AV-2 439

MILITARY TECHNICAL COLLEGE

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SYSTEM IDENTIFICATION USING MODIFIED LMS ADAPTIVE RECURSIVE FILTERING ALGORITHM

BY

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ABSTRACT

The application of the adaptive filtering techniques in system identification and control has received a great interest to model an unknown plant based on the minimisation of the mean square error criterion. Any physical system is well represented by an auto-regressive moving average transfer function. Hence, the infinite impulse response adaptive filter is able to match a zero-pole transfer function with fewer number of coefficients. In this paper a new adaptation algorithm is introduced to update the forward and the backward filter coefficients. The proposed adaptive filter matches efficiently the unknown systems which are modeled by all pole and zero-pole transfer functions. The filter coefficients have converged exactly to the optimal values.

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I. Introduction

Modeling and system identification are very important in the field of control, communication and signal processing[1]. The unknown plant and the adaptive identifier have the same observation inputs. The coefficients of the identifier are adjusted such that a mean square error performance criterion is minimized. Moreover, an adaptation algorithm is used to update the identifier coefficients and to track any variation of the unknown plant [2].

The physical systems are represented by an auto regressive moving average (ARMA) transfer function. Hence, the adaptive recursive filter is the best example to model the physical systems. A proposed least mean square (MS) adaptation algorithm is introduced to update the recursive filter coefficients [3]. In this algorithm, the gradient vector is liberalized and is calculated by a moving average process rather than by an auto regressive one as in White and St earns algorithms [4,5]. Hence, the stability of the proposed algorithm has been improved for a wide range of step size parameter during the adaptation process.The performance measures of the introduced adaptation algorithm are examined by computer simulation.

This paper has five sections. Section two explains the formulation analysis of the adaptive identifier. The proposed MS adaptation algorithm is introduced in section three. Section four demonstrates the simulation results. Conclusions are given in section five

HI- System formulation

The identification of an unknown plant using the adaptive filtering technique is depicted in Figure 1. The coefficients of the adaptive filter are adjusted such that, the mean square of the error signal, ε_k is minimized. In this case, the adaptive filter represents a good model of the unknown system.The unknown system is represented by an auto regressive moving average transfer function, then, the output signal, d_k can be expressed generally as [6]:

$$d_k = \sum_{i=0}^{N} a_i \, x_{k-i} + \sum_{j=1}^{M} b_j \, d_{k-j} \qquad (1)$$

Equation (1) can be written in matrix form as:

$$d_k = A^T X_k + B^T D_k \qquad (2)$$

where X_k and D_k are the input and output observation vectors respectively and are defined as:

$$X_k^T = [\, x_k \; x_{k-1} \ldots\ldots x_{k-N}] \qquad (3)$$

and

$$D_k^T = [d_{k-1}\ d_{k-2} \ldots\ldots d_{k-M}] \tag{4}$$

The forward and the backward coefficient vectors, A and B are defined respectively as:

$$A^T = [\ a_0\ a_1\ a_2 \ldots\ldots\ a_N\ ] \tag{5}$$

and

$$B^T = [b_1\ b_2 \ldots\ldots b_M\ ] \tag{6}$$

The output of the adaptive recursive filter, y_k can be written as:

$$y_k = \sum_{i=0}^{L_f} a_{i,k}\, x_{k-i} + \sum_{j=1}^{L_b} b_{j,k}\, y_{k-j} \tag{7}$$

or in matrix notation as [7]:

$$y_k = \alpha_k^T S_k + \gamma_k^T Y_k \tag{8}$$

where S_k and Y_k are the input and output observation vectors respectively and are defined as:

$$S_k^T = [\ x_k\ x_{k-1} \ldots\ldots x_{k-L_f}] \tag{9}$$

and

$$Y_k^T = [\ y_{k-1}\ y_{k-2} \ldots\ldots\ y_{k-L_b}] \tag{10}$$

α_k and γ_k are the forward and the backward coefficient vectors of the adaptive filter. They are defined respectively as:

$$\alpha_k^T = [\ a_{0,k}\ a_{1,k} \ldots\ldots a_{L_f,k}\ ] \tag{11}$$

and

$$\gamma_k^T = [b_{1,k}\ b_{2,k} \ldots\ldots b_{L_b,k}] \tag{12}$$

The error signal, ε_k is defined as:

$$\varepsilon_k = d_k - y_k \tag{13}$$

Substituting (2) and (8) in (13) yields:

$$\varepsilon_k = (\ A^T X_k - \alpha_k^T S_k\ ) + (\ B^T D_k - \gamma_k^T Y_k\ ) \tag{14}$$

