MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

AN OPTIMIZATION APPROACH FOR ROUTING MILITARY AIRCRAFT

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ABSTRACT

The optimal solution for military aircraft bombers routing model is developed. The proposed mathematical model simulates a military mission in which our missile carrying aircraft located in several airfields have the task to destroy different enemy targets to specified damage percentages and within a given time window.

The problem is to determine the sequence of routing for each aircraft formation and to satisfy the battle objectives. The objectives may be one or more of the following: to minimize the total number of aircraft, to minimize the risk of mission failure, and to minimize the total time for fulfilling the task.

A multi-objective zero-one integer linear programming model is formulated. The proposed algorithm for model solution is a two-phase one. In the first (Feasibility Phase), the enumeration of all possible target combinations is determined. For each possible pair (aircraft formation/target sequence) the following feasibility constraints are examined: the compatibility of aircraft to mission, the fuel range, and the time window. In the second phase (Optimality Phase), the decision variables are determined for the feasible missions, and the formulation of the multi-objective zero-one integer linear programming model is performed. One of the methods for solving such a model is the iterative approach for goal programming.

Finally, an implementation plan for incorporating the proposed algorithm into a sample numerical problem is presented.

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INTRODUCTION :

The problem under consideration is that one of allocating military bomber formations to alternative missions. Certain constraints should be satisfied: First, the aircraft formation should be compatible with the assigned mission from point of view of armament, crew experience, and aircraft protection against enemy antiaircraft means. Second, each target is to be destroyed in a certain predetermined extent. Third, the maximum fuel range for each aircraft should not be exceeded. Finally, the time window within which each target should be destroyed must be respected.

The objectives of the problem may be: to minimize the total number of aircraft fulfilling the task, to minimize the risk of mission failure, and/or to minimize the total time duration of all the assigned missions. The decision maker may be interested in only one of the mentioned objectives, or he may be interested in all the objectives with same or different priorities and weights.

Although there are many efforts -surveyed- in [1]- for routing ships and civil aircraft, only few publications for problems related to the allocation of military aircraft to different missions are available: In [2], a mathematical model for an air-force penetration is constructed based on a Markov chain formulation, the model determines the attrition of the air-force during penetration through the enemy air-defence system and hence finds the final surviving aircraft distribution after penetrating all the layers of the enemy air-defence system. In [3], a structured systems analysis approach has been used to prepare an information flow model which yields the allocation of proper resources and the estimation of the time required for decision making and the completion time for specific missions. In [4], a framework is proposed for a decision support system to help in solving various armament problems. The system is composed of two basic modules: a model base, and a database. The database contains basic data about: troops personnel and organization, armament, weapons technical and tactical specifications, combat operational and environmental factors, and weapon fire power score tables. The fire power score model in the system evaluates the fire power index for a certain formation of military units through estimating the relative power scores of their armament. In [5], the authors solve the problem of finding the optimum route for the aircraft which serves only for minimizing the threat of detection by enemy radars .

The initial data to be prepared for modelling our problem is to know: the needed damage extent for each target, the fuel range for each aircraft, and to determine for each possible pair (aircraft formation /mission sequence) the following

- mission compatibility,
- extent of damage for each target when attacked by different aircraft formations,
- fuel required for each mission,
- total time duration,
- time of hitting each target in the sequence,
- risk of mission failure.

MODEL FORMULATION:

Decision Variables:

Let $x(i,j)$ a zero-one variable, where $x(i,j) = 1$ means that aircraft formation number i will perform the mission number j, $x(i,j) = 0$ means that formation number i will not perform mission number j. A mission means one of the possible combinations of enemy targets. If the number of targets = t, then the number of all possible target combinations:

$$n = \sum_{r=1}^{t} C_r = 2^t - 1$$

(it is known that : $(1+x)^t = \sum_{r=0}^{t} C_r^t . x^r$,

Putting $x = 1$, we can obtain the previous relation).

