MATHEMATICAL MODELING OF THE AIR PRESSURE REDUCER

OPTIMUM NOZZLE DIAMETER OF ITS TESTING PIPE

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ABSTRACT

One of the most important elements in the pneumatic circuits is the pressure reducer which is commonly used to keep a constant downstream pressure, whatever the value of the upstream one. In fact the downstream pressure would anyway vary, but it could be considered to operate with a sufficient accuracy if its outlet pressure varies within a zone of ±5% of its nominal value. That is why every pressure reducer, after being manufactured then assembled; must be tested to verify its function, and whether it works in the prescribed tolerance. The testing bench consists of several pressurized air bottles, branches to the pressure reducer, downstream of which there mounted a short pipe of inner diameter: d_{pipe}; nozzled at its outlet end witha diameter: d_n. In the present research, a complete mathematical model is developed to find out the optimum nozzle diameter of the testing section which is necessary to identify the function of certain reducer.

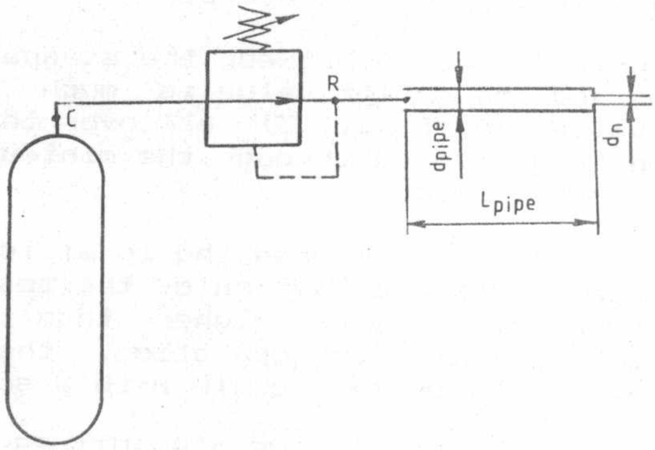

fig.1. LAY OUT OF THE TEST STAND

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1. GENERAL ASSUMPTIONS

A. Regardless of the first few milliseconds during which the inside pressure of the testing pipe passes its transient regime to attain its steady value; the flow can be considered in general as quasi-steady. That is possible because the rates of variation of pressure, temperature and displacement terms are very limited, as it will be shown in the following paragraphs.

B. The pressurized air bottles represent the only part of the system which operates under unsteady conditions.

C. The heat transfer from the external atmosphere to the air bottles is refered to the free convection due to the temperature difference between the inside pressurized air and the outside atmosphere.

D. Because of the short length of the testing pipe; the flow inside is taken as Fanno flow (adiabatic frictional flow in a constant area duct).

E. The throttling section inside the reducer is approximated as a convergent nozzle with an isentropic flow due to its configuration.

F. The outlet nozzle of the testing section is also approximated as a convergent nozzle.

G. As the pressure ratio between the stagnation value inside the bottles and the outlet value is much higher than the critical value (P_c/P_R>1.893) all over the operation, the flow is supposed to pass through the minimum throttling section with a sonic speed

H. As the pressure ratio between the local isentropic stagnation pressure at the end section of the testing pipe and the outer atmosphere is much higher than the critical value (P_{2o}/P_o>1.893) all over the operation, the flow is supposed to pass through the outlet nozzle with a sonic speed.

2. HEAT TRANSFER THROUGH THE AIR BOTTLES WALLS.

The heat transfer through the inner and outer air films from external atmosphere to the enclosed pressurized air inside, is referred to the free convection. For more exact solutions the conduction terms representing the heat transfer through the bottles walls must be added. Anyhow, the later term is relatively much smaller than the convection ones, so in our

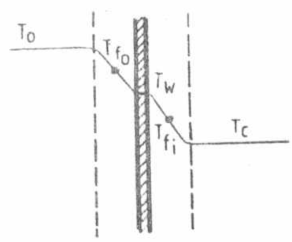


fig.2. TEMPERATURE DISTRIBUTION AROUND THE BOTTLES WALLS

model it is neglected, leading to the assumption that the wall temperature is equal from both sides, and equals T_w. The rate of heat flow through the external boundary layer due to temperature difference (T_o-T_w) is given by:

$$\dot{Q}_{out} = A.\bar{h}_{out}.(T_o-T_w) \tag{1}$$

while taking the rate of heat flow through the internal boundary layer due to a temperature difference (T_w-T_c) as:

$$\dot{Q}_{in} = A.\bar{h}_{in}.(T_w-T_c) \tag{2}$$

where A : The total surface area taken either externally, or internally.

