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# ANALYSIS OF PILOT INDUCED OSCILLATIONS

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## ABSTRACT

Ever since manually controlled flight of powered aircrafts, pilots have experienced pilot induced oscillations (PIO). PIO problem is one in which an oscillation of the aircraft occurs, that is difficult or even impossible for the pilot to stop. A central characteristic of the phenomenon is that the airplane is stable both stick-fixed or stick-free, hence the name pilot induced oscillation. The presented work studies the effect of pilot and aircraft parameters on pitch tracking performance. Also conditions that bring the pilot-aircraft system to instability were discussed. An appropriate pilot model convenient for PIO analysis is selected and a closed loop analysis of pilot-control system-aircraft has been done in time domain. The pilot dynamics was found to be the most significant factor of PIO. When the pilot model includes neuromuscular system dynamics, the pilot gain required to sustain an oscillation was found to be 55% of that value with the pilot as a pure gain.


## INTRODUCTION

Historically, PIO problems have tended to first appear in the final stages of flight test and evaluation, and therefore very expensive to correct. Despite the long history of PIO, a generally valid prediction technique does not exist. Opinion varies widely on causes of PIO. They can be generally classified as follows:

- Feel or control system dynamics and nonlinearities: some examples of these causes are force feedback directly to the pilot due to a bob weight, force feedback through the cables and linkages on reversible system due to control surface balance, and the nonlinear elements in stability augmentation systems, Ref.(1),(2).
- The biodynamical coupling between the vibrating vehicle and the control stick (stick feed through) may have a significant effect on performance, especially if the vehicle has resonance characteristics at the vibrating frequencies, Ref(3).
- Short period inverse time constant ($1/T_{\theta 2}$),Ref.(4).

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It is important to maintain on awareness of how difficult it is to validate any theory for PIO. Therefore it is very important to understand the effect of different parameters on PIO because of potential impact a PIO can have on aircraft development, Ref(4).

## MATHEMATICAL MODEL OF PILOT VEHICLE DYNAMIC SYSTEM

### 1.Pilot Vehicle Dynamic System

In the study of handling qualities , the source of a fundamental problem is that the complete dynamic system is seen only by the pilot. From figure (1), the system elements that affect the pilot's action as a controller, (and therefore affect the closed loop dynamics) are seen. In general, the pilot's characteristics as a controller are adaptive to the aircraft, and his capabilities are significantly affected by training and experience,Ref(5).