The error signal, ε_k in (14) can be written as:

$$\varepsilon_k = \varepsilon_{f,k} + \varepsilon_{b,k} \tag{15}$$

where $\varepsilon_{f,k}$ and $\varepsilon_{b,k}$ are defined respectively as:

$$\varepsilon_{f,k} = A^T X_k - \alpha_k^T S_k \tag{16}$$

and

$$\varepsilon_{b,k} = B^T D_k - \gamma_k^T Y_k \qquad (17)$$

III- The adaptation algorithm

The filter coefficients are updated such that the mean square error E [ε_k^2] is minimized. During the adaptation process, the ensemble average E [ε_k^2] is replaced by the instantaneous squared error signal, ε_k^2 [8]. The gradient based adaptation algorithm is used to update the forward and the backward coefficients. The steep est descent adaptation algorithm is defined in [9] as:

$$\alpha_{k+1} = \alpha_k - \mu \nabla_f \qquad (18)$$

and

$$\gamma_{k+1} = \gamma_k - \mu \nabla_b \qquad (19)$$

where ∇_f and ∇_b are the gradient vectors with respect to the forward and the backward coefficients respectively. μ is the step size parameter that controls the convergence rate and the stability of the adaptation algorithm [10]. The forward gradient vector, ∇_f can be expressed as:

$$\nabla_f = \frac{\partial \varepsilon_k^2}{\partial \alpha_k} \qquad (20)$$

Substituting (15) in (20) the forward gradient vector becomes:

$$\nabla_f = 2 \varepsilon_k \left(\frac{\partial \varepsilon_{f,k}}{\partial \alpha_k} + \frac{\partial \varepsilon_{b,k}}{\partial \alpha_k} \right) \qquad (21)$$

Substituting (16) and (17) in (21) ∇_f can be written as:

$$\nabla_f = - 2 \varepsilon_k (S_k + \Psi_k \gamma_k) \qquad (22)$$

where the derivative matrix Ψ_k is defined as:

$$\Psi_k = \frac{\partial Y_k}{\partial \alpha_k} \qquad (23)$$

Similarly, the backward gradient vector, ∇_b can be expressed as :

$$\nabla_b = -2\,\varepsilon_k\,(Y_k + \Phi_k\,\gamma_k) \qquad (24)$$

where the derivative matrix Φ_k is expressed as:

$$\Phi_k = \frac{\partial Y_k}{\partial \gamma_k} \qquad (25)$$

Substituting (8) and (10) in (23) and (25), the i^{th} columns of the Ψ and Φ matrices are given respectively by :

$$\Psi_{i,k} = S_{k-i} + \gamma_{k-i}^{T}\,\Psi_{k-i} \qquad (26)$$

and

$$\Phi_{i,k} = Y_{k-i} + \gamma_{k-i}^{T}\,\Phi_{k-i} \qquad (27)$$

$$i = 1,2,\ldots\ldots L_b$$

Using the assumption that the dependence of the previous outputs on the current coefficients could be neglected. Hence, the i-th column of the derivnative matrices Ψ_k and Φ_k are approximated by:

$$\Psi_{i,k} \cong S_{k-i} \qquad (28)$$

and

$$\Phi_{i,k} \cong Y_{k-i} \qquad (29)$$

The adaptation algorithm given in (18) and (19) can be rewritten using (22) and (24) as:

$$\alpha_{k+1} = \alpha_k + 2\,\mu\,\varepsilon_k\,(S_k + \Psi_k\,\gamma_k) \qquad (30)$$

and

$$\gamma_{k+1} = \gamma_k + 2\,\mu\,\varepsilon_k\,(Y_k + \Phi_k\,\gamma_k) \qquad (31)$$

