MILITARY TECHNICAL COLLEGE

CAIRO - EGYPT

1- # INVESTIGATION OF THE ACOUSTIC VELOCITY

2- # IN A NONEQUILIBRIUM WET STEAM FLOW

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4- ## ABSTRACT

The acoustic velocity in a nonequilibrium wet steam flow has been investigated theoretically. Modified equations of unsteady one-dimensional wet steam flow are taken to describe the propagation of a plane pressure wave of very small amplitude through the wet steam flow. These equations are derived for the conditions of nucleating and non-nucleating wet steam assuming that the vapour phase obeys the perfect gas law and the water and vapour phases are in disequilibrium both thermally and dynamically. The acoustic velocity is shown to depend mainly on the steam wetness, flow non-equilibrium characteristics (i.e., vapour supercooling and slip ratio) and flow pressure; while it slightly affected by the size of droplets and nucleation occurrence.

INTRODUCTION

Wet steam flow in turbines of both conventional and nuclear power plants is an important parameter for characterizing these turbines. Therefore, the acoustic velocity in wet steam is a widely used concept in describing the flow of wet steam. This problem becomes more complicated in wet steam media than in the case of single phase media due to the presence of the water droplets, phase transition and disequilibrium between the two phases.

This problem has received considerable attention both experimentally and theoretically in the field of two phase flow. Measurements have been presented by several investigators for the propagation speed of pressure and/or rarefaction waves in wet steam flow [1—3] or in vapour-liquid mixtures [4]. Few theoretical researches can be found in the literature concern with predicting the acoustic velocity variation in two phase or wet steam flows. D'Arcy [5] derived a theoretical model to calculate the propagation speed of small pressure disturbances through two phase fluid using the separated flow model. He showed that there are two distinct speeds for propagation at low and at high void fractions. He also suggested from comparisons between his

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theoretical results and the published measurements that the two theoretical speeds fall in the mid-void range. Konorski [6] presented a dynamic model for the behaviour of two phase mist flow in order to analyse the phenomenon of sound propagation in two phase media. He considered in this model that the sound frequencies will be limited to values at which the wave length substantially exceeds the mean free path of gas molecules and the largest droplet diameter. He concluded that for each droplet size there are two ranges of strong sound velocity dispersion, one at lower frequencies caused by heat and mass transfer; the other by momentum transfer. Petr [7] has derived an analytical model for sound dispersion and absorption in wet steam considering all fundamental relaxation phenomena. Petr defined, for a wet steam with restricted values of droplet size; wetness fraction and pressure, two speeds of sound related to the wave frequency as :

i- frozen speed of sound when very high frequency waves propagating through wet steam cantaining large droplets, and

ii- equilibrium speed of sound when very low frequency waves moving through wet steam containing small droplets.

He also showed that the speed and absorption of sound in wet steam are greatly dependent on droplet radius, steam wetness and pressure. An account for sound dispersion in wet steam flows was illustrated by Wegener [8]. He proved that with condensation presence in wet steam, the mixture of droplets and vapour becomes an acoustically dispersive mixture.

This paper presents a mathematical model for the speed of propagation of a small amplitude pressure disturbance in a nonequilibrium wet steam flow. The effects of steam wetness, size of droplets , vapour supercooling, slip ratio, flow pressure and nucleation rate on the acoustic velocity have been considered here. A comparison between theoretical results of the present model and those obtained experimentally by others is offered also.

THEORETICAL ANALYSIS

Assumptions

In order to facilitate the analysis, the following assumptions are applied to all the analytical work presented below :

1- Wet steam consists of a vapour phase as a carrier inviscid medium and fine spherical water droplets suspended in it.
2- The flow is one-dimensional and continuum.
3- Water droplets are assumed to be incompressible and occupy negligable volume.
4- Surface tension effects are insignificant.
5- Effects of capillarity and of droplets coagulation will be neglected.
6- Phase transition does not affect both drag and heat transfer.
7- The vapour and water phases are in thermal and dynamic disequilibrium.