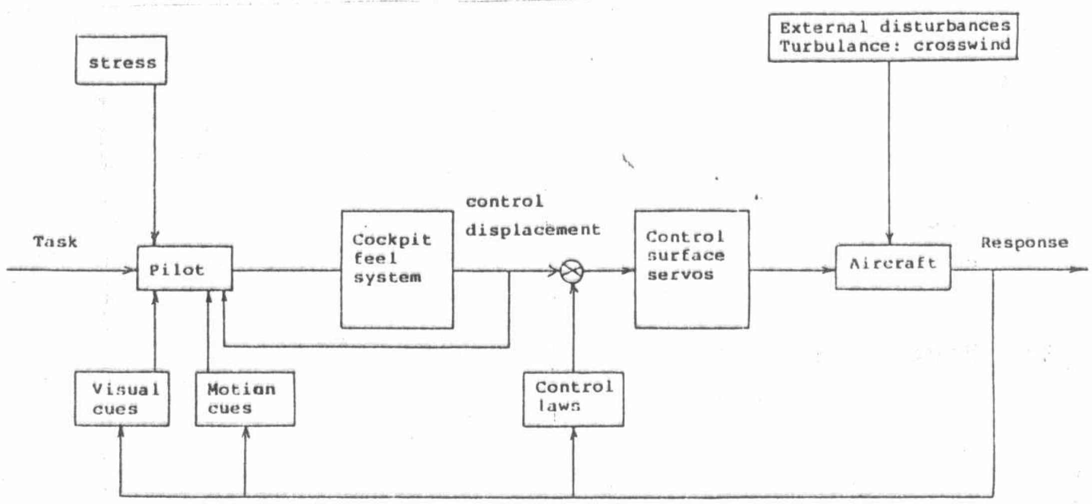


Fig. 1. Pilot Vehicle Dynamic System General Diagram

The pilot vehicle system quality can be assessed analytically and experimentally. In analytical assessment of aircraft handling qualities, the major difficulty is the analytical representation of the human pilot. Experimental methods are the other means of assessing the quality of the pilot-airplane combination. Experimentation involves the combination of the pilot and either the real vehicle or a simulation of the vehicle.

### 2. Aircraft Mathematical Model

The system of equations that has been used is the linearized model for small disturbances about reference steady state. The reference steady state is taken to be symmetric rectilinear flight. The state variables are the longitudinal motion variables $(V,\alpha,\theta,q)$. For the short period approximation in this case, the equations of motion non autonomous linear invariant system. From reference [6], the following system is used:

$$\begin{bmatrix} -(G_{L\alpha}+D_e+mV_e s) & mV_e s-G_{Lq} & 0 \\ -G_{M\alpha} & I_y s-G_{M\alpha} & 0 \\ 0 & 1 & -s \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} = \begin{bmatrix} G_{L\delta e} \\ G_{M\delta e} \\ 0 \end{bmatrix} \delta e \qquad (1)$$

This system represents a two degree of freedom in which only $\alpha$ and $\theta$ change. All derivatives with respect to rate of change of motion variables are neglected except $M_{\dot{\alpha}}$, since it is involved in the damping of the short period mode. Then the system of linear constant coefficient ordinary differential equations with elevator input $\delta_e$ as a forcing function will be

$$\begin{bmatrix} -(L_\alpha+s) & 1 & 0 \\ -(M_\alpha+sM_{\dot{\alpha}}) & s-M_q & 0 \\ 0 & 1 & -s \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} = \begin{bmatrix} L_{\delta e} \\ M_{\delta e} \\ 0 \end{bmatrix} \delta e \qquad (2)$$

It is assumed that the pitch attitude cue is the only pilot in cue, and the pilot controls a single state variable $\theta_o$ through the actuation of a single control input $\delta_e$. The aircraft pitch attitude open loop transfer function can be found as:

$$G_{\theta\delta}(s) = \frac{(M_{\delta e}-M_{\dot{\alpha}}\,L_{\delta e})\left[s+\dfrac{L_\alpha M_{\delta e}-M_\alpha L_{\delta e}}{M_{\delta e}-M_{\dot{\alpha}}\,L_{\delta e}}\right]}{s\;[\;s^2+(L_\alpha-M_q-M_{\dot{\alpha}})s-(L_\alpha M_q+M_\alpha)]} \qquad (3)$$

or

$$G_{\theta\delta}(s) = \frac{K_\theta\;(\;1+1/\;T_{\theta 2})}{s\;[(s/\omega_n)^2+2\xi s/\omega_n+1]} \qquad (4)$$

The longitudinal control is provided by an all movable stabilizer. The deflection angle of the stabilizer is related to pilot's signal by the following transfer function:

$$\frac{\delta_e}{\delta_{ep}} = \frac{1}{0.08\;s+1}$$

## 3. Human Pilot

Pilots may be regarded as links in a closed loop system, sensing the motion and position of the airplane, and actuating controls in response. To carry out analysis of pilot machine combinations, development of an adequate mathematical model of the pilot is required. This model is not constant, either in form or in values of parameters. In general, the pilot's characteristics are affected by an enormous number of physical, psychological and experimental effects. All of these are subsumed under four categories of variables. First, are the task variables, which comprise all the system inputs. The second type of variables is the environment external to the pilot. The third type of variables are procedural including such aspects of experimental procedure as instructions and order of presentation. The final variables to be considered are parameter-centered, including the characteristics of the operator brings to control task.

