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Operating LORAN-C Position Fix with Nonhyperbolic Position Lines

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ABSTRACT

Hyperbolic navigation systems (LORAN-C, DECCA Navigator and OMEGA) represent a widely used class of position fixing systems.Most of their incorporated traditional receivers display position fixes in hyperbolic coordinates for plotting either by hand on navigation charts printed with hyperbolic overlays or they plot tracks automatically, often on distorted charts.This makes them expensive and/or manually operated .Moreover ,it amplifies certain accuracy problems related to the angle of cut of the hyperbolic position lines and geometrical factors relevant to these lines.

Avionics applications as well as automatic vehicle location systems entails low-cost receivers which operate wholly or largely automatically and which display position measurements in geographical coordinates (latitude -longitude) or as headings and distances to given waypoints and/or integrated with readings from dead-reckoning sensors.

Solution issues to some of these problems have been addressed in two published papers by the authors.The underlying idea is to involve modifications in the processing algorithms of on-board receivers.In essence ,that modification was based on generating non-hyperbolic position lines :circles and/or ellipses out of the originally designed hyperbolic systems. Theoretical evaluation of that proposal was the subject of one the authors papers.

In the present paper ,the proposed modification is applied to a real ON-EARTH LORAN-C station to confirm the obtained results. Moreover,an algorithm as well as an implementation proposals are presented .The case of short base-length as in the DECCA chains is presented with its algorithm and the proposal for implementation is highlighted.

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INTRODUCTION

Hyperbolic navigation systems are known to offer a superior combination of accuracy and range compared to any ground based systems [1-4] (LORAN-C, DECCA and OMEGA systems). As to the readout from the receiver itself,measured time differences are to be transfered to corresponding hyperbolic lines on a chart manually or with some form of automated plotting on hyperbolic map Digital computers are used to provide readout in latitude and longitude using one of the standard techniques for converting positions from hyperbolic to rectangular coordinates [8]. Most of these techniques are mainly implemented on mainframe computers and are usually inadequate for implementation in low cost navigation receivers for reasons of speeds, addressing space and memory sizes.

Solutions to such a problem as well as the accuracy problems (relevant to the geometric factors of hyperbolic lines) have been addressed in [5] and highlighted in our previous papers [6,7] .In this work ,we present an algorithm that greatly simplifies the implementation of relevant receivers.The algorithm starts from the idea of modifying the processing algorithms employed in the on-board receivers . This modification enables generating non-hyperbolic position lines;namely: circles or ellipses out of the originally designed hyperbolic systems.

Improvement in position fixing systems was theoretically demonstrated in our paper [7] for the hyperbolic systems , especially DECCA where distances among station are mainly confined within the "planification" of the earth surface .Such planification is ,in essence, a utilization of the *TANGENT PLANE* coordinate system .Section II introduces theoretical evaluation procedure assuming the *REFERENCE SPHERE(not ELLIPSOID)*as the earth model. Calculations are done for real LORAN-C stations where station spans justify the use of this type of earth model. In section III, the practical implementation is outlined . Section IV considers the case of short base length ;i.e.the DECCA chains.Section V is reserved for conclusions and future work .

II. THEORETICAL BASES FOR PROPOSED LORAN FIX ALGORITHM

For the generalized spherical coordinates (Fig.1.), a point P(x,y,z) on the earth-surface with longitude λ and latitude ϕ, the co-ordinate values x, y, z are given by :

$$x = R\cos\phi\cos\lambda$$
$$y = R\cos\phi\sin\lambda$$
$$z = R\sin\phi \quad (1)$$

with the earth radius $R = (x^2 + y^2 + z^2)^{1/2}$ (2)

$$\phi = \sin^{-1}(z/R)\ ,\ \lambda = \tan^{-1}(y/R) \quad (3)$$

Let us define the angle θ (see Fig.2.) :

$$\theta = (\rho/R) \quad [\text{ radian }] \quad (4)$$

Defining the unit vector :

$$\vec{e} = \cos\phi \cos\lambda \,\vec{i} + \cos\phi \sin\lambda \,\vec{j} + \sin\phi \,\vec{k} \qquad (5)$$

Let

$$\begin{aligned} a &= \cos\phi \cos\lambda \\ b &= \cos\phi \sin\lambda \\ c &= \sin\phi \end{aligned} \qquad (6)$$

Now ,we can write a plane equation in terms of this vector and the distance d ($d = R\cos\theta$). From now on ,let us use x,y,z and d as normalized values relative to the earth radius R.The plane equation is given as :

$$ax + by + cz = d \qquad (7)$$

Writing such equations for the three involved planes :master and secondaries ,we have :

