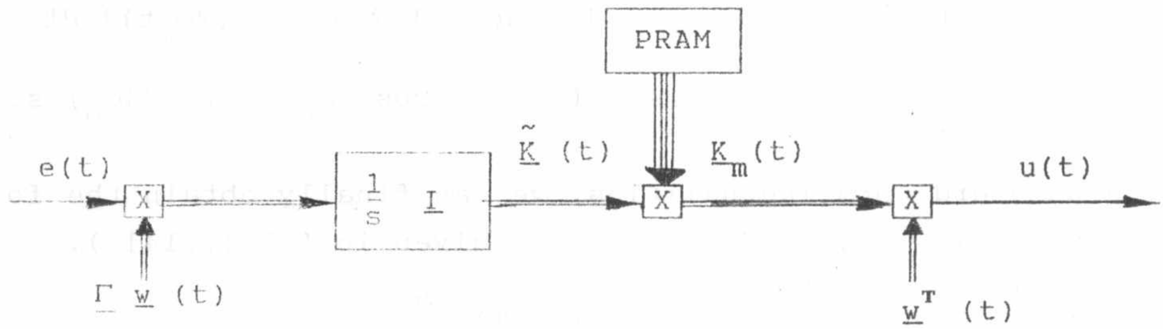


Fig.1. CA1. Modified Adaptive Parameters $\tilde{\underline{k}}_m(t)$ using RRAM.

## 3. ADAPTATION - PERIOD ( $T_a$ )

Using one of the available standard adaptive algorithms (CA1 is chosen for its simplicity), and considering an additive output disturbance of the form

$$d(t) = d_o \sin(\omega_o t) \tag{1}$$

Then, the plant output will be

$$y(t) = y(t) + d(t) \tag{2}$$

The mismatch error between the plant output and the desired model output, e (t), even at full adaptation convergence state, will have a residual sinusoidal component of the form

$$e(t) = a \sin(\omega_o t + \Phi) \cong d_o \sin(\omega_o t + \Phi) \tag{3}$$

For simplification of the mathematical derivation, a zero value for the phase-shift angle ($\Phi$) is considered, and a scalar auxiliary signal w (t) is assumed as

$$w(t) = b + c \sin(\omega_o t) \qquad (4)$$

where a, b, c, $d_o$, $\omega_o$ are assumed to be positive constants.

From equations (3) and (4) we get

$$w(t).e(t) = a \sin \omega_o t \,.\, (b + c \sin \omega_o t)$$

$$= a\, b \sin(\omega_o t) + 1/2\, ac - 1/2\, ac \cos(2\omega_o t) \qquad (5)$$

According to adaptation law of CA1 algorithm, as given in [14] the variation of a single controller parameter [$\tilde{k}$ (t)] will be

$$\tilde{k}(t) = \tilde{k}_o + \int_0^t w(\tau)\, e(\tau)\, d\tau \qquad (6)$$

$$= \tilde{k}_o + \int_0^t [ab \sin \omega_o t + 1/2\, ac - 1/2\, ac \cos(2\omega_o t)]\, dt$$

$$= \tilde{k}_o + 1/2\, act + ab/\omega_o - (ab/\omega_o) \cos \omega_o t - (ac/4\omega_o) \sin 2\omega_o t \qquad (7)$$

Using standard norm inequalities, we can finally obtain the following inequality ( full derivation is given in [ 13],[14] ).

$$\| \tilde{k}(t) \|_0^T \geq (k_3\, T^3)^{1/2} - (k_1\, T)^{1/2} \qquad (8)$$

Similarly, the square of error norm is

$$\left( \| e(t) \|_0^T \right)^2 = a^2 \int_0^T \sin^2 \omega_o t\, dt \leq a^2\, T \qquad (9)$$

According to equation (3), the parameter a can be approximated by $d_o$, at some cases near final convergence state. Using (8) and (9), a ratio between drift of parameters and the error (ratio between the norms) can be obtained as

$$\frac{\| \tilde{k}(t) \|_0^T}{\| e(t) \|_0^T} \geq \frac{(k_3\, T^3)^{1/2} - (k_1\, T)^{1/2}}{d_o\, T^{1/2}} \xrightarrow[T \longrightarrow \infty]{} \infty \qquad (10)$$

$$\text{Where } k_1 = (\tilde{k}_o)^2 + 2\left(\frac{a\,b}{\omega o}\right)^2 + \left(\frac{a\,c}{4\,\omega o}\right)^2 , \qquad (11a)$$

$$k_3 = \left[\frac{a^2 c^2}{12}\right] \tag{11b}$$

Inspecting inequality (10), it is clear that as time (T) increases, the ratio ( $\| \tilde{k}$ t) $\| / \|$ e (t) $\|$ ) will icrease without bound. In this paper, we propose limiting values for adaptation time ( $T_{a_{max}}$ , $T_{a_{min}}$ ); in order to make the drift of parameters bounded within accepted limits.

