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CAIRO - EGYPT

STATE ESTIMATION OF AN INERTIALLY GUIDED AIR VEHICLE

M.H.Abdel-Haleem* M.O.Tantawy** H.S.Tantawy***

ABSTRACT

This paper represents an application of modern control theory to the field of aerospace vehicle navigation system. An attention is paid to the reconstruction of the states of an inertially guided aerospace vehicle via a full-order observer. A mathematical model is derived to represent the motion of the vehicle. Linearization by the perturbation method is made, and the state space equations of the perturbed vertical motion are derived. The controllability and observability of the system are tested . A parametric observer is designed that estimates the states of the system . The Eigenvalues of the observer and the free parameters are used as design variables. Results of such parametric observer show the necessity for optimization to achieve acceptable behaviour of the observer in such application. An optimal observer is designed by solving the algebraic Riccati equation . The performance of the optimal observer is demonstrated by reconstructing the perturbed states of motion about a nominal trajectory of a vehicle dynamic flight simulation.

* Eng.(M.Sc), DSSC, Cairo, Egypt

** Assoc. Prof. DSSC, Cairo, Egypt.

*** Prof. of Automatic Control, Faculty of Engineering, Al-Azhar University, Cairo.

I-INTRODUCTION

In aerospace field , Inertial Navigation Systems (INS) are widely used [1] , [2] and [3] . The importance of such systems has pushed the scientists, to analyse and solve the problems arising from using INS. Because an ideal INS never exists, a lot of researches have been made on its error analysis and a lot of knowledge has been gained from accumulated experience of INS users and analysts. Linear models were developed which describe accurately the behaviour of these errors [4] and [5] . These models were put to use on-line in the mid sixties in the successful implementation of Kalman filter for estimating the INS error outputs and error sources [6] . In 1963 , David G. Luenberger initiated the theory of observers for state reconstruction of linear dynamical systems [7] and [8] . Flight dynamics and automatic flight control is one of the fields that makes use of such theory as it has already made use of Kalman filter [9] and [10] . With the advent of present day digital computer technology, it has become feasible to utilize fast on-board digital computers, which perform mathematical calculations needed for navigation in the state estimation process of a flying vehicle if accurately enough mathematical representation is developed to the vehicle motion, and appropriate estimation algorithm is implemented. The idea of in-flight estimation of an aerospace vehicle states from the parameters measured by inertial sensors is the aim of our present research.

In this paper we adopt the nonlinear mathematical representation of the vertical motion of an aerospace vehicle [11] and [12] . The INS used in such vehicle is the strap down package system (SDP) [13] . The nonlinear system and and output equations are linearized by using the perturbation technique about a nominal state trajectory [14] . The state space representation of the perturbed vertical motion is derived. The system is found to be completely structural controllable and structural observable.

An observer is designed to estimate the states by adapting Ackermann formula to be used in the multivariable system. The eigenvalues of observer and free design parameters are used as design variables [15] , [16] , [17] and [18] . The results of such design method shows the necessity for optimization to achieve acceptable behaviour of the observer in such application.

The well-known method of observer design by solving the algebraic riccati equation is used for designing an optimal observer [19] . Such observer is applied to estimate the states of flight trajectory . the performance of each type of observer has been demonstrated by computer plottings.

II-MATHEMATICAL MODELLING OF THE VEHICLE VERTICAL MOTION

The nonlinear model of the vehicle vertical motion is as follows:

$$\dot{u}(t) = -Q.w - g\sin\theta + \frac{1}{m}\left(c_x\frac{\rho}{2}(u^2+w^2)S + F\cos\eta\right) \quad \text{(a)}$$

$$\dot{w}(t) = Q.u + g\cos\theta + \frac{1}{m}\left(c_z\frac{\rho}{2}(u^2+w^2)S + F\sin\eta\right) \quad \text{(b)}$$

$$(1)$$

$$\dot{Q}(t) = \frac{1}{I_y}\Big((\dot{I}_y + \dot{m}.r_E^2)Q + c_m\frac{\rho}{2}(u^2+w^2)S.d + c_{mq}\frac{\rho}{2}(u^2+w^2)^{1/2}.d^2.Q + F.\,X_F.\sin\eta\Big) \quad \text{(c)}$$

$$\dot{\theta} = Q(t) \quad \text{(d)}$$

And the nonlinear equations describing the measurements of output parameters will be in the form :

$$\underline{Y}(t) = \underline{Y}(\underline{X}, \dot{\underline{X}}, U, t) \quad (2)$$

It is obvious from (1) and (2) that the nonlinear model is of the form :

$$\dot{\underline{X}}(t) = f(\underline{X}(t), U(t), t) \quad \text{(a)}$$

$$\underline{Y}(t) = \underline{Y}(\underline{X}(t), U(t), t) \quad \text{(b)} \qquad (3)$$

which can be linearized by the perturbation method to be as follows:

$$\delta\dot{\underline{X}}(t) = \underline{A}_0(t).\delta\underline{X}(t) + \underline{B}_0(t).\delta U(t)\ ,\ \delta\underline{X}(t_0)=\delta\underline{x}_0 \quad \text{(a)}$$

$$\delta\underline{Y}(t) = \underline{C}_0(t).\delta\underline{X}(t) + \underline{H}_0(t).\delta U(t) \quad \text{(b)} \qquad (4)$$

Matrices $\underline{A}_0$, $\underline{B}_0$, $\underline{C}_0$ and $\underline{H}_0$ are of dimensions nxn , nx1, mxn and mx1 respectively where n and m are number of states and outputs respectively.

$$A_0 = \begin{bmatrix} \dfrac{A_{xu}}{m_0} & \dfrac{A_{xw}}{m_0} - Q_0 & -w_0 & -g\cos\theta_0 \\ \dfrac{A_{zu}}{m_0} + Q_0 & \dfrac{A_{zw}}{m_0} & u_0 & -g\sin\theta_0 \\ \dfrac{M_{Au}}{I_{y0}} & \dfrac{M_{Aw}}{I_{y0}} & \dfrac{M_{Aq}}{I_{y0}} - \dfrac{I_y + mr_E^2}{I_{y0}} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (5)$$

$$B_0 = \begin{bmatrix} -\frac{F}{m_0}\sin\eta_0 \\ \frac{F}{m_0}\cos\eta_0 \\ \frac{F}{I_{y0}}\cos\eta_0 \cdot x_F \\ 0 \end{bmatrix} \quad (6)$$

$$C_0 = \begin{bmatrix} \frac{A_{xu}}{m_0} & \frac{A_{xw}}{m_0} - Q_0 & -w_0 & 0 \\ \frac{A_{zu}}{m_0} + Q_0 & \frac{A_{zw}}{m_0} & u_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (7)$$

$$H_0 = \begin{bmatrix} -\frac{F}{m_0}\sin\eta_0 \\ \frac{F}{m_0}\cos\eta_0 \\ 0 \end{bmatrix} \quad (8)$$

