



DEVELOPMENT OF A PANEL METHOD FOR THE STUDY OF TWO-DIMENSIONAL POTENTIAL FLOW OVER SINGLE AND TWO-ELEMENT AIRFOILS

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ABSTRACT

Panel methods proved a powerful numerical method capable for handling both internal and external flows. In this paper a vortex panel method is employed for studying the potential flow over one and two-element airfoils. A second order vortex method is developed to overcome the instability found in the solution when a first order vortex method is employed. Three different methods for paneling are suggested. The effect of the number of panels is also examined. Comparison of the results obtained by this model and the exact solution of Williams two-element airfoil proved its validity. The effects of the gap and the overlap in the case of two-element airfoil are examined.

INTRODUCTION

The increasingly demand for greater pay-loads in civil transport airplanes as well as the short take-off and landing requirements for faster fighter airplanes necessitates successive advancement of high-lifting devices. These devices are either flaps or slats attached also to either the leading-or-trailing-edges of the wing. Through these devices the wing section is instantaneously and temporarily changed during take-off and landing, leading to an increase in the chord length together with an achievement of a higher lift coefficient.

The complexity of wing design both from aerodynamical, structural and mechanical points of view arises from such high lift devices. Concerning the aerodynamic design, the wing characteristics are determined either experimentally or numerically. In the early design stages numerical procedure is preferred; as all the parametric investigations could be fulfilled simply by changing some input data. Both two and three-dimensional analysis are now visible due to the availability of supercomputers having an endless memory and an extensively fast execution. Experimentation as a counterpart to theoretical and/or numerical analyses is usually

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much more expensive though is necessary prior to the final stage of design and manufacturing.

The present work belongs to the theoretical side, where the flow field around single and two-element airfoils is treated. The governing equations of the two-dimensional potential flow around such a wing section is numerically solved using the vortex panel methods.

Historically, the panel methods was originated by Prager [1] in 1928. However, no progress was achieved in such methods till the arrival of high speed digital computers. In the sixties, non-lifting potential flow problems were treated using surface source methods; refer for example to Smith and Hess [2]. In the seventies, Hess [3] modified his surface source method to include the lift effects using a constant strength vorticity distribution in addition to the distributed sources.

In this paper, the first-order vortex panel method is applied to the potential flow about single-element airfoils. However, such a method proved unsatisfactory with high oscillatory solution originating from the resulting ill-posed system of equation. Thus a second-order vortex panel method was developed, which furnished an accurate solution when compared with other published results. The second-order vortex panel method was applied to the NACA 0012 airfoil and the two-element Williams airfoils. The tangential velocity, pressure and lift coefficients were calculated.

Other panel methods are also available in the open literature. For example, Eppler [4] employed surface singularities of parabolic strengths on curved surface panels.

THEME OF WORK

The surface panel method philosophy for solving arbitrary subsonic potential flow problems involve mating of the classical potential theory with contemporary numerical techniques. Classical theory is used to reduce an arbitrary flow problem to a surface integral equation relating boundary conditions to an unknown singularity distribution. The contemporary numerical techniques are then used to calculate an approximate solution to the integral equation.

The details of the theme of work are as follows:

- 1- Identification and then panelling the surface (s) on which the singularity is to be distributed.
- 2- Decision regarding the choice of singularity.
- 3- Selection of the function that approximates the unknown singularity distribution on the panelled surface.
- 4- Selection of a set points, called control points.
- 5- Developing a mathematical expression for the velocity potential on the airfoil surface.
- 6- Applying the boundary condition to the control points of all the panels.
- 7- Satisfying the kutta condition at the airfoils trailing edge.
- 8- Numerical solution of the simultaneous equations generated in the previous two steps to evaluate the singularity strengths.

- 9- Calculating the tangential velocity, pressure coefficient over the airfoil surface and the resulting lift coefficient.

MATHEMATICAL MODEL

In this section the mathematical model is formulated for the general case of two-element airfoil. The case of a single-element airfoil could then be easily deduced.

Consider a two-element airfoil, as shown in Figure (1), the main airfoil is set at an angle of attack α w.r.t the undisturbed free stream having a speed V_∞ , while the flap is deflected an angle δ_f w.r.t the chord line of the main airfoil. Each airfoil element is modeled by connected straight line segments (panels) on which a piece-wise vortex distribution (either of constant or linearly varying intensity) is positioned. Figure (2) illustrates such a procedure for one element airfoil.