$\bar{h}_{out}$ & $\bar{h}_{in}$: heat transfer coefficients by free convection taken for the fluid properties determined at the average temperature T_{fo} outside and T_{fi} inside the bottles.

$$T_{fo} = (T_o+T_w)/2$$
$$T_{fi} = (T_w+T_c)/2 \tag{3}$$

As: $\dot{Q}_{out} = \dot{Q}_{in}$ (4)

and from (1) & (2):

$$T_w = \frac{T_o.\bar{h}_{out} + T_c.\bar{h}_{in}}{\bar{h}_{out} + \bar{h}_{in}} \tag{5}$$

So; T_w can be determined as function of both $\bar{h}_{out}$ & $\bar{h}_{in}$; these two later parameters are again determined as functions of both T_{fo} & T_{fi}. In other words; they are also functions of T_w in return. That means that the solution of this part must be performed by iteration for given T_o & T_c, to give finally T_w and consequently $\dot{Q}$. William H. McADAMS [1] illustrated that the heat transfer coefficient $h(T_f)$ given as function of the film temperature, can be calculated as:

$h = 0.13(K/L)*(P_r.G_r)^{1/3}$ for: $10^4 < P_r.G_r < 10^9$ (6)

$h = 0.59(K/L)*(P_r.G_r)^{1/4}$ for: $10^9 < P_r.G_r < 10^{12}$ (7)

$P_r = C_p.\mu/k$ (Prandtle number) (8)

$G_r = (L^3.\ \rho_f^2.g.\beta.\ \Delta T)/\mu_f$ (Grashof number) (9)

where: K : air conductivity at T_f, (W/m.C°).

L : equivalent length of each bottle, (m).

ρ_f, μ_f : air density & viscosity at T_f, (kg/m³,kg/m.s)

g : gravity acceleration, (m/s²).

ΔT : temperature difference around the air film.

and

$$\beta = 1/T_f \tag{10}$$
$$L = (L_h.L_v)/(L_h+L_v) \tag{11}$$

where: L_h : equivalent horizontal bottle length ($L_h=D_{cyl}$)

L_v : equivalent vertical bottle length ($L_v=H_{cyl}$)

D_{cyl} & H_{cyl} : diameter & height of each bottle.