IV- Simulation results

The perrformance of the system identification is evaluated by two principal measures; the transient and the steady state responses. The transient response is explained by the learning curve of the adaptation process, while the steady state response is demonstrated by the residual mean square error (RMSE) or adaptation noise after the filter convergence to the optimal solution.The input signal to the unknown plant and the adaptive filter is represented by a white gauss ian signal with unity variance. The unknown plant is represented by both the zero-pole and all pole transfer functions which are defined respectively by:

$$H(z) = \frac{1 - 1.82\,z^{-1} + 1.1\,z^{-2}}{1 - 1.2\,z^{-1} + 0.6\,z^{-2}} \qquad (32)$$

and

$$H(z) = \frac{1}{1 - 1.2\, z^{-1} + 0.6\, z^{-2}} \qquad (33)$$

The learning curve illustrates the change of the instantaneous mean square error versus the iteration number, k (sampling intervals) It is measured by ensemble averaging of 16 individual learning curves. The learning curves of the zero-pole (N_f = 3 and N_b= 2) and all pole (N_f = 1 and N_b = 2) adaptive recursive filters are shown in Figure 2 and 3 respectively. It is concluded that the zero-pole filter has a higher convergence rate than that of the all pole filter. On the other-hand, the all pole filter has smaller adaptation noise (maladjustment) than that of the zero-pole filter after convergence.

The residual mean square error (RMSE) is measured after 5168 iterations. over a period of 2000 samples. The RMSE is defined as:

$$\text{RMSE} = \frac{1}{2000} \sum_{j=5168}^{7168} \varepsilon_j^2 \qquad (34)$$

The RMSE defined in (34) is measured versus a design parameter M that is defined by :

$$M = \mu \, (N_f + N_b) \qquad (35)$$

where N_f and N_b are the numbers of the forward and the backward coefficients respectively. The RMSE versus M for the zero-pole and all pole adaptive filters are depicted in Figures 4 and 5 respectively. Figure 4 illustrates that the zero-pole filter has nearly zero RMSE as $0.02 \le M \le 0.2$. If M increases above 0.2 , the RMSE increases significantly. Moreover, Figure 5 explains that the all -pole filter has RMSE nearly zero as $0.01 \le M \le 0.07$. The RMSE increases as M increases above 0.07. It is concluded from Figure 4 and 5 that the pole-zero adaptive recursive filter has greater range of M (step size) over the all pole filter, to ensure filter convergence and to attain a small adaptation noise. Furthermore, the adaptive filter coefficients converge exactly to the optimal values. The forward and the backward coefficients of both the unknown plant and the adaptive filter after convergence are listed in Table 1.

V- Conclusion

The adaptive recursive filter can match efficiently a physical unknown system . The transient and the steady state responses of the adaptive filter depends on its structure. The zeros addition improves the convergence property and decreases the fluctuations during the transient time.The zero-pole filter converges and remains stable for great range of step sizes while the stability and convergence of the all pole filter is restricted to a smaller range of step sizes.The proposed adaptation algorithm ensures the convergence of the filter coefficients to the optimal values.

VI- References

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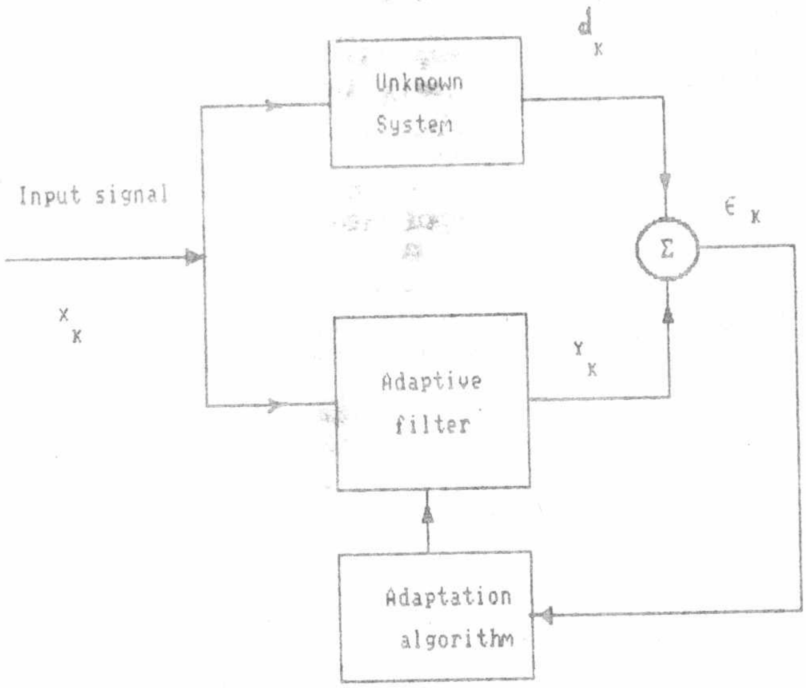