Panelling Procedure

One of the simplest method for panelling the surface is to select a number of points on the airfoil's surface (which will be designated as boundary points). Upon connecting each two neighbouring points by a straight line, then these straight lines are named as panels. There are three types of panelling. The x-coordinates in each type can be defined from either one of these equations.

1st Method: $X_K/C = 1 - \cos \theta_{(N/2-k)}$ or

2nd Method: $X_K/C = (1 + \cos \theta_K)/2$, or (1)

3rd Method: $X_K/C = 1/(N/2)$

Where N is the number of panels (even number),

$$\theta_{(N/2-k)} = (N/2 - k) \pi / N ,$$

$$\theta_K = K \pi / (N/2)$$

$$K = 0, 1, \dots, N/2$$

Figure (3) illustrates the distribution of boundary points using different panelling methods for a ten panel case.

The next step is to find the y-coordinates of these boundary points from the airfoil geometry. Joining each two successive points generate the panels, which furnish a continuous broken line over the airfoil surface. By the increase of the number of nodes such a broken line approaches or even coincides with the curved shape of the airfoil. On the mid of these panels control points are chosen. At these points the boundary conditions are satisfied.

Velocity Potential

The velocity potential (Φ_i) at any control point (x_i, y_i) on either airfoil element is influenced by the vorticity distribution on both elements and the undisturbed free stream. Consequently, it could be expressed as:

$$\Phi_i = V_\infty (x_i \cos \alpha + y_i \sin \alpha) - \frac{1}{2\pi} \sum_{b=1}^2 \sum_{j=1}^{N_b} \int_{b_j} \left\{ \gamma_{bj} + \left(\frac{\gamma_{bj+1} - \gamma_{bj}}{S_{bj}} \right) s_{bj} \right\} \tan^{-1} \left(\frac{y_i - y_{bj}}{x_i - x_{bj}} \right) ds_{bj} \quad (2)$$

Where the integral accounts for the variation of vortex strength over the j th panel of the b th body, while the inner summation for all the panels over one airfoil element, and the outer summation for both elements of airfoil. Figure (4) illustrates the geometry required for evaluating the integral in equation (2) for one element airfoil.

Boundary Conditions

Here two boundary conditions exist; namely, the zero normal velocity over the airfoil surface and the Kutta condition for velocity at the trailing edge. The zero normal velocity at the airfoil surfaces furnishes the following equation derived from equation (1) after some deduction:

$$\sum_{b=1}^2 \sum_{j=1}^{N_b} ((J_{bij} - M_{bij}) \gamma'_{bj} + M_{bij} \gamma'_{bj+1}) = \sin(-\theta_{bi}) \quad (3)$$

Where, $i=1, \dots, N_b$ and $b=1, 2$

and $\gamma'_K = \gamma_K / 2\pi V_\infty$

$$J_{bij} = \int_{b_j} \frac{\partial}{\partial n_{bi}} \tan^{-1} \left(\frac{y_i - y_{bj}}{x_i - x_{bj}} \right) ds_{bj} \quad (4)$$

$$M_{bij} = \int_{b_j} \frac{s_{bj}}{S_{bj}} \frac{\partial}{\partial n_{bi}} \tan^{-1} \left(\frac{y_i - y_{bj}}{x_i - x_{bj}} \right) ds_{bj}$$

J_{bij} and M_{bij} are denoted the influence coefficients, which upon integration yield the following final expressions

$$J_{bij} = -\frac{1}{2} \cos(\theta_i - \theta_{bj}) \ln \left(\frac{S_{bj}^2 + 2A S_{bj} + B}{B} \right) - \sin(\theta_i - \theta_{bj}) \left(\tan^{-1} \frac{S_{bj} + A}{E} - \tan^{-1} \frac{A}{E} \right) \quad (5)$$

and

$$M_{bij} = -\frac{D}{S_{bj}} \left(S_{bj} - A \ln \left(\frac{S_{bj}^2 + 2A S_{bj} + B}{B} \right) + \left(\frac{2A^2 - B}{E} \right) \left(\tan^{-1} \frac{S_{bj} + A}{E} - \tan^{-1} \frac{A}{E} \right) + \frac{1}{S_{bj}} (y_{bi} - y_{bj}) \sin \theta_i + (x_i - x_{bj}) \cos \theta_i \right) \left(-\frac{1}{2} \ln \left(\frac{S_{bj}^2 + 2A S_{bj} + B}{B} \right) \right)$$

$$- \frac{A}{E} \left(\tan^{-1} \frac{S_{bj} + A}{E} - \tan^{-1} \frac{A}{E} \right) \quad (6)$$

