MILITARY TECHNICAL COLLEGE

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ROBUSTNESS OF AN ADAPTIVE POLE PLACEMENT CONTROL ALGORITHM IN THE PRESENCE OF MODELING ERROR AND TIME VARIATION OF PARAMETERS

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ABSTRACT

This paper presents an indirect adaptive robust control scheme for time varying plant in the presence of the modeling error. The robustness is achieved by using a normalizing identification error signal in combination with a dead zone and a projection algorithm in the adaptive law. This modified algorithm is used to estimate the controller parameters so that the closed loop plant will track a certain desired performance closely in some sense and the plant input and output will remain bounded for all time.

The proposed adaptive control algorithm is used to control the behavior of an aircraft system in which a good behavior is obtained. A comparison between the ordinary projection algorithm and the proposed adaptive algorithm in controlling the aircraft system is carried out. The simulation results indicate the effectiveness of the proposed adaptive control algorithm.

INTRODUCTION

Most stability results for adaptive control systems are based on the assumption that the model used in the control structure is an accurate representation of the process. However, most real processes are high order and hence, in general, an approximate model is used in practice. In [1] it was demonstrated that straightforward application of the stable algorithms found in the literature may lead to stability problems when unmodelled plant dynamics or modeling error, due to this model-process order mismatch, are present.

In [2,3,4,5], an adaptive pole placement combined with an adaptive law is used to control a certain plant with bounded disturbance. The main problem is that when the disturbance becomes unbounded this will lead to stability problem.

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In the robustness problem, the distrubance is internally generated and thus depends on the actual plant input and output signals. In particular, if the adaptive control system was unstable and the plant input and output signals were to grow without bound, then the disturbance would also grow without bound. In other words, the robust stability problem becomes the problem of an internal, signal-dependent, and thus potentially unbounded disturbance.

This paper presents an adaptive pole placement control algorithm to control the time varying system in the presence of unmodeled plant dynamics. To achieve the robust stability, a normalized identification error signal is combined with a dead zone in a modified version of projection algorithm. This modified algorithm is used to estimate the controller parameters so that the closed loop system will behave as the desired performance and the system input and output remain bounded for all time. To show the effectiveness of the present work, the proposed adaptive control algorithm is used successfully to control the behavior of the time varying aircraft system in the presence of modeling error.

A comparison between the proposed adaptive algorithm and the ordinary one in controlling the aircraft system is carried out. From the obtained results, the proposed adaptive control algorithm is superior than the ordinary one.

SYSTEM MODELING ASSUMPTIONS AND PROBLEM FORMULATION

It is assumed that the system to be controlled is discrete single input-sigle output time varying associated with a modeling error and that can be represented in the form:

$$A(q^{-1})\ Z(t) = U(t)$$

$$Y(t) = q^{-d}\ B(q^{-1})\ Z(t) + \eta(t) \qquad (1)$$

Where U(t), Y(t) and $\eta(t)$ are input, output and modeling error signals respectively. $A(q^{-1})$ and $B(q^{-1})$ are defined as:

$$A(q^{-1}) = 1 + a_1(t)\ q^{-1} + \ldots + a_{na}(t)\ q^{-na} \qquad (2)$$

$$B(q^{-1}) = b_o(t) + b_1(t)\ q^{-1} + \ldots + b_{nb}\ (t)\ q^{-nb} \qquad (3)$$

q^{-1} denotes the unit delay operator.

Associated with these polynomials define the modeling error $\eta(t)$ as:

$$\eta(t) = A(q^{-1})\ Y(t) - q^{-d}\ B(q^{-1})\ U(t) \qquad (4)$$

Further, we choose $0 < \sigma < 1$, and define the additive modeling error as first order dynamic of the plant [6].

$$E(t) = \sigma\ E(t-1) + |U(t-1)| + |Y(t-1)| \qquad (5)$$

The modeling error is said to be relatively bounded if there exist a finite $\mu > 0$ and $E(0) > 0$ such that:

$$\eta(t) \le \mu E(t) \tag{6}$$

It is assumed that:

A1 : The time delay d is known.

A2 : The relative degree n of the plant is known (n=na-nb).

A3 : $A(q^{-1})$ and $B(q^{-1})$ are coprime and assumed to be arbitrary.

A4 : $a_i(t)$, i=1, ... , na and $b_j(t)$, j=1, ... , nb are time varying parameters.