Governing equations

Figure (1) shows a stationary, infinitesimal pressure disturbance propagating in a constant area duct. In the following, a set of unsteady equations governing the wet steam flow can be written as:

Continuity for vapour phase:

$$\rho_g.\frac{dC}{dt} + C.\frac{d\rho_g}{dt} = - G.\frac{dY}{dt} \qquad (1)$$

Continuity for water phase:

$$\rho_f.\frac{dw}{dt} = G.\frac{dY}{dt} \qquad (2)$$

The combined momentum equation for the two phases:

$$A.\frac{dP}{dt} + N.D.\frac{dx}{dt} + \rho_g AC.\frac{dC}{dt} = o \qquad (3)$$

For a water droplet subjected only to viscous drag , the momentum equation can be take the following form:

$$D = m.w.\frac{dw}{dx} \qquad (4)$$

Combining Eqns.(3) and (4) gives

$$\rho_g C.\frac{dC}{dt} + \frac{N w m}{A} . \frac{dw}{dt} + \frac{dP}{dt} = o \qquad (5)$$

Continuity for vapour phase takes the basic form

$$\rho_g.C = (1 - Y) . G \qquad (6)$$

Using Eqns.(6) and (1) yields to:

$$\frac{dC}{dt} = - \frac{C}{\rho_g}.\frac{d\rho_g}{dt} - \frac{C}{1-Y}.\frac{dY}{dt} \qquad (7)$$

Substituting Eqn.(7) into Eqn.(5) gives

$$\rho_g C.(-\frac{C}{\rho_g}.\frac{d\rho_g}{dt} - \frac{C}{1-Y}.\frac{dY}{dt}) + \frac{Nwm}{A}.\frac{dw}{dt} + \frac{dP}{dt} = o \qquad (8)$$

The mass fraction of water phase or the wetness fraction is given by:

$$Y = \frac{N w m}{C\rho_g A + N w m} = \frac{N w m}{GA} \qquad (9)$$

Therefore, Eqn.(8) can be written using Eqn. (9) to becomes

$$-C^2.\frac{d\rho_g}{dt} - \frac{C^2.\rho_g}{1-Y}.\frac{dY}{dt} + \frac{YC\rho_g}{1-Y}.\frac{dw}{dt} + \frac{dP}{dt} = o \qquad (10)$$

Rearranging Eqn. (10) and putting c = a_m gives

$$a_m^2 = \frac{\frac{dP}{dt}}{\frac{d\rho_g}{dt} + \frac{\rho_g}{1-Y}.\frac{dY}{dt} - \frac{Y}{1-Y}.\frac{\rho_g}{C}.\frac{dw}{dt}} \qquad (11)$$

Assuming that the vapour phase obeys the perfect gas low (i.e., $P/\rho_g = R_g T_g$), this including that the equation of state of the vapour phase can be expressed in differential form as

$$\frac{dP}{dt} = R_g T_g.\frac{d\rho_g}{dt} + R_g \rho_g.\frac{dT_g}{dt} \qquad (12)$$

According to the above equation, the acoustic velocity from Eqn. (11) becomes

$$a_m = \pm \left[\frac{R_g T_g \cdot \frac{d\rho_g}{dt} + R_g \rho_g \cdot \frac{dT_g}{dt}}{\frac{d\rho_g}{dt} + \frac{\rho_g}{1-Y} \cdot \frac{dY}{dt} - \frac{Y}{1-Y} \cdot \frac{\rho_g}{c} \cdot \frac{dw}{dt}} \right]^{\frac{1}{2}} \qquad (13)$$

The plus and minus signs in Eqn.(13) are for the wave travelling upstream and downstream with respect to the flow direction. Putting $Y = o$ in the above equation reduces the acoustic velocity in the wet steam to its value in the gaseous (vapour) phase only (see Appendix A).