Master plane :

$$a_o x + b_o y + c_o z = d_o \qquad (8)$$

Secondary plane of S_1 :

$$a_1 x + b_1 y + c_1 z = d_1 \qquad (9)$$

Secondary plane of S_2 :

$$a_2 x + b_2 y + c_2 z = d_2 \qquad (10)$$

The solution of this linear system of equations yields the intersection point P given by:(see Fig.2.)

$$\begin{aligned} x &= d_o\alpha_x + d_1\beta_x + d_2\gamma_x \\ y &= d_o\alpha_y + d_1\beta_y + d_2\gamma_y \\ z &= d_o\alpha_z + d_1\beta_z + d_2\gamma_z \end{aligned} \qquad (11)$$

where :

$$\begin{aligned} \alpha_x &= (b_1c_2 - b_2c_1) / \det(a,b,c) \\ \beta_x &= -(b_0c_2 - b_2c_0) / \det(a,b,c) \\ \gamma_x &= (b_0c_1 - b_1c_0) / \det(a,b,c) \end{aligned}$$

and $\det(a,b,c) = a_0\alpha_x + a_1\beta_x + a_2\gamma_x \qquad (12)$

similar definition for $\alpha_y, \beta_y, \gamma_y$ and α_z, β_z and γ_z are simple to obtain. As we are interested only on points on the earth surface or at low altitudes (relative to R),we use the following check :

$$x^2 + y^2 + z^2 = 1 \qquad (13)$$

To mechanize the solution in terms of the measurements $\Delta\rho_1$ and $\Delta\rho_2$, we make the transformation described in the appendix. This transformation replaces $\Delta\rho_1$ and $\Delta\rho_2$ by their relevant quantities $\Delta\theta_1$ and $\Delta\theta_2$ respectively. This transforms r^2 to :

$$
\begin{aligned}
r^2 = {} & A + B + C + A\cos 2\theta_o + B\cos 2(\theta_o + \Delta\theta_1) \\
& + C\cos 2(\theta_o + \Delta\theta_2) + D\cos(\Delta\theta_1) + D\cos(2\theta_o + \Delta\theta_1) \\
& + E\cos(\Delta\theta_2) + E\cos(2\theta_o + \Delta\theta_2) + F\cos(\Delta\theta_2 - \Delta\theta_1) \\
& + F\cos(2\theta_o + \Delta\theta_1 + \Delta\theta_2)
\end{aligned}
\tag{14}
$$

Hence the algorithm for finding P (x,y,z) boils down to :

-Given the specific master and secondary stations by their respective longitudes and latitudes λ and ϕ.
-Find the constant quantities A, B, C, D, E and F.
-Measurements $\Delta\rho_1$ and $\Delta\rho_2$ are transformed to $\Delta\theta_1$ and $\Delta\theta_2$.
-Select ρ_o , $\Rightarrow \theta_o$ then $d_o = \cos\theta_o$.
-Find all the cosines given above.
-Substitute in the above equation, and iterate upon the values of ρ_o or equivalently θ_o.
-Terminate the iteration when $J= r^2 = 1$.
-Find x, y and z corresponding to the value of $J = r^2 = 1$.
-Find the value of λ and Φ (instead of x,y and z) corresponding to the value of $J = r^2 = 1$,if needed .

This algorithm was implemented on an IBM-PC and Fig.3.gives a sample result for the LORAN-C chains in the Saudi Arabia .A more useful presentation of the same results is depicted on Fig. 4. In this figure ,the logarithm of the quantity J is plotted .The required solution is attained at the point of crossing the horizontal Zero -line .This reduce the task to that of finding the zero crossing point .

III.LORAN-C ORIENTED IMPLEMENTATION PROPOSAL

To implement systems adequate for application in low cost navigation receivers enjoying convenient speeds,addressing space and memory sizes ,algorithms should be tailored to suit such applications .The proposed algorithm implementation is shown in Fig.5. A precomputed transformation for the coordinates of every Loran-c chain to the corresponding constants is stored in a read-only memory as a lock-up table addressable by the chain number This would surely reduce the on-line computations time .By the same token, the cosine as well as the logarithmic functions are stored with sufficient accuracy in other lock-up tables.
This explains the presence of three ROMs .Thus ,a good share of computations will be reduced merely to the addressing function of the relevant ROMs . Calculating the value J for an initial guess value θ_o then finding the sign of its logarithm .The iteration continues with an increase or decrease of that value by an increment according to the sign of the logarithm.Iteration stops at Zero-crossing point to deliver the position computed in terms of the longitude and latitude coordinates .
For frequent position changes updating, the coordinates along the trajectory is based upon utilizing the previous obtained value of

θ_o as seed for iteration for the new position .A minimum mean square error filtering algorithm may be also implemented for improving the LORAN-C system accuracy and delivering the the vehicle course through time recursion of the type shown in Fig.6. Moreover ,the other navigational quantities such as ground velocity, heading and dead-reckoning instrument errors can be calculated from successive values of found positions .