It is used the structural approach for testing both controllability and observability, which initially converts the system matrices into equivalent boolean forms and uses boolean operations which are much simpler than the ordinary arithmatic operations. To make such structural test, one way is to interpret the state equation coefficient matrices as a digraph, which furnishes " a simple view of structural controllability and structural" observability in terms of reachability properties between digraph states. Based on this interpretation, the structural (potential) controllability and observability can be derived from the system digraph or by a simple set of corresponding boolean matrix operations . Consider that a direct branch exists from node j to node i for every $a_{ij}=0$. Each node is associated with a single state variable. Similarly, elements of matrices B and C connect inputs and outputs to the state variables according to the same conventional node. Illustration of the system structure combined with the input control parameters and the output measurements is shown in Fig.(2.1). The system is found to be completely structural controllable and structural observable [20].

III. PARAMETRIC OBSERVER DESIGN

From the previous section, the vehicle vertical motion is given by a single-input, multi-output analog system described by the equations (4.a and b).
The estimator equation will be

$$\delta\dot{\hat{\underline{X}}}(t) = \underline{A}_0\ \delta\hat{\underline{X}}(t) + \underline{B}_0\ \ \delta U(t) + \underline{G}\ (\delta\underline{Y}(t) - \delta\hat{\underline{Y}}(t)) \tag{9}$$

where $\underline{G}$ is an (nxm) constant matrix with (n.m) unknown elements of $\underline{G}$. So, the problem now is that we have only n roots are to be specified while (n.m) elements of $\underline{G}$ are unknown. It is clarified in [15] that Ackermann formula can be applied to a multivariable system by converting it to an equivalent single-input single-output system. According to such method of design the gain matrix $\underline{G}$ will be as follows:

$$\underline{G} = \begin{bmatrix} g_1 \\ g_2 \\ \cdot \\ \cdot \\ g_n \end{bmatrix} (\ \alpha_1\ \ \alpha_2\ \ \ldots\alpha_m\ ) = \underline{g}\ \underline{\alpha} \tag{10}$$

where $\underline{g}$ is determined by observer pole location, and $\underline{\alpha}$ is optional. The effect of both of them is studied in a computer based simulation and the results are given in Fig. 3.1 , 3.2 , 3.3 and 3.4.

EVALUATION OF PARAMETRIC OBSERVER

The parametric observer is a tunable and reliable observer which gives the freedom to the designer to adjust it. The further the observer poles are located in the L.H.S of complex plane, the bigger the overshoot in error and the higher the speed of error decay to zero (see Fig.3.1 and 3.2) . If a free parameter is affecting usefully in certain direction, both error overshoot and error decay time are decreasing simultaneously (see Fig. 3.3 and 3.4). There should be optimization to such tuners like that method of optimization which is clarified in [21] . Search techniques that can be used in such method need a lot of mathematical work to achieve an optimum parametric observer.

IV. OPTIMAL OBSERVER OF VEHICLE SYSTEM

In this design the observer gain matrix ($\underline{G}$) has to be chosen in such a way that

$$\underline{e} = \delta\underline{X} - \delta\hat{\underline{X}} \rightarrow 0 \qquad \text{as } t \rightarrow \infty$$

where the error $\underline{e}(t)$ satisfies the equation

$$\dot{\underline{e}} = (\underline{A}_0 - \underline{G}\ \underline{C}_0)e\ ,\quad \underline{e}(0) = \underline{e}_0 \tag{11}$$

Here, however , instead of seeking a gain matrix to achieve specified closed-loop pole locations, we now seek a gain matrix to minimize a specified performance criteria J expressed as the integral of a quadratic form in the error $\underline{e}$.

$$J = \int_0^{\infty} \underline{e}^T \underline{q}\, \underline{e} \quad dt \qquad (12)$$

where

$\underline{q}$ is a symmetric matrix.

It has already been clarified in [19] a very clear and systematic procedure to solve such problem in optimal controller design case where the algebraic riccati equation

$$\underline{P}\,\underline{A}_0 + \underline{A}_0^T \underline{P} - \underline{P}\,\underline{B}_0 \underline{R}^{-1} \underline{B}_0^T \underline{P} + q = 0 \qquad (13)$$

and the optimum controller gain in the steady state is given by

$$\underline{K} = \underline{R}^{-1} \underline{B}_0^T \underline{P} \qquad (14)$$

By applying the duality between controller and observer, the optimum gain of observer is given by

$$\underline{G} = \underline{P}\,\underline{C}_0^T \qquad (15)$$

where $\underline{P}$ can be calculated by the following procedure:

1. Build up Hamilton-Matrix $\underline{H}$

$$\underline{H} = \begin{bmatrix} \underline{A}_0^T & \underline{C}_0^T \underline{R}\, \underline{C}_0 \\ \underline{q} & -\underline{A}_0 \end{bmatrix} \qquad (16)$$

2. Calculate the Eigenvalues λ_{Hi} of the $\underline{H}$ matrix

$$|\lambda_H \underline{I} - \underline{H}| = 0 \qquad (17)$$

3. Calculate the Eigenvectors $\underline{X}_i$ of the $\underline{H}$ matrix

$$(\lambda_{Hi} - \underline{H})\underline{X}_i = 0 \qquad (18)$$

4. Build up $\underline{X}$ matrix such that

$$\underline{X} = \begin{bmatrix} \underline{X}_{11} & \underline{X}_{12} \\ \underline{X}_{21} & \underline{X}_{22} \end{bmatrix} \qquad (19)$$

where Eigenvectors corresponding to stable igenvalues are shifted to the L.H. half of $\underline{X}$ matrix.

5. A verification test can be made by performing the multiplication $\underline{X}^{-1} \underline{H}\, \underline{X}$ which should be

$$\underline{X}^{-1} \underline{H} \, \underline{X} = \begin{bmatrix} \underline{J}_1 & \underline{0} \\ 0 & \underline{J}_2 \end{bmatrix} \tag{20}$$

6. Calculate $\underline{P}$ matrix as follows:

$$\underline{P} = (-\underline{X}_{21} \, X_{11}^{-1}) \tag{21}$$

The behaviour of the designed optimal observer is studied in a computer based simulation and the results are as follows(see Fig.4.1 , 4.2 , 4.3 , 4.4 and 4.5).