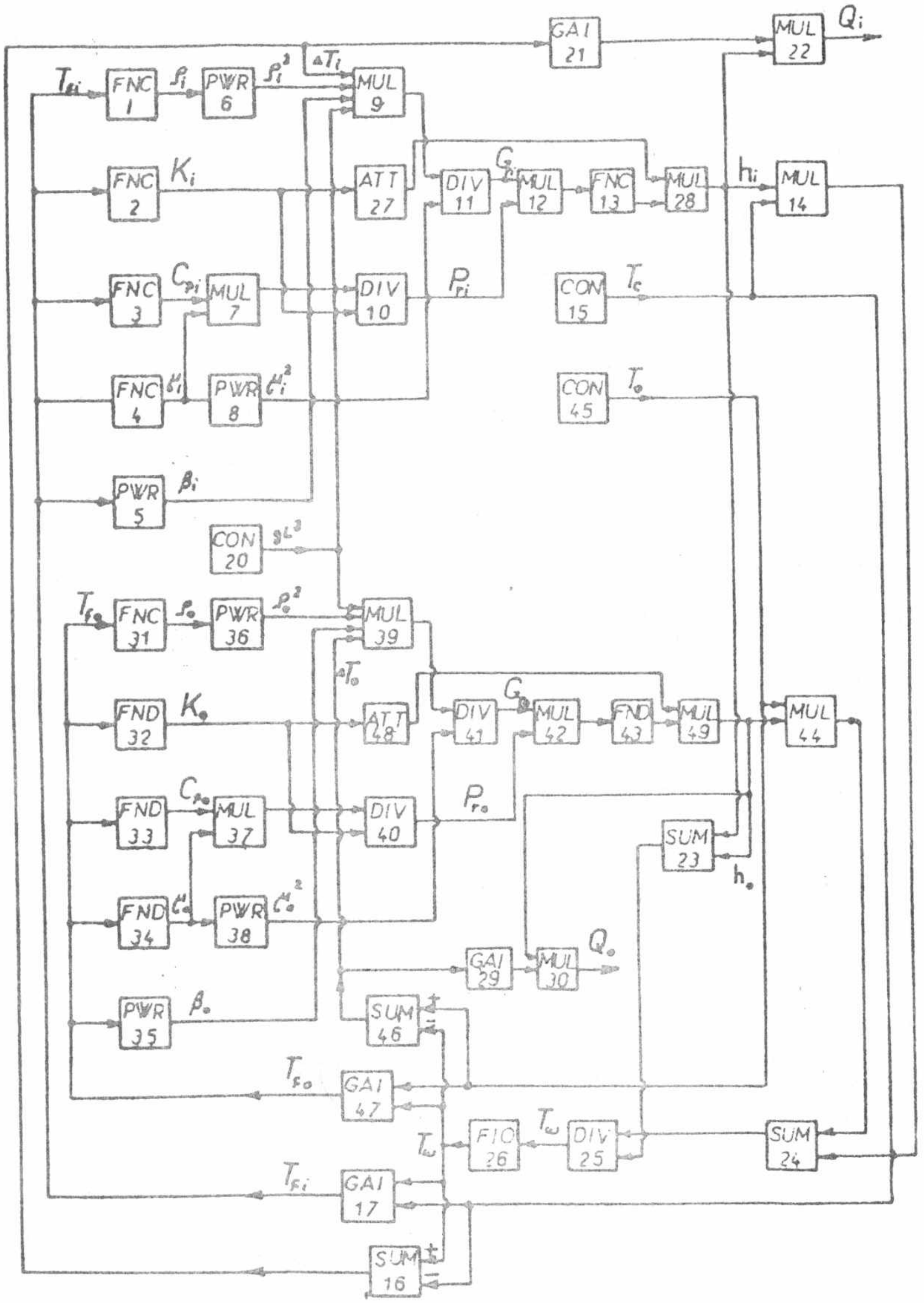


fig.3. TUTSIM BLOCK DIAGRAM - SIMULATION OF HEAT FLOW

For $P_r.G_r < 10^4$, William H. McADAMS [1] did not put any expression to calculate h; he has only put some experimental results, by the aid of which we found that they fit the following equation with error < 0.09 %.

$$h = (K/L)[1.384+5.635E-2(P_r G_r)-4.817E-4(P_r G_r)^2+...$$
$$+4.279E-6(P_r G_r)^3 \qquad (12)$$

According to the previous equations (1):(12), the equations describing the heat transfer to the compressed air bottles during the expansion process are treated on the computer using the TUT-SIM simulation program. This program is a powerful tool for the simulation of dynamic systems, represented by a system of non-linear differential and algebraic equations. Herein, we have applied this program for the solution of the present mathematical model, including only nonlinear-implicit algebraic relations. The solution is based upon a block diagram representation of the mathematical model in terms of the function blocks of the TUTSIM program. A first order element of small time constant is introduced to eliminate the algebraic loop, (block 26, fig.3). The relation between the heat transmitted & temperature of air in the bottle had been calculated point by point. The inner temperature T_c, is introduced by the block 15, where the constant atmospheric temperature is given by block 45, (fig.3).

3. AIR FLOW FROM THE AIR BOTTLES

Applying the first law of thermodynamics in its unsteady general form on a control volume containing the air bottles inside:

$$\dot{Q} = dU/dt + \dot{m}_c(h_c' + v_c'^2/2) \qquad (13)$$

where : U : internal energy of enclosed air inside the air bottles.

h_c', v_c': enthalpy & velocity of air flowing out of the bottles.

$\dot{m}_c$: mass flow rate of air leaving bottles.

As:

$$U = m_c . C_v . T_c \qquad (14)$$
$$h_c = h_c' + v_c'^2/2 = C_p . T_c \qquad (15)$$

where : m_c : the enclosed mass inside the bottles.

C_v, C_p : specific heats at constant volume and constant pressure.

T_c, h_c : stagnation temperature & enthalpy of enclosed air.

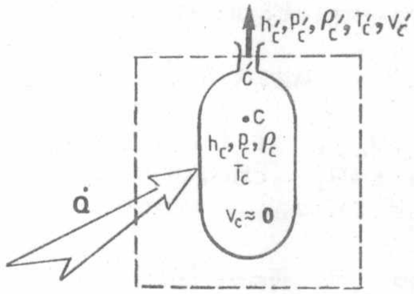


fig.4. AIR BOTTLE HEAT BALANCE

From (14):

$$dU/dt = m_c C_v . dT_c/dt + m_c T_c . dC_v/dt + C_v T_c . dm_c/dt \qquad (16)$$

where : $dm_c/dt = -\dot{m}_c \qquad (17)$

As the specific heat at constant volume C_v is not strongly pressure dependent and may be used over a fairly wide range of pressure, then:

$$C_v = C_v(T_c) \tag{18}$$

and: $$dC_v/dt = (dC_v/dT_c).(dT_c/dt) \tag{19}$$

also as:

$$m_c = m_{co} - \int_0^t \dot{m}_c.dt \tag{20}$$

where : m_{co} : the initial mass of the enclosed air inside the bottles.

from (16),(17),(18),(19) & (20):

$$\frac{dU}{dt} = \left[m_{co} - \int_o^t \dot{m}_c.dt \right]\left[C_v + T_c.\frac{dC_v}{dT_c} \right]\left[\frac{dT_c}{dt} \right] - \dot{m}_c C_v T_c \tag{21}$$

from paragraph (3):

$$\dot{Q} = A_{cyl}(T_o - T_c) \Big/ \left[\frac{1}{\bar{h}_{out}} + \frac{1}{\bar{h}_{in}} \right] \tag{22}$$

Substituting by (22),(21) and (15) in equation (13), then arranging we find:

$$\frac{dT_c}{dt} = \left[\frac{A_{cyl}.(T_o - T_c)}{\left[\frac{1}{\bar{h}_{out}}\right] + \left[\frac{1}{\bar{h}_{in}}\right]} - \dot{m}_c.C_v.T_c(\gamma - 1) \right] /\ldots$$

$$\ldots \left[(m_{co} - \int_o^t \dot{m}_c.dt).(C_v + T_c.\frac{dC_v}{dT_c}) \right] \tag{23}$$

from which, the stagnation temperature of the enclosed air inside the bottles is determined.

