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RESIDUAL STRESS MEASUREMENT FOR A FINITE PLATE UNDER TENSION USING TWO DOUBLE EDGE CRACKS METHOD

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ABSTRACT

Making use of stress distribution in a finite plate with the presence of cracks, residual stress and actual applied stress can be obtained. The finite element technique is used to determine stress intensity factor and stress distribution in a plate having two double edge cracks with different crack depth. Experimental measurement has been carried out in a finite plate subjected to tensile stress. Strain relaxation due to making two double edge cracks is given, residual stress can also be estimated.

The effect of crack depth and its location for measuring residual stress is investigated under same tensile loading condition for three different engineering materials. A good agreement between theoretical and experimental determination of stress relaxation is obtained. An engineering chart for obtaining the actual applied stress in a tensile loaded plate is implemented.

INTRODUCTION

Residual stress is one of the most important components which has great influence on the fatigue life, corrosion and surface wear of metals . During machining processes and heat treatment, residual stresses can be generated by non-uniform plastic deformation, thermal gradients and phase transformation. The basic concept adopted by [1],[2] for determination of residual stresses on the surface of an arbitrarily shaped part is to release the residual stress on that surface by proper cutting or drilling operations. Then released strain can be measured by the aid of electrical resistance strain gages .

The finite element method can provide a general approach to analysis of stress and strain for a complex crack configurations and loading conditions. In addition, it gives values of stress intensity factors near crack tip with acceptable accuracy for engineering applications. It is found that the crack tip stress field will make relatively some effect to the overall deformation of the body, so that adequate representation of crack tip stress field is important.

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In this paper, an application of the mathematical model developed by Tong el at [3] to calculate the stiffness matrix for the super-element at the crack tip is carried out with conjunction of the regular finite element, the displacement model. The validity of that program was confirmed by comparing the results of the super-element stiffness matrix and stress intensity factor with their results. Stresses and strains were determined at each discreet element for cracked and uncracked plate. Strain relaxation was calculated theoretically by taking strain difference at same elements location in cracked and uncracked plate.

Actual strain relaxation of a tensile plate due to making two double edge cracks is measured using four strain gages (Tokyo Sokki Kenkyujo Fla-3-11-120, Sg=2.12). These strain gages were mounted on the front and back side of the plate edges . The effect of crack depth and its location with respect to the strain gages location on the strain relaxation has been investigated. Residual stress is estimated experimentally for the case of uniaxial state of stress.

Engineering chart for different engineering materials to determine the actual applied stress in a tensile loaded plate has been suggested.

Method of Analysis

Finite element method is used to determine the stress and strain distribution on the plate having two double edge cracks. The strain relaxation can be determined by the principal of super-position .

In this investigation, the finite element mesh is considered for a quarter plate due to the symmetry of cracks Fig.(1). A typical quarter plate of the finite element network is shown in Fig.(2). This quarter was divided into 77 quadrilateral isoparametric 4-node elements and one singular super-element with total number of degrees of freedom 183. The number of elements and degrees of freedom were slightly changed when changing either crack depth or crack location.

Stresses and strains based on the plane stress elasticity problem were determined from the finite element code at nine Gaussian point in each element. Stress and strain distributions have been calculated using two different finite element models.

The first one is the conventional finite element displacement model [4] which based on the potential energy functional in terms of displacements and strains as in the following functional:

$$\pi_p = \int_V \frac{1}{2} E_{ijk\ell} \, \varepsilon_{ij} \, \varepsilon_{k\ell} \, dV - \int_{S_\sigma} \bar{T}_i \, u_i \, dS \qquad (1)$$

Where the volume integral represents the work of internal strain energy and the variation of the surface integral represents the external work done by the specified surface traction (Ti) during displacement. Then by choosing a continuous function for the displacement field (shape function), the compatibility conditions will be enforced and the equilibrium conditions will be the automatic consequence of the stationary conditions. This conventional finite element displacement model is well explained with FORTRAN programming in Hinton's book [4]. The authors picked up the necessary subroutines and joined them together with a main program to solve the plan stress plate problem. After some modification in that FORTRAN finite element computer program, the authors were able to run it on the IBM PC 640K RAM memory and get exactly the same answer for the same worked example given in [4].

