



## THERMAL ANALYSIS OF ROCKET NOZZLE

BY THE FINITE ELEMENT METHOD

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## ABSTRACT

In the rocket motor design, there arises an acute need for a method of thermal analysis that could be directly coupled with the stress analysis. To describe the included thermal gradients with sufficient accuracy, the heat transfer coefficient together with the initial and boundary conditions should be well defined. Three-dimensional axisymmetric finite element method that accounts for transient heating and material non-linearities is presented. A computer program based on the finite element technique was implemented to determine the temperature distribution across the walls at any time instant. In order to verify the validity of the method it was firstly applied to the analytically known case of a thick hollow cylinder heated by convection on its internal surface. The method was then applied to a submerged nozzle made of composite material. The comparison with the results obtained from a general purpose thermal code has proven the applicability of the method to real problems. Practical measurements were made on an 80 mm testing motor for the temperature-time variation at different points through the nozzle wall. Numerical calculations using the developed program showed satisfactory agreement with test data.

## 1. INTRODUCTION

Thermal analysis of a solid rocket nozzle requires the definition of heat transferred from the exhaust gases to the nozzle liner materials and the calculation of thermal response of these materials. Usually, convection heat transfer coefficient is determined using the well established Bartz equation, [1] while the effective values of emissivity for radiation are determined using charts according to the propellant type and the nozzle geometry, [2]. Numerical techniques are available for predicting response of materials exposed to convection and/or radiation, [3]. The objective of this paper is to present the application of finite element method to determine the temperature distribution through the wall of a solid rocket motor nozzle. The basic equations of axisymmetric transient heat transfer are introduced in brief for the sake of simple further treatment.

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Different boundary conditions which can be encountered in rocket nozzle including fixed temperature, convection and radiation are considered. Nonlinear variation of material thermal properties with temperature due to severe environment of rocket exhaust gases is considered. Moreover, for nozzles made of ablative materials, the heat and the temperature of ablation must be included, [4]. This can be accomplished artificially in the finite element solution by increasing the specific heat of the material over the charring temperature range.

A sample problem of internal convection in a hollow cylinder was solved by the developed program and the results were compared with the analytical solution. A case of submerged nozzle made from composite material was solved by the developed program and a general purpose thermal code to test the validity of the program for dealing with real problems.

Finally, an experimental program has been conducted to measure the temperature profile across the nozzle structure of an 80mm test motor. Numerical calculations showed satisfactory agreement with test data.

The given procedure was implemented using FORTRAN 77 on VAX11/785 computer in M.T.C. The output of temperature distribution with time is linked to another finite element program for thermo-mechanical stress analysis.

## 2. FINITE ELEMENT FORMULATION

The partial differential equation governing the transient heat conduction in solids is obtained from the principle of conservation of energy and the application of Fourier's law, in the form:

$$\rho C [\partial T(r,t) / \partial t] = \nabla \cdot [k \cdot \nabla T(r,t)] + Q(r,t) \quad (1)$$

The finite element formulation of this problem can be achieved using the weighted residual (Galerkin) approximation, [5]. Another approach consists in the determination of a functional for which stationary value is to be sought. In this work, the following form of Gurtin's functional  $\mathfrak{F}$ , [6] is used for obtaining the governing finite element equations:

$$\mathfrak{F} = \frac{1}{2} \int \left[ \rho C T^2 + \frac{\partial T}{\partial x} K_x \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} K_y \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z} K_z \frac{\partial T}{\partial z} - 2QT - 2\rho C T_0 T \right] (x,y,z,t) dv - \int_S (q_x n_x + q_y n_y + q_z n_z) T(x,y,z,t) ds \quad (2)$$

The thermal equilibrium condition in the discretized solid requires that the functional  $\mathfrak{F}$  in each individual element be kept minimum with respect to the nodal temperature  $[T]$ .

$$\delta \mathfrak{F}_e / \delta t = 0$$

This will lead to the establishment of the element equation as follows:

$$[C_e][\dot{T}(t)] + [K_e][T(t)] = [F_e] \quad (3)$$

where

$[C_e]$  = element heat capacitance matrix

$$= \int_{V_e} \rho_e C_e [N(r)] T [N(r)] dv_e \quad (4)$$

$[K_e]$  = element thermal conductivity matrix

$$= \int_{V_e} [B(r)] T [D_e] [B(r)] dv_e \quad (5)$$

$[F_e]$  = element thermal force vector

$$= \int_{V_e} Q_e(r) [N(r)] dv_e - \int_{S_e} [q_e(r)] T [n] [N(r)] ds_e \quad (6)$$

$[T(t)]$  = temperature derivative w.r.t. time.