The selection of the weighting matrix $\underline{q}$ is an important factor in determining the behaviour of observer. Matrix $\underline{q}$ is selected systematically with an initial guess using the rule of thumb described in [22] .

V.DYNAMIC OBSERVER OF VEHICLE SYSTEM.

Estimating the perturbed states of the vehicle in one operating point on the nominal trajectory is not actually the final target of our study. The actual benifit of an observer in our application is to estimate the whole state trajectory of the vehicle motion dynamically, i.e while the vehicle is flying. The vehicle flight is substituted by a computer based simulation program which simulates the nonlinear vertical motion represented by equations (1) and (2). Such program includes the perturbed model represented by Eq.(4) and the whole equations calculating the linearized system matrices (equations 5,6,7 and 8). In our case an initial condition error in the state estimation is inserted to test the behaviour of the observer dynamically.Figures 5.1 , 5.2 , 5.3 and 5.4 show that the observer has tracked the actual states although there were initial condition errors after about three seconds.

VI.CONCLUSION

1. Parametric observer is tunable explicitly by both Eigenvalues and free parameters. There is no way to use it , in our application, without optimizing its tuners. Because too much mathematical work is needed to optimize them, another known method is chosen in our application. Optimization of the parametric observer is an important direction for future researches.
2. The optimal observer design method used is a known method, and it is successfully used in our application because it is a direct, an easy applied and a deterministic method that does not need any iterations or search techniques. Such reasons make it suitable to real time implementation. Applying such method dynamically is acceptable after choosing a suitable weighting matrix.
3. It can also be concluded that such design method is dependent on implicit parameters (weighting matrix elements) and there is a

relation between choosing the weighting elements and the behaviour of observer. The relation between the weighting matrix and both of optimal observer behaviour and optimal observer pole location is recommended to be another point of research in future.

4. In spite of the fact that we have limited our analysis to a simplified vehicle system represented by planer motion, we know well that this design of observer can be extended straight forward to more realistic vehicle system represented by its space motion (motion in two planes with the corresponding coupling).

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Nomenclature

F	engine thrust force .
m	vehicle mass .
g	acceleration of gravity .
u, v and w	components of velocity in body fixed coordinate system .
I_x, I_y and I_z	moments of inertia about x,y and z .
P, Q and R	components of angular velocity about x,y and z.
θ	pitch angle .
η	thrust deflection angle .
C_x, C_y, C_m, C_{mq}	aerodynamic coefficients .
S	reference area .
d	reference diameter .
x_F	the distance between point of application of thrust force and vehicle C.G. .
ρ	density of air .