$$T_c = T_{co} + \int_o^t dT_c/dt \tag{24}$$

Applying equation of state on air inside bottles:

$$P_c.V_{cyl} = m_c.R.T_c \tag{25}$$

where : V_{cyl} : total volume of bottles.
At time t=0; the pressure is P_{co}, the temperature is T_{co} while the mass is m_{co}, hence:

$$P_{co}.V_{cyl} = m_{co}.R.T_{co} \tag{26}$$

From (25),(26)(20) and (24):

$$P_c = P_{co}.\left[1 - \frac{\int_o^t \dot{m}_c.dt}{m_{co}} \right].\left[1 + \frac{\int_o^t dT_c/dt}{T_{co}} \right] \tag{27}$$

from which the pressure P_c is determined at any instant.

4. GENERAL DYNAMIC EQUATION OF THE REDUCER

Considering that the three springs shown in figure (5) are tightened by the forces: F_{10}, F_{20} & F_{30}, and taking into account that X must be greater than zero during operation, then the induced forces F_1 & F_3 are less than their initial values F_{10} & F_{30}, while the force F_2 is greater than F_{20}:

$$F_1 = F_{10} - K_1 . X \tag{28}$$
$$F_2 = F_{20} + K_2 . X \tag{29}$$
$$F_3 = F_{30} - K_3 . X \tag{30}$$

where : K_1, K_2 & K_3 : stiffness of the three springs.
Studying the applied forces on the moving element, it can be found that they may be divided into the following groups:

a. the spring forces F_1, F_2 & F_3.

b. The forces due to the input pressure: P_c which is acting in the middle chamber on the diameters: D_p & d_v; then:

$$F_{pc} = P_c [A_p - A_v] = P_c . \pi . [D_p^2 - d_v^2]/4 \tag{31}$$

c. The forces due to the outlet pressure: P_R which is acting in both the lower and the upper chambers, then:

$$F_{pR.L} = P_R . A_v = \pi . P_R . d_v^2/4 \tag{32}$$
$$F_{pR.U} = P_R (A_m - A_p) = \pi . P_R (D_m^2 - D_p^2)/4 \tag{33}$$
$$F_{pR} = F_{pR.L} + F_{pR.U} \tag{34}$$

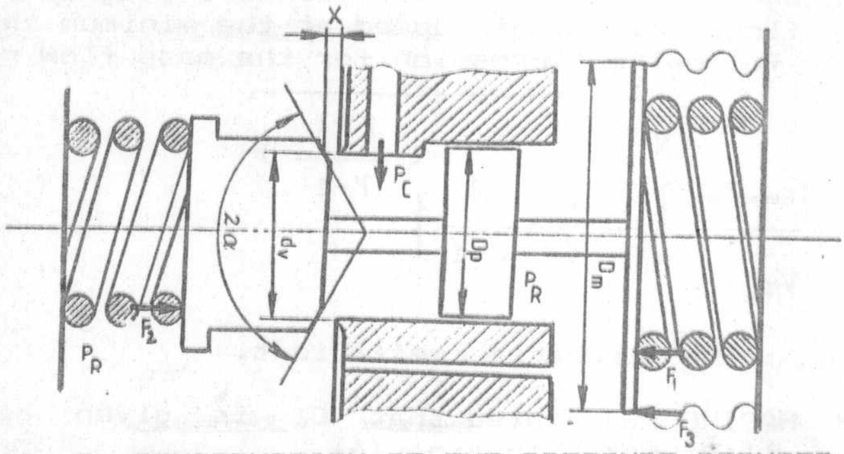


fig.5. CONSTRUCTION OF THE PRESSURE REDUCER

d. the dynamic reaction force due to the jet leaving the middle chamber through the clearance: X, then:

$$F_D = C_d . \rho_{th} . A_{th} . v^2 . \cos\alpha \tag{35}$$

where : C_d : discharge coefficient of the throat.
ρ_{th} : air density at the throat (through clearance : X)
A_{th} : throttling area.
v : flow velocity through the throat.

Then, the general equation can be put as:

$$F = m.\ddot{X} + B.\dot{X} \tag{36}$$

where : m : equivalent mass of the moving element.
B : equivalent dashpot coefficient.

Hereafter, it would be found that the following two assumptions must be put:

1. The right hand side of equation (36) can be neglected ($m.\ddot{X}+B.\dot{X}=0$) with respect to the terms of left hand side.
2. The dynamic reaction force F_D would be also neglected.

