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AN ADAPTIVE MODEL REFERENCE CONTROLLER WITH BOUNDED GAINS

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ABSTRACT

It has been established that currently available adaptive control algorithms may become unstable in the presence of high frequency unmodeled dynamics and additive sinusoidal disturbances. In such nonlinear dynamic adaptive control system ,there exist two infinite-gain operators which can cause the loop gains to increase without bound. Such unbounded parameters would likely cause instability .

These problems can be alleviated by using a proposed model reference adaptive algorithm with bounded gains. In this paper, the mechanism of disturbance instabilities is investigated. The modified algorithm uses a limiter for the controller gains to prevent the unbounded drift. The limiting values of the proposed limiter, can be estimated analytically by applying stability criteria for linear control systems,to the linearized model of the controlled plant.

The proposed adaptive controller is tested by simulation. Simulation results show the algorithm to be capable of ensuring stability of the adaptive control system , in the presence of sinusoidal disturbances.

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1. INTRODUCTION

Stability proofs of currently available adaptive controllers require that the relative degree of the plant, and an upper bound on the order of the plant be known. These requirements can not be realized in practice. It was demonstrated (Rohrs, 1982), that there exist two mechanisms of instability in the available adaptive control systems. One candidate of the proposed solutions for the existing problems is presented in this paper.

In section 2, the general structure of standard MRAC system is depicted . The infinite gain operators are discussed, briefly, in section 3. In section 4, a Lyapunov-based stability analysis is derived. The modified algorithm with choice of its bounds, is presented in sections 5,and 6. In section 7, effectiveness of the proposed controller is verified by simulation. Section 8 contains conclusions.

2. GENERAL STRUCTURE OF MRAC ALGORITHM

The simplest prototype for a model reference adaptive control algorithm in continuous - time has its origins to at least as far back as 1974, in the paper by Monopoli [14]. This algorithm has been proved asymptotically stable only for the case when the relative degree of the plant is unity or at most two. The algorithms published by Narendra and Valavani [1] and Feuer and Morse [2] reduce to the same algorithm for the pertinent case. This algorithm is referred to as CA1 (continuous - time algorithm NO.1) in [15] , [17].

The following equations summarize the dynamical equations that describe it; see also Fig. 1. The presented equations pertain to the case where a unity relative degree has been assumed, r(t) is the command reference input, and the disturbance d(t) in Fig.1 is equal to zero.

plant: $$y(t) = \frac{g_p B(s)}{A(s)} [u(t)] \tag{1}$$

auxiliary variables: $$Wui = \frac{S^{i-1}}{P(s)} [u(t)]; \quad i=1,2,..,n-1 \tag{2}$$

$$Wyi = \frac{S^{i-1}}{P(s)} [y(t)]; \quad i=1,2,..,n \tag{3}$$

$$\underline{W}(t) \triangleq \begin{bmatrix} r(t) \\ Wu(t) \\ Wy(t) \end{bmatrix} ; \quad \underline{K}(t) \triangleq \begin{bmatrix} Kr(t) \\ Ku(t) \\ Ky(t) \end{bmatrix} \tag{3a}$$

model : $$Y_M(t) = \frac{g_M B_M(s)}{A_M(s)} [r(t)] \tag{4}$$

Control input : $$u(t) = \underline{K}^T(t)\, \underline{W}(t)$$

$$= \underline{K}^{*T}(t)\, \underline{W}(t) + \underline{\tilde{K}}^T(t)\, \underline{W}(t) \tag{5}$$

output error : $$e(t) = y(t) - y_M(t) \tag{6}$$

parameter adjustment law: $$\dot{\underline{K}}(t) = \dot{\underline{\tilde{K}}}(t) = \Gamma \underline{W}(t)\, e(t); \quad \Gamma = \Gamma' > 0 \tag{7}$$

nominal controlled plant : $$\frac{g^* B^*(s)}{A^*(s)} = \frac{K_r g_p B(s) P(s)}{A(s) P(s) - A(s) K^* u(s) - g_p B(s) K^* y(s)} \tag{8}$$

error equation: $$e(t) = \left[\frac{g^* B^*(s)}{A^*(s)} - \frac{g_M B_M(s)}{A_M(s)}\right] [r(t)]$$

$$+ \frac{g^* B^*(s)}{A^*(s)} \left[\frac{\underline{\tilde{K}}^T(t)\, \underline{W}(t)}{K_r^*}\right] \tag{9}$$