The second finite element model is based on the modified complementary energy principle (hybrid stress) which known as the two field principles. One for the assumed stress field within the element and the other for the independent assumed displacement on its boundary. This modified complementary energy functional can be written as

$$\pi = \int_{\partial A} \{T\}^T \{\tilde{u}\} \, dS - \frac{1}{2} \int_{\partial A} \{T\}^T \{u\} \, dS \qquad (2)$$

Where,

$$T_i = \sigma_{ij} \, \nu_j \qquad \text{on } \partial A$$

$$(u_{i,j} + u_{j,i})/2 = C_{ijk\ell} \, \sigma_{k\ell} \qquad \text{in } A \qquad (3)$$

This second model is used in the element located at the crack tip and called the stress hybrid singular super element. The concept of that super element was first developed by Tong el at [3]. This stress hybrid singular super element was checked as a single stiffness subroutine with the results obtained in ref.[3].

After results confirmation with the same worked example in that reference we properly joined this stiffness matrix to the other regular stiffness matrix in to a global stiffness matrix in a subroutine called STIFPS.
This program was compiled using Microsoft FORTRAN Optimizing Compiler [5] installed on IBM PC. Stress and strain relaxation have been determined using these models for the plate under uniform loading condition.

Experimental work

In order to measure residual stress experimentally, the stress must be relieved in some fashion. Strain gages can measure the change in strain caused by the removal of the stressed material. This can be done by cutting and sectioning the structural parts [6] or by blind hole drilling method [7].
In the present work, stress relieving is made by using two cracks in each side of the stretched plate. This method is considered as a semi-distructive testing method .

Test specimen

A procedure similar to that used in the blind hole drilling method was applied for measuring residual stresses. This procedure can be used to measure strain relaxation on the plate edges by strain gages with some modifications.

Some consideration for designing the test specimen :

1- The applied stress must be uniform and sufficient to provide measurable changes in strain relaxation around cracks without producing plastic deformation in this zone.
3- The crack depth (a) must be small compared to the specimen width (w).
4- In order to avoid the undesired residual stresses produced from cracks, the crack location should be chosen in an adequate distance from the strain gage position.

To fulfill these requirements, tests were carried out on standard specimens having dimensions 6x60 mm. in cross section and 600 mm. in long with crack depth 2,4,6,8 & 12 mm. Fig.(3a). A photographic of the test specimen after making cracks is shown in Fig.(3b).

Test procedure

The specimen was loaded gradually, using the tensile and compression testing machine Fig.(3c) "VEB type ZDM machine", with load 50 KN. Test procedure was repeated six times with different loading values P=50,45,35 KN. for three identical specimens of steel 37.

The loading cycle for one test is shown in Figs.(4a),(4b) for the left and right hand side of the plate edge. Similar curves have been plotted [2] for the blind hole drilling methods.
The strain gage readings were recorded at every incremental load, so that the line oa Figs.(4a),(4b) can be plotted. At constant load of 50 KN., four edge cracks were made by hand saw; two cracks cut at each side Fig.(3). These cracks allow the material to be relaxed with certain value depending on the crack depth.

The cracks were made with equal limited depth "a" and then increased gradually from a= 1 to 12 mm. Before taking the strain gage readings at each crack depth, the material was allowed to be relaxed for a short time (10-15 minutes). Then points b1,b2,...,bn were plotted and line ab was drawn. The specimen was unloaded gradually and strain gage reading was recorded. So that line bc could be drown as in Fig.(4).

Discussion of results

Residual strain can be determined directly from the curves in Figs.(4). It can be seen that, changing in strain relaxation due to making cracks above and below strain gage position during application of the load is a function of both residual and applied stresses. Separation of strain caused by these two stress components can be obtained as shown in Fig.(4) where the strain relaxation is

$$\xi_{rel} = \Delta\xi_1 = \xi_a - \xi_b \qquad (4)$$

and the strain caused by the applied stress is given by

$$\Delta\xi_2 = \xi_{rel} - \xi_c \qquad (5)$$

The value of the strain relaxation is directly proportional to the crack depth, i.e the stress relaxation increased as crack depth increases. The points b1,b2,b3,...,bn in Figs.(4a),(4b) show the strains recorded when the crack depths a=1,2,3,.....,12 mm., so that the strain relaxation at each crack depth is equal to

$$\xi_{rel1} = \xi_a - \xi_{b1}, \quad \xi_{rel2} = \xi_a - \xi_{b2}, \quad \xi_{rel3} = \xi_a - \xi_{b3}$$

Finite element solution provides stress distribution for a plate with and without double edge cracks. Using the superposition method between these two cases, gives the stress relaxation at each element.