The human operator is a multi input, multi output device. The inputs are signals derived from the task variables and the environmental variables while the outputs include the physiological and psychological activities as well as control actions.

When the key variables are approximately time stationary over an interval of interest, the operator-vehicle system can be modeled as a quasi-linear system. In such system, the response for a given input is derived into two parts, one component $Y_p(s)$ which corresponds to the response of an equivalent linear element driven by that input, and the other component n(t) a "remnant" which represents the difference between the actual system and an equivalent system based on the linear element, Ref(7).

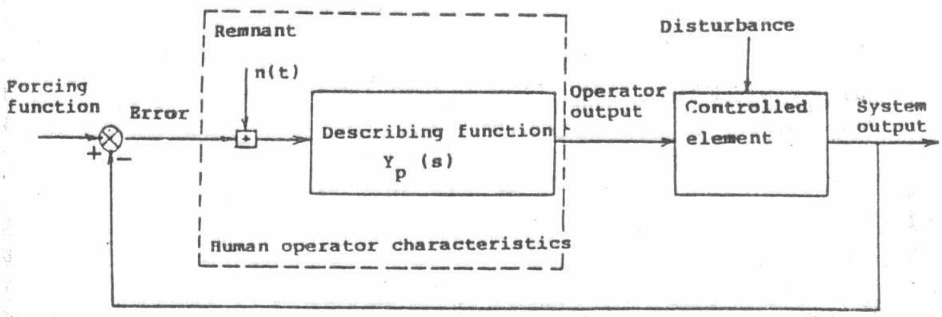


Fig.(2),Operator-vehicle control system with quasi-linear human operator model

The following form for the describing function of the pilot is selected to cover the single degree of freedom compensatory tracking task with only visual cue.

$$Y_p(s) = \frac{K_p \, e^{-\tau s}}{0.1\, s + 1} \tag{5}$$

Where, $e^{-\tau s}$ represents the pure transmission time delay within the pilot associated with nerve conduction and stimulation.
$K_p$ is the pilot gain [ N/deg ]
and $1/(0.1s + 1)$ represent the dynamics of neuromuscular system.

4. Closed loop analysis with pilot in the control loop

The system is a closed loop with unity feed back, for which the pilot is the comparator of what's happening versus what he wants to happen and the supplier of corrective inputs to the aircraft controls to achieve his desired mission. The model is linear and the nonlinear effects on system dynamics are excluded ( such as friction, hysteresis, and break out ). The input to the pilot is the pitch attitude error appears on the cockpit display. In case of precise control, the pilot attempts to zero this error at specified pitch command.

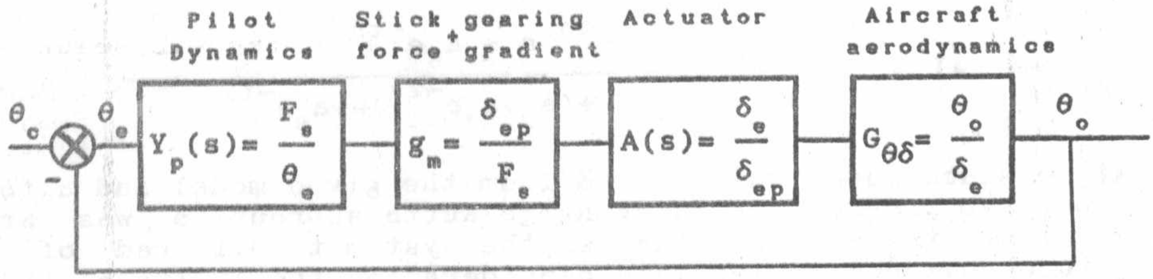


Fig. (3) Linear descriptive model with pilot in the loop

The stick force displacement characteristics is a linear ratio which is multiplied by a linear gearing represents, the elevator to stick deflection giving the control system gain $g_m$ . The open loop transfer function will be:

$$G(s) = Y_p(s).g_m.A(s).G_{\theta\delta}(s) \tag{6}$$

and the closed loop transfer function will be

$$\frac{\theta_o}{\theta_c} = \frac{G(s)}{1 + G(s)} \tag{7}$$

## ANALYSIS AND COMPUTATION OF THE PROBLEM

Sensitivity analysis in the time domain was made for both the aircraft and parameters and human pilot parameters. The aircraft was considered to be normally loaded and with 50% fuel. Variation of aircraft static margin from the forward to after position of the center of gravity has been done. The second aircraft parameter that was studied is the inverse time constant (1/To2) of the short period mode. Human pilot parameters that are varied in the sensitivity analysis are the pilot gain, and the pilot pure time delay (0.0 and 0.2 sec) with and without nueromuscular system dynamics. This combination has been done for two Mach numbers 0.4 and 0.9.

For the analysis including neuromuscular system dynamics, the closed loop transfer function will have the form:

$$\frac{\theta_o}{\theta_c}(s) = \frac{a_5 e^{-\tau s} s + a_6 e^{-\tau s}}{s^5+a_1 s^4+a_2 s^3+a_3 s^2+(a_4+a_5 e^{-\tau s})s+a_6 e^{-\tau s}} \qquad (8)$$

In the case of the analysis without neuromuscular system dynamics the closed loop transfer function will be

$$\frac{\theta_o}{\theta_c}(s) = \frac{a_4 e^{-\tau s} s + a_5 e^{-\tau s}}{s^4+a_1 s^3+a_2 s^2 +(a_3+a_4 e^{-\tau s})s+a_5 e^{-\tau s}} \qquad (9)$$

A state space model was derived from the given model and a fortran 77 computer program including Runge-Kutta subroutine was written to evaluate the step response of the system to 0.1 rad of pitch attitude command, taking into consideration the limits of the gain that heve evaluated using Routh's criterion.

## RESULTS AND DISCUSSIONS

### 1. Pilot Gain K:

To evaluate the effect of the pilot gain upon the system response, the pilot has been represented by "synchronous" model in which the pilot output is stick deflection equal to pilot gain K multiplied by the pitch attitude error.

The system response characteristics are highly affected by gain variation. That is , while there is no overshoot at K= -0.2, it reaches 35.53 at K = -0.88, and the settling time changes from 11.22 sec to a nearly sustained oscillation respectively. At pilot gain -0.6 a small amplitude oscillation was found to persist for several cycles, which may be a case precise control such as in the case of formation, photography, aiming and refueling missions. A large amplitude oscillation seeded to occur at K = -0.88, which may be a case of abrupt maneuver or tight control, see figure (4).

### 2. Pilot time delay:

In this case the pilot was represented by a gain with a transportation lag associated with nerve conduction and stimulation $Y_p(s) = K e^{-\tau s}$. It was found that a pilot time delay of 0.2 seconds was highly affecting the system performance and stability. That is, while a satisfactory response obtained with "synchronous pilot model", K=-0.4, a diverging large amplitude oscillation could be encountered for the same pilot gain but with pilot time delay of 0.2 sec the system exhibited oscillation before reaching the input value, see fig.(5).

### 3. Neuromuscular system dynamics:

The system response was evaluated in this case for both values of tome delay ($\tau$= 0 or 0.2 sec). In case of zero tome delay ($\tau$=0), instability occurs at much lower pilot gain(K=-0.49) for M=0.4 and C.G. at forward position. For the same condition of flight Mach number and C.G. position, diverging oscillation occurs for 0.2 sec pilot time delay at K = -0.3 , see fig.(6).

4. Static Margin:

For Mach number M = 0.4 the static margin was varied from 10.9% (at forward position of C.G.) to 4.3% MAC (at the aft C.G.position) respectively. In case where the airplane involved had large static stability, the settling time increased for small pilot gain (k = -0.2), but in case where the airplane had small static stability, the small pilot gain was in normal maneuver. Since for static margin = 4.3% MAC an overshoot of 6.56% at 1.5 sec occurred, while for static margin = 10.9% MAC a dead beat response occurred.

