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STRESS AND STRENGTH ANALYSIS OF ROCKET NOZZLE BY THE FINITE ELEMENT METHOD

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ABSTRACT

Rocket motor design is based on the use of maximum capability of each component. To fully realize this goal in the case of rocket nozzle, a more precise structural analysis has to be performed. The finite element method provides one of the powerful and commonly used techniques for such type of analysis. In this work, the stress analysis on rocket nozzle is investigated. A finite element computer program was implemented for stress analysis of axisymmetric structures. The program would assist in thermo-structural verification of rocket nozzles made of composite materials with non-linear thermal and mechanical properties.

For a case study on a submerged nozzle, exploration of critical loading zones was made possible, by the analysis of results from running this program. The efficiency of the program had been proved by the good agreement with results obtained from theory and other validated codes.

1. INTRODUCTION

One of the important problems in nozzle design is the determination of stress distribution and structural integrity under thermo- mechanical loading conditions either in steady or transient state. As generally, the coupling between the state of stress and the temperature distribution is weak, the later is solved independently and then used as input to stress analysis. The thermo-structural analysis in axisymmetric solids under axisymmetric loading is of practical interest in the industry of aerospace. For this type of problem the circumferential strain is uniquely determined with radial displacement.

Currently used structural analysis technique is based on the finite element method. The stress analysis of axisymmetric solids by the finite element method was tried by many investigators [1-4]. It became so accurate and so sophisticated that optimization of a design is limited only by the available material property data and time. Analysis procedure accuracy can lead to a minimum number of tests required for the qualification of the

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given nozzle design and consequently a minimum cost. An accuracy enhancement may be obtained by increasing either the number of nodes of element or the number of degrees of freedom of each node. The use of cubic polynomial for radial and axial displacements leads to a good estimation of stresses by using small number of elements which means a reduction of effort done in discretization. In this work only nodes at the vertices of triangle are considered, but every node has six degrees of freedom representing nodal displacements and displacement derivatives.

Since composite materials give rise to important weight savings, they have been used in nozzle structure for quite some time [5]. In case of composite structure, the nozzle can be divided into components of different composite materials according to each component requirements. For example the critical throat area is subjected to the most severe environment. Therefore, less erosive versions of composite materials are required in the inlet and throat areas than those required in the exit cone. The general treatment of composites requires the use of anisotropic material models. As a first step we consider a simple axisymmetric case which is denoted as stratified or transversely isotropic.

In this work, A computer program is implemented in FORTRAN 77, which is called "FESAX" (Finite Element Stress analysis of AXi-symmetric structures). This computer program has the following capabilities:

(1) Boundary conditions of zero and non-zero displacements.
(2) Concentrated and distributed (pressure) loads.
(3) Initial strains representing the thermal loads.
(4) Centrifugal and gravititional loads.
(5) Stratified material with transversely isotropic properties.
(6) Variation of material properties with temperature.
(7) Structure contains more than one material.

2. STRESS ANALYSIS

The element stiffness matrix and nodal force vector are calculated using the displacement method according to the procedure indicated by Zienkiewiez [6]. Figure 1 shows the triangular element of an axisymmetric solid with its three nodes i,j,k. The nodal parameters of the needed element are represented by a vector column. The displacement at the interior of the element is defined by the following complete cubic polynomial.

$$u = a_1 + a_2.r + a_3.z + a_4.r^2 + a_5.rz + a_6.z^2 + a_7.r^3 + a_8.r^2z + a_9.rz^2 + a_{10}.z^3$$

$$v = a_{11} + a_{12}.r + a_{13}.z + a_{14}.r^2 + a_{15}.rz + a_{16}.z^2 + a_{17}.r^3 + a_{18}.r^2z + a_{19}.rz^2 + a_{20}.z^3 \qquad (1)$$

Then, the element displacement is defined as,

$$[\ \delta\ ]^T = [\ \delta_i \quad \delta_j \quad \delta_k] \tag{2}$$

The nodal displacements of node i are defined as:

$$[\ \delta_i]^T = [\ u_i \quad u_{ri} \quad u_{zi} \quad v_i \quad v_{ri} \quad v_{zi}] \tag{3}$$

where u_r, u_z and v_r, v_z are the first derivative of u,v w.r.t. the radial and axial coordinates.