In the above equations the following definitions apply :

P (s) is the characteristic polynomial for the state variable filters,

$$\underline{K}(t) \triangleq \underline{K}^* + \underline{\tilde{K}}(t) \tag{10}$$

where, $\underline{K}^*$ is a constant 2n vector

$\underline{\tilde{K}}$ is a parameter misalignment 2n vector

In (8) , [$g^* B^*(s) / A^*(s)$] represents the closed-loop plant transfer function that would result if $\tilde{K}$ were identically zero, i.e., if a constant control law $\underline{K} = \underline{K}^*$ were used.

In general, existing continuous - time algorithms [1] - [9] can be classified into four groups labeled in [15] as CA1, CA2, CA3, CA4, for continuons - time algorithms 1 , 2 , 3 , 4. Fig.3 represents a generalized error structure which can be particularized to describe the error loop of any one of the existing adaptive algorithms, both in continuous time and by its discrete analog) in discrete time as well. In Fig.3, the forward loop consists of a positive real transfer function, while the feedback path comprises the adaptation mechanism (parameter adjustment), which contains the infinite gain operotor(s).

The above - mentioned four classes of adaptive algorithms have in common the error feedback loop structure and the basic ingredients of the parameter update mechanism, i.e., multiplication - integration - multiplication, which forms the feedback part of the loop and is shown [15] to constitute the infinite gain operators present in all existing adaptive algorithms. The four mentioned classes differ in the specific paraneterization that realizes the positive real transer function in the forward path, and in the particular details of the parameter adjustment laws [choice of C, D, M, F(s)].

3. THE INFINITE GAIN OPERATORS;

The time - varying feedback operator, shown in Fig.2, is reproduced in Fig.4 for the case where w is a scalar and $\Gamma=1$. It can be represented by:

$$\tilde{u}(t) = G_{w(t)}[e(t)] = \tilde{u}_0 + W(t)\int_0^t W(\tau)\; e(\tau)\; d\tau \qquad (11)$$

In order to demonstrate the infinite gain nature of the feedback operator of the error system of CA1 , it is assumed that a component of $\underline{W}(t)$ has the form

$$Wi\ (t) = b + c\ \sin w_0 t \qquad (12)$$

and that the error has the form

$$e(t) = a \sin (w_0 t + \Phi) \qquad (13)$$

if e(t) and a component of $\underline{W}(t)$ have distinct sinusoids at a common frequency, the operator $G_{w(t)}$ of (11) will have infinite gain. Rohrs demonstrated [15] two possibilities for e(t), w(t) to have the forms (12), (13) ;

Case I: if the reference input consists of a sinusoid and a constant. e.g.,

$$r(t) = r_1 + r_2 \sin w_0 t \qquad (14)$$

Case II: if a sinusoidal disturbance, d(t), at frequency w_0 enters the plant output as shown in Fig.1, the sinusoid will appear in W(t) through the following equation, which replaces (3) in the presence of an output disturbance:

$$Wyi\ (t) = \frac{s^{i-1}}{P(s)}\,[\, y(t) + d(t)\,]\ ; \quad i=1,2,..,\ n \qquad (15)$$

The following equation replaces (6) when an output disturbance is present

$$e(t) = y(t) + d(t) - y_M(t) \tag{16}$$

Any sinusoid in d(t) will also enter e(t) through equation (16), so the signals e(t) and w(t) will contain sinusoids of the same frequency.