### 3.1 Higher Limit ( $T_{a_{max}}$ ) :

In order to make the ratio $\left[ \| \tilde{k}(t) \| / \| e(t) \| \right]$ not higher than a finite positive constant (L) ; higher limit for the adaptation - period ( $T_{a_{max}}$ ) can be evaluated,

$$\frac{\| \tilde{k}(t) \|_o^{T_{a_{max}}}}{\| e(t) \|_o^{T_{a_{max}}}} = \frac{\left( k_3 T^3_{a_{max}} \right)^{1/2} - \left( k_1 T_{a_{max}} \right)^{1/2}}{a \left( T_{a_{max}} \right)^{1/2}} = L \tag{12}$$

$$\text{then,} \quad T_{a_{max}} = \frac{(k_1)^{1/2} + a L}{(k_3)^{1/2}} \tag{13}$$

substituting for $k_1$ and $k_3$ , we can get

$$T_{a_{max}} = \frac{(12)^{1/2} \left( a L + \left[ (\tilde{k}_o)^2 + 2 \left(\frac{ab}{\omega_o}\right)^2 + \left(\frac{ac}{4\omega_o}\right)^2 \right]^{1/2} \right)}{ac} \tag{14}$$

### 3.2 Lower Limit ( $T_{a_{min}}$ )

A zero value for the lower limit of adaptation time, must be avoided because this prevents any sort of adaptive control to go on. The reasonable lower limit for adaptation-time can be considered as the minimum time after which a nearly zero drift of gain parameters is attained, i.e., when $\| \tilde{k}(t) \| \cong 0$ .

Referring to inequality (10), we may have,

$$\left[ k_3 \, T^3_{a_{min}} \right]^{1/2} \qquad \left[ k_1 \, T_{a_{min}} \right]^{1/2} = 0 \qquad (15)$$

then,

$$T_{a_{min}} = \left( \frac{k_1}{k_3} \right)^{1/2} \qquad (16)$$

Substituting for $k_1$ and $k_3$ , we get

$$T_{a_{min}} = \frac{\left[ 12 \left( \tilde{k}_o \right)^2 + 24 \left( \frac{ab}{\omega_o} \right)^2 + \frac{3}{4} \left( \frac{ac}{\omega_o} \right)^2 \right]^{1/2}}{ac} \qquad (17)$$

## 4. COMMENTARY ANALYSIS ON ADAPTATION-TIME LIMITS

(i) From eqn. (14), one can notice that as frequency ($\omega_o$) increases, ( $T_{a_{max}}$ ) would decrease. This means that instability of a standard MRAC system would occur sooner for higher frequency of disturbance.

(ii) Investigating inequality (10) and eqns.(11a,b), we note that as sinusoidal disturbance amplitude $d_o$ decreases; the ratio ($\| \tilde{k}(t) \| / \| e(t) \|$) will increase. To make it more clear, consider a very low extent of disturbance ( $d_o = 1 \times 10^{-6}$ ) ; then (10) can be approximated by ( neglecting higher orders of $d_o$ )

$$\| \tilde{k}(t) \|_o^T \,/\, \| e(t) \|_o^T \cong \frac{k_o \, T^{1/2}}{d_o T^{1/2}} \cong \frac{\tilde{k}_o}{d_o} \qquad (18)$$

Thus, decreasing ($d_o$) the ratio of parameter drift to error signal is increased; thereby pushing the adaptive system faster to instability. This analysis gives a clear mathematical interpretation for the simulation results first obtained by(Rohrs, 1982 [13]), and which were amazing at that time.

(iii) One may also note that the developed higher limit for adaptation time ( $T_{a_{max}}$ ), can be viewed, in some sense, as a stability indicator for the adaptive system subjected to mentioned practical conditions.