Maximum value of the stress relaxation is located at the measuring points between the two cracks on each side of the plate. Figs.(5a),(5b) show change in strain relaxation as a function of crack depth to plate width ratio for tensile loads of P=50,45 KN. These figures present experimental and theoretical results.
It is found that the maximum deviation between theoretical and experimental results is about 20%. Each of these figures show that, the strain relaxation increases as the crack depth increases. This deviation of results may be caused due to the following reasons :

1- In finite element solution, the crack tip is considered to be sharp while it is not that in the actual experimental case.
2- Small misalignment due to cutting four cracks mechanically using hand saw.
3- Symmetry error due to the difference in crack depth.

Strain relaxation of loaded plate in tension with two double edge cracks can be written as

Stain Relaxation = Strain due to the applied load +
Strain due to residual stresses

The effect of crack location (l) to the relaxed strain has been investigated theoretically and verified experimentally. The results are shown in Fig.(6) in which it is clear that the strain relaxation decreases as the crack location increases, where (l) is the distance between the center line of the crack and the center line of the strain gage.

Engineering charts for estimating actual applied stress

Strain relaxation, crack depth to plate width ratio (a/w) and actual applied load is presented in engineering chart Fig.(7) for steel 37. Similar charts for Aluminum and Brass are presented in Figs.(8),(9). These charts enable the inspector to predict actual applied load in the loaded plate.

Conclusion

The strain relaxation has been investigated for two double edge cracks in a plate subjected to a uniform tensile stress using finite element method. The experimental verification has been done, where strain relaxation was measured by strain gages. A deviation of analytical and experimental results is around 20%.
Residual stresses was determined in the plate by making two double edge cracks as shown in Figs.(3),(4). This method can also be used to determine residual stresses for unloaded member.

Making crack by hand saw and using single strain gage is easier than drilling a fine hole in the blind hole method with using special rosette strain gages.
The actual applied load can be obtained with a reasonable accuracy by using engineering charts.

Acknowledgments

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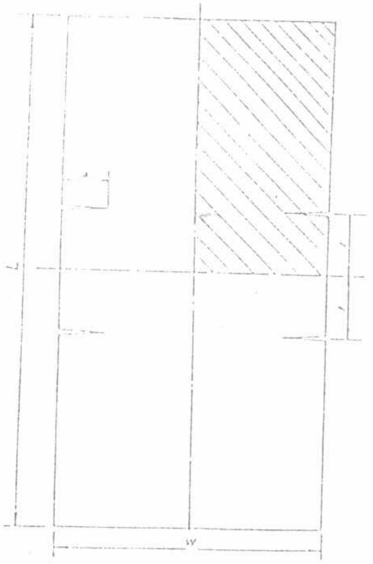