IV. DECCA ORIENTED IMPLEMENTATION PROPOSAL

As far as Decca navigation system is considered ,distances among stations are reduced to about 100 km .The 3-planes seen in LORAN-C chains degenerate totally to practically one plane .Hence ,for such conditions ,a separately developed algorithm is to be used. We consider the limit case ,where the whole chain lies in one plane.Such planification of the earth surface is ,in essence, a resort to the *TANGENT PLANE* coordinate system
The idea of the following algorithm was outlined in our previous papers [6,7] and the derivation of the used algorithm is given in the appendix . On the basis of Fig.7 ,the iterative process is started by selecting the radius ρ_o for :

$$\rho_o = \max \{(d/2 - \Delta\rho_1) , (d/2 - \Delta\rho_2)\} \tag{15}$$

For this value of ρ_o, we find $\rho_1 = \rho_o + \Delta\rho_1$, and $\rho_2 = \rho_o + \Delta\rho_2$, these circles intersect the master circle at point P_{o1} and point P_{o2} shown in the figure.
From these two points we find the angles θ_o and δ_o (as start point shown in the figure θ_o = zero) as initial angles for the iteration process :

$$\rho_k \cos\theta_k - \rho_{k-1} \cos\theta_{k-1} = S\Delta_1 \tag{16}$$

$$\rho_k \cos\delta_k - \rho_{k-1} \cos\delta_{k-1} = S\Delta_2 \tag{17}$$

where S = ($\Delta\rho$ /d) as defined in the appendix.
As far as $\theta_o + \delta_o \neq \Psi$, we increase the radius ρ_1 by a step $(\Delta\rho)$ and check $\theta + \delta = \Psi$, iteration stops at iteration k when

$$\theta_k + \delta_k = \Psi . \tag{18}$$

To implement this iteration, we have to simplify the computations and choose a sharp parameter to stop iteration.This is again realized by the logarithm function .Hence the algorithm is as follows :

-Select $\rho = (\rho_o /d)$, compute $\Delta_1 = (\Delta\rho_1 /d)$ and $\Delta_2 = (\Delta\rho_2 /d)$

-Using the iteration formula ,Find $\rho_k \cos\theta_k$ and $\rho_k \cos\delta_k$

-Find $\rho_k \cos(\psi - \theta_k)$

$$\rho_k \cos(\psi-\theta_k) = (\rho_k \cos\theta_k) \cos\psi - \sin\psi \{\rho^2_k - (\rho_k \cos\theta_k)^2\}^{0.5}$$

-Find $L = \text{Log}\{\rho_k \cos(\psi-\theta_k)/\rho_k \cos\delta_k\}$

-Find the zero crossing point of L ,hence ρ_k

Implementation of this algorithm is given in Fig .8

V. CONCLUSIONS

The present work introduces an implementation proposal aiming at a solution to some long lasting problems inherent in the hyperbolic navigation systems (DECCA,LORAN-C and low-cost OMEGA). The proposed modification,while affording better position fixing, it is appended to the currently existing conventional systems simply as an "Add-on" product without any other modification .One of the very interesting merits of that implementation is that it offers a striking simplicity along with remarkably real time operability.We are working in an additional software for selecting best combination of the now-available lines (circles and hyperbolas)that would greatly improve the performance and open a survivability issue to DECCA , LORAN-C and low-cost OMEGA systems.

Appendix

A. LORAN-C CHAIN RELATIONS

$$
\begin{aligned}
r^2 &= x^2 + y^2 + z^2 \\
&= d_o^2(\alpha_x^2+\alpha_y^2+\alpha_z^2) + d_1^2(\beta_x^2+\beta_y^2+\beta_z^2) + d_2^2(\gamma_x^2+\gamma_y^2+\gamma_z^2) \\
&\quad + 2d_od_1(\alpha_x\beta_x+\alpha_y\beta_y+\alpha_z\beta_z) + 2d_od_2(\alpha_x\gamma_x+\alpha_y\gamma_y \\
&\quad +\alpha_z\gamma_z) + 2d_1d_2(\gamma_x\beta_x+\gamma_y\beta_y+\gamma_z\beta_z)
\end{aligned} \tag{A1}
$$

Since $d_o = \cos\theta_o$, then $d_o^2 = 0.5\,(1+\cos 2\,\theta_o)$. This is equally correct for d_1 and d_2 .