$[N(r)]$  = the interpolation function [7].

$$T_e(r, t) = [N(r)] [T(t)] \quad (7)$$

$[B(r)]$  = the partial derivative of the interpolation function with respect to the coordinate axes.

$[D_e]$  = matrix which expresses element conductivities.

Assembly of elements equations gives an equation governing the whole domain similar to equation (3).

#### Transient Heat Conduction in Axisymmetric Solids

Nozzle structure has a geometry which is axisymmetric. The basic element considered in the present analysis (and in most of axisymmetric finite element applications) is the triangular torus element. A typical element is shown in Fig.1, with  $r$  and  $z$  being respectively the radial and axial coordinates of an arbitrary point inside the element. The quadrilateral element can be created by combining four triangular elements.

The first step in the analysis is the discretization of the continuum into a finite number of triangular torus elements. The temperature of the element  $T_e(r, z, t)$  can be related to its nodal values  $[T(t)]$  via an interpolation function  $[N(r, z)]$ . This interpolation function  $[N(r, z)]$  can be determined by an assumed temperature distribution inside the element. The simplest form of such function is a linear polynomial of the coordinate. For the case described in Fig.1, the temperature at the three nodes can be expressed by the substitution of the respective nodal coordinates into the following equation:

$$T_e(r, z, t) = a_1(t) + a_2(t).r + a_3(t).z \quad (9)$$

The arbitrary constants  $a_1(t)$ ,  $a_2(t)$  and  $a_3(t)$  are parameters of time and can be obtained under the form:

$$[a(t)] = [h] [T(t)] \quad (10)$$

The matrix  $[h]$  contains elements of the specified nodal coordinates. Substituting  $[a(t)]$  from equation (10) into equation (9) and rearranging the terms we get:

$$T_e(r, z, t) = \begin{bmatrix} h_{1i} + rh_{2i} + zh_{3i} \\ h_{1j} + rh_{2j} + zh_{3j} \\ h_{1k} + rh_{2k} + zh_{3k} \end{bmatrix}^T \begin{bmatrix} T_i(t) \\ T_j(t) \\ T_k(t) \end{bmatrix} = [N(r, z)] [T(t)]$$

Then the matrix  $[B(r, t)]$  can be evaluated as:

$$[B] = \begin{bmatrix} \partial N_1 / \partial r & \partial N_2 / \partial r & \partial N_3 / \partial r \\ \partial N_1 / \partial z & \partial N_2 / \partial z & \partial N_3 / \partial z \end{bmatrix}$$

The  $[D_e]$  matrix for an axisymmetric solid has the form:

$$[D_e] = \begin{bmatrix} k_{rr} & k_{rz} \\ k_{zr} & k_{zz} \end{bmatrix}$$

The general expression for the element thermal conductivity matrix  $[K_e]$ , heat capacitance matrix  $[C_e]$  and the thermal force vector  $[F_e]$  can be used for obtaining the explicit expressions for these matrices in the axisymmetric case. For details of derivation, interested readers are referred to [8].

#### Initial and Boundary Conditions

Initial conditions are required when dealing with transient heat transfer problems in which the temperature field in a solid changes with elapsing time. Because the differential equation is first order in time, only one initial condition is required  $T(r, 0) = T_0$ . Specific boundary conditions are, however, required in the analysis of all transient or steady-state problems involving solid of finite shape. The commonly used types of boundary conditions include the prescribed surface temperature (Dirichlet type), and the prescribed heat flux (Neumann type), [7].

There are two ways of incorporating these boundary conditions in the computation. The first method is to include these effects in the element thermal force matrix given in equation (3). The second method is to deal with these boundary conditions for those elements constituting the boundaries of the discretized model. The final results from the two approaches are, of course, identical.