(iv) Investigating the mathematical condition for computing ($T_{a_{min}}$), namely $\| \tilde{k}(t) \| = 0$, this state is satisfied when $k(t)$ is nearly constant, i.e., when the variation (drift) of adaptive parameters is nearly zero. From practical experience, supported by simulation results, we know that the drift of parameters will not approach zero until, at least, the transients of the controlled system are damped out. The transient is assumed to be over when the response error has become below certain minimum level.
Note: settling time for a second order system, when response error is reduced below 2% of its nominal value, is approximately equal to four time constants of the envelope of the damped sinusoidal oscillations.

Thus, the lower limit ( $T_{a_{min}}$ ) may be well approximated by

$$T_{a_{min}} \cong 4\,\tau_m \tag{19}$$

Where ($\tau_m$) is assumed to be equal to the time constant of the desired reference model specified in the used MRAC algorithm. One may consider that the lower limit obtained by eqn.(19) is more practical than the former one given by eqn. (17).

## 5. OPTIMAL ADAPTATION - PERIOD ( $T_{a_{opt}}$ )

The derived equations (14) and (17) prove that, in principle, there is a feasible proper range of adaptation time, within which all signals in the adaptive system can be kept bounded in practical applications subjected to unmodeled dynamics, and sinusoidal disturbances.

As most items of eqns. (14) and (17) such as, (a,b,c, $\omega_o$, and $d_o$ ) can not be known a priori. Besides, such items are not positive constants as what has been assumed in section.4, but they are rather variables affecting one another. Therefore, we need to develop here an approach to determine an optimal value for the parameter $T_a$( in some sense ) by minimizing an assumed performance criterion, on - line.

Suppose that the system is described in discrete-time form by

$$y\,(t) = \frac{g_p \; q^{-dp} \; B^{(m)}}{A^{(n)}} \; [\, u\,(t)\,] + \upsilon\,(t) \tag{20}$$

Where $y\,(t)$ and $u\,(t)$ are the output and input, respectively at time t and $\upsilon\,(t)$ is a disturbance term, and

$$B^{(m)} = 1 + b_1\, q^{-1} + b_2\, q^{-2} + \ldots + b_m\, q^{-m} \tag{21}$$

$$A^{(n)} = 1 + a_1\, q^{-1} + a_2\, q^{-2} + \cdots + a_n\, q^{-n} \tag{22}$$

Where $q^{-1}$ is the backward shift operator, $g_p$ and $d_p$ are the gain and delay of the system respectively. Assuming the system (20) is controlled by a conventional adaptive controller with PRAM modification, then the adaptive gain, the input, and the output of the system will of course depend on the modification parameter $T_a$, and will be denoted by $k(t, T_a)$, $u(t, T_a)$ and $y(t, T_a)$.

The parameter $T_a$ is to be chosen so that a criterion of the form

$$V(T_a) = E \quad g\Big(y(t, T_a), u(t, T_a)\Big) \qquad (23)$$

is minimized. The details of a similar approach can be found in [3], where expectation is done over the stochastic signals in the system, such us $\{ \upsilon(t) \}$ which was assumed in [3] to be stationary stochastic process, with zero mean and arbitrary correlation properties. Minimizing (23) could be replaced by the solution of

$$0 = V'(T_a) = E \left[ g'_y \Big(y(t, T_a)\Big) y'(t, T_a) + g'_u \Big(y(t, T_a), u(t, T_a)\Big) u'(t, T_a) \right] \qquad (24)$$

Here $y'(t, T_a)$ and $u'(t, T_a)$ denote the output and input differentiated with respect to $T_a$, and $g'_y(y,u)$ is the derivative of $g$ with respect to its first argument $y$, and so forth. The idea of this approach is to update $T_a$ in a negative gradient direction of the criterion [4]. The parameter $T_a$ is adjusted on-line in what is believed to be descent direction of a quadratic criterion V. The parameter $T_a$ will converge to, at least, a local minimum of the criterion V at $(T_{a_{opt}})$.

Without loss of generality, a quadratic criterion can be assumed as

$$V(T_a, t) = \frac{1}{2} E \left\{ F_e e^2 + F_{\tilde{k}_r} \tilde{k}_r^2 + F_{\tilde{k}_y} \tilde{k}_y^2 + F_u u^2 \right\} \qquad (25)$$

where $F_e$, $F_{\tilde{k}_r}$, $F_{\tilde{k}_y}$, and $F_u$ are arbitrary weighting factors.