By adding the displacements of the element centroid (u_c, v_c) we get system of 20 equation in 20 unknowns of arbitrary constants. Arranging and solving we get:

$$[\ \delta\ ]_e = [\ A\ ]_e [\alpha]_e$$

$$[\alpha]_e = [\ A\ ]_e^{-1} [\ \delta\ ]_e = [\ B\ ]_e\ [\ \delta\ ]_e \tag{4}$$

The usual technique of construction of element stiffness matrix is to express the strain energy of the e^{th} element in quadratic matrix form:

$$U_e = 0.5 \int_{v_e} [\ \epsilon\ ]_e^T\ [\sigma]_e\ dv_e \tag{5}$$

where:

$$[\ \epsilon\ ] = [\ \epsilon_z \quad \epsilon_r \quad \epsilon_\theta \quad \gamma_{rz}]^T$$

$$[\sigma] = [\ \sigma_z \quad \sigma_r \quad \sigma_\theta \quad \tau_{rz}]^T$$

for stratified material (transversely isotropic)(Fig. 2):

$$\epsilon_z = \frac{\sigma_z}{E_2} - \frac{\nu_2 \sigma_r}{E_2} - \frac{\nu_2 \sigma_\theta}{E_2} \qquad \epsilon_r = \frac{-\nu_2 \sigma_z}{E_2} + \frac{\sigma_r}{E_1} - \frac{\nu_1 \sigma_\theta}{E_1}$$

$$\epsilon_\theta = \frac{-\nu_2 \sigma_z}{E_2} - \frac{\nu_1 \sigma_r}{E_1} + \frac{\sigma_\theta}{E_1} \qquad \gamma_{rz} = \tau_{rz}/G_2$$

Let: $n = E_1/E_2$ and $m = G_2/E_2$

Hence, we get the stress-strain relation as:

$$[\sigma] = [D]\ [\epsilon\ ] \tag{6}$$

where: [D] is the elasticity matrix,

$$[D] = \frac{E_2}{(1+\nu_1)(1-\nu_1-2n\nu_2^2)} \begin{bmatrix} (1-\nu_1^2) & n\nu_2(1+\nu_1) & n\nu_2(1+\nu_1) & 0 \\ & n(1-n\nu_2^2) & n(\nu_1+n\nu_2^2) & 0 \\ & & n(1-n\nu_2^2) & 0 \\ \text{SYMMETRIC} & & & M \end{bmatrix}$$

and $M = m(1+\nu_1)(1-\nu_1-2n\nu_2^2)$

By substituting equation (6) in equation (5) we get

$$U_e = 0.5 \int_{v_e} [\epsilon]_e^T [D]_e [\epsilon]_e \, dv_e \tag{7}$$

Applying the strain displacement relations

$$[\epsilon] = [S][\alpha] = [S][B][\delta] \tag{8}$$

where [S] depends on the nodal coordinates (r,z), we get,

$$U_e = 0.5 [\delta]^T [K_e][\delta] \tag{9}$$

where the elment stiffness matrix,

$$[K] = \int_{v_e} [B]^T [S]^T [D] [S] [B] \, dv_e \tag{10}$$

The equivalent nodal forces $[F_e]$ are calculated by the principle of virtual work [7]. In this paper the axisymmetric structure is studied when loaded by axisymmetric loads which is one or more of the following loads: concentrated loads, distributed loads (pressure), thermal loads and body loads (centrifugal and/or gravitational). Since the element stiffness matrix $[K_e]$ and the force vector $[F_e]$ are obtained, the global matrix and vector can be evaluated as:

$$[K] = \sum_{e=1}^{N} [K_e] \quad ; \quad [F] = \sum_{e=1}^{N} [F_e]$$

Then, the equation of equilibrium takes the form

$$[K][\delta] = [F] \tag{11}$$

The present triangular element poses no problem for calculation of stresses at nodes. As indicated before, one of element advantages is that all necessary informations are available for every node under form of nodal parameters to have a direct evaluation of the components of nodal strains and accordingly the stresses at nodes. Once the parameters of displacements are resolved, equations (8) and (6) will give directly the strains and the stresses at each node of the model.