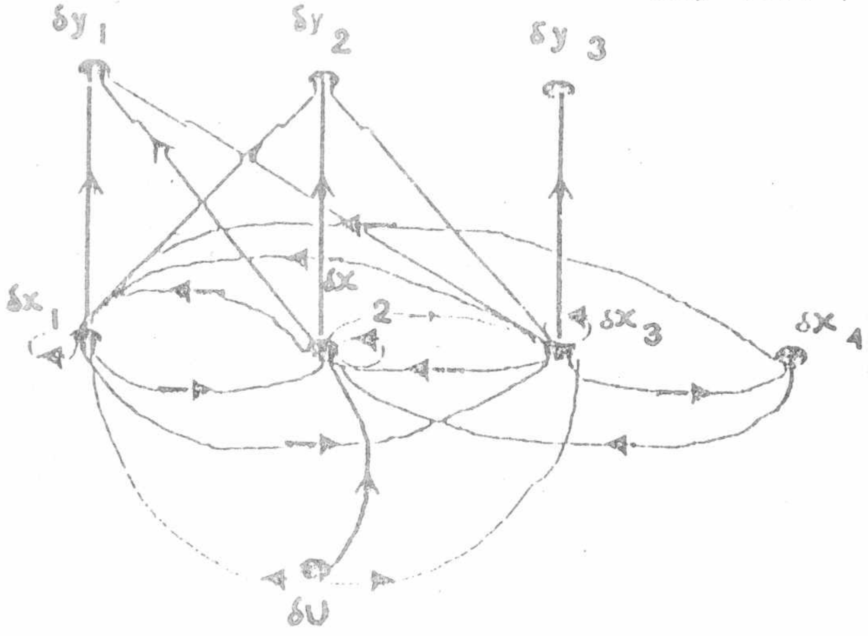

Fig. 2.1. Combined system digraph

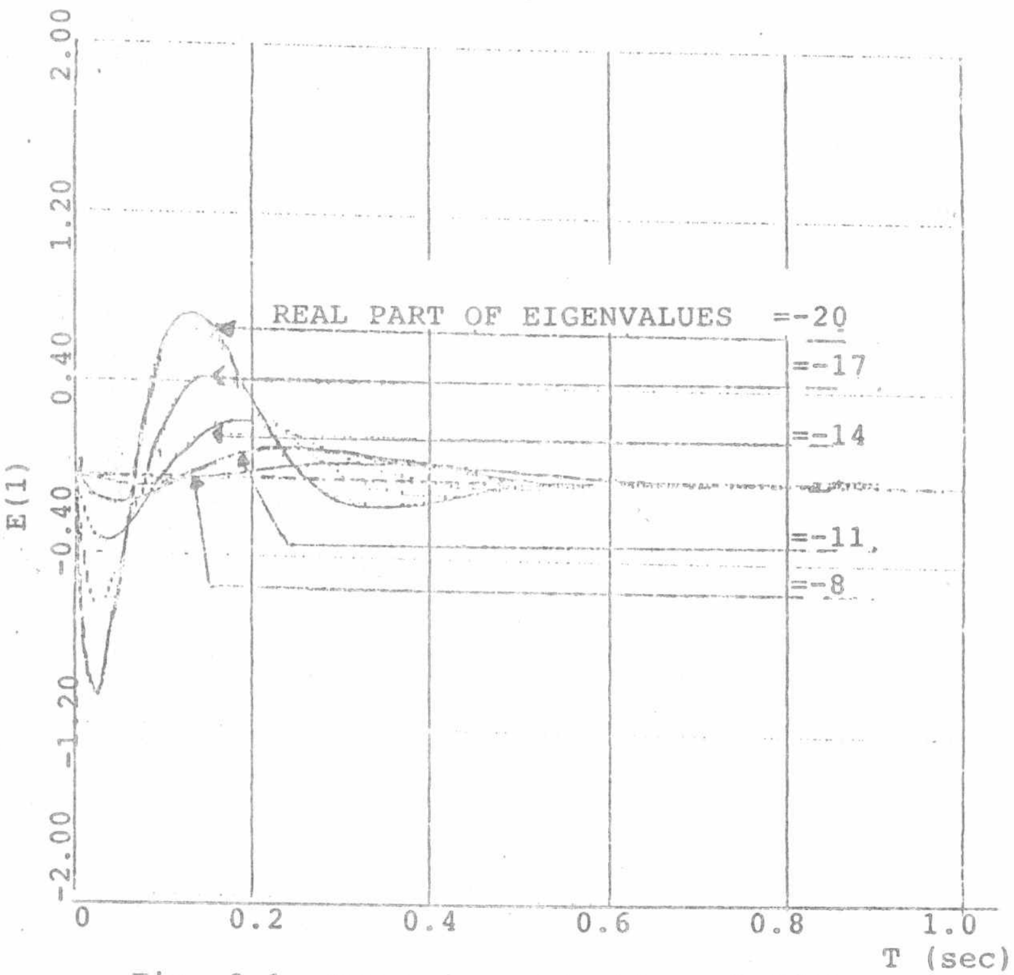

Fig. 3.1 Error in estimation of X(1)

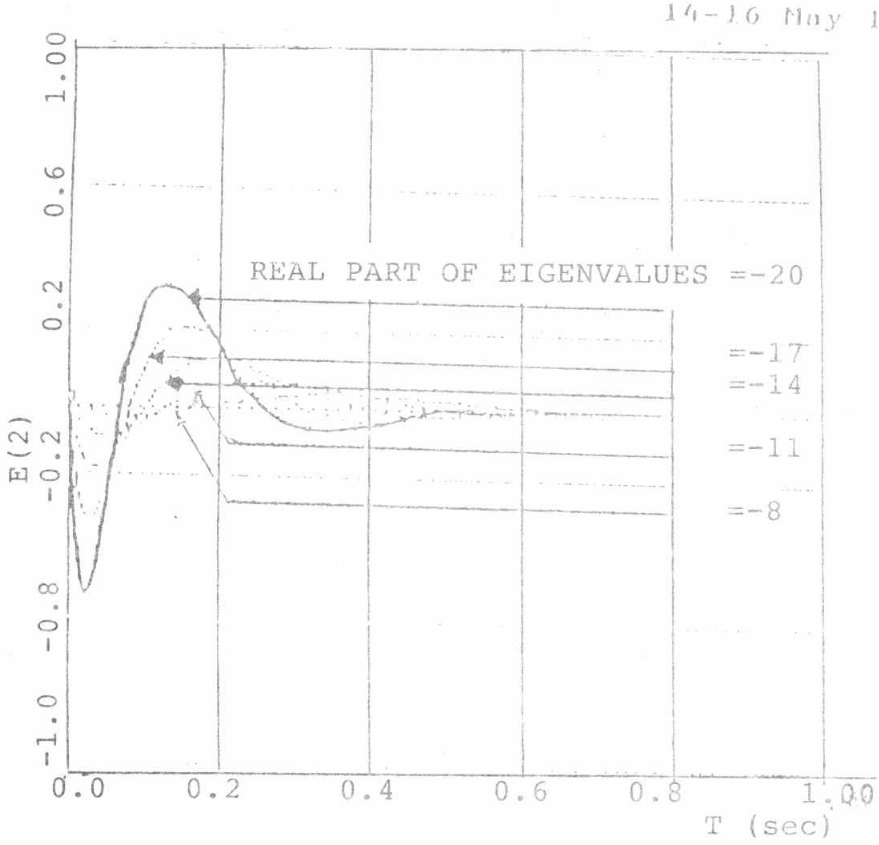

Fig. 3.2 Error in estimation of X(2)