For pilot gain of -0.4 the overshoot was varied from 2.48% at the forward C.G. position to about 38% at the aft C.G. position. The settling time for this pilot gain was approximately the same. At higher pilot gain of -0.88, the overshoot was varied from 45.68% at the forward C.G. position to about 81.88% at the aft. C.G. position. In spite of this large change in overshoot, the converging large amplitude oscillation at the forward C.G. position still not diverge at the aft. C.G. position, fig.(7).

5. Inverse Time Constant $1/T_{\theta 2}$:

In the evaluation of the aircraft mathematical model, it was found that, this parameter has very slight change along the aircraft static margin. but, it is slightly affected by the lift curve slope and flight speed. Since, we have postulated that the high lift devices for the two flight Mach numbers have the same position, so it is possible to say that $1/T_{\theta 2}$ depends mainly on flight Mach number.

The inverse time constant was varied from 0.89 to 0.92 for M= 0.4, and from 2.37 to 2.48 for M=0.9 along the aircraft static margin. For synchronous pilot , the critical pilot gain related to stick deflection,varies from -.089 at M=0.4 to -0.868 at M=0.9, for the largest static margin. AS the time constant increased, the maximum overshoot also increased, while the settling time decreased from 8.72 sec to 1.9 sec, for the same pilot gain. It was also found that the tome to the first peak and effective time delay decreased by about 50%. At higher pilot gain (K=-0.88), a large amplitude nearly sustained oscillation for M = 0.4 becomes diverging oscillation for M = 0.9. From the step response (fig.8), frequency of the system response was higher than 1 cps for M = 0.9, while less than 1 cps for M = 0.4.

Finally it was found that, increasing $1/T_{\theta 2}$ decreases the critical pilot gain. But for lower pilot gain, the response characteristics was improved.

## CONCLUSION

A PIO analysis has been presented that is believed to unify divergent viewpoints regarding the significance to PIO of airframe and pilot dynamics. The pilot dynamics was found to be the most significant to initiation of PIO. Small pilot gain related to stick deflection gives a stable system response, which corresponds to the case of normal maneuver. Higher pilot gain results in system oscillation which persists for several cycles. Sustained oscillations have been found to occur for critical pilot gain, which is used in case of precise control. A pilot pure time delay of 0.2 sec reduces the critical gain by about 50% of that value when the pilot is only pure gain. For lower pilot gain a pilot time delay of 0.2 sec causes the system to oscillate before reaching its input value. When pilot model includes the neuromuscular system dynamics, the pilot gain required to sustain an oscillation was reduced to about 55% of that value with pilot as pure gain.

Variation of aircraft static margin affects the system overshoot and other characteristics for the same pilot gain. sustained oscillation at the forward C.G. position, tend to diverge at the aft. C.G. position.Increase of $1/T_{02}$ reduces the critical pilot gain. But for lower values of pilot gain, the step response characteristics are modified at higher value of $1/T_{02}$.

Generally, it is found that, the pilot model and its parameters are of predominant effect. So, it is reasonable to expect that, more understanding and developing pilot dynamics during PIO situation will be necessary for revealing the causes of PIO.

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## NOMENCLATURE

| | |
|---|---|
| $F_s$ | Stick force, [N] |
| $G(s)$ | Transfer function |

| | |
|---|---|
| $G_{L\alpha}$ | Lift to angle of attack transfer function |
| $G_{Lq}$ | Lift to pitch rate transfer function |
| $G_{L\delta_e}$ | Lift to elevator angle deflection transfer function |
| $G_{M\alpha}$ | Pitching moment to angle of attack transfer function |
| $G_{Mq}$ | Pitching moment to pitch rate transfer function |
| $I$ | Moment of inertia refered to lateral axis |
| $L$ | Lift force [N] |
| $M$ | Pitching moment [N.m] |
| $q$ | Pitch rate [rad/sec] |
| $s$ | Laplace operator |
| $V$ | Flight speed |
| $\alpha$ | Angle of attack |
| $\theta$ | Pitch angle |
| $\theta_e$ | Pitch attitude error |
| $\delta_e$ | Elevator deflection angle |
| $\tau$ | Pilot time delay [sec] |