Constraints :

1. Compatibility Constraints :

Several compatibility conditions should be satisfied for each combination (i, j), for example: the aircraft armament must be suitable for the specified mission, the aircraft crew should be trained for such missions, aircraft protection against enemy antiaircraft weapons should be taken into consideration ....etc. These compatibility constraints should force the decision variables corresponding to the incompatible missions (i, j) to take the value of zero:

$$x(i,j) = 0 \quad , \text{ for all the incompatible missions } (i,j).$$

2. Target Constraints:

Each target k should be destroyed to a certain predetermined extent:

$$\sum_{i=1}^{m} \sum_{\substack{j \\ k \in j}} x(i,j) . d_{ij}^{k} \geq D_K \quad , \text{ for all } k ,$$

Where: m = the number of aircraft formations,

n = the number of all possible combinations of targets,

d^k_{ij} = the extent of damage for target k when aircraft formation i performs the mission number j,

D_K = the required extent of damage for target k.

3. Aircraft Formation Constraints:

Each aircraft formation can stay idling, or it can perform only one of the possible missions:

$$\sum_{j=1}^{n} x(i,j) \leq 1 \quad , \text{ for all } i.$$

4. Fuel Range Constraints:

Each aircraft formation can not exceed its maximum fuel range:

$$x(i,j).f_{ij} \leq F_i \quad , \text{ for all } i, j \; ,$$

Where:

f_{ij} = the fuel required by aircraft formation i to fulfill mission j,

F_i = the maximum amount of fuel available for aircraft formation i.

5. Time Window Constraints:

The time window constraints can be expressed as:

$$\left.\begin{array}{l} x(i,j).h^k_{ij} \leq H^k_{max}, \\ x(i,j).H^k_{min} \leq h^k_{ij}. \end{array}\right\} \quad \text{for all } i, j, k.$$

$$\text{Where: } h^k_{ij} = \begin{cases} \left.\begin{array}{l}\text{the time at which aircraft formation i} \\ \text{will hit target k if assigned mission j}\end{array}\right\} \text{for } k \in j \\ \infty \; , \quad \text{for } k \notin j \end{cases}$$

H^k_{min} = the required earliest time for hitting target k,

H^k_{max} = the required latest time for hitting target k.

6. Integrality Constraints:

$$x(i,j) = 0 \text{ or } 1 \quad , \text{ for all } i, j.$$

Objective Functions:

The decision maker may be interested in achieving only one of the following objectives, or he may be interested in all of them with same or different priorities and weights: To fulfill the whole task with minimum number of aircraft, with minimum risk of mission failure, and/or in the shortest time period. These objectives may be expressed mathematically as follows:

1. Minimizing the total number of aircraft:

$$\text{Min.} \sum_{i=1}^{m} \sum_{j=1}^{n} x(i,j).n_i ,$$

Where: n_i = the number of aircraft formations i.

2. Minimizing the risk of mission failure:

$$\text{Min.} \sum_{i=1}^{m} \sum_{j=1}^{n} x(i,j).p_{ij} ,$$

Where: p_{ij} = probability of failure in fulfilling the mission j by aircraft formation i, it depends on navigation skill, protection against enemy antiaircraft means, aiming accuracy to targets and aircraft maneuverability.

3. Minimizing the total time duration:

$$\text{Min.} \sum_{i=1}^{m} \sum_{j=1}^{n} x(i,j).t_{ij} ,$$

Where :

t_{ij} = the time duration to fulfill the mission j by aircraft formation i.

ALGORITHM OF SOLUTION:

From the model formulation, it is clear that the number of decision variables corresponding to the number of possible target combinations n and the number of aircraft formation will be very large. The proposed algorithm to decrease the number of decision variables and hence the problem size and to obtain the efficient solution for the problem is a two phase one [6], Figure 1.

In the first phase (Feasibility Phase) the enumeration of all possible aircraft-formation/target- sequence (i, j) is determined. The Travelling Salesman problem technique is used to rearrange each cycle so that to obtain the shortest route, [7]. Then for each pair (i, j), the compatibility, the fuel range and the time window constraints are examined. In the second phase (Optimality Phase) the decision variables x(i, j) are defined only for feasible pairs (i, j) and the remaining system constraints will be the target, the aircraft formation, and the integrality constraints.