The implemented program "TH" can deal with different types of problems including transient or steady state, different boundary conditions (forced or free convection, radiation, heat flux and constant surface temperature) which can take place simultaneously. In addition, the variation of boundary conditions with time as well as the variation of material properties with temperature can be considered.

### 3. SAMPLE PROBLEM

A thick hollow cylinder heated by convection on the internal surface as shown in Fig.2. is considered. Boundary conditions and material properties were selected such that an exact analytical solution can be obtained and used as a basis for comparison. Convection at internal surface takes place from a fluid having bulk fluid temperature of  $100^{\circ}\text{C}$ . Results are calculated over a period of 200 seconds. The structure has initial temperature of  $0^{\circ}\text{C}$ . The forced convective coefficient is  $864.7 \text{ J/sec.m}^2\text{C}$ . Different time steps were considered in each run. Values of 5, 10 and 20 seconds were used.

The output of thermal analysis is nodal temperature at each time step. The temperature history of a node on the external surface was slightly altered as incremental time step was changed. The solution converged to the exact one as the time increment was decreased. The 5-sec. time step gave the smallest error compared to analytical solution. Fig.3 shows the temperature history of the external surface obtained using the "TH" program as opposed to the analytical solution. Agreement of results indicates the efficiency of the present implemented program.

### 4. APPLICATION ON ROCKET NOZZLE

The selected nozzle, shown in Fig.4., is made from carbon/carbon composite material and has a submerged configuration. It is heated along the back side in addition to heating along the internal flow contour. Both convection and radiation were considered. Calculations were made based on the specified motor pressure  $p = 137.94 \text{ bar}$ , mass flow rate  $\dot{m} = 45.37 \text{ kg/sec}$  and combustion chamber temperature  $T = 3200^{\circ}\text{C}$ . The burning time was about 20 seconds.

The temperature profile (isotherms) were obtained at different time steps. However, two instants (1 and 20 sec.) which were considered critical for thermal stress analysis are given in Fig.5. The profile at one second is characterized by sharp temperature gradients, while the profile at twenty seconds is characterized by high temperatures. Table 1. gives a comparison between the "TH" program output and that obtained by a general purpose thermal code [8]. The results indicate a good agreement and confidence in the use of the implemented program for thermal analysis of practical problems in solid-rocket- motor design.

Finally different runs using convection only and convection with radiation were performed. As expected, Table (2) indicates that convection is the most important source in nozzle heating. However, the contribution of radiation is important in the combustion chamber (i.e., nozzle backside and convergent) and becomes less important or even negligible in the nozzle divergent.

14-16 May 1991 , CAIRO

Table 1. Comparison of Temperature Results

node	1 second			20 seconds		
	"TH"	Ref. 8	err %	"TH"	Ref. 8	err %
* A	2281	2026	11.18	2897	*****	
B	429	386	9.75	2362	2293	2.92
* C	2643	2360	10.67	2933	*****	
D	269	250	7.06	2100	2010	4.28
E	2233	2021	9.49	2826	2583	8.60
F	767	695	9.38	2387	2296	3.81
G	902	821	8.98	2199	2068	5.96
H	706	668	5.38	1960	1908	2.65

\* Nodes at ablation zone

\*\*\*\*\* Erosion

Table 2. Representation of Radiation Effect

node	1 second			20 seconds		
	CON+RD	CON.	RD%	CON+RD	CON.	RD%
A	2643	2360	10.71	2897	2677	7.59
C	2281	2066	9.42	2933	2721	7.23
E	2233	2096	6.13	2826	2703	4.36
I	2401	2100	12.49	2860	2619	8.43

CON+RD ... Convection &amp; Radiation.

CON. ... Convection only.

## 5. EXPERIMENTAL DETERMINATION OF NOZZLE THERMAL PROFILE

An experimental program had been conducted to measure the temperature at different points on the structure of the nozzle used with a test rocket motor developed in the Military Technical College. The experiments were carried out at the testing facility of the Technical Research Center of the Armed Forces. Static firings were performed to measure temperature at different point locations on the nozzle, within the time domain of motor operation which was 1.1 sec.

The tested convergent-divergent nozzle, has the dimensions as shown in Fig.6. Three thermocouples are welded at the bottom of the three holes at the indicated locations.