For the quadratic criterion (25) to be minimized, we seek the solution of the next eqn.

$$0 = V'(T_a, t) = E \left\{ F_e e \frac{\partial e}{\partial T_a} + F_{\tilde{k}_r} \tilde{k}_r \frac{\partial \tilde{k}_r}{\partial T_a} + F_{\tilde{k}_y} \tilde{k}_y \frac{\partial \tilde{k}_y}{\partial T_a} + F_u u \frac{\partial u}{\partial T_a} \right\} \qquad (26)$$

During searching process, the adaptation-time parameter $(T_a)$ is considered the real time t which is always updated by the sampling-period $(T_s)$ of the discrete-time system; till the optimal value $T_{a_{opt}}$ is reached. Hence, $V^{`}$ can be approximated as,

$$V^{`}(t) = E\left\{ F_e\, e(t)\,\frac{e(t) - e(t-1)}{T_s} + F_{\tilde{k}_r}\,\tilde{k}_r(t)\,\frac{\tilde{k}_r(t) - \tilde{k}_r(t-1)}{T_s} + F_{\tilde{k}_y}\,\tilde{k}_y(t)\,\frac{\tilde{k}_y(t)-\tilde{k}_y(t-1)}{T_s} + F_u\, u(t)\,\frac{u(t)-u(t-1)}{T_s}\right\} \quad (27)$$

Since the signals of the system are assumed to be deterministic, and the disturbance is a sinusoid with distinct frequency; then the expectation is replaced by the current values of signals and parameters.

Being interested in the instant when the gradient changes its direction, rather than getting the precise value of that gradient, $V^{`}$ can be rescaled as

$$V^{`}(t) = F_e\, e(t)\left[ e(t)-e(t-1) \right] + F_{\tilde{k}}\,\tilde{k}_r(t)\left[ \tilde{k}_r(t) - \tilde{k}_r(t-1) \right] + F_{\tilde{k}}\,\tilde{k}_y(t)\left[\tilde{k}_y(t)- \tilde{k}_y(t-1)\right] + F_u u(t)\left[F_u u(t)-u(t-1)\right] \quad (28)$$

In this approach the time $T_{a_{opt}}$ corresponds to the gradiant first conversion of direction from negative to positive one, just after the assumed transient-time is over(see Fig.2.). This on-line computed value $(T_{a_{opt}})$ is the local optimum choice of adaptation-time that would ensure local minimization of performance criterion (V) in the feasible operating conditions.

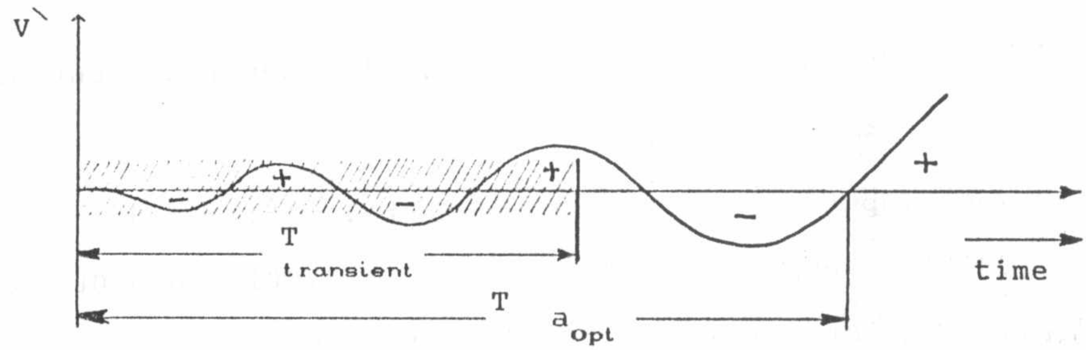


Fig.2 $T_{a_{opt}}$ corresponds to first conversion of $V^{`}$ direction. ( from negative to positive after transient is over)

## 6. SIMULATION RESULTS

To illustrate the effectiveness of the proposed PRAM-modification presented in the previous sections; consider the following example with a plant described by,

$$Y(t) = \frac{2}{(s+1)} \cdot \frac{229}{(s^2 + 30s + 229)} [u(t)] \qquad (29)$$

The adaptive scheme CA1 [14] is used, with a first order reference model given as

$$Y_m(t) = \frac{3}{s+3} [r(t)] \qquad (30)$$

The simulations were all initialized with

$$k_Y(0) = 0.65 \; ; \; k_r(0) = 1.14 \qquad (31)$$

which yield a nominal controlled plant with the following transfer function

$$\frac{g^* B^*(s)}{A^*(s)} = \frac{527}{s^3 + 31s^2 + 259s + 527} \qquad (32)$$

Remark: $\frac{g^* B^*}{A^*}$ is the closed loop transfer function that would result if $\underline{\tilde{k}}$ were identically zero, i.e., if a constant control law $\underline{k} = \underline{k}^*$ were used.