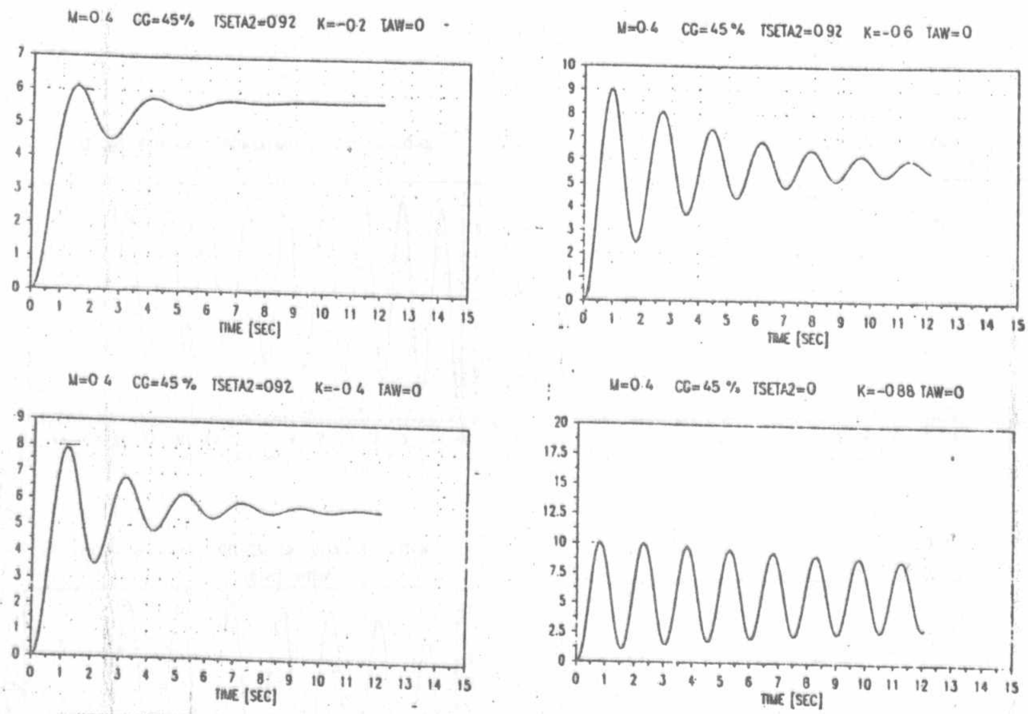


Fig. (4) Effect of pilot gain on system response

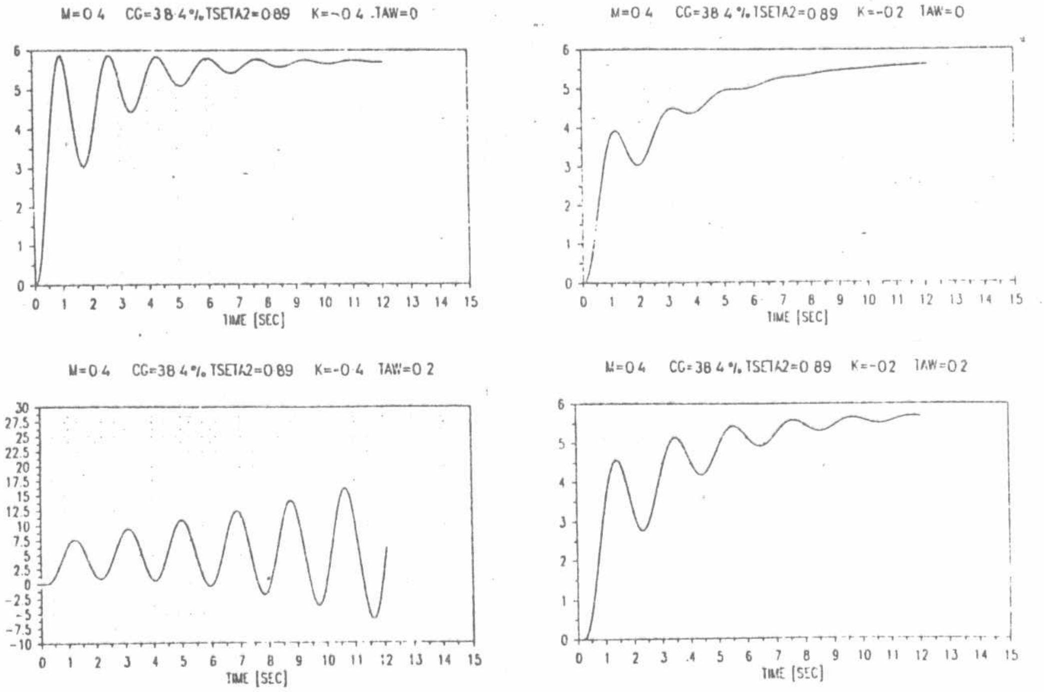


Fig. (5) Effect of pilot time