Let us transform d_1 and d_2 as follows :

$$
\rho_1 = \rho_o + \Delta\rho_1 \quad\Rightarrow\quad \theta_1 = \rho_1 / R = \theta_o + \Delta\theta_1 \quad\Rightarrow d_1 = \cos(\theta_o+\Delta\theta_1)
$$

$$
\rho_2 = \rho_o + \Delta\rho_2 \quad\Rightarrow\quad \theta_1 = \rho_2 / R = \theta_o + \Delta\theta_2 \quad\Rightarrow d_2 = \cos(\theta_o+\Delta\theta_2)
$$

Let

$$
\begin{aligned}
A &= 0.5\,(\alpha_x^2+\alpha_y^2+\alpha_z^2), \quad & B &= 0.5\,(\beta_x^2+\beta_y^2+\beta_z^2) \\
C &= 0.5\,(\gamma_x^2+\gamma_y^2+\gamma_z^2), \quad & D &= \alpha_x\beta_x+\alpha_y\beta_y+\alpha_z\beta_z \\
E &= \alpha_x\gamma_x+\alpha_y\gamma_y+\alpha_z\gamma_z, \quad & F &= \gamma_x\beta_x+\gamma_y\beta_y+\gamma_z\beta_z
\end{aligned} \tag{A2}
$$

Substituting this transformation in the above equation, we have :

$$
\begin{aligned}
r^2 &= A + B + C + A\cos 2\theta_o + B\cos 2\,(\theta_o+\Delta\theta_1) \\
&\quad + C\cos 2\,(\theta_o+\Delta\theta_2) + D\cos(\theta_o-\theta_1) + D\cos(\theta_o+\theta_1) \\
&\quad + E\cos(\theta - \theta_2) + E\cos(\theta_o+\theta_2) + F\cos(\theta_2-\theta_1) \\
&\quad + F\cos(\theta_2+\theta_1)
\end{aligned} \tag{A3}
$$

Substituting for θ_1 and θ_2 in terms of θ_o,$\Delta\theta_1$ and $\Delta\theta_2$, we have :

$$r^2 = A + B + C + A\cos 2\theta_o + B\cos 2(\theta_o + \Delta\theta_1) + C\cos 2(\theta_o + \Delta\theta_2) + D\cos(\Delta\theta_1) + D\cos(2\theta_o + \Delta\theta_1) + E\cos(\Delta\theta_2) + E\cos(2\theta_o + \Delta\theta_2) + F\cos(\Delta\theta_2 - \Delta\theta_1) + F\cos(2\theta_o + \Delta\theta_1 + \Delta\theta_2) \tag{A4}$$

B. DECCA CHAIN RELATIONS

The relation between ρ_1 and θ is given by :

$$\rho_1^2 = \rho_o^2 + d^2 + 2\rho_o d\cos\theta \tag{A5}$$

but : $\rho_1^2 = \rho_o^2 + 2\rho_1\rho_o + \Delta\rho_1^2$

so : $2\rho_1\rho_o + \Delta\rho_1^2 = d^2 + 2\rho_o d\cos\theta$ (A6)

$$2(\rho_o/d)\cos\theta = [(\Delta\rho_1/d)^2 - 1] + 2(\Delta\rho_1/d)(\rho_o/d)$$

Let $(\rho_o/d) = \rho$ and $(\Delta\rho_1/d) = \Delta_1$

and at the start point $\theta = \theta_o$

then $\therefore 2\rho\cos\theta_o = (\Delta_1^2 - 1) + 2\Delta_1\rho$ (A7)

iterate by $S = (\Delta\rho/d)$ then (A8)

$$\rho_k = \rho_{k-1} + S$$

$$2\rho_k\cos\theta_k = (\Delta_1^2 - 1) + 2\rho_k\Delta_1$$

$$2\rho_{k-1}\cos\theta_{k-1} = (\Delta_1^2 - 1) + 2\rho_{k-1}\Delta_1$$

$$\therefore \rho_k\cos\theta_k - \rho_{k-1}\cos\theta_{k-1} = S\Delta_1 \tag{A9}$$

Similar relations are obtained for the other stations i.e.:

$$\rho_k\cos\delta_k - \rho_{k-1}\cos\delta_{k-1} = S\Delta_2 \tag{A10}$$

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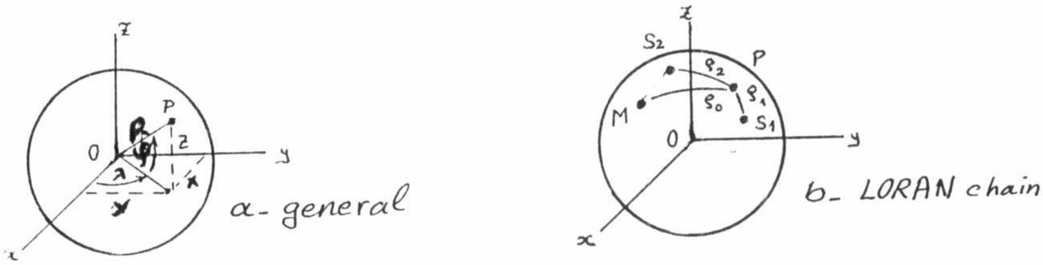