The reference input signal was chosen to be: $r(t) = 0.3$ (33)

with an additive output disturbance: $d(t) = 0.001 \sin 5.0t$ (34)

The adaptaion gains were set equal to four.i.e $\Gamma = 4$, (35)

but the amplitude and frequency of the sinusoidal disturbance were varied for different cases to study their effects. The relatively large value of adaptation gain in (35) was chosen so that the unstable behavior would occur over a reasonable simulation time, for such conditions.

All simulations were carried out with the discretized equivalent system. In order to obtain a discrete-time system which is equivalent to the system (29), the standard technique of discrate-time control system analysis was used (see [1], section 3.4). A sampling period of $T_s$ = 0.04 seconds was used. This represents fairly fast sampling, since it is approximately ten times as fast as the fastest dynamics in the plant [13] .

It is seen from simulation results depicted in Figs.3 and 4 that the stability of MRAC system using CA1, is more deteriorated when increasing the frequency of the sinusoidal disturbance and / or decreasing its amplitude. It is shown also that PRAM-modification does provide a stable closed-loop control system with a reasonable performance, at conditions where none of the available standard adaptive control schemes can provide.

Fig.5 indicates that PRAM-modification, using the computed optimal value of $T_a$ ( here, $T_{a_{opt}}$ = 1.8 sec.) gives better response than any other one, in such conditions.

## 7. CONCLUSIONS

In this paper, a modified MRAC algorithm with Periodic Reset of Adaptation Mechanism (PRAM) has been developed. Such a controller can provide stability with better performance when the system operates in the presence of unmeasured and possibly persistant sinusoidal disturbances.

The proposed scheme alleviates the dangerous problems of the unbounded parameter drift present in all available standard adaptive schemes, in such practical conditions. The analytical arguments and improvements provided by the proposed algorithm has been verified by simulation results.

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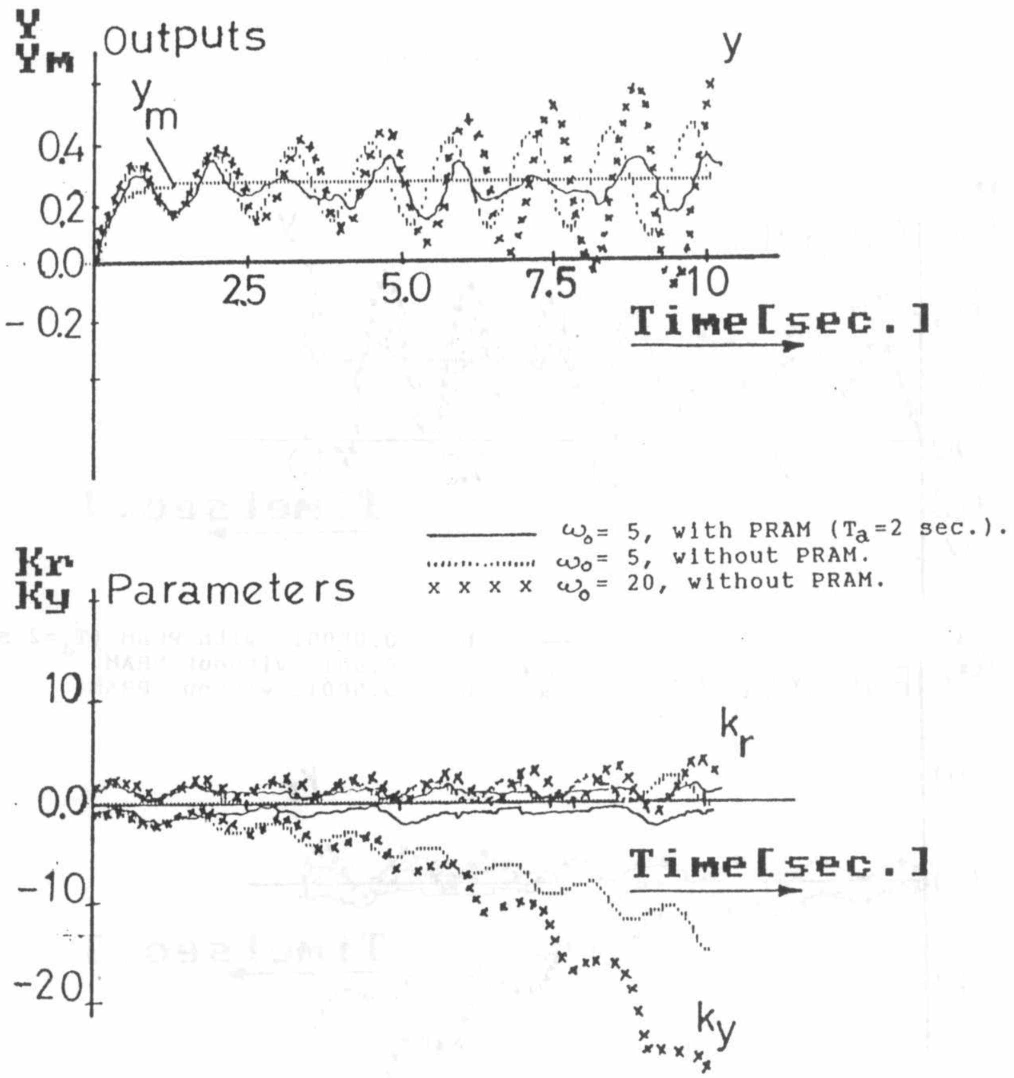