Qualitative Explanation of the Infinite Gain of G_w and H_w :

If w(t) and e(t) contain sinusoids of the same requency, and phase difference of a value (not exactly 90°); the multiplication of w(t) and e(t) will produce a constant correlation term. The integration of this constant correlation produces the infinite gain. Since the integral of a constant is a ramp, the output signal of the integrator increases in amplitudes indefinitely with time.

4. LYAPUNOV STABILITY ANALYSIS

Assume that the plant is actually first order with the simple transfer function (for disturbance free case),

$$y_p(t) = \frac{g_p}{s+a}[u(t)] \quad ; \; g_p > 0 \tag{17}$$

Assume also that the adaptive controller is designed using CA1 with:

$$\underline{W}(t) = \begin{bmatrix} r(t) \\ y(t) \end{bmatrix} , \qquad \underline{K}(t) = \begin{bmatrix} Kr(t) \\ Ky(t) \end{bmatrix} \tag{18}$$

$$y_M(t) = \frac{g_M}{s+a_M}[r(t)] \quad ; \; g_M , a_M > 0 \tag{19}$$

$$\text{let } \underline{\Gamma} = \gamma \underline{I} \; , K_r^* = \frac{g_M}{g_p} , \quad K_y^* = \frac{a - a_M}{g_p}$$

Considering the existence of additive output disturbance (Fig.1)

$$\text{where, } y(t) = y_p + d(t) \tag{20}$$

a Lyapunov function of the form,

$$V(e,\tilde{k}) = e^2 + \underline{\tilde{K}}^T \frac{1}{\Gamma} \underline{\tilde{K}} , \tag{21}$$

and deriving the time trajectory of the adaptive scheme ; following equations could be obtained,

$$\dot{e}(t) = -a_M e(t) + g_p [\tilde{K}r\, r(t) + \tilde{K}y\, y(t)] + ad(t) + \dot{d}(t) \tag{22}$$

$$\dot{\tilde{K}}r = -\gamma\, r(t) \; e(t) \tag{23}$$

$$\dot{\tilde{K}}y = -\gamma\, y(t) \quad e(t) \tag{24}$$

Substituting from (22), (23), (24) to get $\dot{V}(t)$ as,

$$\dot{V}(t) = -2\, a_M e^2 + 2e\, [\, (ad+\dot{d}\,) - (1-g_p)\, (\tilde{K}_r r + \tilde{K}_y y)\,] \tag{25}$$

In order to have $\dot{V}$ negative (to ensure asymptotic Lyapunov stability); the second term of equation (25) has to be $\leqslant 0$

Comments:

a) From eqn. (25), to satisfy ($\dot{V} \leqslant 0$), each parameter drift ($\tilde{K}r$,$\tilde{K}y$) is related to :

1. Input reference command r(t)
2. Plant parameters (g_p ,a).
3. Disturbance magmitude and its time evolution [d(t) , $\dot{d}$(t)].

b) For sinusoidal disturbance ($d(t) = d \sin w_0 t$), the error e(t) would always contain the sinusoid d(t) , so the term 2e(t) [a d (t) + $\dot{d}$ (t)] would always contain positive quantity (Squares of sinusoids). This positive term in Lyapunov function derivative will have a destabilizing effect with increase of disturbance magnitude.

5. MODIFIED ALGORITHM WITH BOUNDED GAINS

Analysis of the previous results concerning study of MRAC systems, obtained by Rohrs and others [15-17], and carrying out further simulations for many different conditions, indicated the following:

1. Adding sinusoidal disturbances, with any frequency, either to the input or to the output (which are extremely common in practice) would lead (much probable) to unstable response.
2. In all cases of unstable response,such persistent disturbances will cause, after certain time, the controller adapted parameters to drift without bound very quick. This unbounded drift of parameters derives the system to instability.
3. The proposed modified algorithm aims to prevent the undefinite drift of controller parameters [$\underline{K}$(t)], by assuming limiting bounds ($\underline{K}$min ,$\underline{K}$max), the parameters would not exeed. So a limiter is introduced in the adaptive scheme as shown in Fig.5.