Since the zeros of $B(q^{-1})$ are arbitrary, the plant may be nonminimum phase.

From the above description, the problem is to devise an adaptive controller for the plant so that, inspite of the modeling error $\eta(t)$, the adaptive control system is globally stable, and the output of the plant follows the bounded desired reference signal closely in some sense.

ADAPTIVE CONTROLLER STRUCTURE

Consider the following feedback control system associated with integral action such that:

$$F(q^{-1})\ U(t) = R(q^{-1})\ \varepsilon(t) \tag{7}$$

with

$$F(q^{-1}) = (1 - q^{-1})\ (\ 1 + f_1 q^{-1} + \ldots + f_{nf}\ q^{-nf}) \tag{8}$$

$$R(q^{-1}) = 1 + r_o + r_1 q^{-1} + \ldots + r_{nr}\ q^{-nr} \tag{9}$$

and

$$\varepsilon(t) = Y_r(t) - Y(t) \tag{10}$$

where $\varepsilon(t)$ is the tracking error, $Y(t)$ is the system output, and $y_r(t)$ is the desired bounded set point sequence to be followed.

Applying the control law (7) to the system (1) results the following closed loop system:

$$[A(q^{-1})\ F(q^{-1}) + q^{-d}\ B(q^{-1})\ R(q^{-1})\ ]\ Z(t) = R(q^{-1})\ Y_r(t)$$
$$Y(t) = q^{-d}\ B(q^{-1})\ Z(t) + \overset{*}{\eta}(t) \tag{11}$$

Assume that $C(q^{-1})$ is a stable polynomial of degree nc whose zeros represent the desired closed loop location. From equation (11) it can be verified that:

$$A(q^{-1})\ F(q^{-1}) + q^{-d}\ B(q^{-1})\ R(q^{-1}) = C(q^{-1}) \tag{12}$$

where:

$$C(q^{-1}) = 1 + c_1 q^{-1} + \ldots + C_{nc} q^{-nc} \tag{13}$$

$$\overset{*}{\eta} = \eta\ F(q^{-1})\ A(q^{-1}) / C(q^{-1})$$
$$nc = na + nb$$
$$nf = nb + d - 1 \tag{14}$$
$$nr = na$$

MODIFIED PROJECTION ALGORITHM:

In this section, modifications are introduced to the projection algorithm [3]. These modifications include a normalized identification error combined with a dead zone in which the dead zone acts on a suitably normalized relative identification error.

The system which is represented by the equation (1) can be put in more compact form as:

$$Y(t) = \theta(t-1)^T \phi(t-1) + \eta(t) \tag{15}$$

where $\theta(t)$ contains the actual system parameters and defined as:

$$\theta(t-1) = (-a_1(t),\ldots, a_{na}(t)\ b_1(t),\ldots, b_{nb}(t))^T \tag{16}$$

$$\phi(t-1) = (Y(t-1),\ldots, Y(t-na)\ U(t-d),\ldots,U(t-d-nb))^T \tag{17}$$

If the plant parameters are assumed to be unknown and time invariant, the following projection algorithm [1] is used to estimate the plant parameters $\hat{\theta}(t)$.

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \alpha\, \phi(t-1)\, e(t)/(1 + \phi(t-1)^T \phi(t-1)) \tag{18}$$

where e(t) is defined as:

$$e(t) = Y(t) - \hat{\theta}(t-1)^T \phi(t-1) \tag{19}$$

In order to deal with time varying plant and additve modeling error, the algorithm (18 -19) should contain a normalized identification error such that the relative modeling error signal is within the dead zone i.e. the modeling error becomes bounded.

Defining a normalizing factor N(t) and a relative error signal (normalized error signal) $E_1(t)$ as

$$N(t) = \gamma_o + E(t) \quad , \quad \gamma_o > 0 \tag{20}$$

$$E_1(t) = e(t) \,/\, N(t) \tag{21}$$

and the adaptive law becomes

$$\hat{\theta}(t) = \hat{\theta}(t-1) + [\alpha\, \phi(t-1)\, N(t-1)\, D(E_1(t-1)]/r(t-1) \tag{22}$$

$$r(t) = r(t-1) + \phi(t)^T\phi(t) \tag{23}$$

where

$$\alpha > 0 \quad , \quad r(0) > 0 \tag{24}$$

$D(E_1(t))$ is a function defined by (see also fig. (1)).