Where

$$A = -(X_i - X_{bj}) \cos \theta_{bj} - (Y_i - Y_{bj}) \sin \theta_{bj}$$

$$B = (X_i - X_{bj})^2 + (Y_i - Y_{bj})^2$$

$$D = (Y_i - Y_{bj}) \sin \theta_i + (X_i - X_{bj}) \cos \theta_i$$

$$E = (B - A^2)^{1/2}$$

$$\text{For } j=i \quad J_{bij} = 0.0, \quad M_{bij} = -1.0$$

The second boundary condition; namely the Kutta condition, which emphasises the equality of the tangential velocities at 1st and Nth control points of each element.

$$V_{1b} = V_{Nb} \quad b = 1, 2 \quad (7)$$

Equations (3) and (7) furnish a set of (N_b+2) simultaneous equations (where $b=1,2$), which will have the form:

$$[A_{ij}] \{ \gamma_j \} = \{ B_i \} \quad (8)$$

The Tangential Velocity

Once the singularity strengths are known, the solution of the tangential velocity at the control point of each panel on either elements can be calculated from the relation:

$$V_i = \frac{\partial}{\partial t_{bi}} \Phi_i \quad (9)$$

where t_{bi} is the unit vector tangent to the surface of the b th panel. Then from equations (2) and (9) we get the following relation

$$\frac{V_i}{V_\infty} = \cos(\alpha - \theta_i) - \sum_{b=1}^2 \sum_{j=1}^{N_b} ((J'_{bij} - M'_{bij}) \gamma'_{bj} + \gamma'_{bj}) \quad (10)$$

$$\text{with } i=1, \dots, N_b \\ b=1, 2$$

The new influence coefficients J'_{bij} and M'_{bij} are similar in principle to J_{ij} and M_{ij} , and due to the limited length of paper will not be given here. However, it is worthy to identify the special case when $j=i$, then

$$J_{bij} = -\pi \quad \text{and} \\ M_{bij} = -\pi / 2$$

Pressure and Lift Coefficients

Once the tangential velocity at the control point of each panel is determined, then using Bernoulli's equation, the pressure coefficient is evaluated from the well-known relation:

$$C_{p_i} = 1 - \left(\frac{V_i}{V_\infty} \right)^2 \quad (11)$$

Assuming that the tangential velocity is uniform on each panel, then the total lift is evaluated from the relation:

$$C_L = 2 \sum_{b=1}^2 \sum_{i=1}^{N_b} \left(\frac{V_i}{V_\infty} \right) \left(\frac{S_{bi}}{C_b} \right)$$

where C_b is the chord of the element number b of the airfoil.

NUMERICAL SOLUTION

The mathematical model yielded a set of linear equations expressed by equation (8) with the vortex strengths as their unknown. To solve these equations, the well-known and effective computer package known as LLSQ is employed.

RESULTS AND DISCUSSION

In this section, the previously described panel method was applied to two main problems; the single-element and two-element airfoils. Concerning the single-element airfoil several parameters were investigated namely:

- 1- Vortex Distribution
- 2- Number of Panels
- 3- Panelling Method

Concerning the two-element airfoils, two parameters were examined. These are:

- 1- The overlap between the airfoil and flap.
- 2- The gap for constant overlap.

One-element Airfoil

The first-order vortex panel method is applied to NACA 0012 airfoil at different angles of attack (0° , 2° and 9°) and variable number of panels ($N=12, \dots, 42$). Such a vortex distribution causes oscillations in the C_p distribution, and the minimum oscillations occurs when $N=40$. Figure (5) clarifies such conclusion. The presence of such oscillations could be interpreted by examining equation (8). The matrix A for a first-order vortex distribution is ill-posed as its elements are so small and the corresponding elements of its inverse A^{-1} are very large. The maximum value of the determinant A occurs at $N=40$ which leads to minimum oscillations. Furthermore, the control point and vortex center are coincident which gives zero normal velocity component due to vortex distribution over the i th panel. Thus an off-diagonal coefficient matrix is obtained, causing the ill-conditioned problem; Nobel 5. When a second order vortex panel method is employed, the oscillations in the pressure distribution has been removed, and stability dominates with the increase of the number of panels, as shown in Figure (6).