Figure 1 The adaptive system identification model

Table 1 The forward and the backward coefficients after convergence (μ = 0.02)

The filter coefficients	Unknown system pole zero	Unknown system all pole	Pole-zero filter	All pole filter
a_0	1.0	1.0	0.999999	1.0
a_1	-1.82		-1.81999	
a_2	1.1		1.099999	
b_1	1.2	1.2	1.199999	1.2
b_2	-0.6	-0.6	0.599999	-0.6

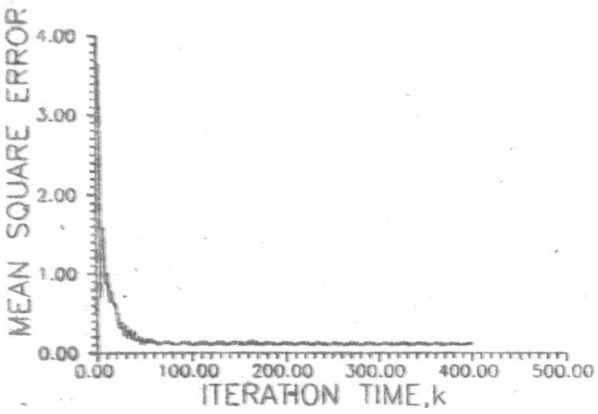

Figure 2 The learning curve of the zero-pole filter (μ = 0.02)

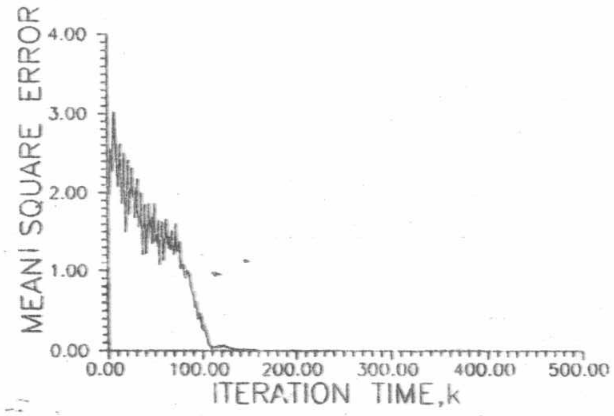

Figure 3 The learning curve of the all pole filter (μ = 0.02)

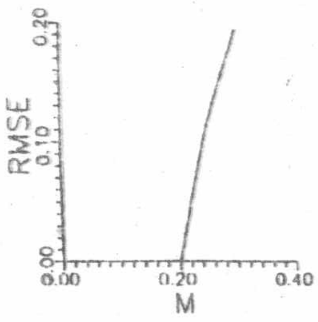

Figure 4 The RMSE of the zero-pole filter versus misadjustment, M

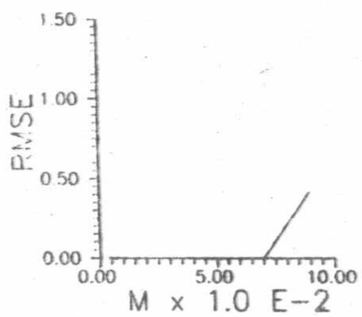

Figure 5 The RMSE of all pole filter versus misadjustment, M