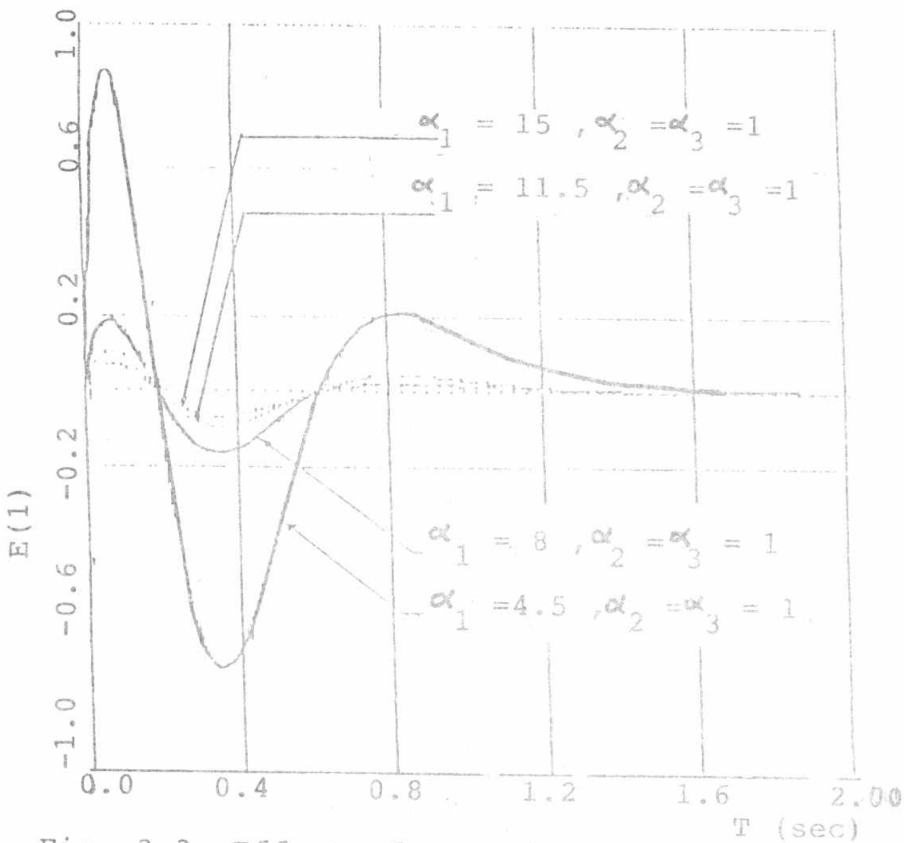

Fig. 3.3 Effect of variation on E(1) dynamics

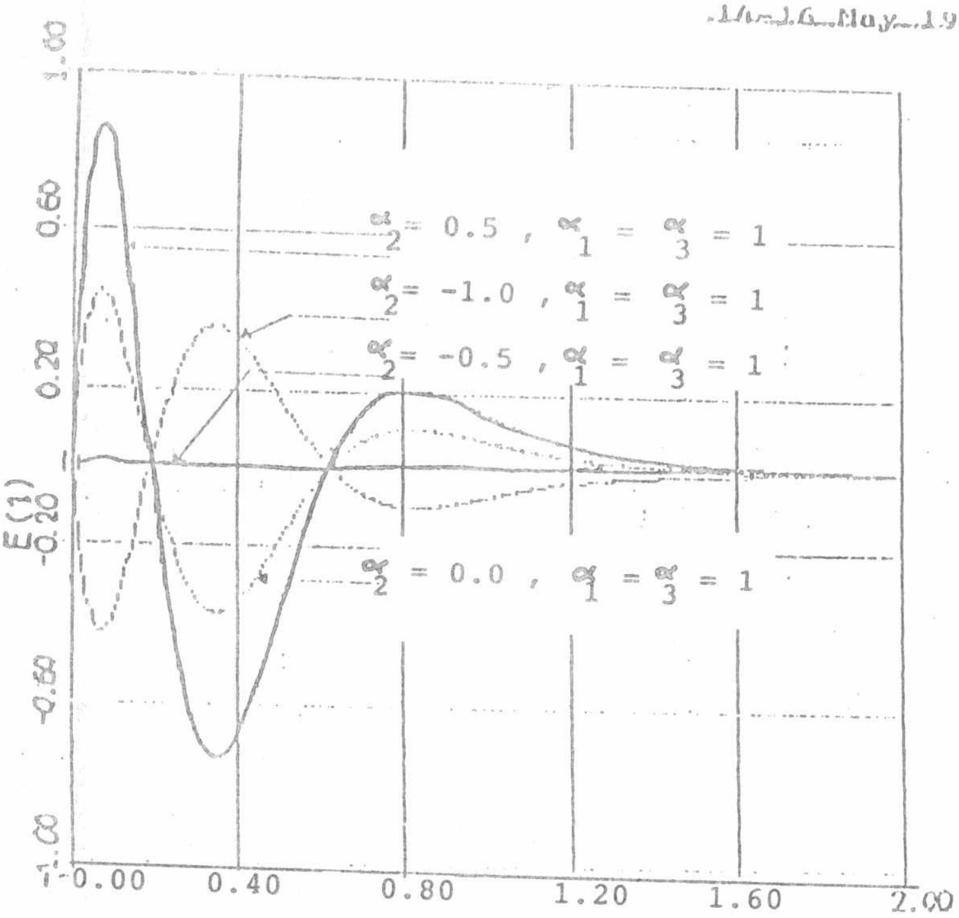

Fig. 3.4 Effect of $_2$ variation on E(1)