In order to solve Eqn.(13), the differential terms in this equation can be calculated analytically using the following set of equations [9, 10]:

1- Variation of wetness fraction:

a- for non-nucleating wet steam:

In this case, wet steam is assumed to be a homogeneous mixture of vapour with a temperature T_g and a constant number of water droplets of size d and temperature T_s. The wetness fraction is therefore given by

$$Y = n \cdot \frac{\pi}{6} d^3 \rho_f \qquad (14)$$

The generally used form of the wetness fraction variation is that presented by Gyarmathy [9] and corrected by Young [10] as

$$\frac{dY}{dt} = \frac{(1-Y) \cdot C_{pg}}{h_{fg}} \cdot \frac{T_s - T_g}{\tau} \qquad (15)$$

where τ is the thermal relaxation time defined by

$$\tau = \frac{(1-Y) \cdot C_{pg} \cdot d^2 \cdot \rho_f \cdot (1+3.78\ Kn/Pr_g)}{12\ Y \cdot \lambda_g}$$

b- for nucleating wet steam:

Here, the wet steam being a homogeneous mixture of vapour and discrete i^{th} groups of droplets. Each i^{th} group contains n_i droplets of size d_i per unit mass of wet steam. Therefore, the wetness fraction is given by

$$Y = \Sigma Y_i = \Sigma n_i \cdot \frac{\pi}{6} d_i^3 \cdot \rho_f \qquad (16)$$

and the variation of the wetness fraction is defined by

$$\frac{dY}{dt} = \Sigma \frac{dY_i}{dt} \qquad (17)$$

where , $\Sigma \frac{dY_i}{dt} = \Sigma \frac{(1-Y_i) \cdot C_{pg}}{h_{fg}} \cdot \frac{T_s - T_g}{\tau_i}$

, $\tau_i = \frac{(1-Y_i) \cdot C_{pg} \cdot d_i^2 \cdot \rho_f \cdot (1+3.78\ Kn_i/Pr_g)}{12\ Y_i \cdot \lambda_g}$

2- Drag force exerted on water droplets is that due to Gyarmathy [9]

$$D = C_D \cdot (\pi d^2) \cdot \frac{1}{8} \rho_g \cdot (c-w)^2 \qquad (18)$$

where C_D is the drag coefficient given by

$$C_D = 0.292 \left(\frac{9.06}{\sqrt{Re_d}} + 1\right)^2$$

3- Variation of droplets velocity is expressed as

$$\frac{dw}{dt} = \frac{1}{m.S} \cdot D + \frac{c-w}{S} \cdot \frac{1}{Y} \cdot \frac{dY}{dt} \qquad (19)$$

4- Variation of the specific internal energy of the vapour phase is given for the above approximations by

$$\frac{du_g}{dt} = (u_g - u_f) \cdot \frac{1}{1-Y} \cdot \frac{dY}{dt} - \frac{P}{\rho_g CS} \cdot \frac{dw}{dt} \qquad (20)$$

5- Variation of temperature of the vapour phase is obtained from

$$\frac{dT_g}{dt} = \frac{1}{C_{vg}} \cdot \frac{du_g}{dt} \qquad (21)$$

6- Variation of the vapour phase density is given by

$$\frac{d\rho_g}{dt} = - \frac{\rho_g}{1-Y} \cdot \frac{dY}{dt} - \frac{\rho_g}{C.S} \cdot \frac{du}{dt} \qquad (22)$$

RESULTS AND DISCUSSION

To calculate the acoustic velocity in a wet steam flow, Eqn. (13) can be solved with the aid of Eqns.(14) through (22). Solving these equations revealed the dependence of the acoustic velocity on the vapour supercooling, slip ratio, size of droplets, nucleation rate and pressure of a nonequilibrium wet steam flow. Results presented in Figs. (2-7) are obtained for the following restricted area on wet steam.

$$0 \leq Y \leq 0.5$$
$$0.1\ °K \leq \Delta T \leq 20.0\ °K$$
$$0.7 \leq S \leq 0.99$$
$$0.01\ \mu m \leq d \leq 20.0\ \mu m$$
$$0.1\ bar \leq P \leq 30\ bar$$

These figures show the variation of the acoustic velocity , in dimensionless value of a_m/a_g using Eans.(13) and (29), for different values of vapour supercooling; slip ratio;droplet diameter; nucleation rate and pressure of a wet steam flow within the range of data presented above. As shown in these figures, acoustic velocity increases as the steam wetness decreases. This is due to the creation of number of droplets with decreasing the steam wetness fraction.