3. STRENGTH ANALYSIS

Having performed the stress analysis by finite element method, the axial, radial, circumferential and shear stresses are determined at each node of the finite element model. To determine the status, and margins of safety (or integrity number), for the nozzle structure it is required to use one of the known failure criteria. For isotropic materials subjected to simple stress states such as tension, compression and shear, the structural integrity can be determined by comparing the applied stress with some fraction of yield or ultimate strength of material. For more complex stress states and orthotropic materials which are encountered in rocket motor nozzle, it is not physically possible to conduct tests which represents every complex stress state. It is, therefore, necessary to have a failure criterion to predict the strength and structural integrity from a limited amount of principal material direction strength data.

A comprehensive survey of failure theories for both isotropic and anisotropic materials is given by Sandhu [8]. It would be convenient to be able to apply one of the existing theories to nozzle analysis. There are many criteria which indicate the integrity number such as Hill's criterion used for isotropic materials. This criterion was modified by Azzi-Tsai [9] to be used with the stratified materials. The form of the structure integrity is:

$$\left(\frac{\sigma_z}{F_z}\right)^2 + \left(\frac{\sigma_r}{F_r}\right)^2 + \left(\frac{\sigma_\theta}{F_\theta}\right)^2 - \left[\left(\frac{1}{F_z^2} + \frac{1}{F_r^2} - \frac{1}{F_\theta^2}\right)\sigma_z\sigma_r + \left(\frac{1}{F_z^2} + \frac{1}{F_\theta^2} - \frac{1}{F_r^2}\right)\sigma_z\sigma_\theta + \left(\frac{1}{F_r^2} + \frac{1}{F_\theta^2} - \frac{1}{F_z^2}\right)\sigma_r\sigma_\theta + \left(\frac{\tau_{rz}}{F_{rz}}\right)\right] \leqslant 1$$

where:

σ_z, σ_r, σ_θ and τ_{rz} are the nodal applied stresses

F_z, F_r, F_θ and F_{rz} are the nodal allowable stresses

From the nodal temperatures (output of thermal analysis), nodal stresses (output of stress analysis) and material mechanical properties versus temperature, the structure integrity can be calculated and the structure critical zones (zones containing nodes with integrity number exceeding the value of one) can be determined at any time.

4. SAMPLE PROBLEMS

4.1. Stress Analysis of Stratified Axisymmetric cylinder Under Internal Pressure.

This example is chosen to check the efficiency of the program in dealing with stratified material properties. The model of this example is shown in Fig. 3.a. The material properties are:

$E_r = 1554$, $E_z = 155.4$, $G_{rz} = 140$

$\nu_{rz} = 0.05$, $\nu_{zr} = 0.005$, $p = 1$

The results of radial and circuferential stresses are given in Fig. 3.b and 3.c. Their agreement with theoretical results [2,10] is excellent inspite of using simple FE model.

4.2. Stress Analysis of Hollow Cylinder Under Thermal Load.

The hollow cylinder (Fig. 4) heated by convection on the inetrnal surface is solved thermally [7]. Now, we shall consider the transient thermal stress analysis at different increments of time. The data required for this calculation based on the finite element method consists of structure idealization, material properties, thermal profile at each time step.

The thermal stress analysis of this model is done by using "FESAX" program to determine the nodal stresses at each time step. The nodal stress (Von-Misses stress) history represented in Fig. 5 indicate that the maximum stress is at the first time step, where sharp thermal gradient are expected. From the comparison of the nodal stresses, it is found that the circumferential component of stress is the greatest one. The circumferential isostresses at the first time step are presented in Fig. 6.

5. APPLICATION ON ROCKET NOZZLE.

The selected nozzle is of submerged configuration, which is heated along the back side of the nozzle as well as along the internal flow contour. The nozzle material is carbon/carbon composite. The main dimensions of the nozzle is shown in Fig. 7. These were calculated based on the specified motor pressure (P = 137.94 bar), mass flow rate ($\dot{m}$ = 45.37 kg/sec), and the gas temperature (T = 3200 °C). The nozzle is discretized into finite elements. The data required for this calculation consists of: nozzle idealization, nodal temperature, and material properties versus temperatures. The operating time of this nozzle is about 20 seconds.