The two previous assumptions would be checked at the end of the analysis. The final dynamic equation would be written as:

$$F_2 - F_1 - F_3 + F_{PC} + F_{PR} = 0$$

in other words:

$$(F_{20} - F_{10} - F_{30}) + X(K_1+K_2+K_3) + \ldots$$
$$\ldots + P_c.\pi(D_p^2 - d_v^2)/4 + P_R.\pi(D_m^2 + d_v^2 - D_p^2)/4 = 0 \tag{37}$$

The pre-tightening forces ($F_{20} - F_{10} - F_{30}$) are very important for determination of the downstream pressure, they are chosen such that for maximum P_c, and prescribed P_R, the valve closes completely (X=0).

5. THE MASS FLOW RATE THROUGH THE PRESSURE REDUCER: $\dot{m}_c$

For an enough high value of the cylinders stagnation pressure P_c, the air flows with sonic speed at the minimum throttling area, giving the following expression for the mass flow rate:

$$\dot{m}_c = C_w.\frac{A_{th}.P_c}{\sqrt{T_c}}\sqrt{\frac{\gamma}{R}\left[\frac{2}{\gamma+1}\right]^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \tag{38}$$

where : C_w : the discharge coefficient.

Ascher H.SHAPIRO [2] stated that C_w is given as function of (P_R/P_c) as illustrated in fig. (6).

A_{th} : minimum throttling area.

Equation (38) indicates that the air mass flow rate is always given as function of pressure, temperature inside the bottles, regardless of the downstream conditions.
As for the throttling area, it is easy to find out its relation to the spool displacement (X) as follows:

$R_v = d_v/2 = x.\cos\alpha.\sin\alpha$
$R_1 = R_v - \delta$

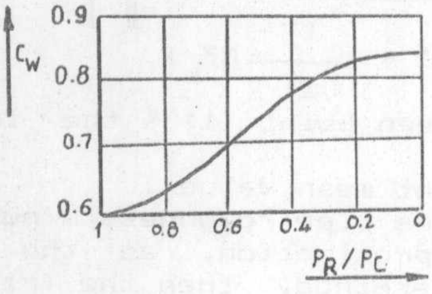


fig.6. DISCHARGE COEFFICIENT C_w AS FUNCTION OF PRESSURE RATIO

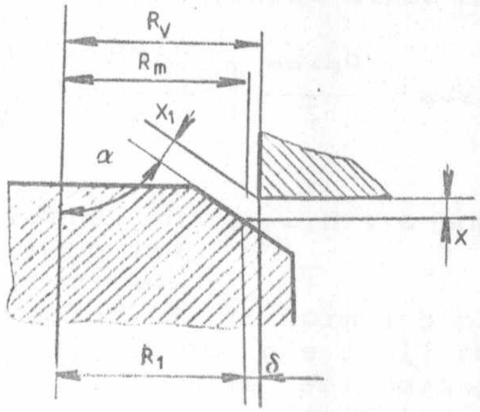


fig.7. ANALYSIS OF THROTTLING AREA

$R_m = ½\ (R_v + R_1) = R_v - ½\ x.\cos\alpha.\sin\alpha$

$x_1 = x.\sin\alpha$

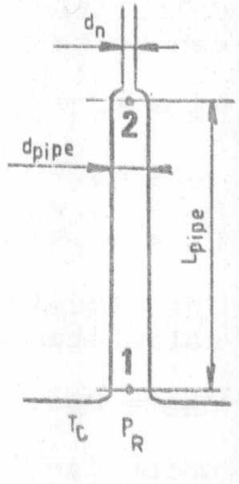
then:

$$A_{th} = 2.\pi.R_m.x = x.(\pi d_v.\sin\alpha) - x^2(\pi\sin^2\alpha.\cos\alpha) \quad (39)$$

6. ANALYSIS OF FLOW INSIDE THE TESTING TUBE

Suppose that the stagnation pressure and temperature just before entering the testing pipe are given as: P_R, T_C.

Suppose also that the pressure at the inlet point of the test tube (just inside) is given, then:



$$M_1 = \sqrt{\frac{2}{(\gamma-1)}\left[(P_R/P_1)^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right]} \quad (40)$$

$$T_1 = T_C \,/\, \{1 + (\frac{\gamma-1}{2}).M_1^2\} \quad (41)$$

$$\rho_1 = P_1 \,/\, R.T_1 \quad (42)$$

$$v_1 = M_1.\sqrt{\gamma.R.T_1} \quad (43)$$

From the equations (40:43), it is clear that the basic parameter to be solved is the mach number at point (1): M_1, from which temperature T_1, density ρ_1 and the flow velocity v_1 can be solved.

Assuming no heat exchange in the testing pipe, it is clear that the flow inside is of Fanno type.

If taking an immaginary point (3) at which the flow speed tends to sonic speed, then:

$$L_{1-3} = \frac{d_{pipe}}{\bar{f}} \cdot \left[\frac{1-M_1^2}{\gamma . M_1^2} + \frac{\gamma+1}{2\gamma} . \ln \left[\!\left[\frac{(\gamma+1).M_1^2}{2(1+\frac{\gamma-1}{2}.M_1^2)} \right]\!\right] \right] \quad (44)$$

where : L_{1-3} : the length of pipe between point (1) & the imaginary point (3).