delay on system response

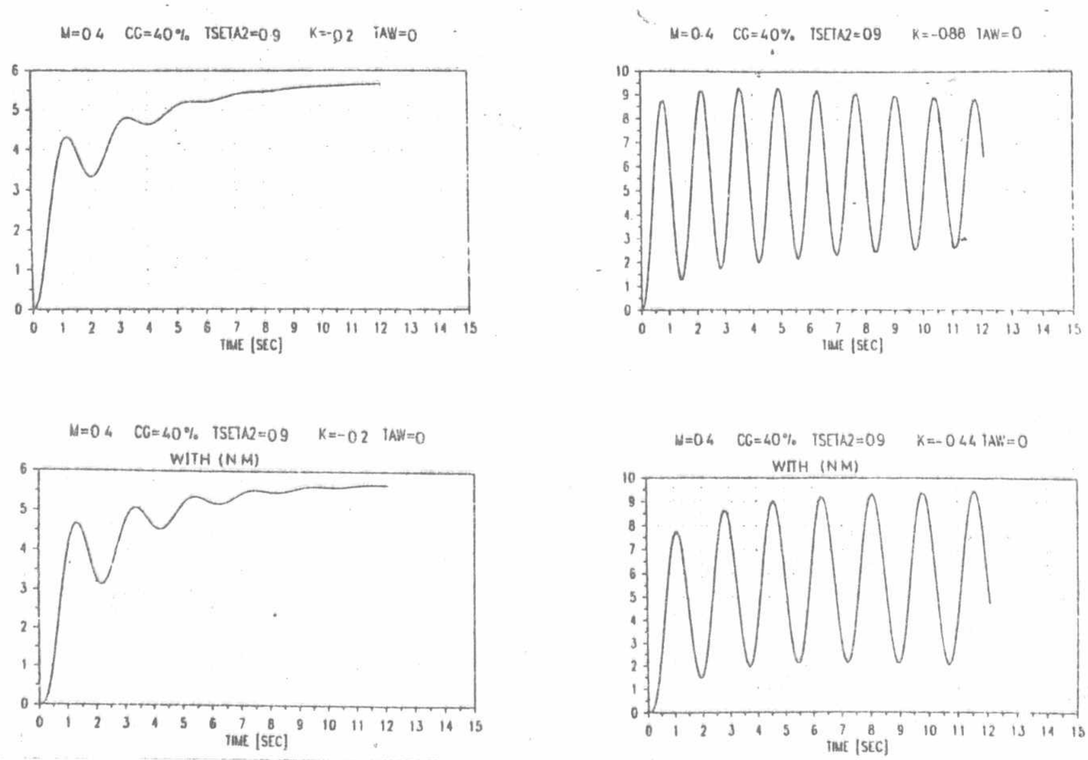


Fig. (6) System response with and wiyhout neuromuscular parameter

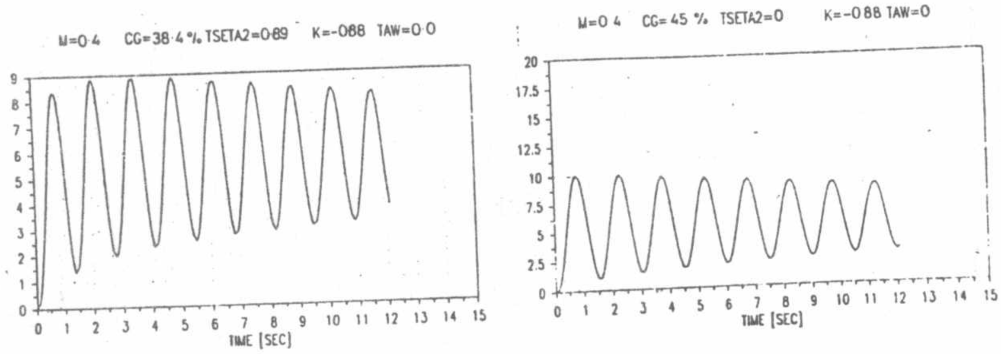