Fig. 1 A test specimen with four cracks

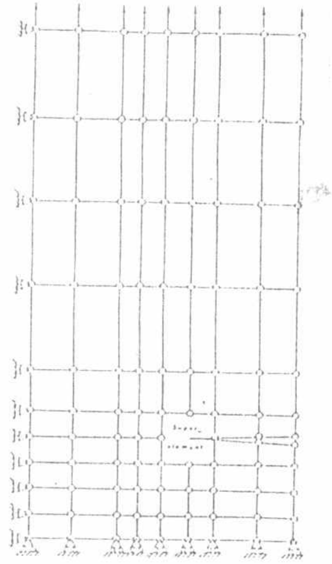


Fig. 2 Mesh for one quarter specimen

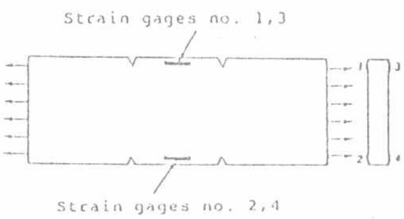


Fig. 3a Specimen test with two double edge cracks

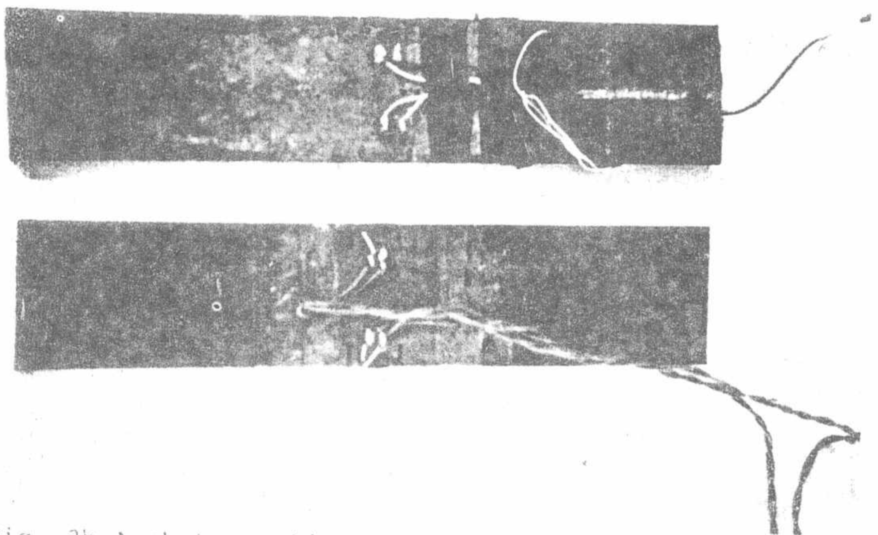


Fig. 3b A photographic of the test specimen after making cracks

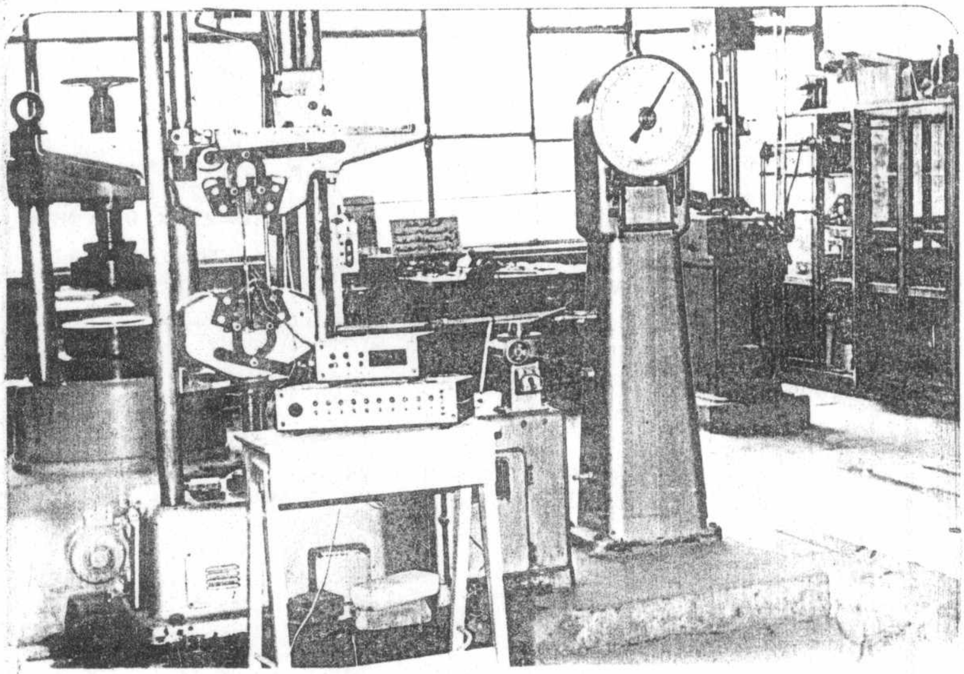


Fig. 3c The tensile and compression testing machine

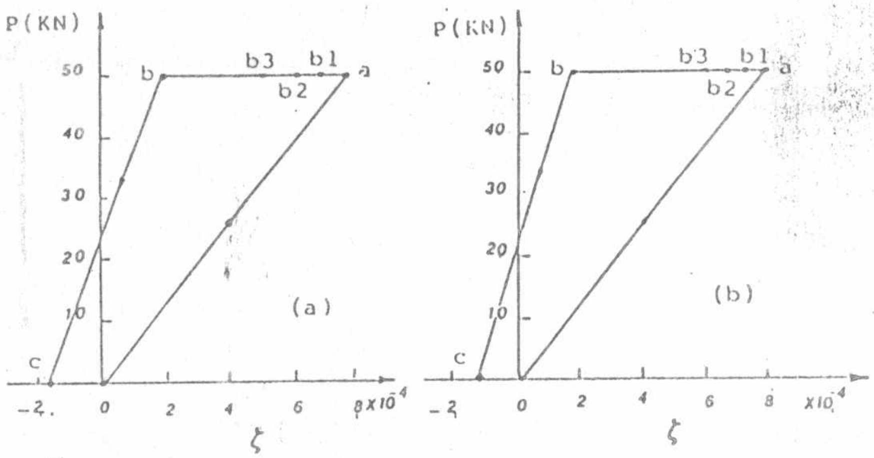
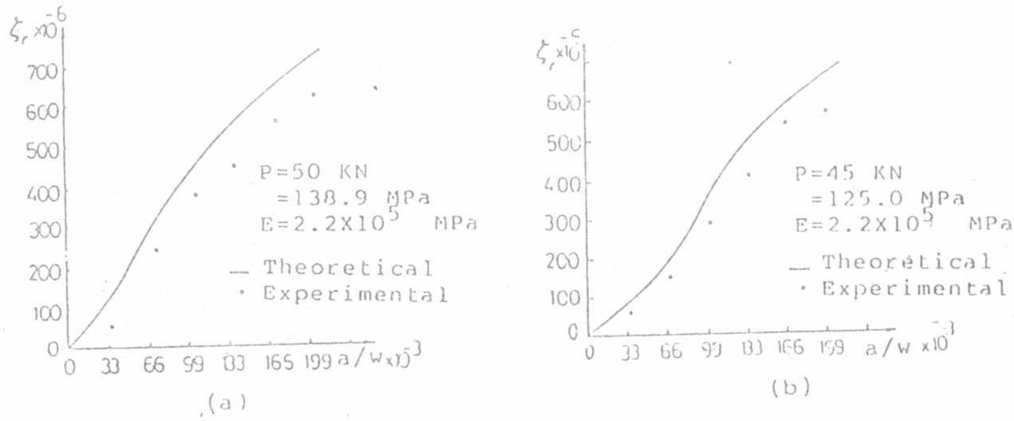