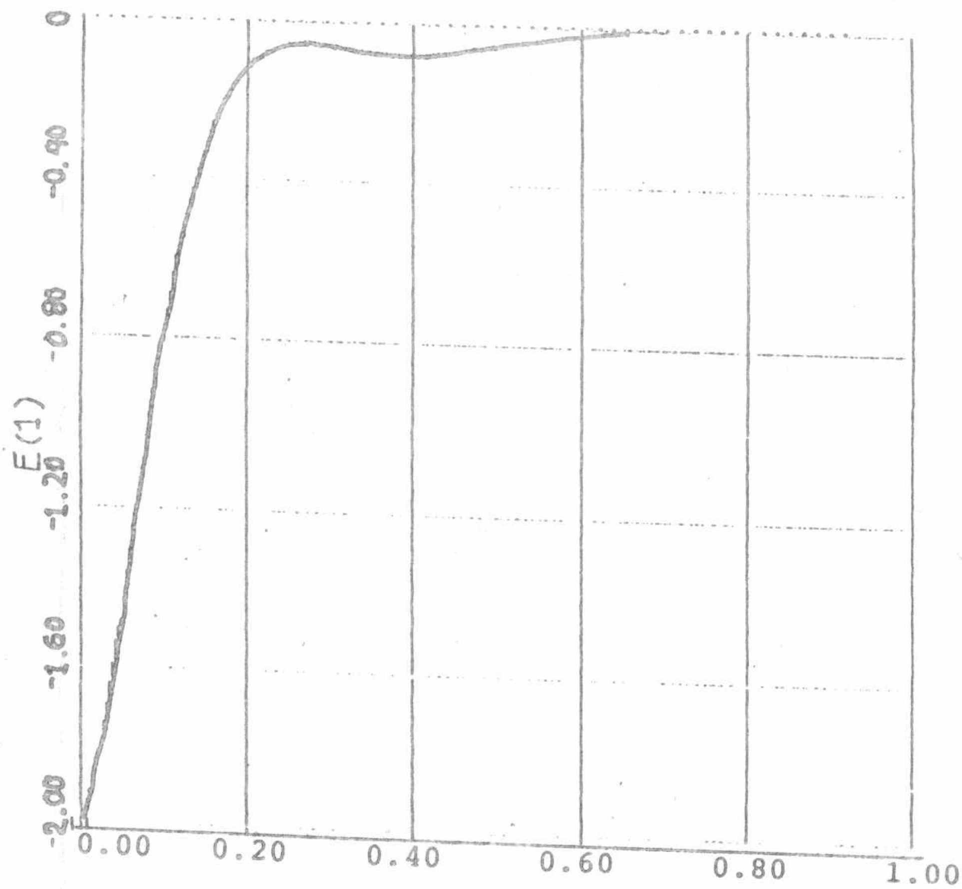

Fig. 4.1 Effect of first state estimation by optimal observer

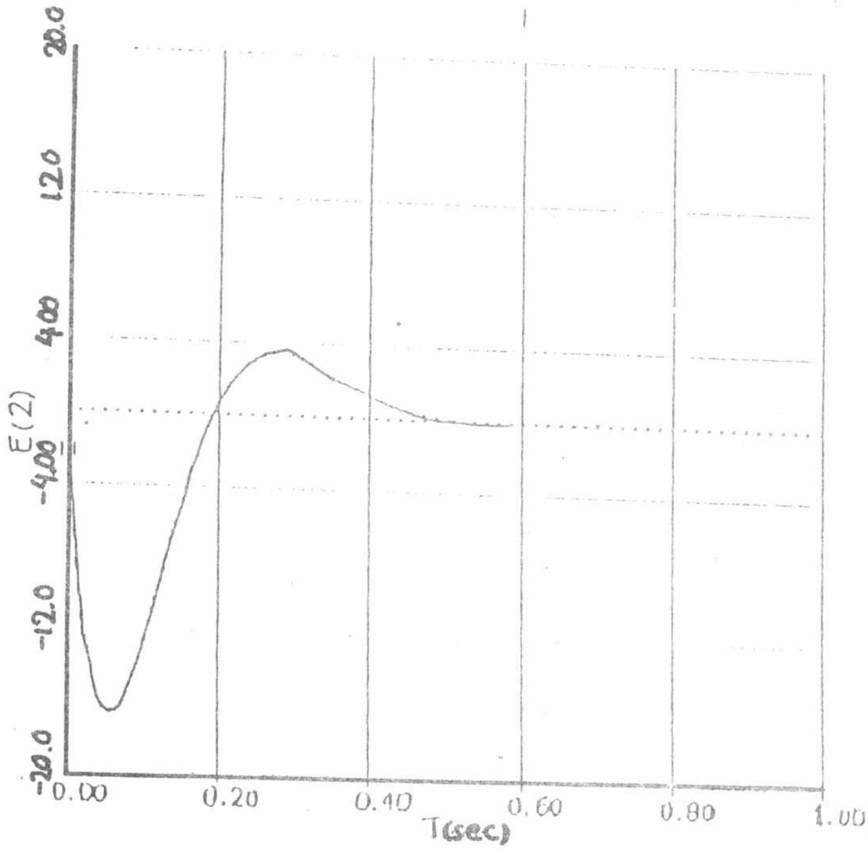

Fig. 4.2 Error behaviour of second state estimation by optimal observer

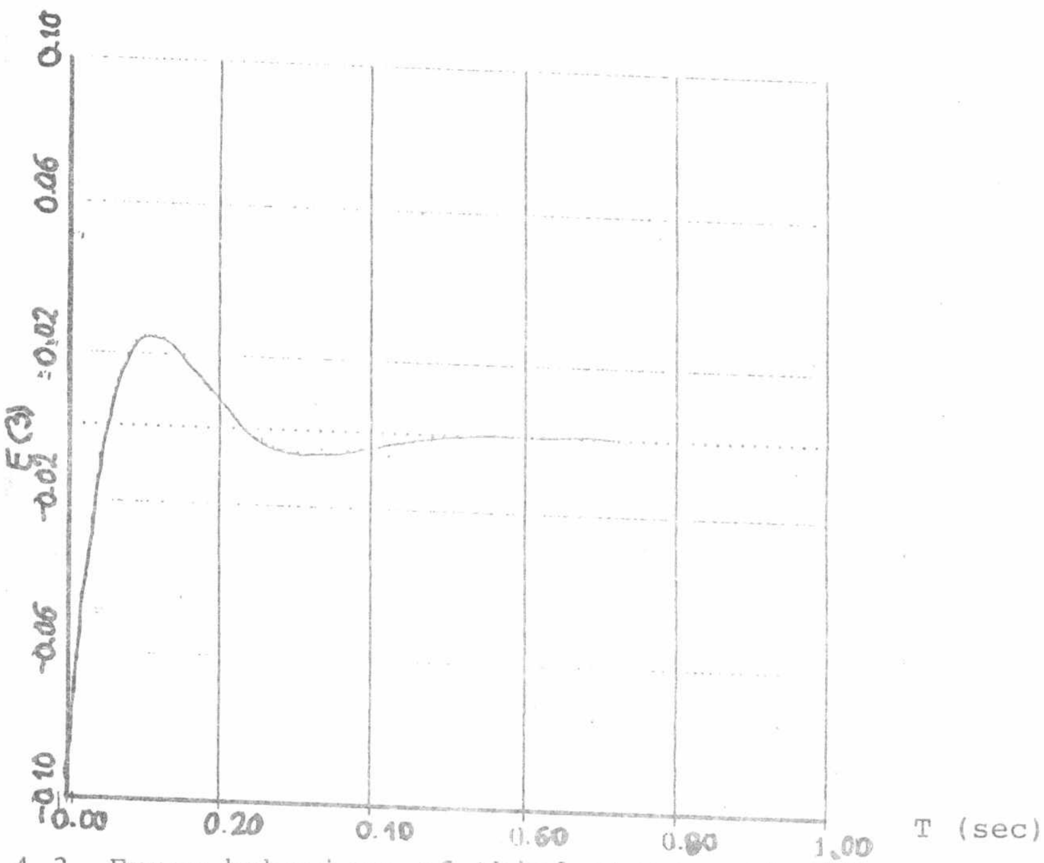

Fig. 4.3 Error behaviour of third state estimation by optimal observer