Results of stress and strength analyses at one second after propellant ignition indicate two zones of interest (zones having nodes of maximum integrity number). Figure 8 indicates the location of these zones on the nozzle structure. Zone No. 1 is stressed in compressive axial and circumferential modes. The axial stresses are caused by the high pressure loading on the nozzle backside which causes it to bend, while the circumferential stresses result from the pressure load as well as the effects of the end support. Zone No. 2 is stressed in a compressive circumferential mode. In the throat region, compressive axial stresses are also prevalent. Both are caused by the high thermal gradient which causes the weakened hot material to expand and the cooler stronger backup material to act as restraint.

The circumferential isostresses at one sec. are illustrated in Fig. 9, as an example of stress distribution. As a quantitative description of the contribution of different loading conditions in the total stresses, we give in Table (1) the circumferential component (which is always the highest component) at two location A (convergent part) and B (backside) due to 4 different loading conditions: thermal, pressure, axial acceleration (10 g), and combined loading. Table (1) shows that the thermal stresses are the most important component, followed by the stress caused by internal pressure and then that due to axial acceleration.

Table (1) Contribution of different loadings

	Circumferential stress due to:						
	Thermal	%	Pressure	%	A. accel.	%	Combined
A	78.885	69.30	34.358	30.00	0.797	0.70	113.834
B	90.911	81.55	19.24	17.26	1.325	1.19	111.484

6. CONCLUSION

From this paper, one can get the following conclusions:

- The sample problem of simple ring with stratified material properties has shown the increased accuracy due to the use of higher order element.
- The stress analysis of submerged rocket nozzle has indicated that there were two critical loading conditions: the first is sharp temperature gradients at the begining of combustion period (1 second) and the second is high temperatures attained with loss of mechanical properties at the end of combustion period (20 seconds).
- The thermal stresses are the most important component followed by the stresses caused by internal pressure and then that due to axial load.
- The strength analysis indicated that the critical zones should be redesigned using different types of materials with better capabilities in order to carry the working thermal and mechanical loads.
- Finally, the implemented FESAX program is capable of dealing with real problems of stress and strength analysis of rocket nozzles using the finite element method.

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NUMENCLATURE

a ..... polynomial coefficient.
E ..... Young's modulus.
F ..... nodal force.
G ..... shear modulus.
K ..... stiffness.
p ..... pressure.
r ..... radial coordinate.
T ..... temperature.
t ..... time.
U ..... strain energy.
u ..... radial displacement.
v ..... axial displacement.
z ..... axial coordinate.
γ ..... shear strain.
δ ..... element displacement.
ϵ ..... strain.
ρ ..... density.
σ ..... stress.
τ ..... shear stress.
ν ..... Poisson's ratio.

Subscripts

1 ..... radial
2 ..... axial
c ..... centroid
e ..... element number e
i ..... node i.
j ..... node j.
k ..... node k.
o ..... initial
r ..... radial component
z ..... axial component
rz ..... shear component
θ ..... circumferential

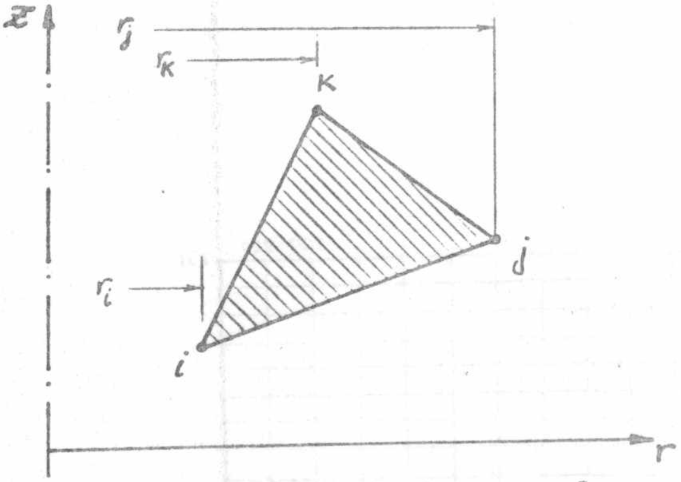


Fig. 1 Triangular element of axisymmetric solid.