If only one objective is considered, then we use the usual zero-one integer programming to obtain the optimal solution. If more than one objective is to be considered, then according to the situation , the decision maker will determine his preferred priorities and weights for the objectives, one of the possible methods to obtain the corresponding optimal solution is the iterative approach for solving the goal programming [8] after altering the objectives into goals by estimating their aspiration levels.

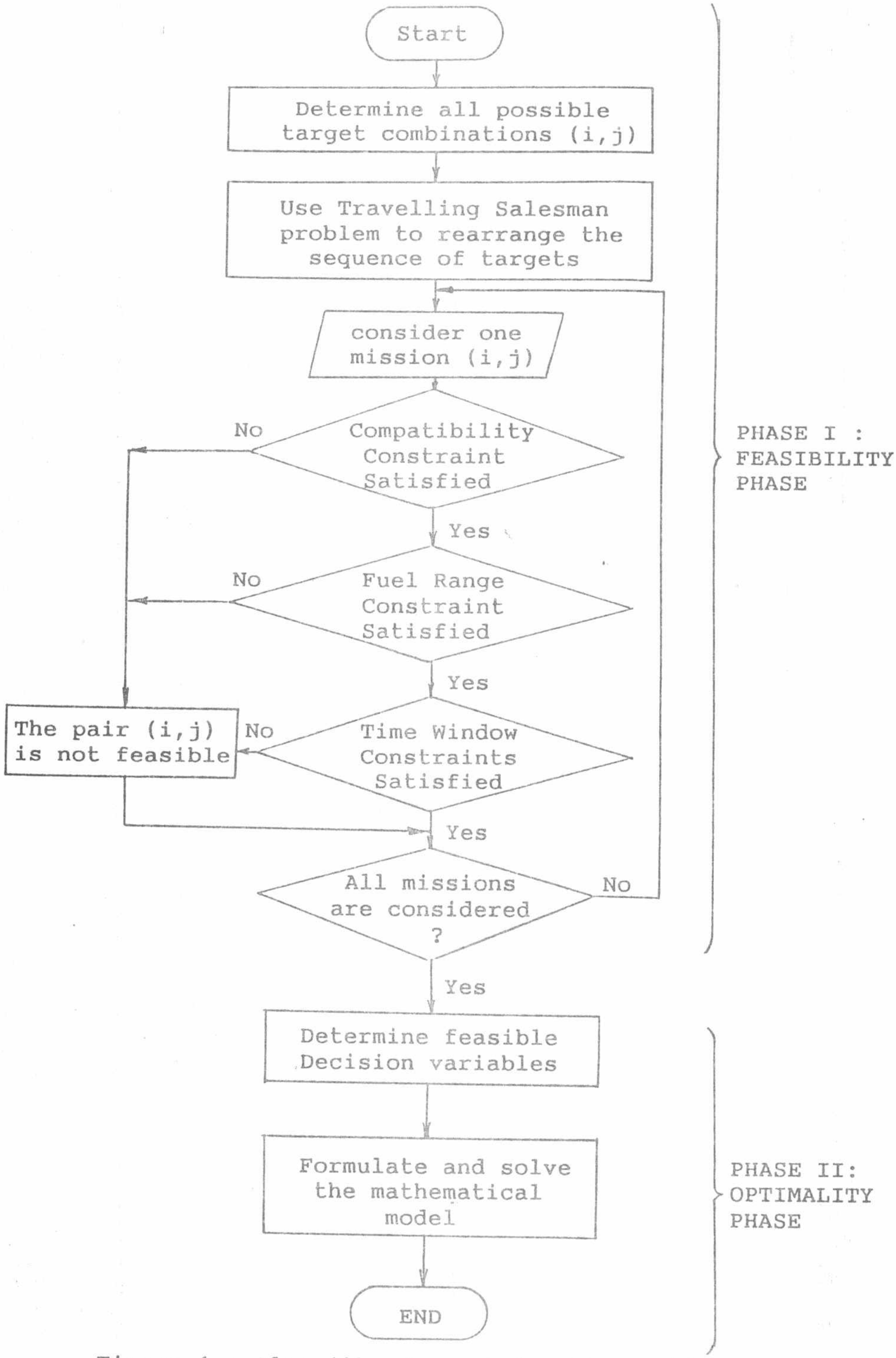

Figure 1 : Algorithm for solving the problem

CASE STUDY:

Consider that we have 6 aircraft formations located in 4 different airfields, the type of aircraft and their numbers are shown in table 1:

Airfield No.	Formation No. (i)	Aircraft Type	Number of Aircraft
1	1	A	4
2	2	B	4
	3	A	4
3	4	C	8
	5	B	4
4	6	B	4
TOTAL			28

Table 1 : Available Aircraft formations

The task is to fulfill the military mission of destroying t=5 enemy targets with specified damage extent. The percentage of damage of each target when attacked by each aircraft formation type and the required damage extent for targets are shown in Table 2:

Type of Aircraft \ Target No.	1	2	3	4	5
A	60	50	60	70	40
B	50	40	50	60	30
C	80	60	70	90	50
Damage Extent	100	70	85	70	25

Table 2 : Damage extent for enemy targets

A special computer program is made to enumerate all possible target combinations j, table 3. The total number of possible target combinations $n = 2^5 - 1 = 31$. Each cycle (airfield/mission/airfield) is rearranged using the Travelling Salesman problem to obtain the shortest cycle, then each pair (aircraft-formation i / mission j) is tested for feasibility. Table 4 presents a list of all the feasible missions for different aircraft formations and the corresponding decision variables .

Targets to be Hitted				
1-target	2-targets	3-targets	4-targets	5-targets
1	1-2	1-2-3	1-2-3-4	1-2-3-4-5
2	1-3	1-2-4	1-2-3-5	
3	1-4	1-2-5	1-2-4-5	
4	1-5	1-3-4	1-3-4-5	
5	2-3	1-3-5	2-3-4-5	
	2-4	1-4-5		
	2-5	2-3-4		
	3-4	2-3-5		
	3-5	2-4-5		
	4-5	3-4-5		

Table 3 : All possible target combinations

Aircraft Formation (i)	Attacked Targets (j)	Decision Variable No.	Aircraft Formation (i)	Attacked Targets (j)	Decision Variable No.
1	1	1	4	1	26
1	2	2	4	2	27
1	3	3	4	3	28
1	4	4	4	5	29
1	1-3	5	4	1-2	30
1	1-4	6	4	1-3	31
1	2-3	7	4	1-5	32
1	2-4	8	4	2-3	33
1	3-4	9	4	2-5	34
1	1-2-3	10	4	3-5	35
1	1-2-4	11	5	1	36
1	1-3-4	12	5	2	37
2	1	13	5	3	38
2	2	14	5	5	39
2	3	15	5	1-2	40
2	1-2	16	5	1-3	41
2	1-3	17	5	1-5	42
2	2-3	18	6	1	43
2	1-2-3	19	6	2	44
3	1	20	6	3	45
3	2	21	6	4	46
3	3	22	6	5	47
3	1-2	23	6	1-2	48
3	1-3	24	6	1-3	49
3	2-3	25	6	1-4	50

Table 4 : All feasible missions and corresponding decision variables

Assuming that the probability coefficients of mission failure and the time duration corresponding to each decision variable are calculated [2, 3], Table 5 presents these coefficients beside the number of aircraft :

Dec. Var. x_j	Prob. of Fail. p_j (%)	No. of Planes n_j	Time Dur. t_j
1	10		40
2	12		50
3	15		45
4	8		30
5	20		52
6	18		42
7	22		52
8	18		53
9	20		47
10	30		55
11	26		55
12	22	4	50
13	8		42
14	10		52
15	12		46
16	15		54
17	17		55
18	20		55
19	25		58
20	10		45
21	12		55
22	8		50
23	15		57
24	20		52
25	18		57

Dec. Var. x_j	Prob. of Fail. p_j (%)	No. of Planes n_j	Time Dur. t_j
26	15		45
27	17		55
28	18		50
29	25		35
30	27		57
31	28		52
32	30	8	47
33	25		52
34	30		57
35	35		52
36	12		47
37	15		57
38	16		52
39	22		37
40	25		60
41	26		55
42	38		49
43	12	4	47
44	15		57
45	16		52
46	20		60
47	15		37
48	25		60
49	26		55
50	30		64

Table 5: Coefficients of the objective functions for different decision variables.