### Thrust and Pressure Measurement

Thrust-time and pressure-time records are obtained during the time of motor operation, Fig.7. Analysis of these records gave the main parameters of the motor as follows:



14-16 May 1991 , CAIRO

- main thrust at effective time	1987.173	N
- main pressure at effective time	67.846	bar
- effective burning time	1.095	sec
- specific impulse	1593.759	m/sec
- characteristic velocity	1074.163	m/sec

### Temperature Measurement

Three thermocouples are used to measure the temperature at three different locations of the nozzle structure. The installation of a thermocouple through the wall of a rocket nozzle introduces a temperature error which is peculiar to the individual installations. These errors cannot be compensated by thermocouple calibration external to the installation, hence they must be accounted for by calculation, [9]. In general, thermocouple installation errors may be attributed to heat leakage and position or location errors. For this experiment the thermocouple installation error is primarily caused by difficulty of welding at the bottom of holes, difference between thermocouple diameter and cavity diameter, difference between thermal properties of thermocouple material and nozzle material, and distance between hole bottom and internal surface.

In Fig.8 is presented a comparison between the measured temperature history and that calculated by the "TH" program and "ABL" program, [10] for two specified points on nozzle structure. The differences found between the measured and the calculated nodal temperatures may be attributed to:

- (1) Errors in measurement.
- (2) Errors in material thermal properties used in calculation.
- (3) Error in the boundary conditions used in the calculations; Their values were obtained from the nozzle theory based on various assumptions [11].

### 6. CONCLUSION

A two dimensional axisymmetric finite element thermal analysis was written as a FORTRAN 77 program. This program is capable to deal with steady or transient heat transfer under different boundary conditions. It is also capable to handle different materials, where the variation of their thermal properties with temperature is taken into consideration.

A real case of thermal analysis on a submerged nozzle was studied. The nozzle temperature profile throughout the operating time domain was obtained using "TH" program. For the case under study, the thermal analysis indicated two critical time instants: 1 sec and 20 seconds. The results from "TH" program were compared with the results from another code. From the comparison, it was found that, the "TH" results were slightly higher. However, this can be explained by the fact that "TH" program does not account for the effect of ablation of nozzle surface which was considered in the general purpose code.

The submerged nozzle was solved with convective boundary condition and with convective and radiative boundary conditions.

14-16 May 1991 , CAIRO

The analysis of results indicated that the radiation caused rise in nodal temperature for nodes on the nozzle backside and convergent surface by about (9-13)% and for nodes inside the nozzle structure by about (4-7)% (Table 2).

An experimental program including different static firings was performed, and the test results of temperature at some locations were compared to those calculated by programs "TH" and "ABL". The general trends of temperature results were in a good agreement.

## NOMENCLATURE

a	.... polynomial coefficient	t	.... time
C	.... specific heat	$\Delta t$	.... incremental time
k	.... thermal conductivity	v	.... volume
N	.... interpolation function	z	.... axial coordinate
n	.... directional cosine	$\rho$	.... specific mass
Q	.... heat generation	$\Phi$	.... Gurtin's functional
q	.... heat flux		
r	.... radial coordinate	Subscripts	
s	.... surface area	i, j, k	nodes
T	.... temperature	r	.... radial
$\dot{T}$	.... temp. variation w.r.t. time	z	.... axial
$\Delta T$	.... temperature difference	o	.... initial

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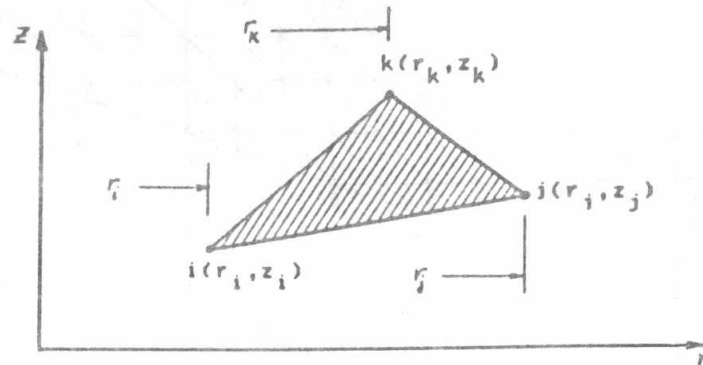