Fig. (7) Effect of static margin on system response

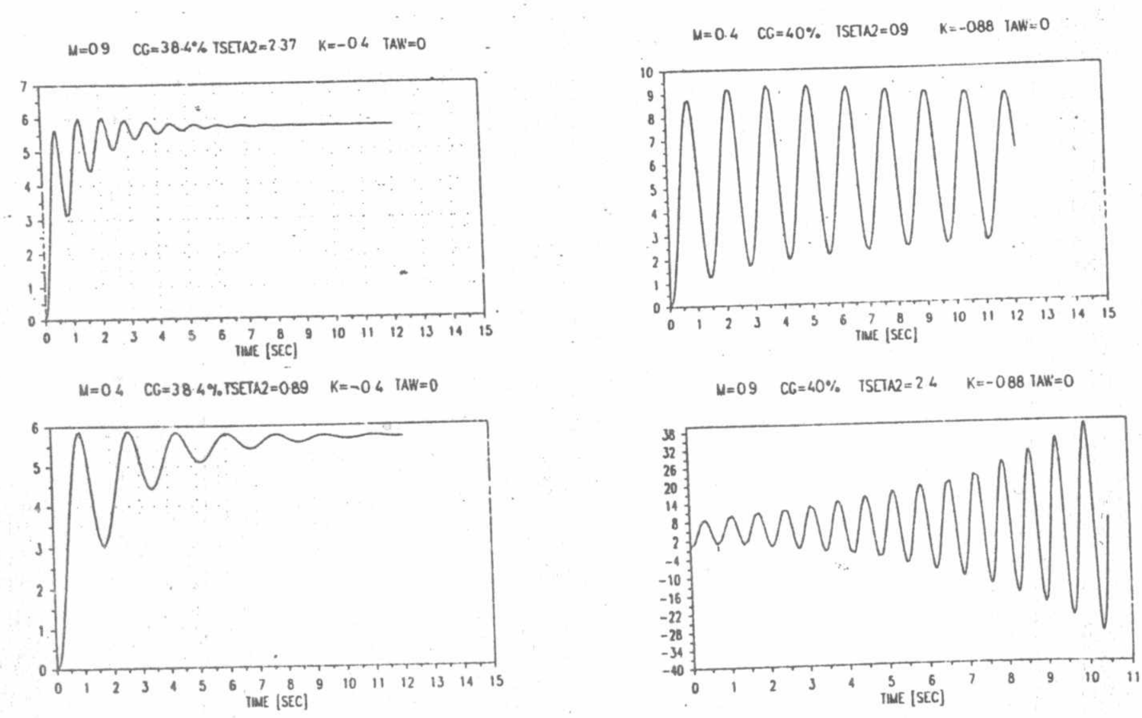


Fig. (8) Effect of inverse time constant on system response