Fig.1. Genelarized spherical coordinate system

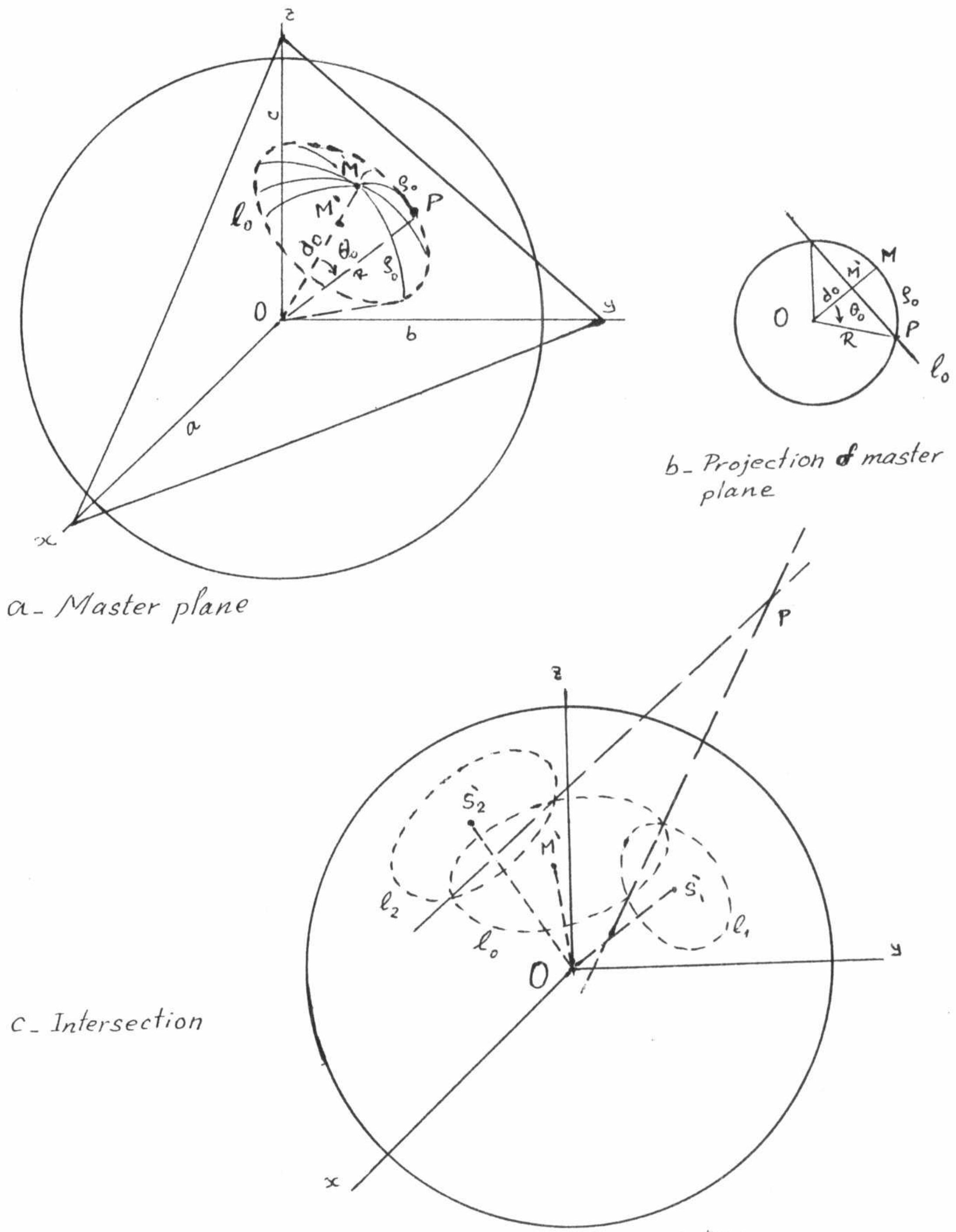
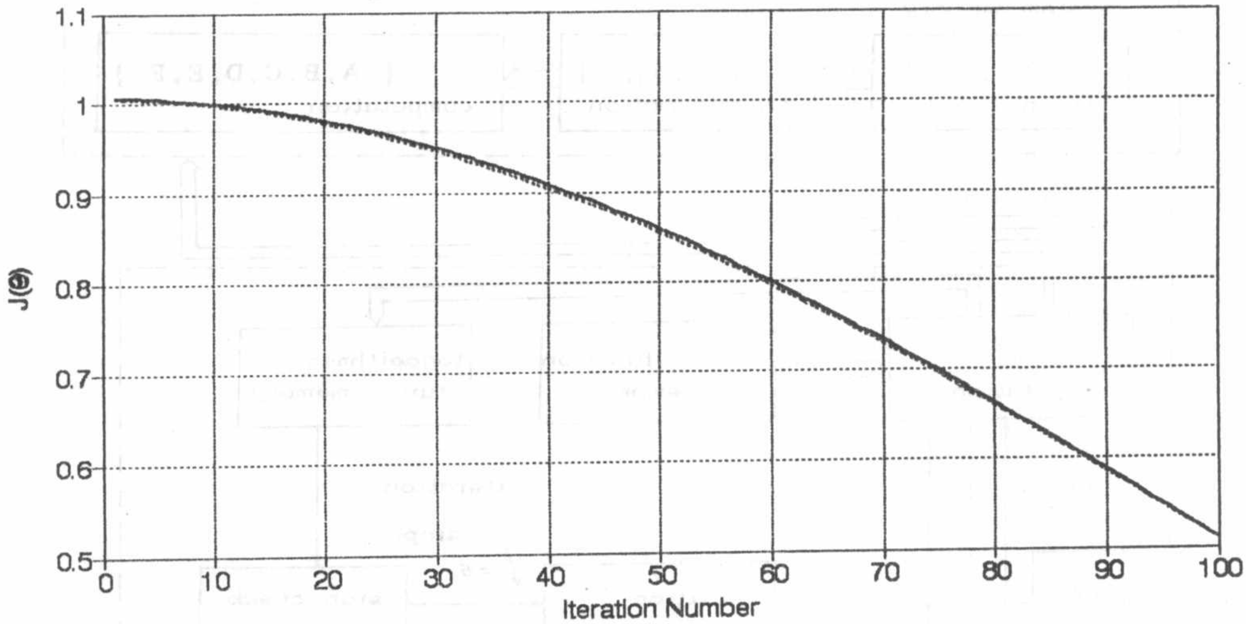