6. CHOICE OF LIMITING VALUES

For estimating proper values for the limiters, an additional analysis (besides the original Lyapunov - based stability analysis) is performed using a linearization technique. The immediate consequence is a linear, time - varying system. By assuming further that the reference input and, therefore, the model output, are constant the system is transformed into a linear time - invariant (LTI) system. Thus, the well known analysis techniques for LTI systems, such as root - locus, Nyquist, Routh - Hurwitz, etc., can be used for the adaptive systems [15].

That the linearization analysis is valid only locally, is a fact of life that one must accept. This shortcoming can be dealt with, by performing a set of Linearizations around different operating conditions to gain more global insight.

In most practical applications, the range of change of plant parameters are fairly known a priori, for different operating conditions. Performing set of Linearizations, especially for the limiting sets of the typical plant parameters, and applying one of stability criteria, a set of values for the proposed limiter can be calculated. These calculated values can be tested by simulations. However, exact computation is not so easy ;and the analyst must use his practical experience, and engineering judgement to select the proper values of the proposed controller parameters bounds ($\underline{K}min$,$\underline{K}max$).

7. SIMULATIONS OF THE BOUNDED ADAPTIVE CONTROLLER

The proposed model reference adaptive controller with bounded gains was tested by simulation studies. The following situation is representative of the results obtained. The plant used in the simulations is represented by the following transfer function: (the same example used by Rohrs [15], [16], [17]) .

$$y(s) = \frac{2}{s+1} \; \frac{229}{s^2 + 30s + 229} \; [u\,(s)] \qquad (26)$$

The adaptive controller is designed using a reference model:

$$\frac{g_M \, B_M}{A_M} = \frac{527}{s^3 + 31s^2 + 259s + 527} \qquad (27)$$

which represents the transfer function of the controlled plant at nominal operating conditions.

Simulations were run with a step reference input, $r(t) = 0.1$, a sinusoidal output disturbance, $d(t) = 0.015 \sin 8t$, and adaptation gain $\gamma = 1$. Simulations were made with all initial conditions set to zero.

Applying the Routh - Hurwitz stability test, for the chosen case; the following condition was obtained :
$-17.03 < Ky < 0.5$. The limiter's values for the presented simulations were selected to be :

$Krmin = Kymin = -0.4$,
$Krmax = Kymax = 0.4$.

The simulations were made by converting the system to discrete time. A sampling period of T= 0.01 sec. is used. In this paper, the model was chosen of the same order of the plant to highlight the effect of output disturbance without unmodeled dynamics.

Fig.6 displays the output error and Fig.7 displays the controller parameters, Kr (t) and Ky (t), for the standard adaptive system. Due to the infinite gain operator in the feedback loop of Fig.2 , and the persistant sinusoidal disturbance, d(t), the parameters drift without bounds deriving the system to instability. Fig. 8 shows the output error and Fig. 9 shows the controller parameters for the bounded adaptive system. It is clear from Fig. 8, that output error approaches steady state value, and the system stability is ensured.

8. CONCLUSIONS

An adaptive controller with bounded gains has been developed. Such controller can provide increased robustness to high frequency unmodeled dynamics and additive output disturbances. The feasibility of the bounded controller has been demonstrated with simulation example. A bounded controller is a practically appealing issue. However, minimization of drawbacks of the adaptive performance, and theoritical establishment of the proper design of the limiter; are topics for further research.

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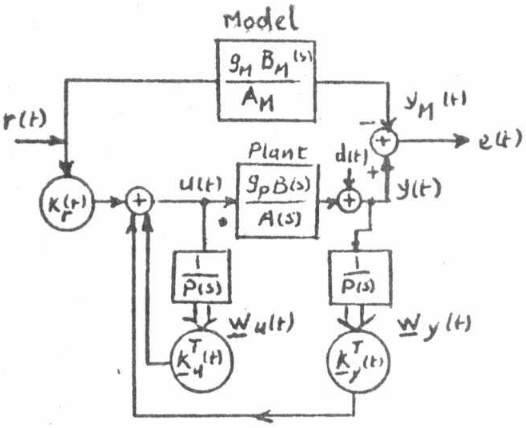

Fig.1. Controller Structure of CA1 with additive output disturbance d(t).