Figures (2—3) indicate the effect of changing the nonequilibrium characteristics of wet steam (i.e., vapour supercooling and slip ratio) on the acoustic velocity depression. It is evident from Fig.(2) that the depression of the acoustic velocity increases with increasing the value of supercooling. This tendency can be explained as the supercooling increases the vapour temperature decreases and then the vapour density increases and consequently this tends to attenuate the wave propagation through the mixture. depression of acoustic velocity decreases as shown in Fig.(3) with the slip ratio to be descreased. This is attributed to less interaction between the two phases as the degree of dynamic disequilibrium increases ahead of the pressure wave.

Effect of changing the size and distribution of water droplets in wet steam on the acoustic velocity is shown in Figs.(4—5). The effect of droplet size on the acoustic velocity variation is illustrated in Fig.(4). From this figure it can be observed that, the effect of changing the size of water droplets in wet steam on the depression of acoustic velocity is relatively small. This is because the number of droplets increases with decreasing the droplet size for a constant value of steam wetness. Thus the number of droplets , i.e. distribution of droplets in the mixture, gives a small variation on depression of the acoustic velocity. A similar feature is shown in Fig.(5) which presents the effect of considering the rate of nucleation on the acoustic velocity depression. This supports the discussion of Fig.(4), since variation of droplet size distribution result in weak effect on the variation of the acoustic velocity within a flow of wet steam.

Figure (6) illustrates the effect of pressure changing on depression of the acoustic velocity through a nonequilibrium wet steam flow. It shows that, with the pressure increases the acoustic velocity depression increases also in two modes. In the first, the acoustic velocity decreases; for constant wetness fraction; with pressure increasing from 0.1 bar to 10 bar. In the second, the same characteristics are shown when the pressure increases from 20 bar to 30 bar. Another important notice can be seen in this figure that the curves of the second mode having higher values of the acoustic velocity within the range of wetness $0 \leqq Y \leqq 0.3$ and intersect the curves of the first mode at wetness fraction $Y = 0.3$. This result was confirmed previously by Petr [7]. Results of Petr [7] showed that; as the pressure increases within an angular frequency range of $10^2 \leqq \omega \leqq 10^4$ the acoustic velocity increases also, while this velocity decreases with pressure increasing within the range $10^6 \leqq \omega \leqq 10^7$ and in the intermediate range $10^4 \leqq \omega \leqq 10^6$; pressure increasing gives intersecting curves for the corresponding acoustic velocity variations.

A comparison between theoretical results from the present model and the experimental one, which was obtained by DeJong and Firey [2] and England, et al.[3] at a pressure of 3.0 bar, is illustrated in Fig.(7). From this figure, it is clear that the theoretical results have a reasonable agreement with the experimental one within the wetness range of $0 \leqq Y \leqq 0.2$. The difference between the experimental and theoretical results is more pronounced at the higher values of steam wetness. This difference can be explained as being due to the formulas of heat transfer and drag coefficients, in Eqns.(15) and (18), were recommended for wet steam conditions of low wetness fractions.

CONCLUSIONS

This paper presents an analytical model to predict the velocity of small amplitude pressure disturbances in a nonequilibrium wet steam flow. Theoretical results obtained here show that the acoustic velocity depends greatly on the steam wetness, nonequilibrium characteristics (i.e., vapour supercooling and slip ratio) and flow pressure; while the size of droplets and nucleation occurance have a slight effect on this velocity. Reasonably good agreement between theory and experiments could be obtained if the correlations of heat transfer and drag coefficients of the droplets and/or the thermodynamic behaviour of wet steam are adapted to be suitable for all the range of wetness used.