Fig. 1 Nodal coordinates of triangular torus element

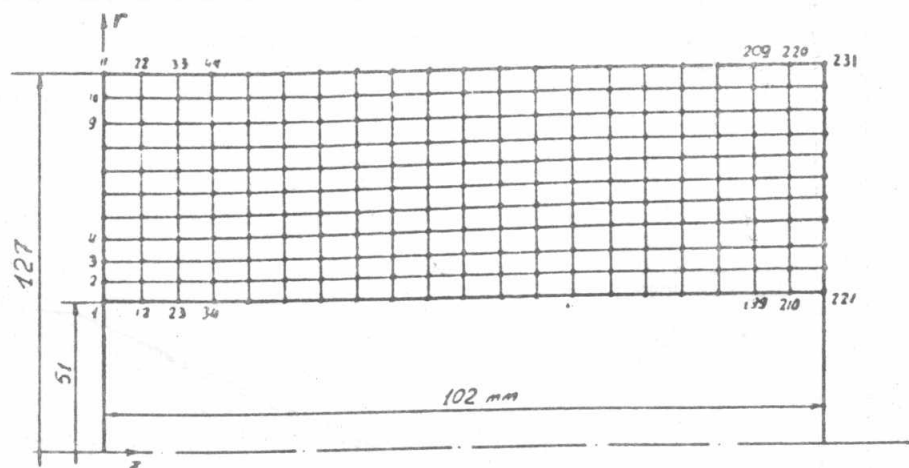


Fig. 2 Sample problem, dimensions and discretization

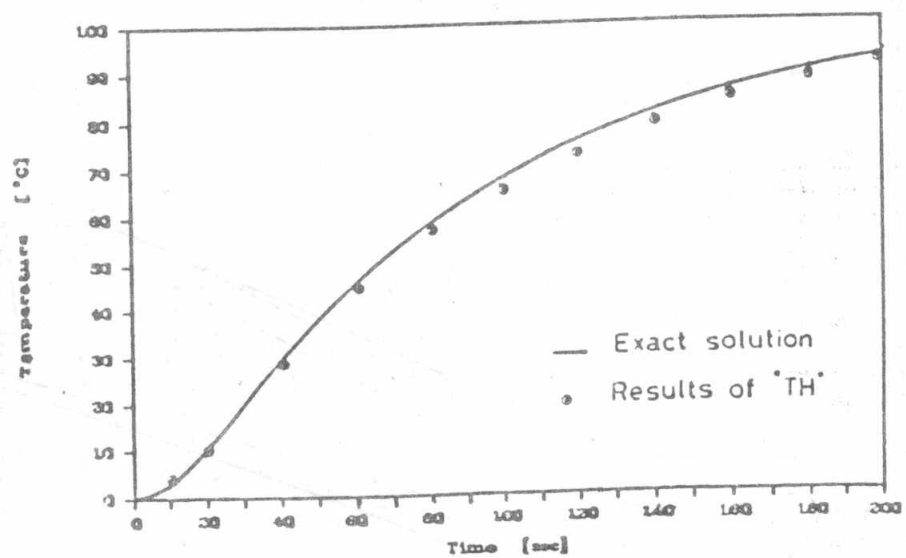


Fig. 3 Comparison of the "TH" results with the analytical solution