Fig.2. Generation and intersection of the three planes

Fig.3.Sample Result for S.Arabia Chain



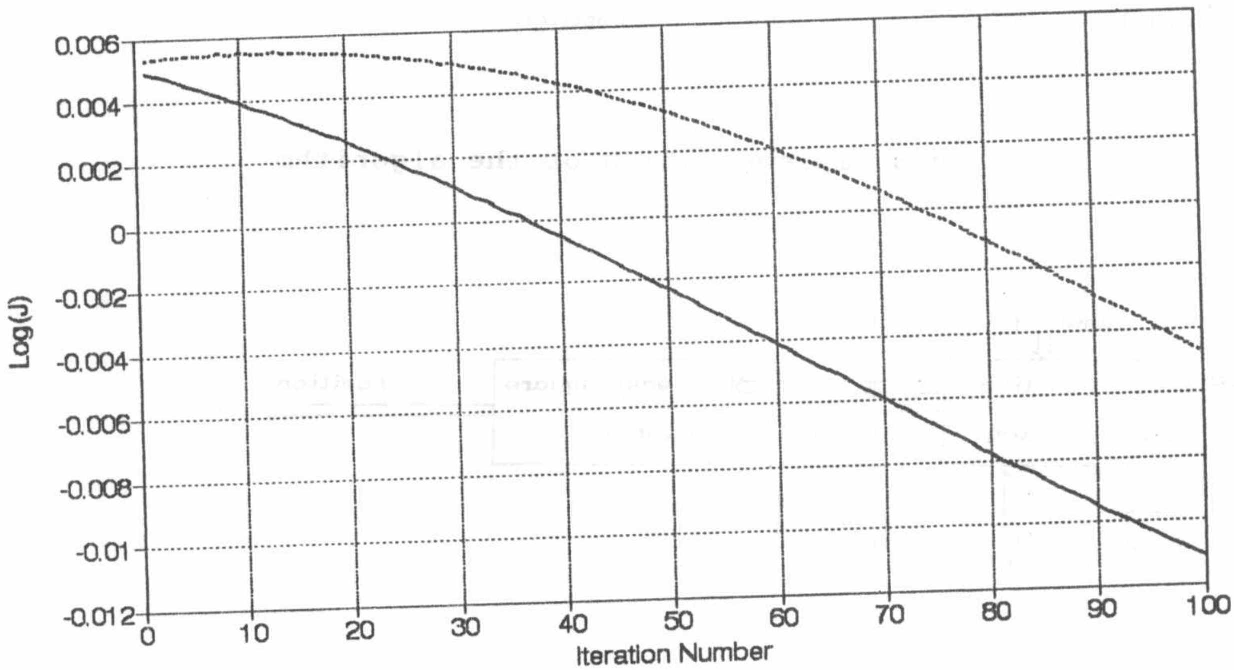


Fig.4. Zero crossing of the Logathimic function