Fig. 4 Relationship between strain relaxation & applied load, (a) Left hand side (b) reight hand side



Figs. 5 Relationship between relaxed strain & crack depth to plate width ratio for the two cases a & b

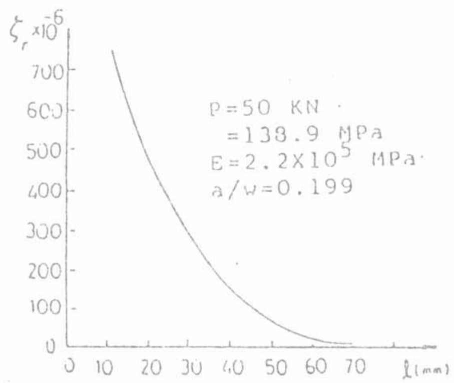


Fig. 6 Relationship between relaxed strain & crack location

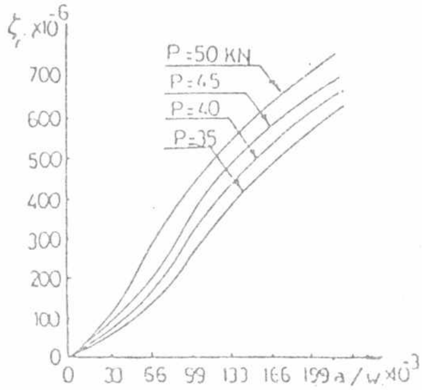


Fig. 7 Engineering chart for Steel

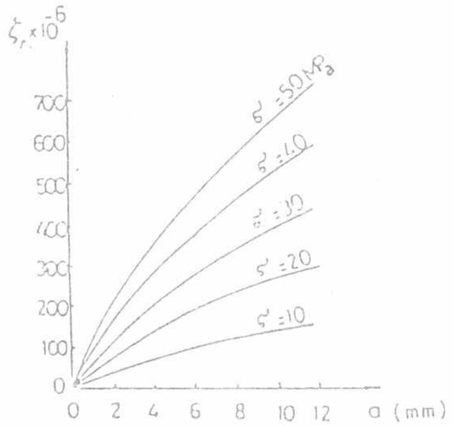


Fig. 8 Engineering chart for Aluminum

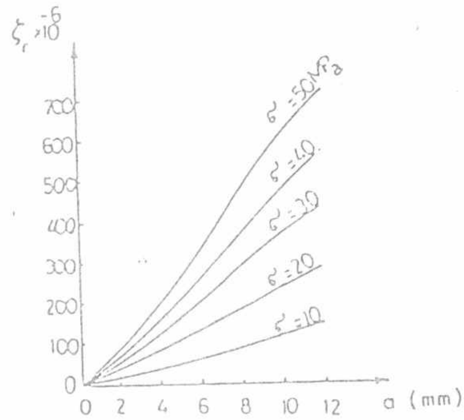


Fig. 9 Engineering chart for Brass