Fig.3. Simulation of CA1 with and without PRAM, $r(t)= 0.3$ and $d(t)= 0.001 \sin \omega_o t$.

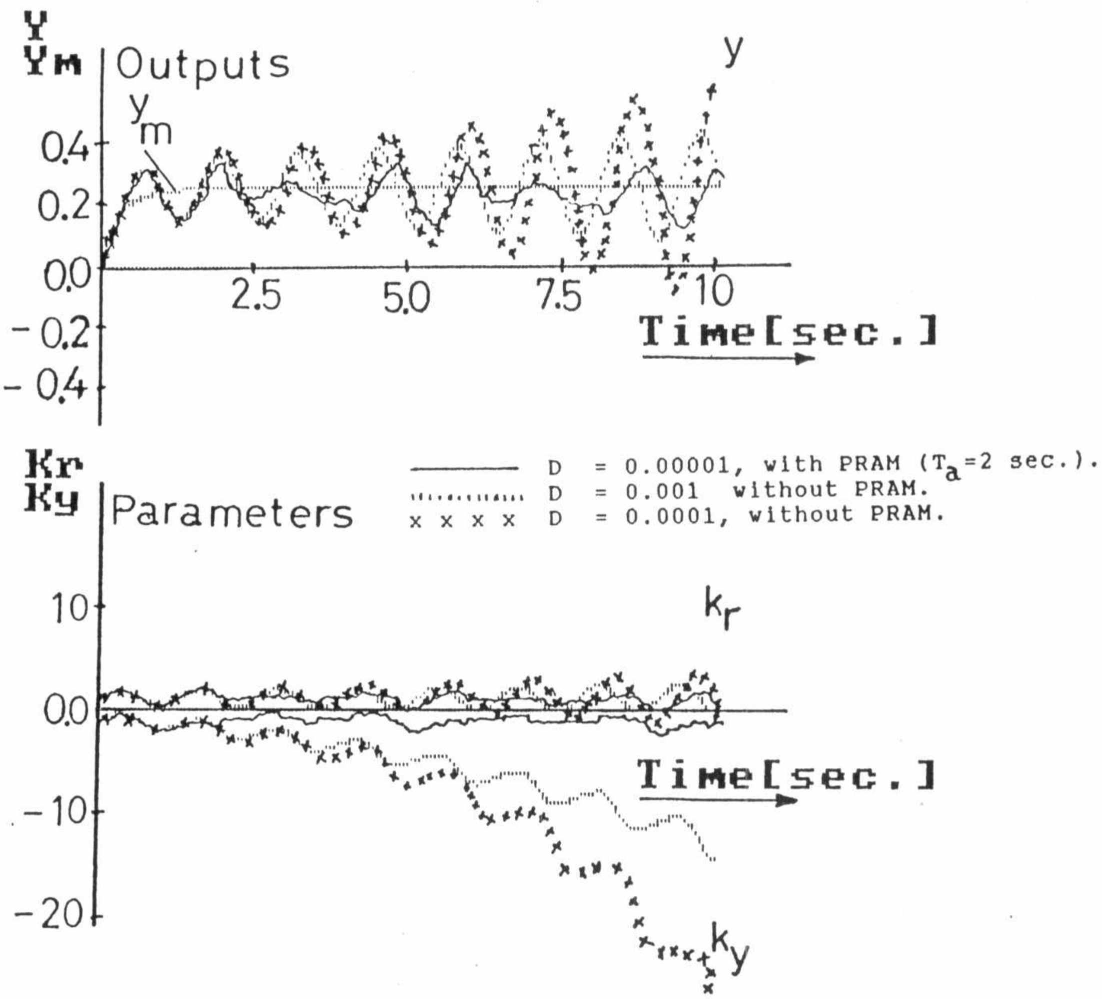


Fig.4. Simulation of CA1 with and without PRAM,