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For the second-order vortex panel method, the control point and vortex panel center are no longer coincident, thus the ill-condition situation vanishes and a stable solution is obtained.

The pressure distribution over the NACA 0012 airfoil at an angle of attack of 9° was calculated and plotted in Figure (6) for a successively increasing number of panels ($N=18, 24, 30, 36, 42$ and 48). In all cases, a stable solution is obtained. Increasing the number of panels improves the accuracy of solution particularly at the leading edge.

For examining the effect of panelling on the accuracy of solution, the NACA 0012 airfoil was again examined using the three types of panelling techniques for a constant number of panels ($N=36$) and zero angle of attack; refer to Figure (7). It is clear from this Figure that the first type of panelling predicts the C_p distribution at the leading edge better than the two other types of panelling as the first type of panelling represents the leading edge with large number of panels compared to the others. In regard with the third type of panelling, it is worthy mentioning here that there is a need for representing the leading and trailing edges by more number of panels than the mid portion of the airfoil due to the large velocity and pressure gradient in both domains.

Two-Element Airfoils

The two-element Williams airfoil of Figure (8) represents a real design that achieves high lift coefficients. Since its exact solution is available, refer to Williams (6), the case was treated using the prepared model of the 2nd order vortex panel method and the obtained results were compared to that exact solution. An excellent agreement is noticed from Figure (8). A parametric study for examining the effects of overlap and gap on the pressure distribution and lift coefficient for both elements have been performed.

Figure (9) examines the effect of overlap for nearly constant gap. It is noticed that as the flap approaches the main airfoil, the lift force generated by both the airfoil and flap decrease.

Figure (10) illustrates the contribution of gap to both C_p and lift for constant overlap. It is noticed that an increase in the gap furnishes an increase in the lift for both airfoil and flap.

The CPU time on PC machines for the two-element Williams airfoil using 122 panels is approximately 60 second.

CONCLUSIONS

- 1- The first-order vortex panel method provides oscillatory solutions due to the resulting ill-conditioned system of equation.
- 2- The second-order vortex panel method yields satisfactory solutions of high accuracy provided a large number of panels is adopted.
- 3- The first type of panelling is the best for predicting the pressure distribution at the leading edge of airfoils.

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- 4- The developed second-order vortex panel method could handle not only two-element airfoil but also airfoils having a number of elements larger than two.

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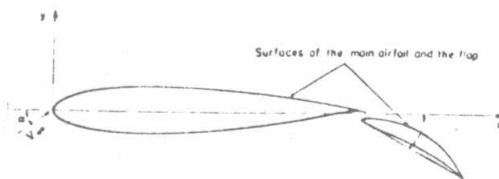
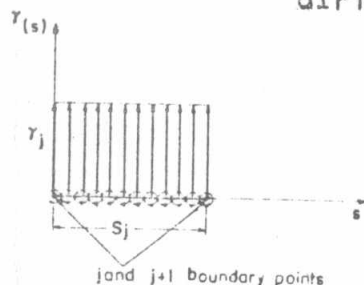
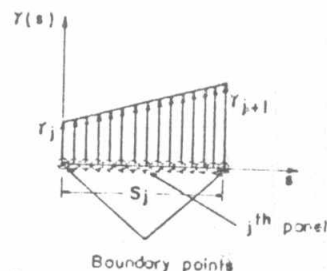


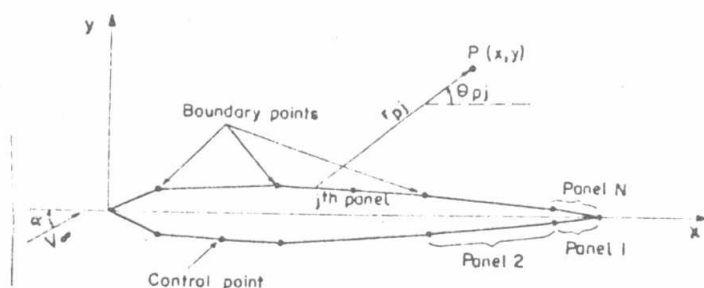
Fig. (1) Geometry of an arbitrary two-element airfoil.