$$D(E_1(t)) = \begin{cases} 0 & \text{if } |E_1(t)| \le d_o \\ K(E_1(t) - d_o) & \text{if } E_1(t) > d_o \\ K(d_o - E_1(t)) & \text{if } E_1(t) < -d_o \end{cases} \tag{25}$$

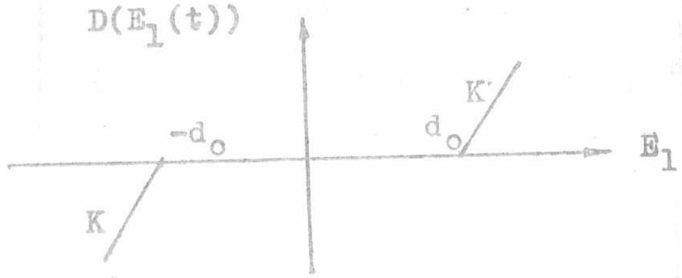

Fig.(1) The dead zone function D(.) of the normalized identification error $E_1(t)$

In the algorithm (22) the identification error e(t) is replaced by N(t) $D(E_1(t))$, where $D(E_1(t))$ is the dead zone function of the normalized identification error and d_o is the size of the dead zone. The slope K and the dead zone size d_o can be obtained through studying their effect on the mean square of the tracking control error $\varepsilon(t)$ and the accurate values of K and d_o are chosen based on achieving a least value of $\varepsilon(t)$.

After estimating the time varying parameters of the unknown system, they put in the following form as:

$$\hat{\theta}(t) = (-\hat{a}_1(t) \; ... \; -\hat{a}_{na}(t) \; \hat{b}_o(t) \; ... \; \hat{b}_{nb}(t))^T \quad (26)$$

$$\hat{A}(t,q^{-1}) = 1 + \hat{a}_1(t)\, q^{-1} + ... + \hat{a}_{na}(t)\, q^{-na} \quad (27)$$

$$\hat{B}(t,q^{-1}) = \hat{b}_o(t) + \hat{b}_1(t)\, q^{-1} + ... + \hat{b}_{nb}(t)\, q^{-nb} \quad (28)$$

The control signal is calculated as:

$$\hat{A}(t,q^{-1})\, \hat{F}(t,q^{-1}) + q^{-d}\, \hat{B}(t,q^{-1})\, \hat{R}(t,q^{-1}) = C(t,q^{-1}) \quad (29)$$

$$\hat{F}(t,q^{-1})\, U(t) = \hat{R}(t,q^{-1})\, \varepsilon(t) \quad (30)$$

The closed loop adaptive control system thus established comprises the plant (1), the controller (29-30) and the adaptive law (20-25).

AIRCRAFT TIME VARYING SYSTEM

The main properties of the above design algorithm will be illustrated by computer simulation results. One of the models used for the experimental analysis under study represents the longitudinal dynamics of an aircraft time varying system [7]. It is given by the following equation:

$$\dot{X}(t) = A(\beta(t))\, X(t) + B(t)\, U(t) + \eta(t) \quad (31)$$

$$, \quad X(0) = 0$$

and

$$A(\beta(t)) = \begin{vmatrix} \beta_1(t) & -.4085 & 0 \\ 1 & \beta_2(t) & 0 \\ 1 & 0 & 0 \end{vmatrix} \quad (32)$$

$$B(t) = \begin{vmatrix} 2.85700 \\ .061581 \\ 0.0000 \end{vmatrix} \quad (33)$$

The state variable $X_1(t)$ is taken as an output of the aircraft system. A two dimensional vector of unknown parameters $\beta(t)$ is assumed to be time varying obeying the relations.

$$\beta_1(t) = \begin{cases} -0.2778 & \text{for} \quad 0 \le t < 8 \\ -0.06444(t-3.689) & \text{for} \quad 8 \le t < 13 \\ -0.6 & \text{for} \quad 13 \le t \le 22 \end{cases} \quad (34)$$

$$\beta_2(t) = \begin{cases} -0.4075 & \text{for} \quad 0 \le t < 10.5 \\ -0.065(t-4.23) & \text{for} \quad 10.5 \le t < 15 \\ -0.7 & \text{for} \quad 15 \le t \le 22 \end{cases} \quad (35)$$

This system is discretized at sampling T = 0.05 second, and by applying the proposed adaptive control algorithm, the following simulation results will obtain.