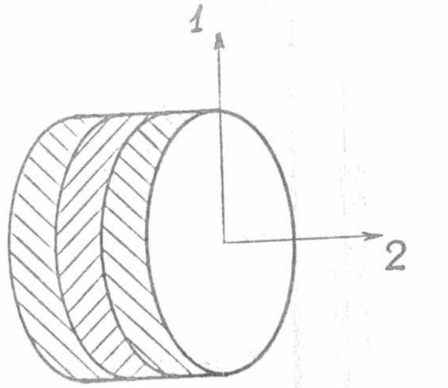
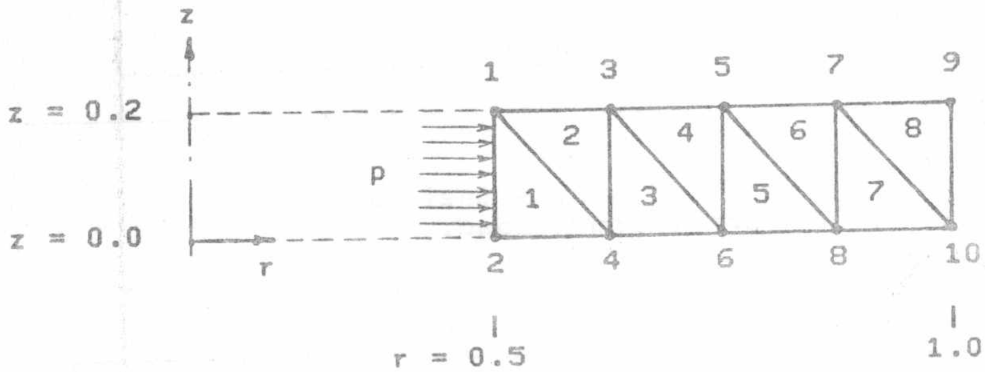
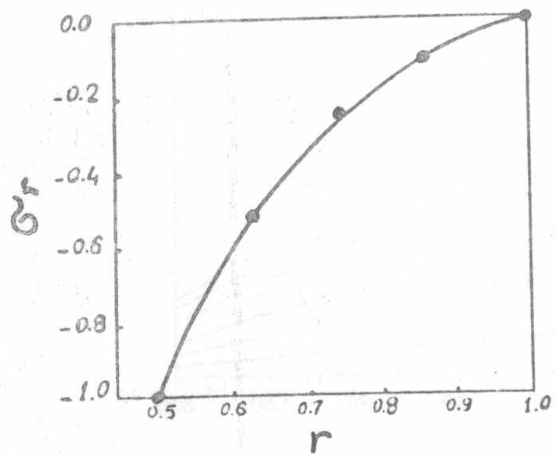


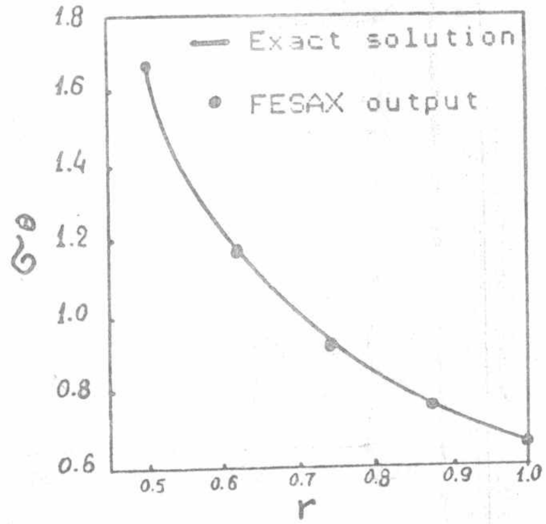
Fig. 2 Stratified material configuration.