(a) 1st order vortex method.

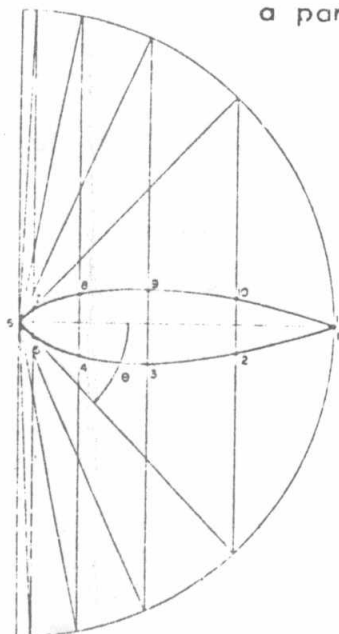


(b) 2nd order vortex method.

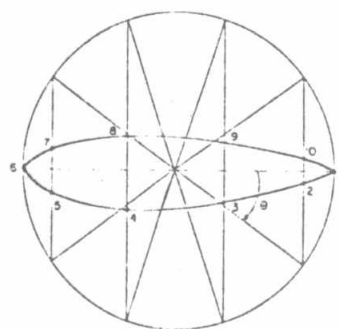


(c) Panelling method.

Fig. (2) First and Second order vortex distribution over a panelled airfoil.



1st Panelling Method.



2nd Panelling Method.



3rd Panelling Method.

Fig. (3) Distribution of boundary points using different panelling methods for $N = 10$.

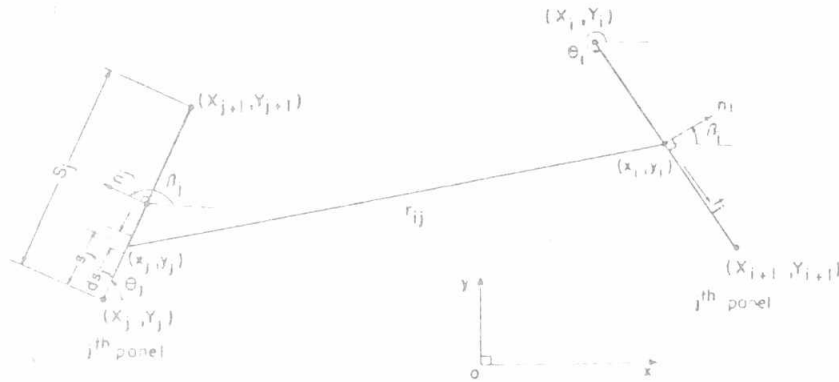


Fig. (4) Geometry required for the evaluation of the Integrals.

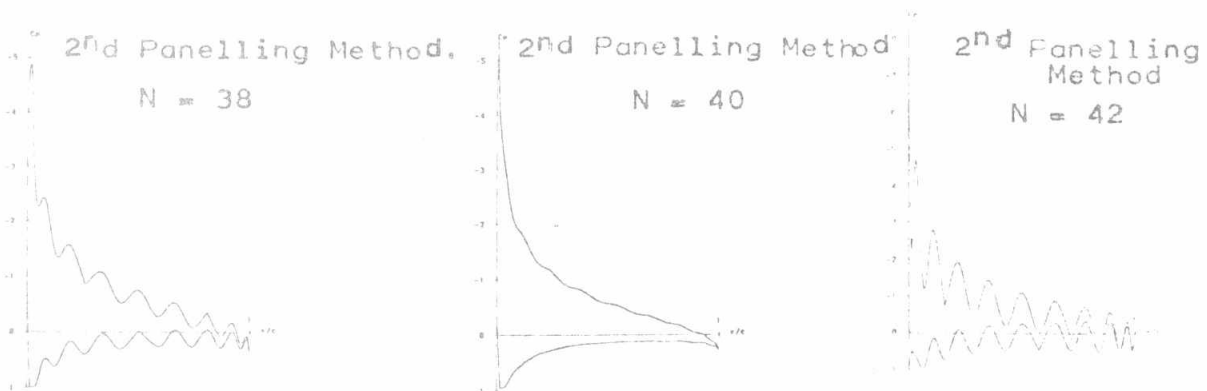


Fig. (5) Pressure distribution using 1st order vortex panel method on airfoil of NACA 0012 at $\alpha = 9^\circ$.