$\bar{f}$: The friction coefficient mean value .

To detrmine $\bar{f}$, Reynolds no. $\overline{Re}$,and the pipe roughness K must be at first evaluated, for the first approximation, as the flow properties at point (1) are now determined, then the friction coefficient at point (1): f_1 can be calculated.

$$Re_1 = \rho_1 . V_1 . d_{pipe} / \mu_1 \quad (45)$$

$$0 = \frac{0.25}{\sqrt{f_1}} + \log_{10} \left[\frac{K}{3.71\ d_{pipe}} + \frac{1.26}{Re_1 \sqrt{f_1}} \right] \quad (46)$$

from which f_1 can be calculated and taken as f, then substituted in (44), L_{1-3} is determined. It must be denoted that equation (44) is applicable again between points (2) & (3) giving:

$$L_{2-3} = \frac{d_{pipe}}{\bar{f}} \cdot \left[\frac{1-M_2^2}{\gamma . M_2^2} + \frac{\gamma+1}{2\gamma} . \ln \left[\!\left[\frac{(\gamma+1).M_2^2}{2(1+\frac{\gamma-1}{2}.M_2^2)} \right]\!\right] \right] \quad (47)$$

where: $L_{2-3} = L_{1-3} - L_{pipe}$ (48)

Calculating L_{2-3} from (48), then being substituted in (47); M_2 can be calculated, and:

$$T_2 = T_c \,/\, \{ 1 + (\frac{\gamma-1}{2}).M_2^2 \} \quad (49)$$

$$V_2 = M_2 . \sqrt{\gamma R . T_2} \quad (50)$$

$$\rho_2 = \rho_1 . V_1 / V_2 \quad (51)$$

$$P_2 = \rho_2 . R . T_2 \quad (52)$$

The local stagnation isentropic pressure P_{20} at point (2) can be calculated as:

$$P_{20} = P_2 \,.\, \{ 1 + (\frac{\gamma-1}{2}).M_2^2 \}^{[\gamma/(\gamma-1)]} \quad (53)$$

which can considered as the stagnation pressure responsible for the flow through the testing nozzle.

$$\dot{m}_n = C_w . \frac{A_n . P_{20}}{\sqrt{T_c}} \sqrt{ \frac{\gamma}{R} \left[\frac{2}{\gamma+1} \right]^{(\frac{\gamma+1}{\gamma-1})} } \quad (54)$$

It's very important to take into consideration the continuity equation:

$$\dot{m}_c = \dot{m}_n \qquad (55)$$

which will lead in return to the checking condition for the calculation completeness.

7. RESULTS AND ANALYSIS

A computer program was constructed to solve the system quasi-steady response in order to evaluate the different operational parameters. For the case study, the pressure reducer is adjusted at $P_{nominal}$ of value 8.4 bars, & is tested with high pressure air bottles of initial pressure P_{co}= 211 bars. A short test tube of length = 25cm, and nozzle diameter d_n= 6.5 mm was assumed to be mounted in order to represent the actual load.

7.1. EFFECT OF THE DEGREE OF INSULATION

When applying the principle of complete free convection and with taking the conduction into consideration, the temperature showed that it would not drop rapidly (fig 8.c). That can be refered to the low rate of discharge leaving the bottles, which permits good compensation of temperture drop due to rapid expansion inside the bottles from the external atmosphere. Most of roughly measured values indicate that the temperature would drop in a way more severe than the calculated case.

This can be explained if taking into account that the commercially used air bottles are - in most of cases - externally painted and also covered by a layer of insulating dust that would not allow complete perfect convection taken in the mathematical model. That is why, a degree of reduction of the heat transmitted would be added to the mathematical model in order to representthe actual case.

The program was run for different degrees of reduction ranging from 1 (corresponding to complete free convection) to zero (corresponding to complete insulation), and gave the group of results demostrated in figures (8.a : 8.f). It is easy to observe that for higher insulation, the duration of the testing period would decrease. The reducer oultlet pressure would vary during the test in a smaller zone leading to a better accuracy of measurement. The air bottles would decrease in both pressure and temperature more rapid, while on the other hand, the mass flow rate and the reducer piston velocity would increase leading again to higher value of unsteadiness of flow.

7.2. EFFECT OF THE NOZZLE DIAMETER

Another group of program runs were performed with different testing tube nozzle diameters ranging from 0.2:0.4 of the testing tube inner diameter and with same degree of insulation.