REFERENCES

1. Collingham, R.E. and Firey, J.C., "Velocity of Sound Measurements in Wet Steam", I & EC Process Design and Development 2, 197-202 (1963).
2. DeJong, V.J. and Firey, J.C., "Effect of Slip and Phase Change on Sound Velocity in Steam-Water Mixtures and the Relation to Critical Flow", I & EC Process Design and Development 7, 454-463 (1968).
3. England, W.G., Firey, J.C. and Trapp, O.E., "Additional Velocity of Sound Measurements in Wet Steam", I & EC Process Design and Development 5, 198-202 (1966).
4. Chen, G., Thompson, Ph. A. and Bursik, J.W., "Soundspeed Measurements in Vapour-Liquid Mixtures Behind Shock Waves" Experiments in Fluids 4, 279-282 (1986).
5. D'Arcy, D.F., "On Acoustic Propagation and Critical Mass Flux in Two-Phase Flow", Trans. ASME, J. of Heat Transfer 93, 413-421 (1971).
6. Konorski, A. , "Non-equilibrium Effects in Rapidly Expanding Two-Phase Media", Proc. of NATO Advanced Study Institute on "Two-Phase Flows and Heat Transfer", Istanbul 1247-1261 (1976).
7. Petr, V., "Variation of Sound Velocity in Wet Steam",Instn Mech. Engrs, Conference Publication 3, 33-37 (1973).
8. Wegener, P.P. (Ed.), " Nonequilibrium Flows", Marcel Dekker Co., New York (1969).
9. Moore, M.J. and Sieverding, C.H. (Eds.), "Two-Phase Steam Flow in Turbines and Separators ", McGraw-Hill Co., Washington (1976).
10. Moore, M.J. and Sieverding, C.H. (Eds.), "Aerothermodynamics of Low Pressure Steam Turbines and Condensers" , Hemisphere Publishing Co., Washington (1987).

APPENDIX (A)

Acoustic velocity for the vapour phase:

In order to use Eqn.(13) to calculate the acoustic velocity for the vapour phase only, the following are considered:

$Y = o$ and $dY = o$

This consideration yields Eqn.(13) to become

$$a_m = \pm \left[\frac{R_g T_g \cdot \frac{d\rho_g}{dt} + R_g \rho_g \cdot \frac{dT_g}{dt}}{\frac{d\rho_g}{dt}} \right]^{\frac{1}{2}} \tag{23}$$

Simplifing the above equation gives

$$a_m = \pm \left[R_g T_g \cdot (1 + \frac{\rho_g}{T_g} \cdot \frac{dT_g}{d\rho_g}) \right]^{\frac{1}{2}} \tag{24}$$

Assuming perfect gas behaviour for the vapour phase, the continuity and energy equations for steady inviscid vapour flow can be written as

$$\frac{1}{\rho_g} \cdot d\rho_g + dC = 0 \qquad (25)$$

$$, \; du_g + \frac{P}{\rho_g} \cdot dC = 0 \qquad (26)$$

Combination of equations (25) and (26) gives

$$d\rho_g = \frac{\rho_g^2}{P} \cdot du_g \qquad (27)$$

Substituting Eqn.(27) into Eqn.(24) makes the acoustic velocity to take the form

$$a_m = \pm \left[R_g\, T_g \cdot \left(1 + \frac{P}{\rho_g T_g} \cdot \frac{dT_g}{du_g}\right)\right]^{\frac{1}{2}} \qquad (28)$$

By using the following ideal approximations of the isochoric specific heat capacity for the vapour phase and the isentropic exponent

$$C_{vg} = \frac{du_g}{dT_g}$$

$$, \; k = 1 + \frac{R_g}{C_{vg}}$$

the equation of acoustic velocity takes the final form

$$a_m = \pm\, (k\, R_g\, T_g)^{\frac{1}{2}}$$
$$= a_g \qquad (29)$$

NOMENCLATURE

A	Flow cross-sectional area.
a_g	Frozen speed of sound in vapour phase.
a_m	Acoustic velocity in wet steam.
C	Absolute velocity of vapour phase.
C_D	Drag coefficient.
C_{pg}	Isobaric specific heat capacity of vapour phase.
C_{vg}	Isochoric specific heat capacity of vapour phase.
D	Drag force on a droplet.
d	Droplet diameter.
G	Rate of mass flow per unit area.
h	Specific enthalpy.
Kn	Knudsen number.
k	Index of isentropic expansion.
m	Mass of a droplet.
N	Number of droplets per unit length of flow passage.
n	Number of droplets per unit mass of steam.
P	Pressure.
P_{rg}	Prandtl number of vapour phase.
Re_d	Reynolds number of a droplet.
R_g	Gas constant (= 461.51 J/Kg.°K).
S	Slip ratio (= w/c)