a) Model of sample problem



b) radial stress



c) circumferential stress

Fig. 3 Stratified axisymmetric cylinder under internal pressure

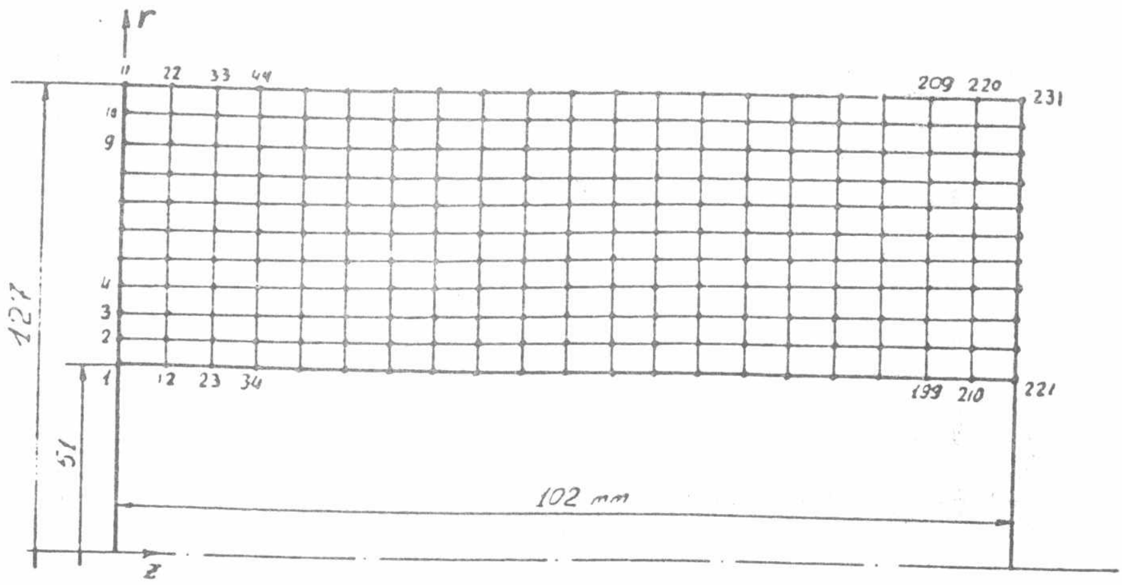


Fig. 4 Model of stress analysis under thermsl load.

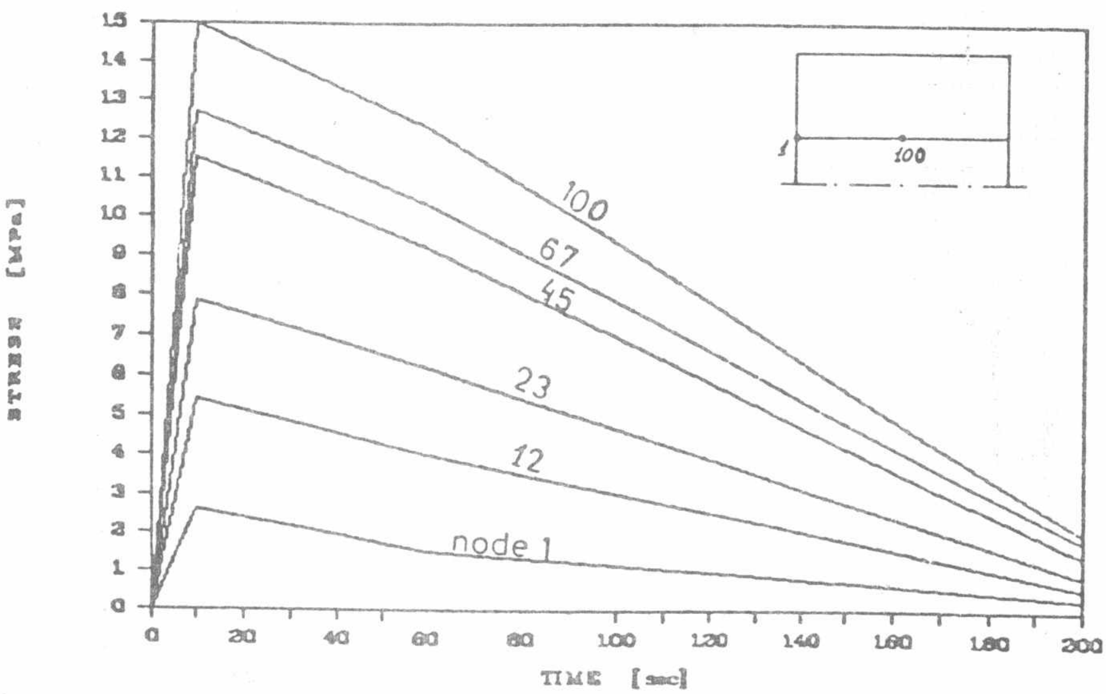
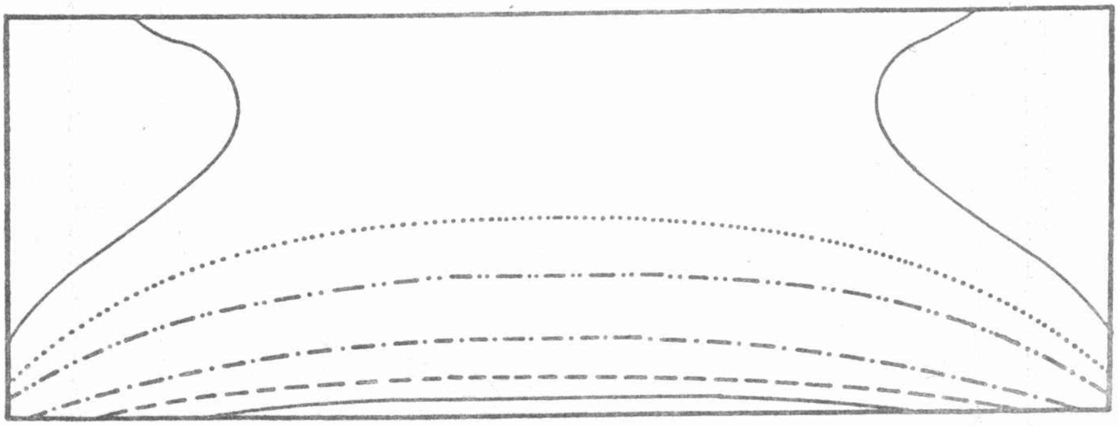