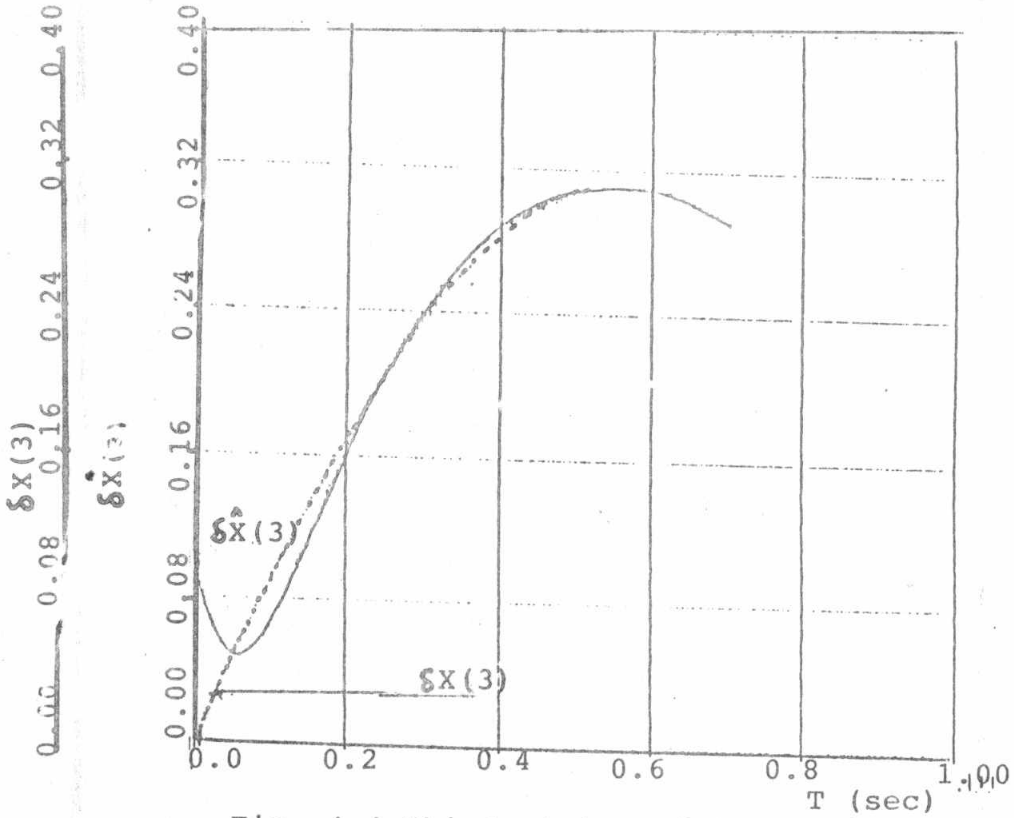

Fig. 4.4 Third state and its estimate dynamic behaviour

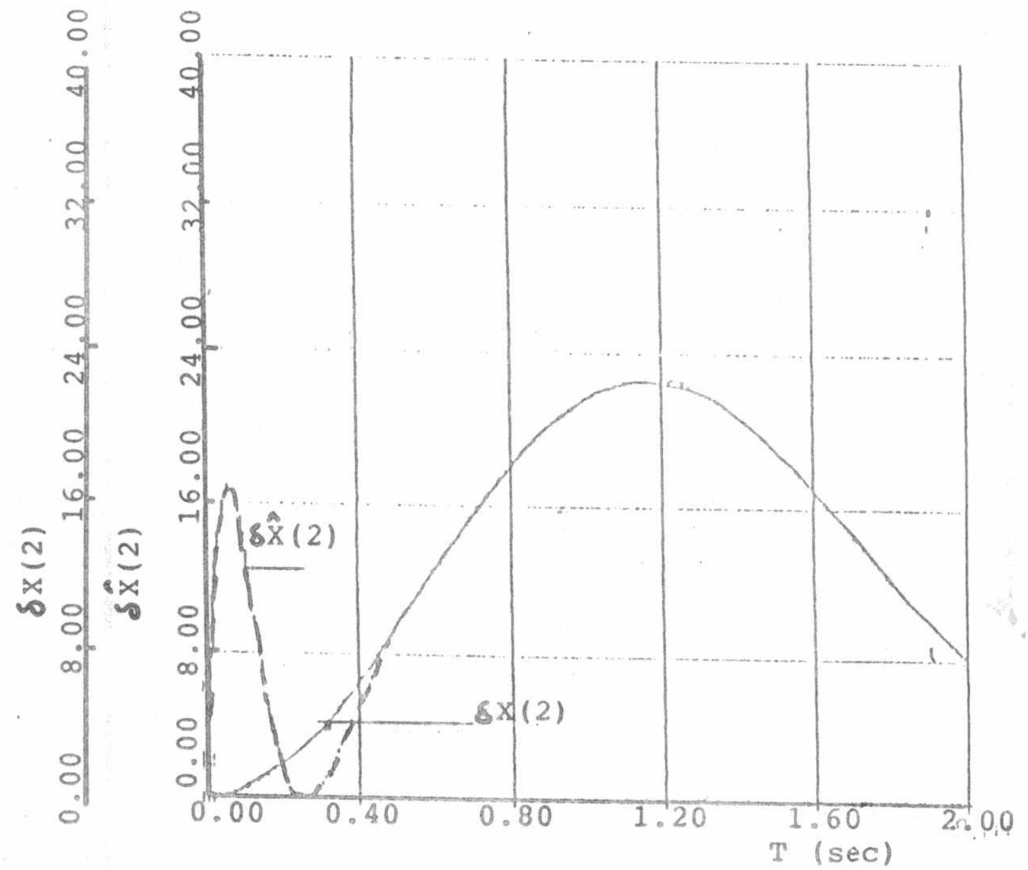

Fig. 4.5 Second state and its estimate dynamic behaviour

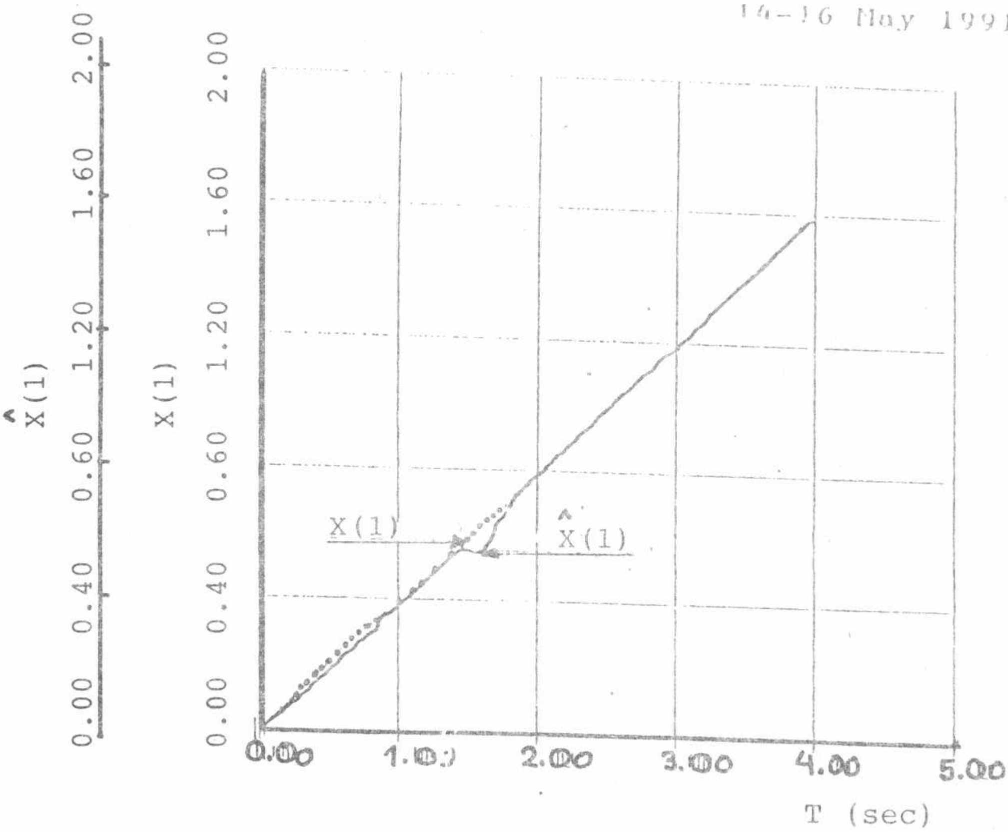

Fig. 5.1 First State and its Estimate (Global State)