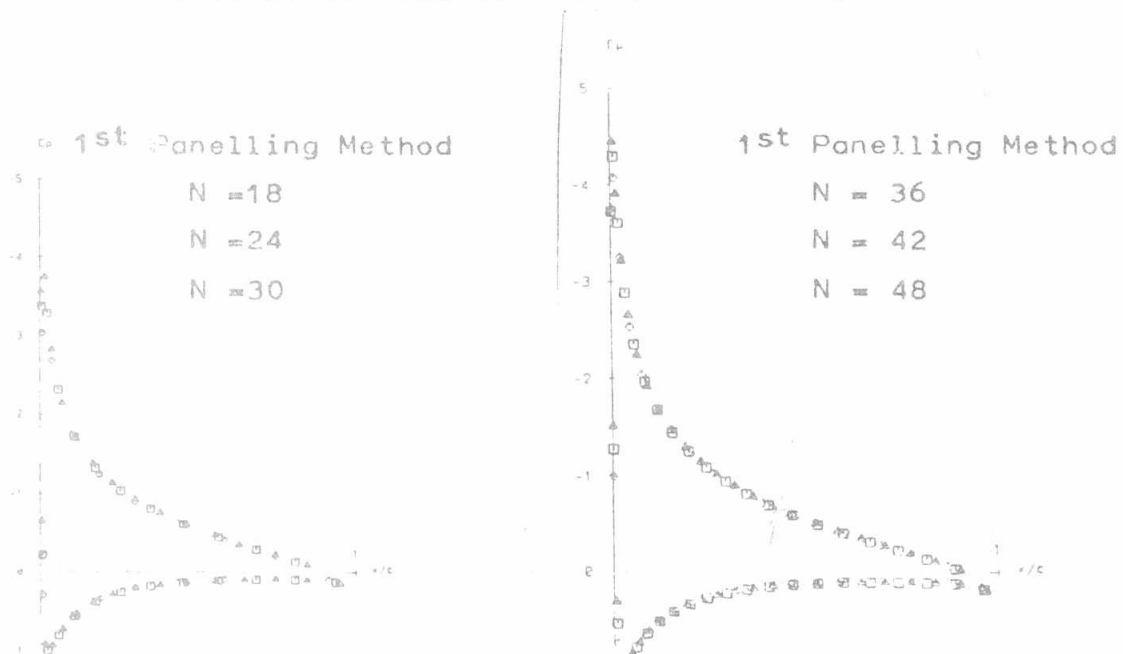


Fig. (6) Pressure distribution using 2nd order vortex panel method on airfoil of NACA 0012 at $\alpha = 9^\circ$.

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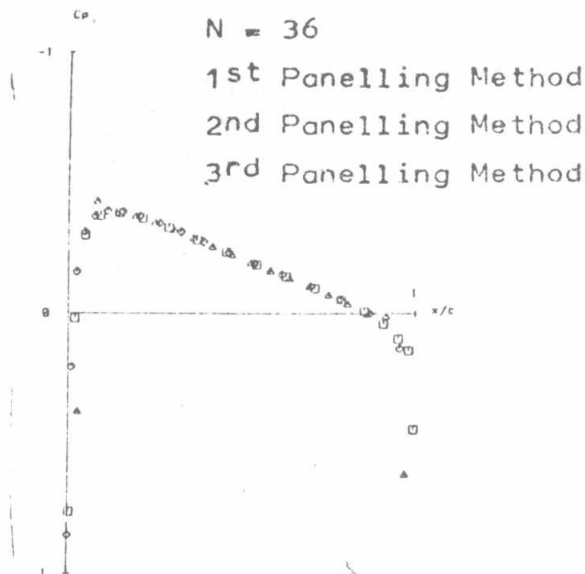


Fig. (7) Pressure distribution using 2nd order vortex panel method on airfoil of NACA 0012 at $\alpha = 0^\circ$.