Fig. 5 Nodal Von-Misses stress -vs- time.



- 19.25 MPa.
- 13.50 MPa.
- 7.50 MPa.
- 1.50 MPa.
1.23 MPa.
4.16 MPa.

Fig. 6 Circumferential isostress at first time step.

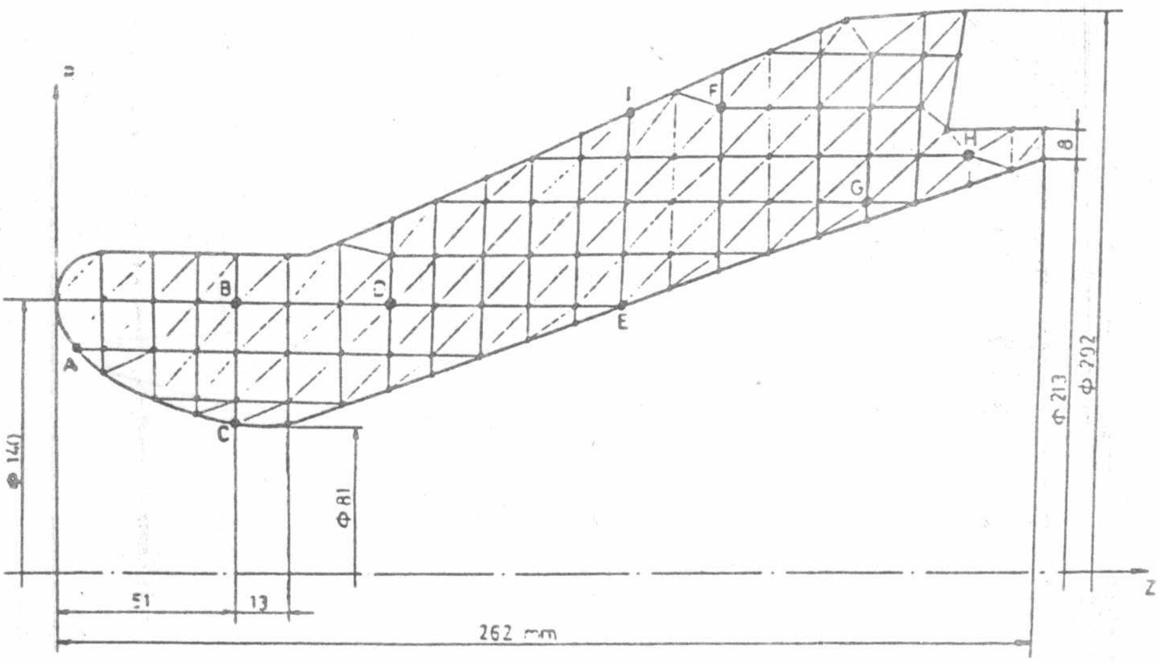


Fig. 7 Submergid nozzle configuration and dimensions.