SIMULATION RESULTS

The aircraft system is represented by a second order difference equation as:

$$Y(t) = - a_1(t)\ Y(t-1) - a_2(t)\ Y(t-2) + q^{-d} [\ b_1(t)\ U(t) + b_2(t)\ U(t-1)\ ] \quad (36)$$

The open loop time varying parameters of equations (34-35) are plotted in fig.(2) and the time delay d is chosen equal to 1. At the begining of operation, the coefficients of d_o, K, r(0) and α are found to be 0.2, 1., 65 and 3.5 respectively. These obtained values can be chosen through studying their effects on the mean square control error and their reasonable values correspond to the least value of the mean square control error.

The values of γ_o, σ and μ are chosen as 0.1, 0.1, and 0.2 respectively. At the starting, the adaptive law is initialized by

$\hat{\theta}^T(0)$ = [0.0 0.0 0.6 0.6]. The resulted closed loop control system through the proposed adaptive control algorithm behaves as shown in fig.(3) for the desired tracking sequence, fig.(4) for control signal, fig.(5) for the auxiliary parameters, fig.(6) for controller parameters and fig.(7) for additive modeling error. The tracking error between the aircraft system and the desired performance is shown in fig.(8).

From the obtained results, a robust behavior of the closed loop aircraft system is obtained inspite of modeling error and time variation of process parameters. By comparing the results obtained by the proposed modified adaptive control algorithm and the control algorithm based on the ordinary projection type algorithm, the proposed algorithm is superior than the others. This is clearly demonstrated in fig.(9) which shows the instability of ordinary one especially when dealing with time varying systems.

CONCLUSION

This paper presents an adaptive robust control algorithm for controlling time varying plants in the presence of plant uncertainties. The proposed agorithm ensures the robust stability of the resulting closed loop adaptive control system. The robustness is achieved by using a normalized identification error combined with a dead zone and a projection type algorithm in the adaptive law. Some factors such as d_o, α and r(0) affect on the proposed adaptive algorithm and the best combination of these factors has been found based on achieving adequate performance of the closed loop plant.

To show the effectiveness of the proposed algorithm, it is used successfully to control the performance of the time varying aircraft system in the presence of disturbance. A comparison between the modified adaptive algorithm and the ordinary projection type in controlling the aircraft system is carried out. From the obtained results, the behavior of the proposed adaptive control algorithm is robust.

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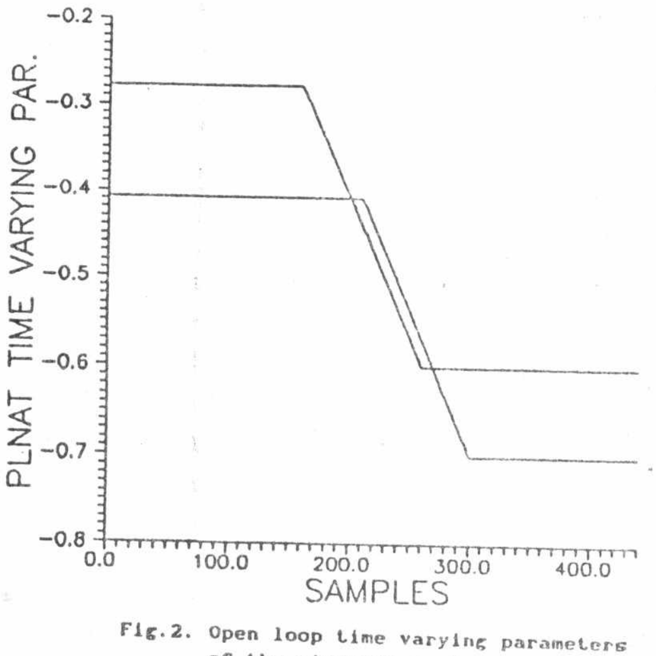