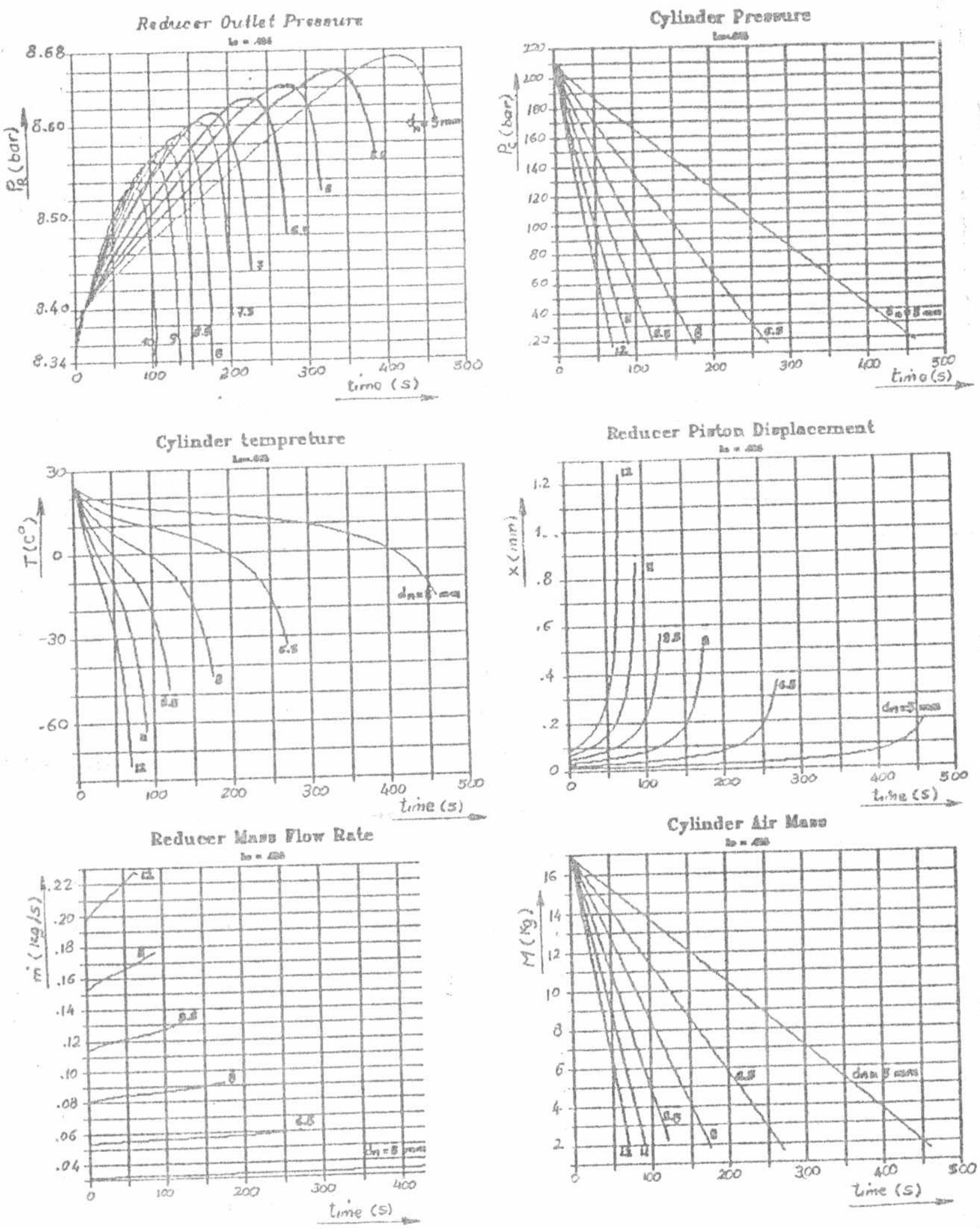


fig.9. EFFECT OF TESTING TUBE DIAMETER ON THE REDUCER RESPONSE

The results shown in fig.(9.a : 9.f) indicate that enlarging the nozzle diameter would reduce the testing experiment duration and