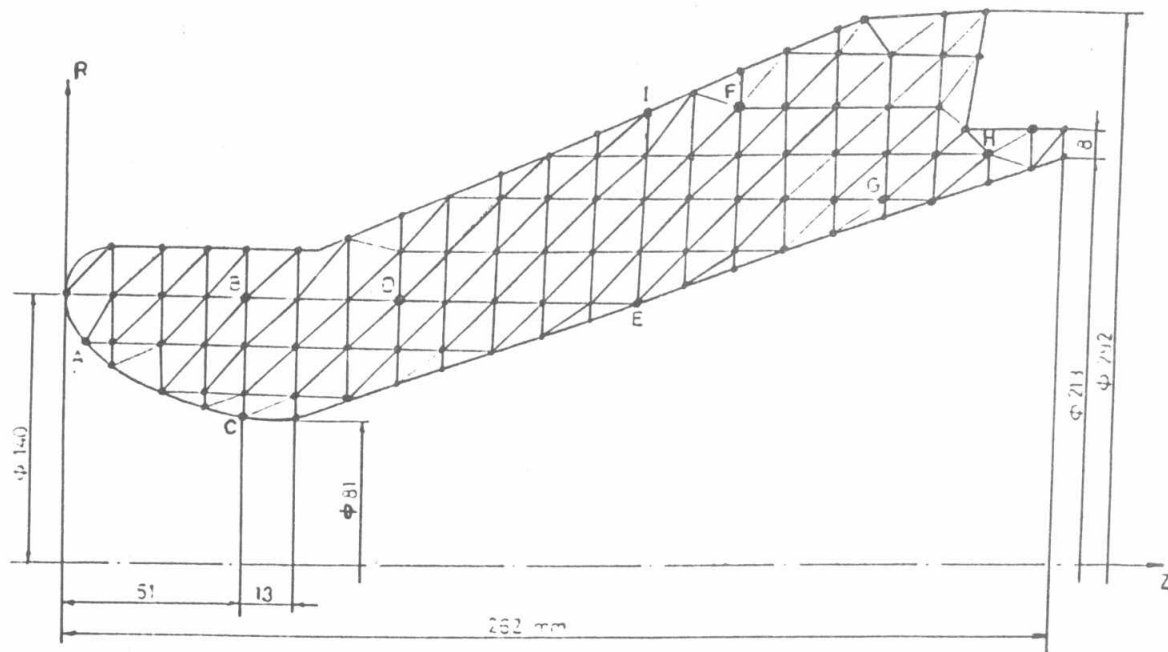
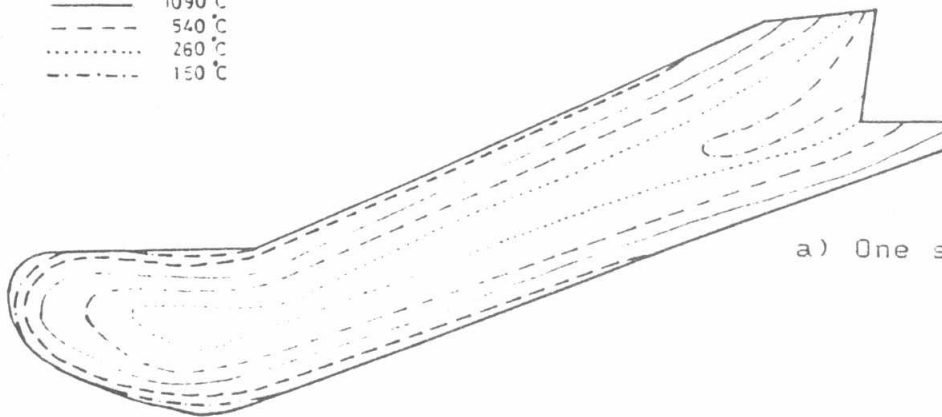
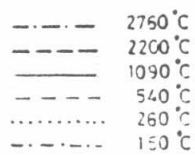
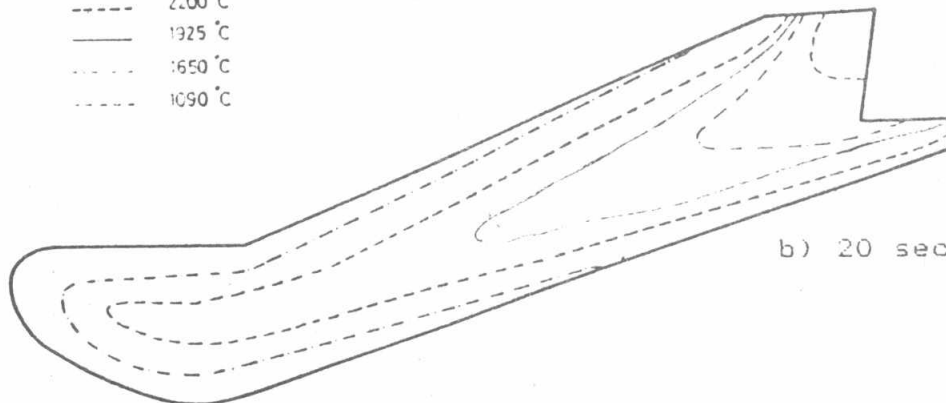
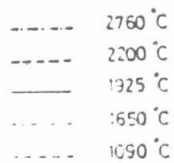