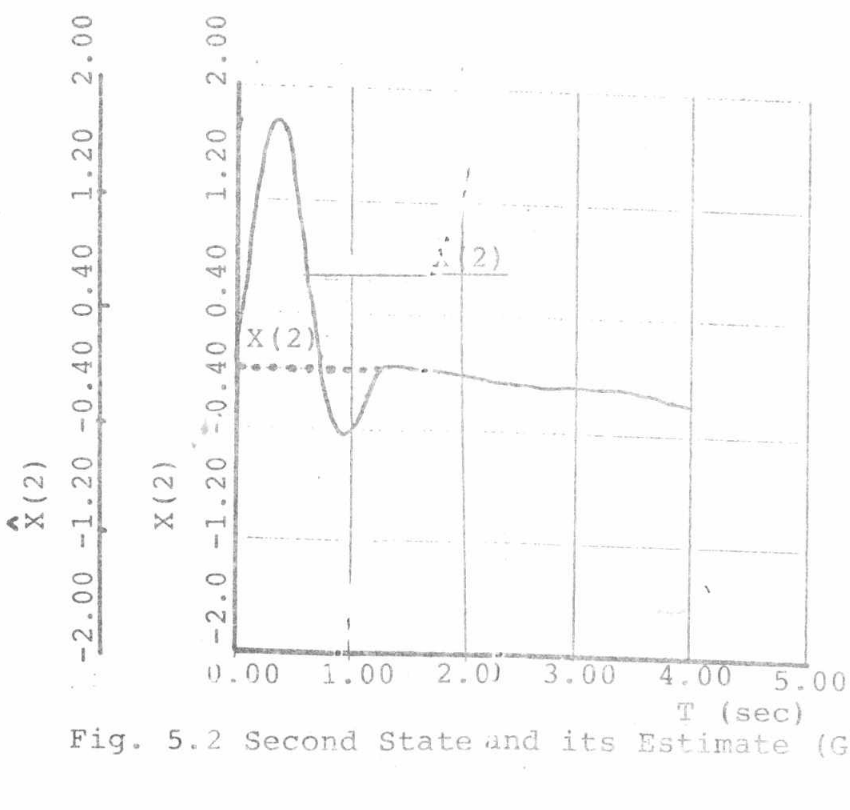

Fig. 5.2 Second State and its Estimate (Global State)

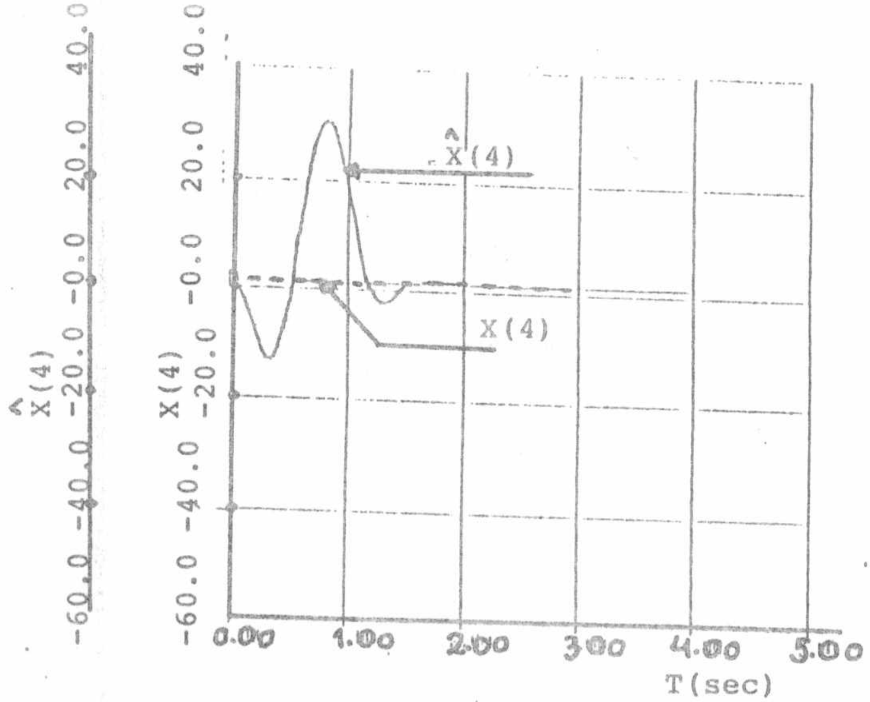

Fig. 5.3 Fourth State and its Estimate (Global State)

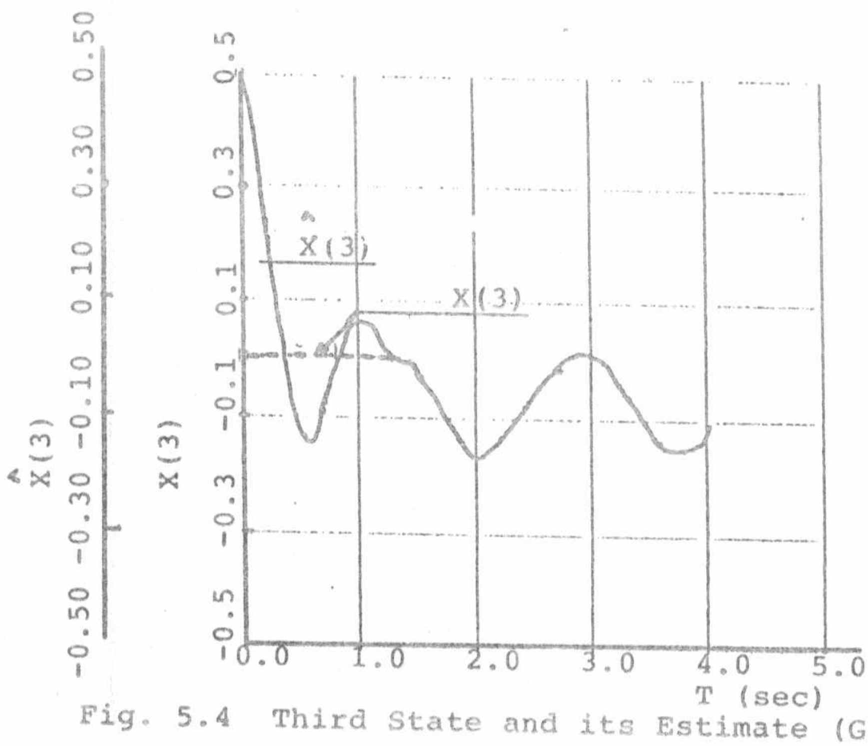

Fig. 5.4 Third State and its Estimate (Global State)