$r(t) = 0.3$ and $d(t) = D \sin 5t$.

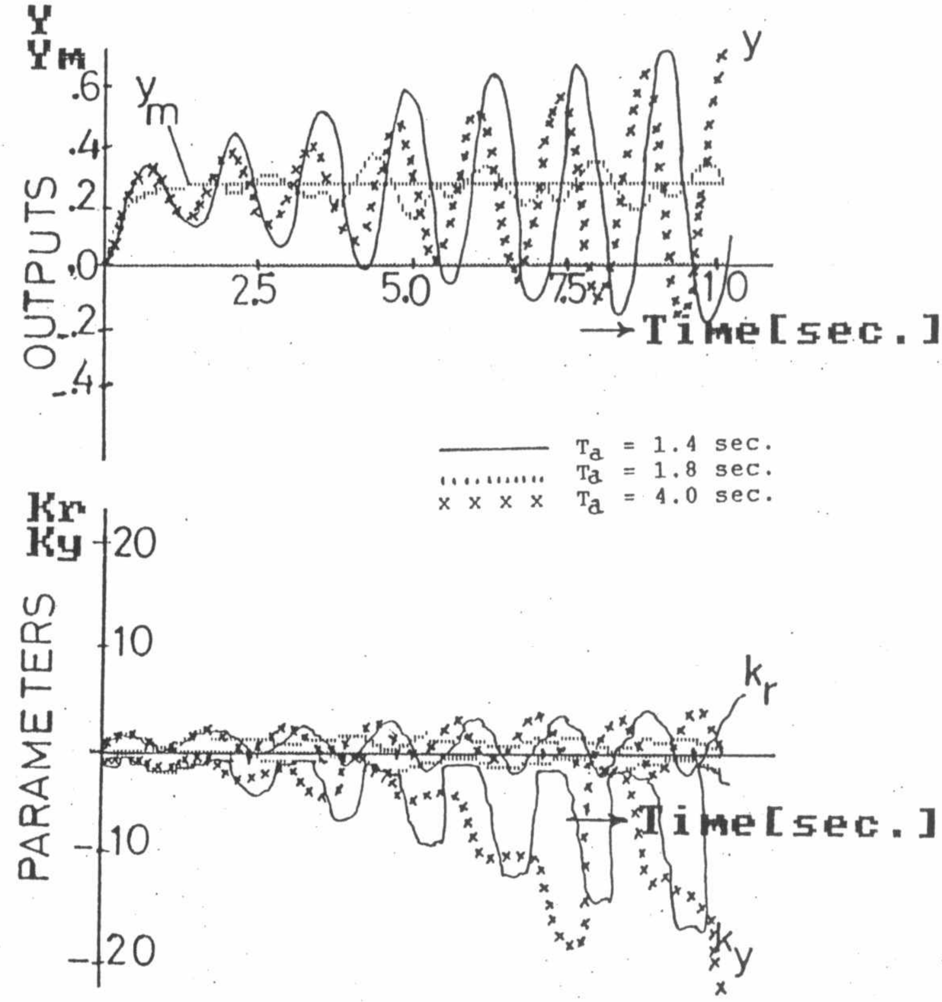


Fig.5. Simulation of CA1 with PRAM, r(t)= 0.3, and d(t)= 1 x $10^{-5}$sin 5t, at different $T_a$.