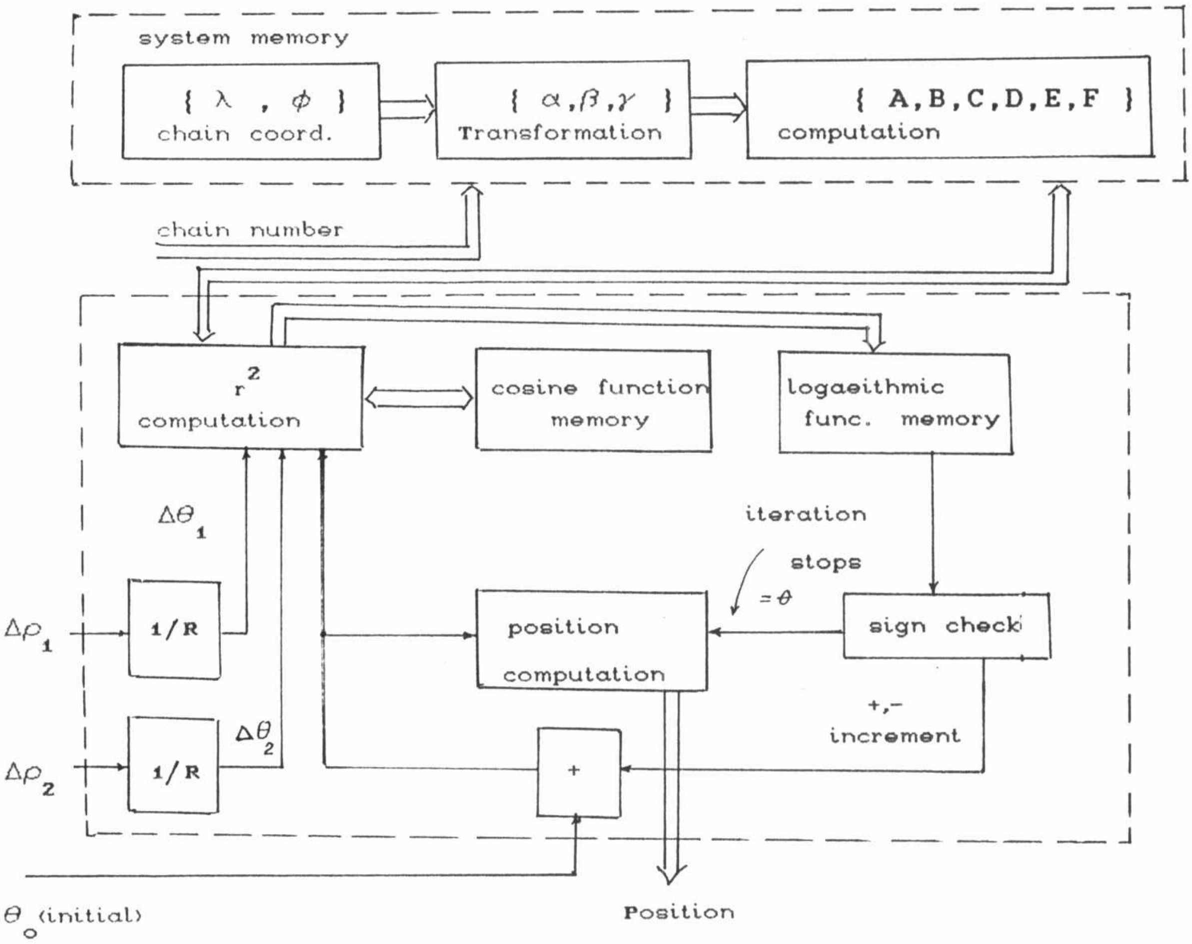


Fig.5.Implementation of the algorithm

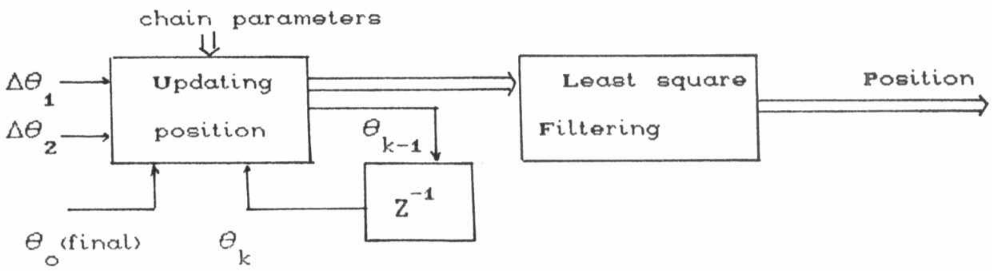


Fig.6. Updatind with least-square filtering