T Temperature.
t Time.
u Specific internal energy.
w Droplet velocity.
x Distance in flow direction.
Y Wetness fraction.
ΔT Amount of vapour supercooling ($= T_s - T_g$).
λ Thermal conductivity.
ρ Density.
τ Thermal relaxation time.

Subscripts

d Droplet.
f Water phase.
fg Phase transition.
g Vapour phase.
i Number of droplets group.
s Saturation.

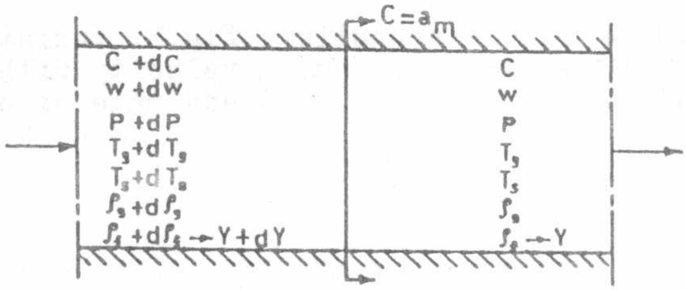

Fig.1. Wet steam conditions in the flow passage upstream and downstream the pressure disturbance.

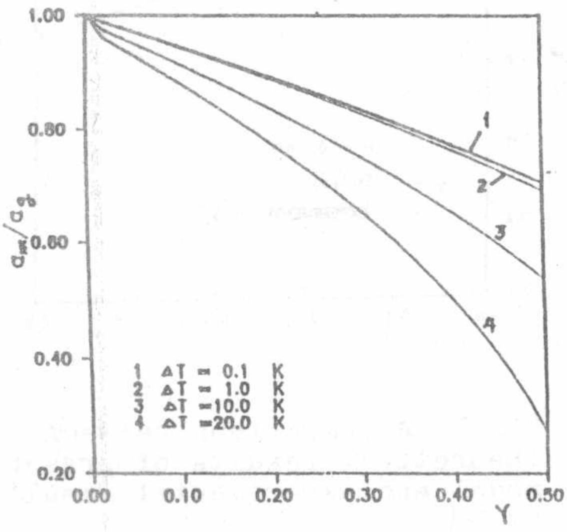

Fig.2. Variation of the acoustic velocity with steam wetness and vapour supercooling.

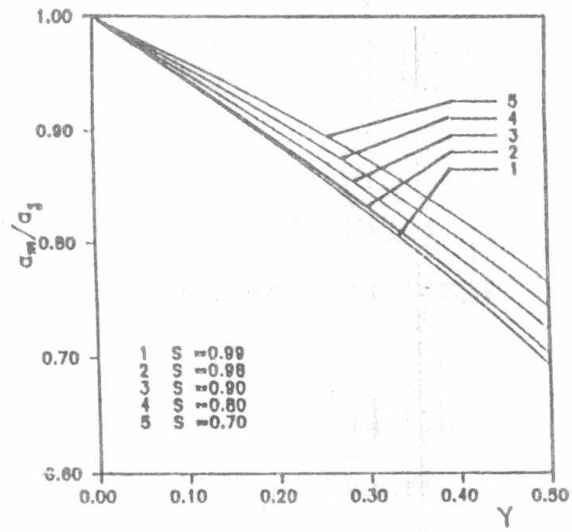

Fig.3. Variation of the acoustic velocity with steam wetness and and slip ratio.