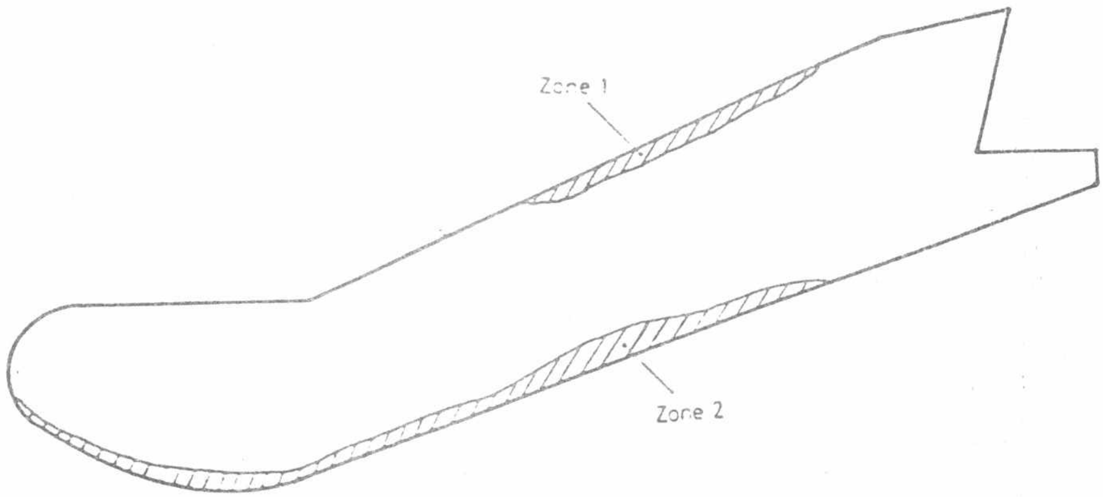


Fig. 8 Critical zones after 1 sec. from ignition

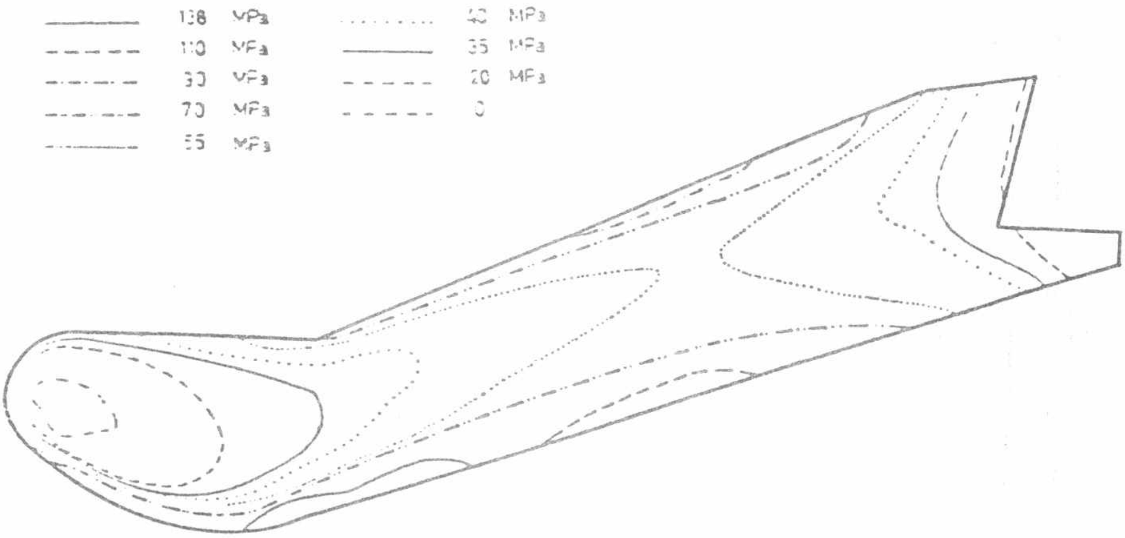


Fig. 9 Circumferential isostresses