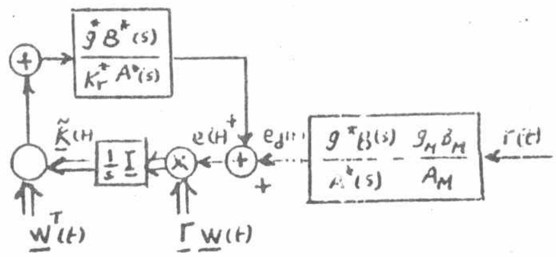

Fig.2. Error System for CA1.

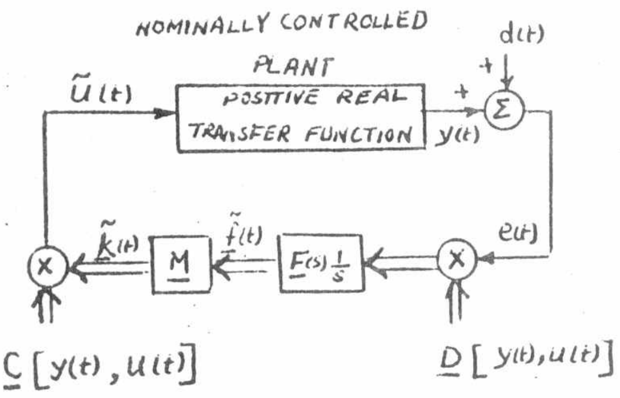

Fig.3. Block diagram of a generic controller

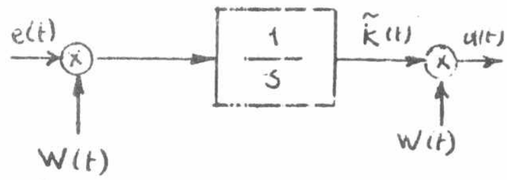

Fig.4. Infinite gain Operator of CA1

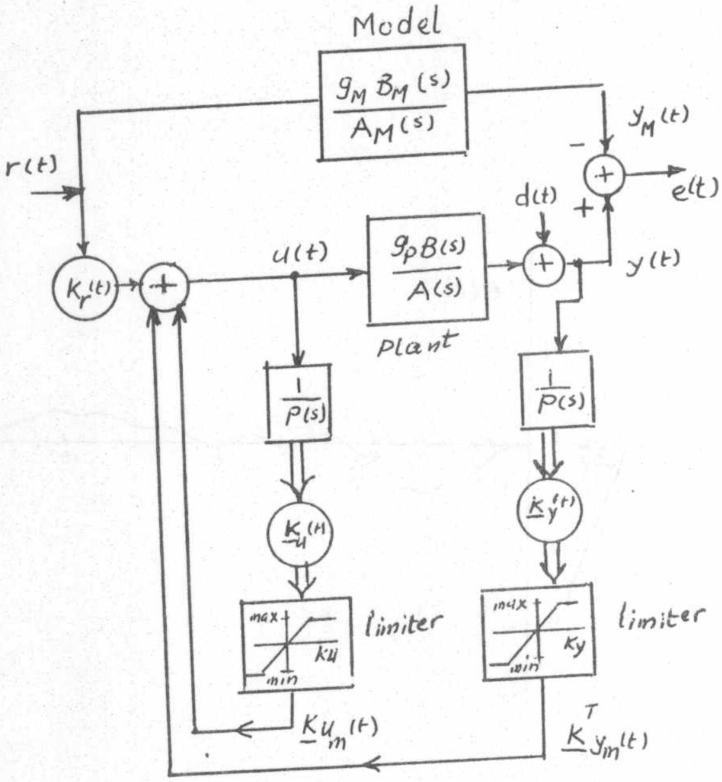

Fig. 5. Modified Controller Structure with Bounded Gains.