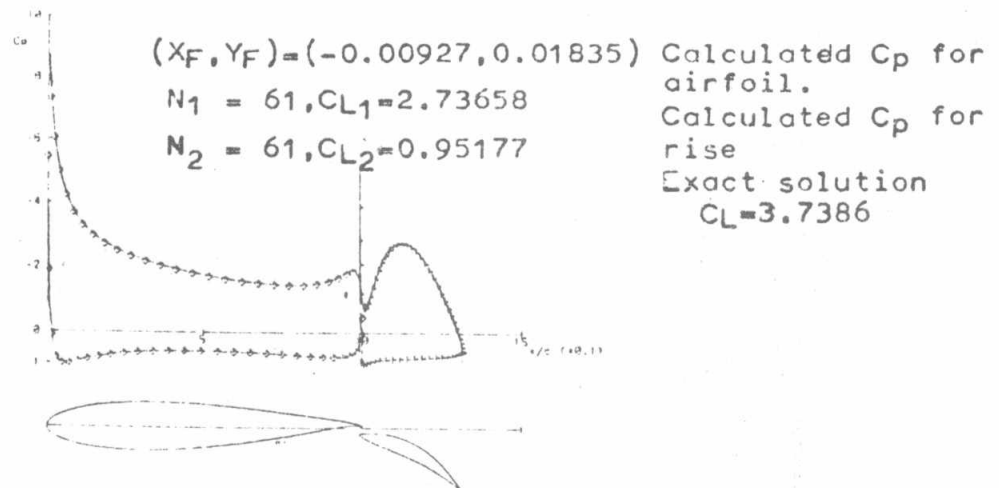


Fig. (8) Comparison of analytic and calculated (using 2nd order method) pressure distribution on two-element Williams airfoil at $\alpha = 0^\circ, \delta_f = 30^\circ$.

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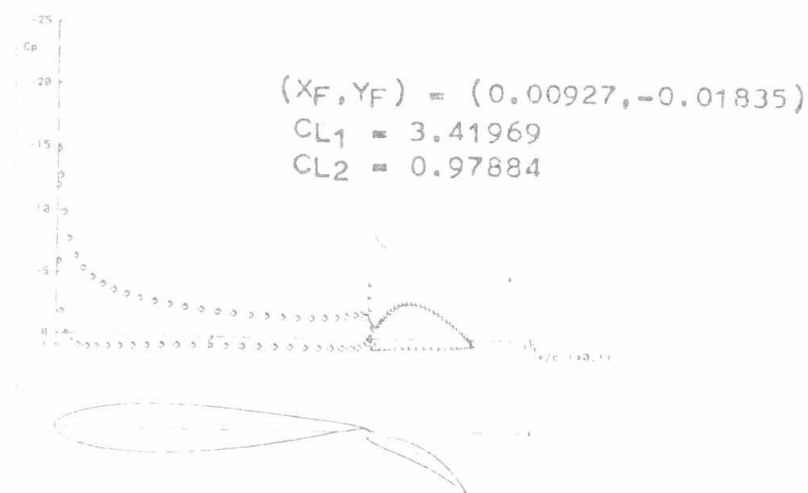
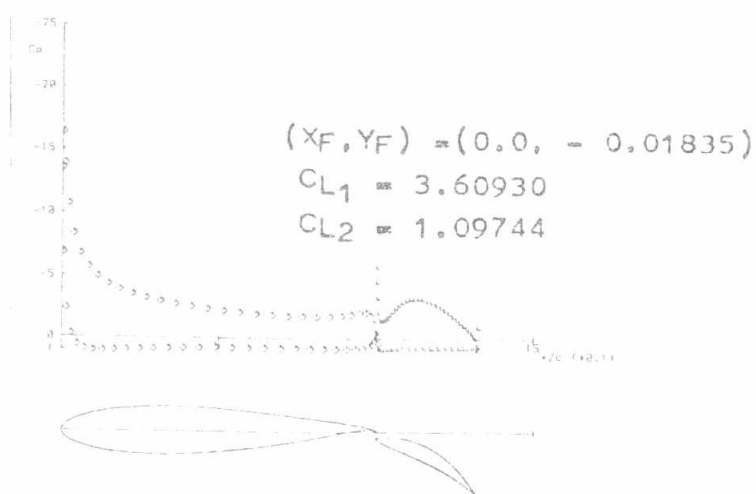
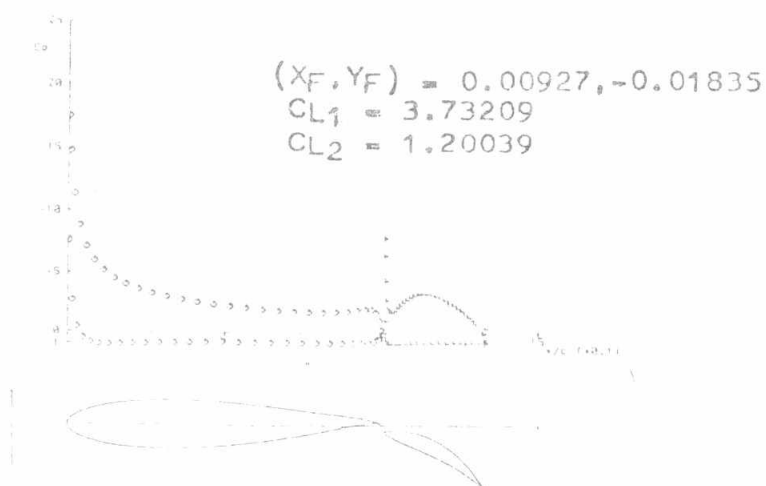