Fig. 4 Submerged nozzle



a) One second



b) 20 seconds

Fig. 5 Submerged nozzle isotherms by "TH"

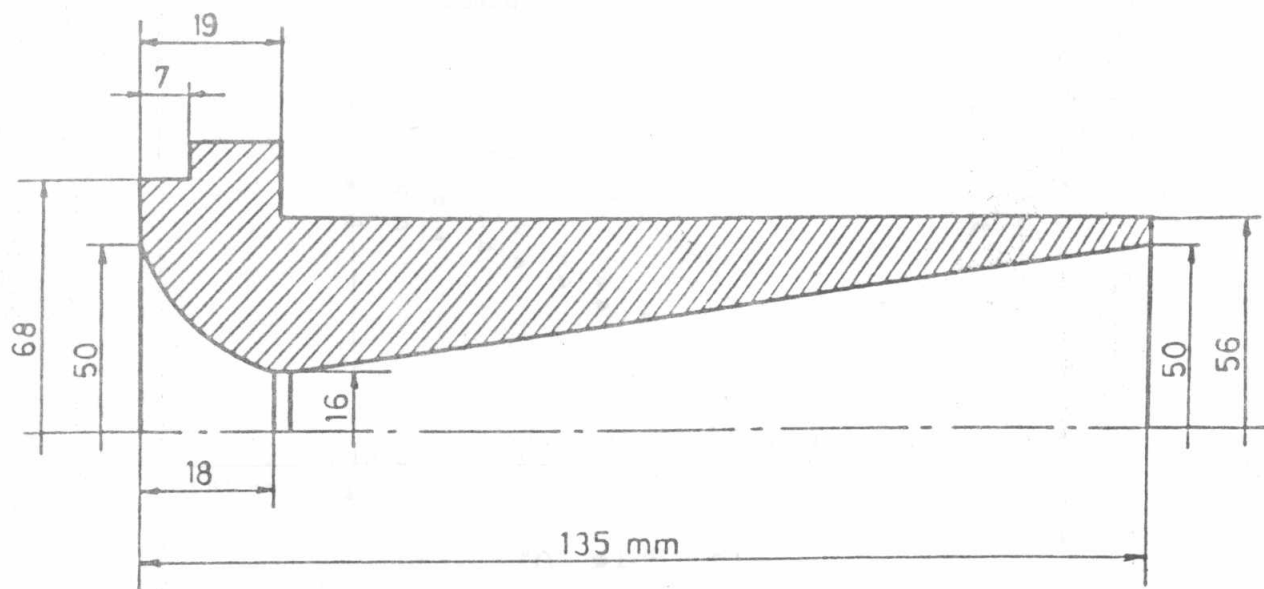


Fig. 6 Tested nozzle

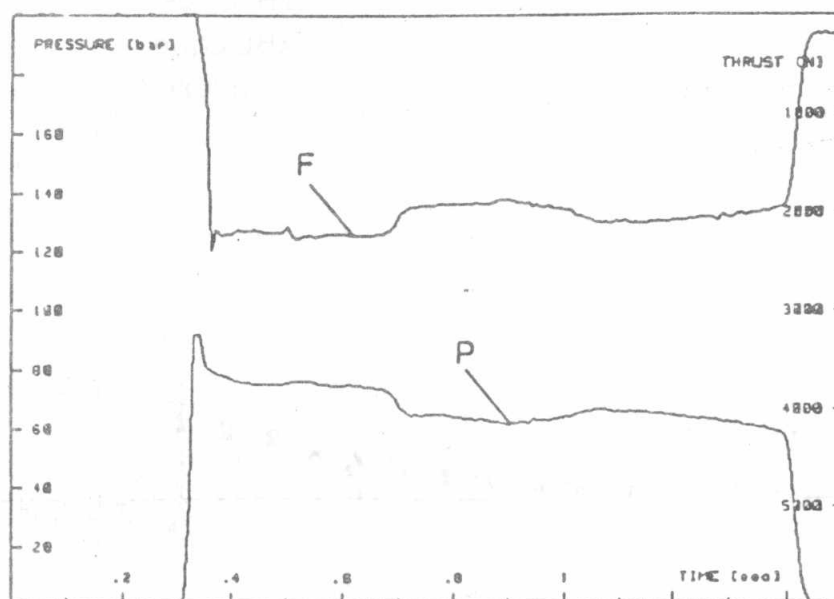
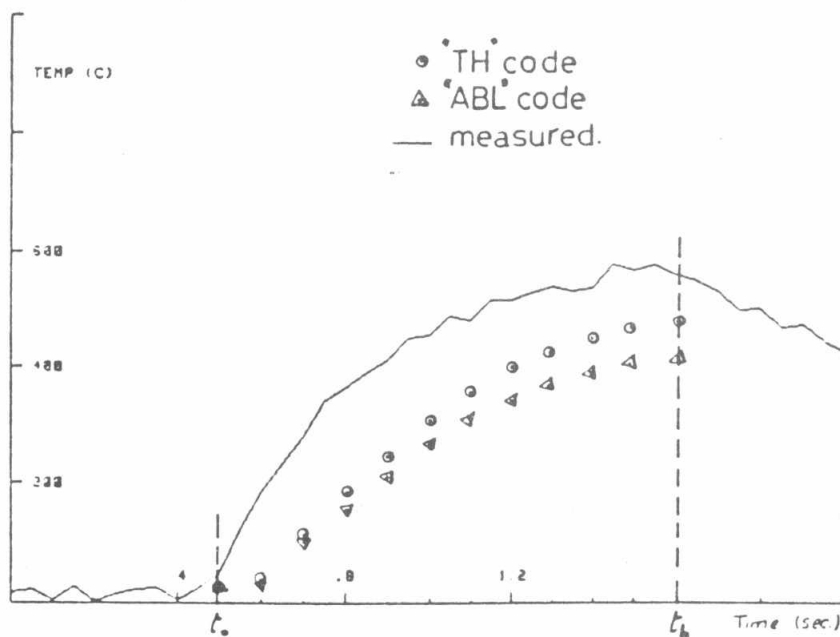