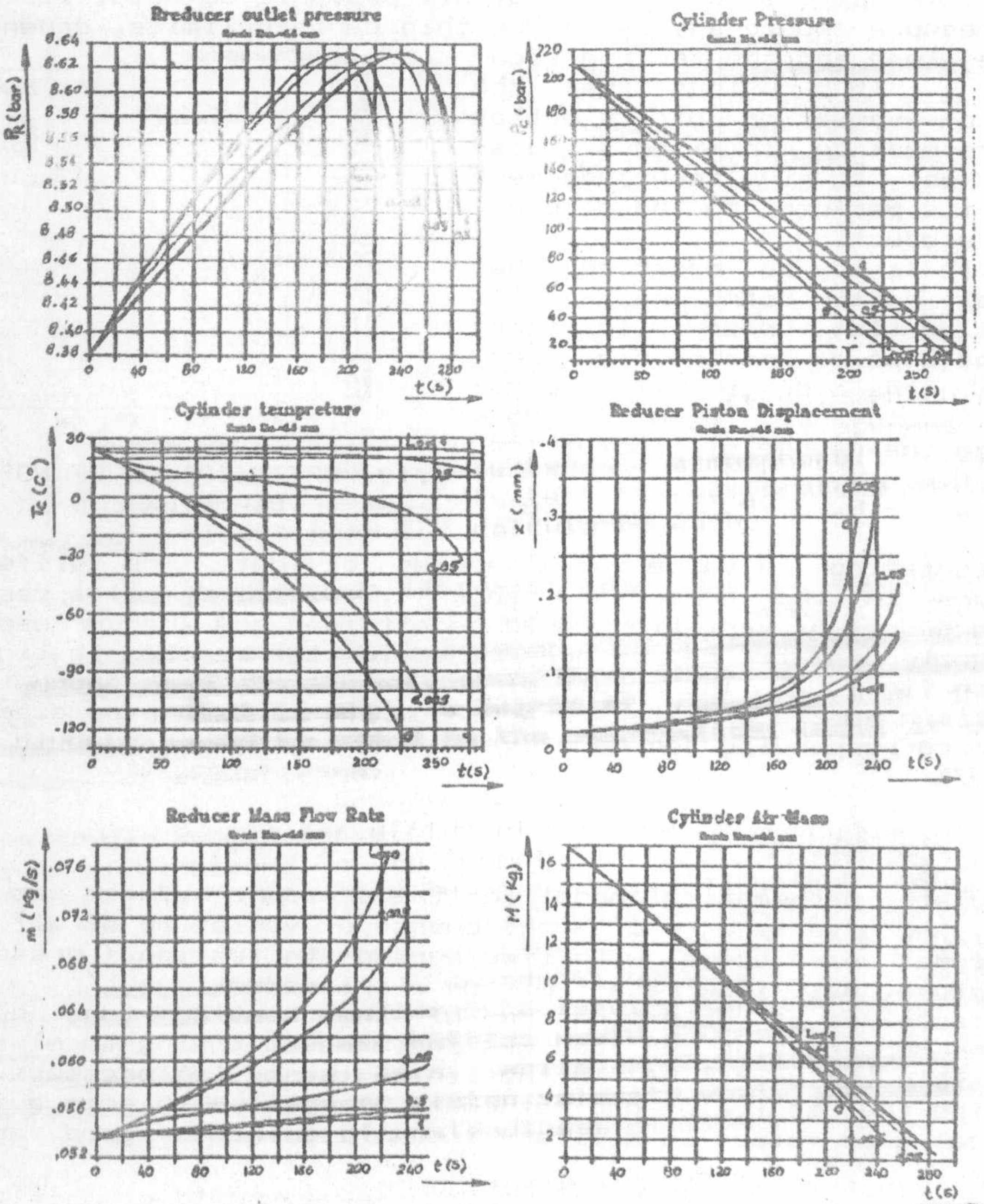


fig.8. EFFECT OF DEGREE OF INSULATION ON REDUCER TESTING RESPONSE

is accumpanied by a sharp drop of both pressure & temperature of the air bottles (fig. 9.b & 9.c). The mass flow rate and the valve piston velocity would also increase (fig. 9.d & 9.f).

8. OPTIMIZATION OF THE TESTING TUBE NOZZLE DIAMETER

It was shown in the previous paragraphs that any pressure reducer, in spite of being free of all probable defects, its outlet pressure would anyway vary within certain limits, depending on the load and the testing technique. It's obvious that the range of variation of its outlet pressure must be minimized as possible just to give true judgement on the its performance and accuracy of its production.

In fig.9a it may be observed that for small nozzle diameters, the maximum outlet pressure is higher than the case of greater diameters, however, the average outlet pressure would be also higher. Defining the degree of fluctuation as:

$$\sigma = (P_{max} - P_{min})/P_{average} \quad (56)$$

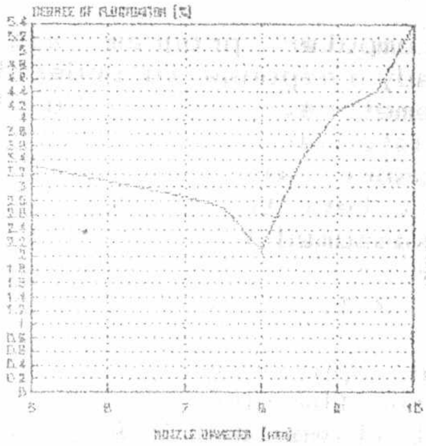


fig.10. OPTIMIZATION OF NOZZLE DIAMETER

and calculating its value for a series of data for different diameters in the precedent range (0.2:0.4 of d_{pipe}) a certain minimum degree of variation could be attained and the corresponding nozzle diameter may be prescribed for the reducer testing as shown in fig.10 The result of the case study had shown that the optimum nozzle diameter is ≈ 0.3 of the testing pipe diameter and the corresponding degree of variation in this case would be: $\sigma \approx 2.07\%$

9. CONCLUSION

A complete mathematical model for the pressure reducer was performed, to give as a results the course of variation of all its operational parameters with time during its testing (pressure, temperature, mass flow rate & piston displacement).

A new technique of determination of the testing tube nozzle diameter representing the load was performed, trying to minimize as possible the degree of fluctuation of the outlet pressure.

Comparing the results of our mathematical model with some guiding practical used values in military industries showed good agreement.

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