Fig. (9) Effect of overlap (for a constant gap) on pressure distribution and lift coefficient using 2nd order vortex panel method on two-element Williams airfoil at $\alpha = 5^\circ$, $\delta_f = 30^\circ$.

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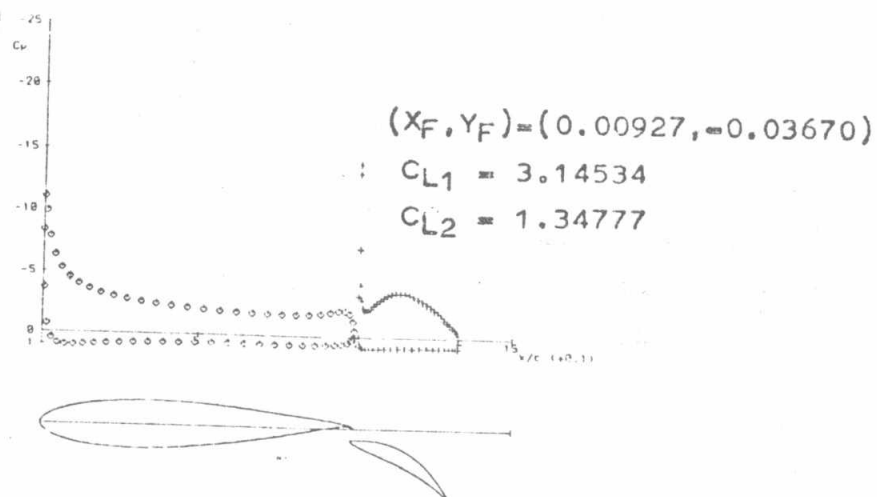
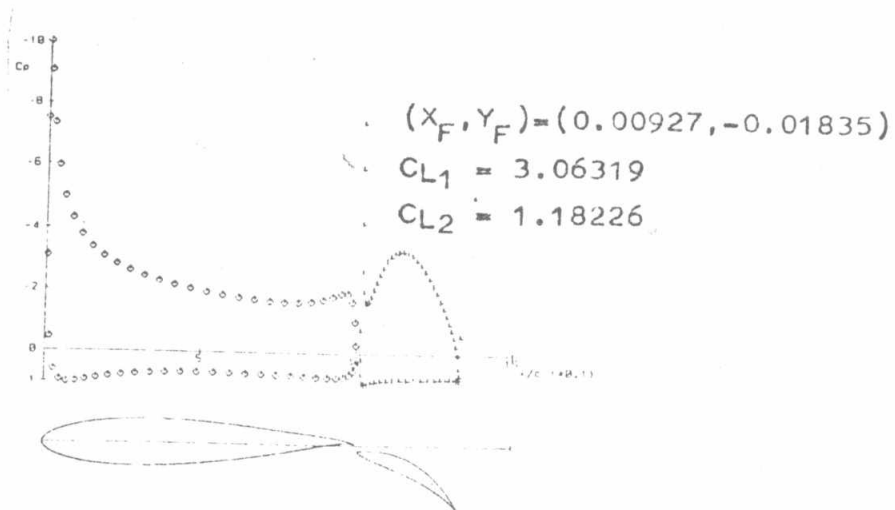


Fig. (10) Effect of gap (for a constant overlapping) on pressure distribution and lift coefficient using 2nd vortex panel method on two-element Williams airfoil at $\alpha = 0^\circ$, $\delta_f = 30^\circ$.