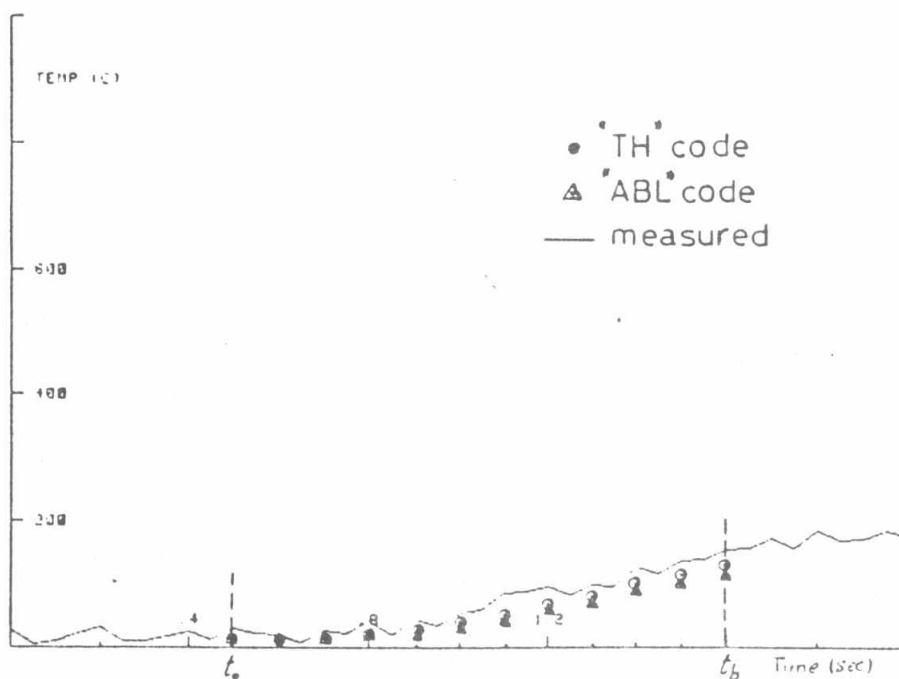


Fig. 7 Thrust and pressure histories

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(a) Hole "A"



(b) Hole "B"

Fig. 8 Comparison of measured and calculated temperatures at the bottoms of cavities