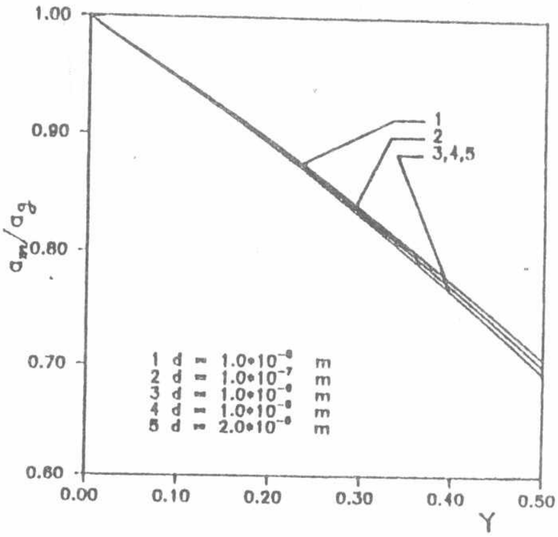

Fig.4. Variation of the acoustic velocity with steam wetness and size of droplets.

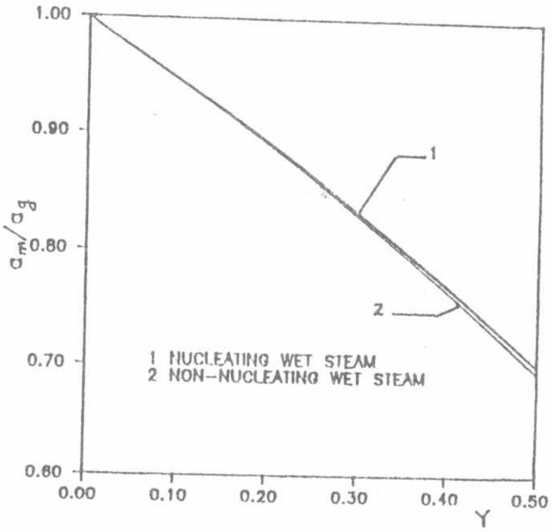

Fig.5. Variation of acoustic velocity with steam wetness and rate of nucleation.

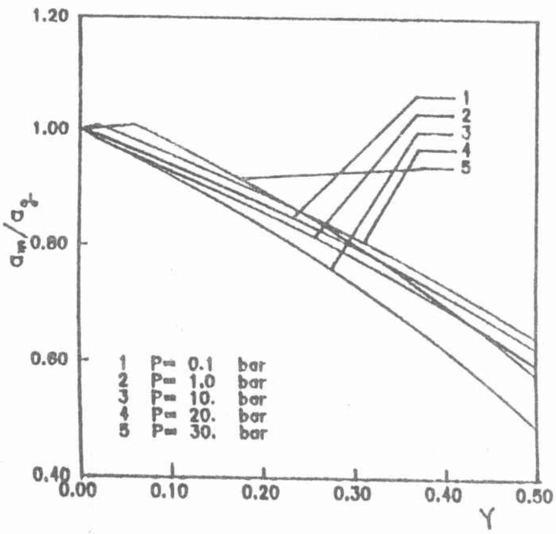

Fig.6. Variation of acoustic velocity with steam wetness and pressure.

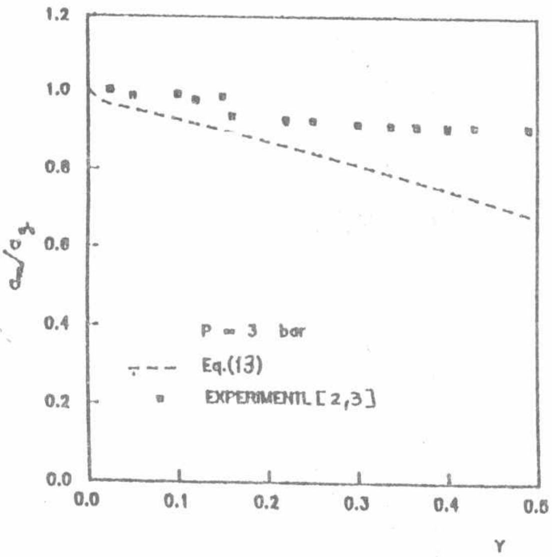

Fig.7. A comparison between theoretical results of present model and experimental results [2,3].