The corresponding mathematical model will take the form:

Objective functions :

First priority: $$\text{Min.} \sum_{j=1}^{50} x_j . p_j$$

Second priority: $$\text{Min.} \sum_{j=1}^{50} x_j . n_j$$

Third priority: $$\text{Min.} \sum_{j=1}^{50} x_j . t_j$$

Target Constraints:

The coefficients of damage extents dij are obtained directly from Table 2 when mission j contains only one target, if mission j contains more than one target then it is assumed that the extent of damage for each one is a percentage of the full damage extent (according to Table 2) depending upon the number of targets in the discussed mission.

1) $60x_1 + 30x_5 + 30x_6 + 20x_{10} + 20x_{11} + 20x_{12} + 50x_{13} + 25x_{16} + 25x_{17} + 17x_{19} + 60x_{20} + 30x_{23} + 30x_{24} + 80x_{26} + 40x_{30} + 40x_{31} + 40x_{32} + 50x_{36} + 25x_{40} + 25x_{41} + 25x_{42} + 50x_{43} + 25x_{48} + 25x_{49} + 25x_{50} \geq 100$

2) $50x_2 + 25x_7 + 25x_8 + 17x_{10} + 17x_{11} + 40x_{14} + 20x_{16} + 20x_{18} + 13x_{19} + 50x_{21} + 25x_{23} + 25x_{25} + 60x_{27} + 30x_{30} + 30x_{33} + 30x_{34} + 40x_{37} + 20x_{40} + 40x_{44} + 20x_{48} \geq 70$

3) $60x_3 + 30x_5 + 30x_7 + 30x_9 + 20x_{10} + 20x_{12} + 50x_{15} + 25x_{17} + 25x_{18} + 17x_{19} + 60x_{22} + 30x_{24} + 30x_{25} + 70x_{28} + 35x_{31} + 35x_{33} + 35x_{35} + 50x_{38} + 25x_{41} + 50x_{45} + 25x_{49} \geq 85$

4) $70x_4 + 35x_6 + 35x_8 + 35x_9 + 23x_{11} + 23x_{12} + 60x_{46} + 30x_{50} \geq 70$

5) $50x_{29} + 25x_{32} + 25x_{34} + 25x_{35} + 30x_{39} + 15x_{42} + 30x_{47} \geq 25$

Aircraft-formation Constraints :

1) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} \leq 1$

2) $x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} \leq 1$

3) $x_{20} + x_{21} + x_{22} + x_{23} + x_{24} + x_{25} \leq 1$

4) $x_{26} + x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \leq 1$

5) $x_{36} + x_{37} + x_{38} + x_{39} + x_{40} + x_{41} + x_{42} \leq 1$

6) $x_{43} + x_{44} + x_{45} + x_{46} + x_{47} + x_{48} + x_{49} + x_{50} \leq 1$

Using a computer program [9], and applying the iterative approach for goal programming considering the mentioned objective priorities, the following the nonzero decision variables are obtained : $x_4, x_{18}, x_{22}, x_{26}, x_{34}, x_{37}$.

CONCLUSIONS :

This paper presents an approach to solve the military aircraft bombers routing problem with several objectives. A route is considered as a sequence of ordered enemy targets to be destroyed with one of our aircraft formations. The task is to be completed within a specified time window and each target to be destroyed in a predetermined damage extent.

The objectives of the problem may be to minimize one or more of the following: the total number of aircraft fulfilling the military task, the risk of mission failure, and the total time duration of all the assigned missions.

The proposed algorithm is a two phase one: the feasibility phase and the optimality phase. In the feasibility phase, the formation/mission compatibility, fuel range, and the time window constraints are examined. The infeasible missions are eliminated from further considerations that reduces the size of the problem considerably. In the optimality phase, the decision variables are defined corresponding only to the feasible missions, the mathematical model for the problem is formulated as a multi-objective zero-one integer programming, then stated problem is solved using the iterative approach for goal programming.

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