Fig.2. Open loop time varying parameters of the aircraft system

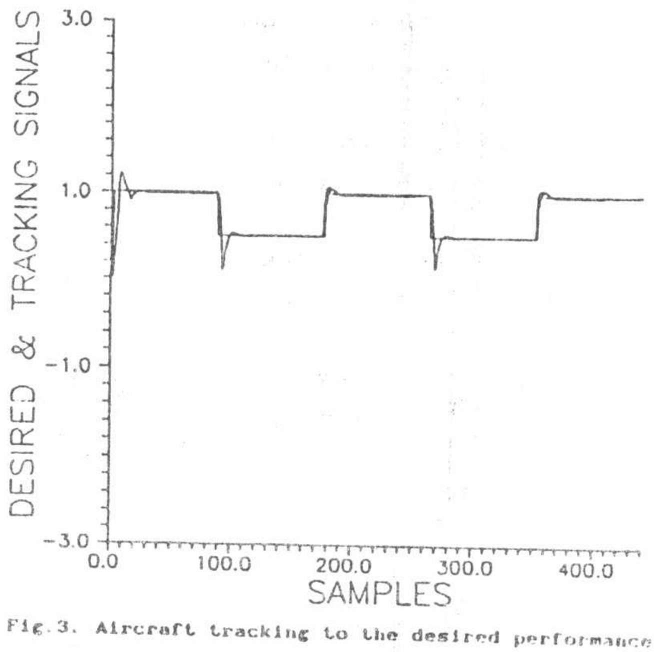

Fig.3. Aircraft tracking to the desired performance

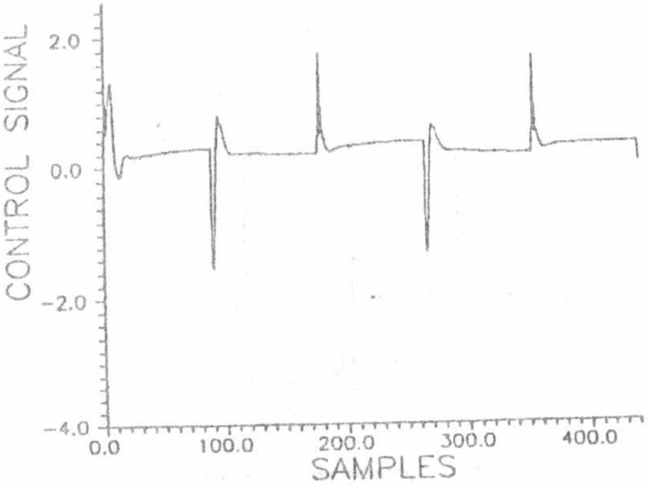

Fig.4. Control signal

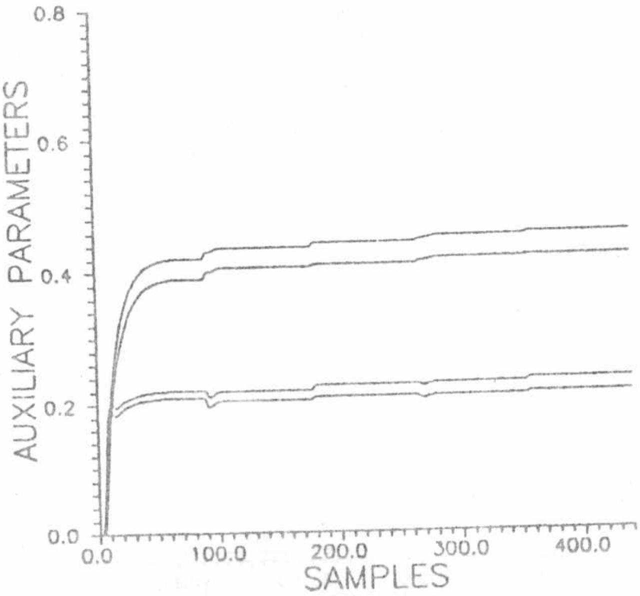

Fig.5. Auxiliary parameters