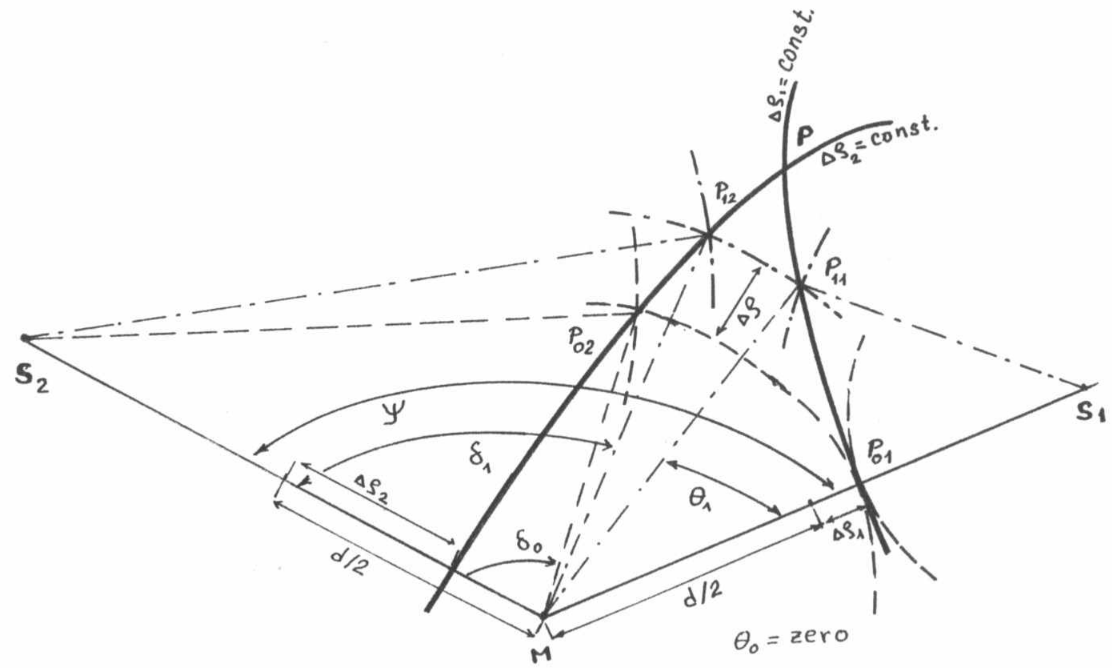


Fig.7.Iteration Algorithm for DECCA System

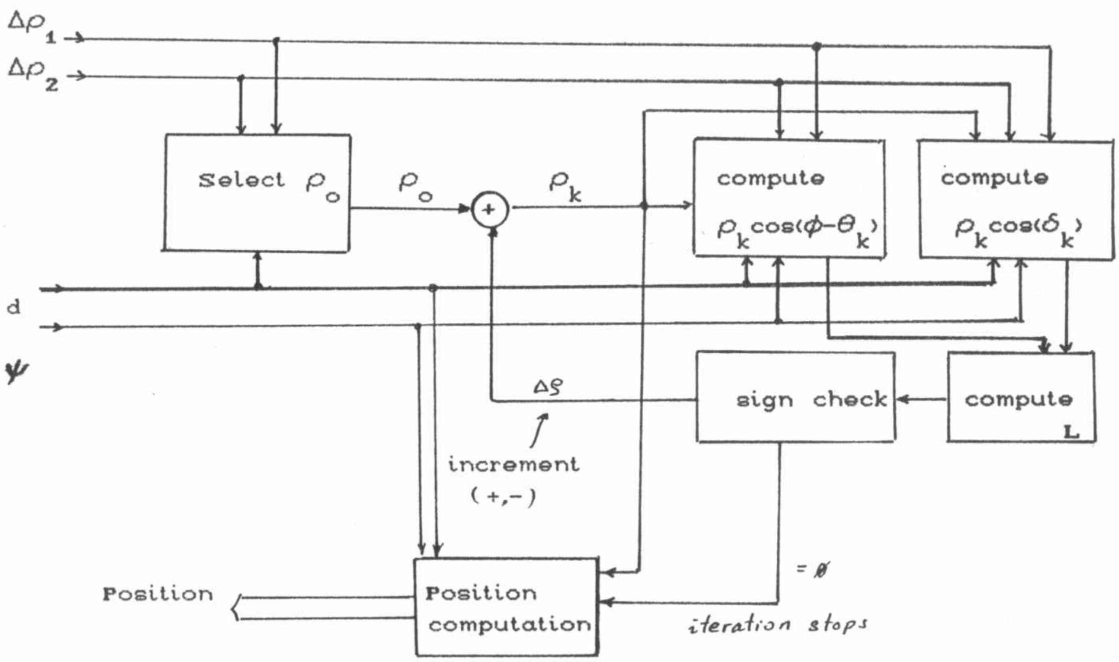


Fig.8.Implementation of the algorithm for DECCA System