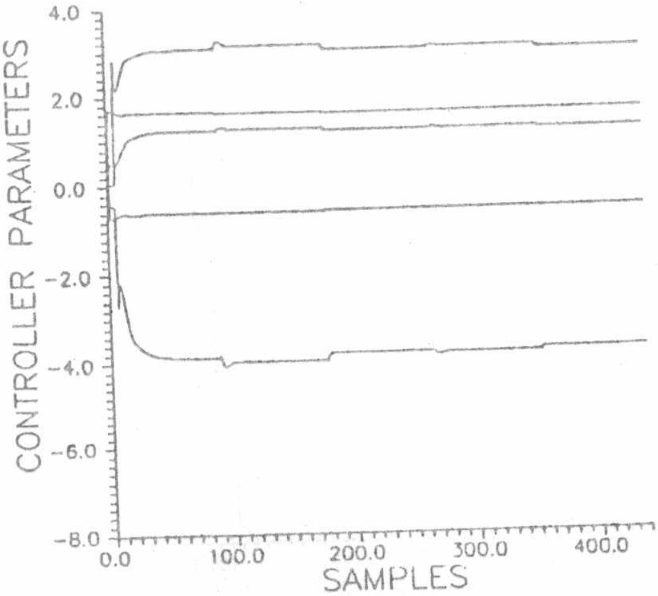

Fig.6. Controller parameters

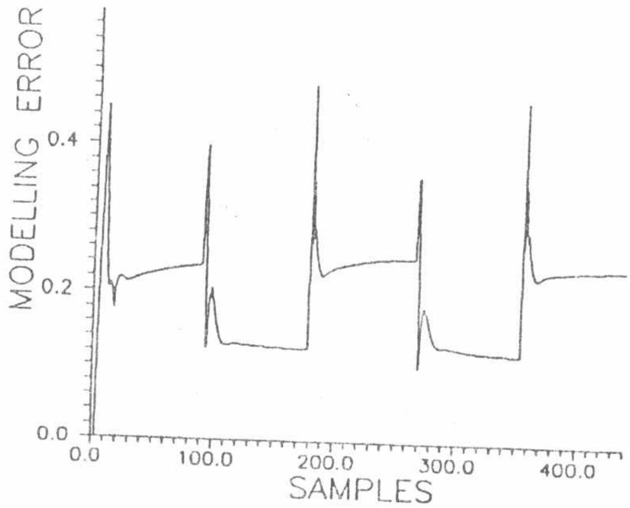

Fig.7. Additive modeling error signal

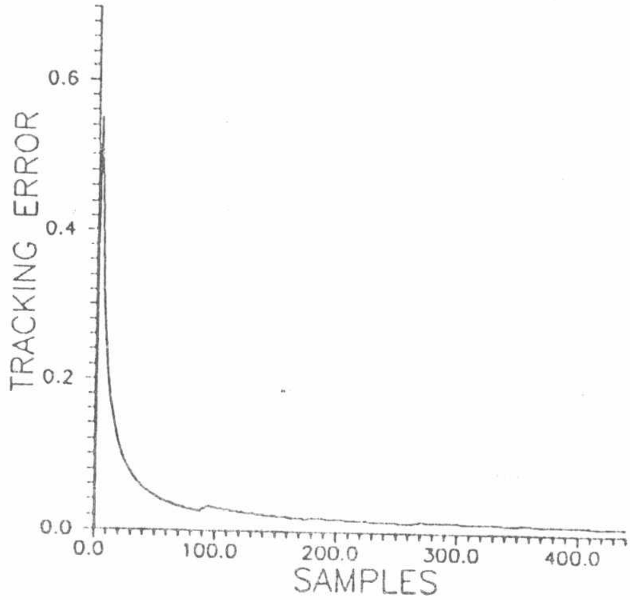

Fig.8. Tracking error signal

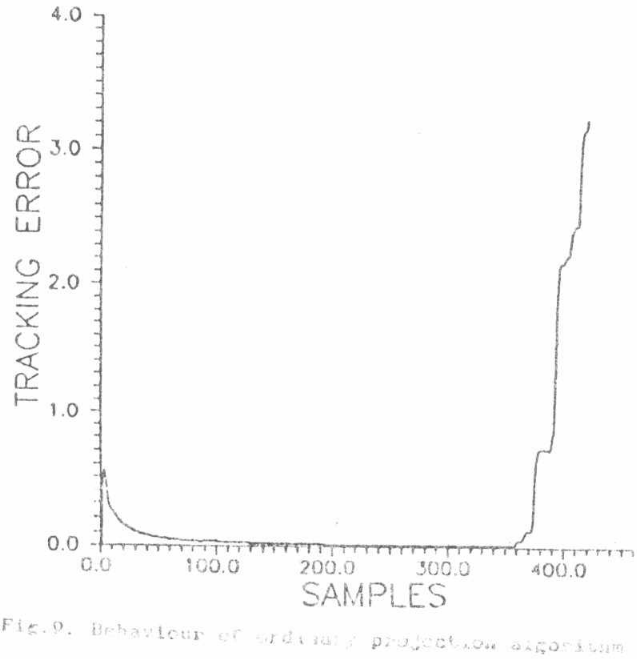

Fig.9. Behaviour of ordinary projection algorithm