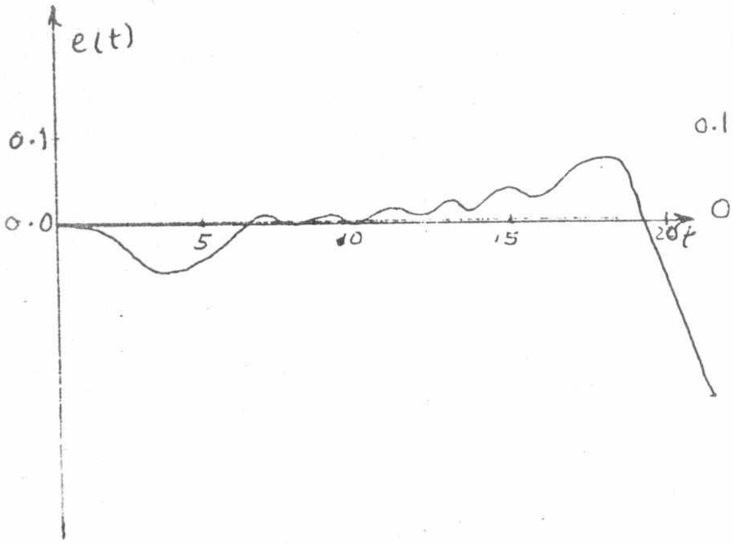

Fig.6. Output error for the standard adaptive system.

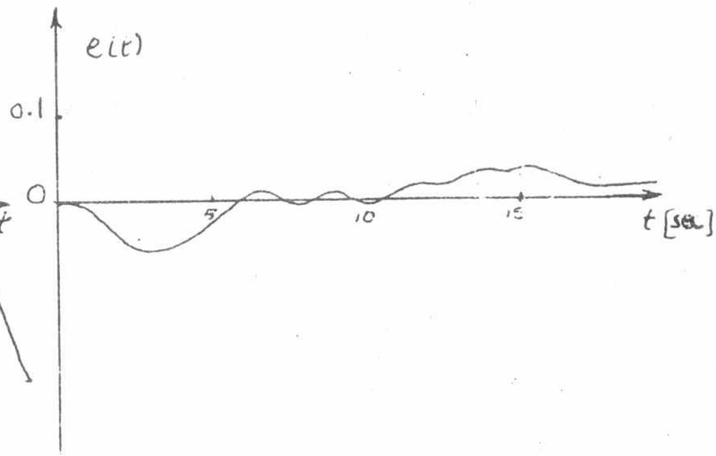

Fig.8. Output error for the bounded adaptive system.

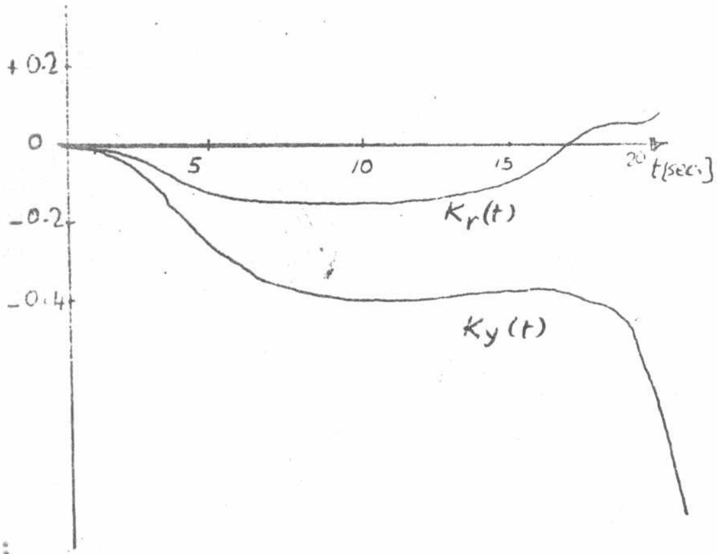

Fig.7.Controller parameters for standard adaptive system.

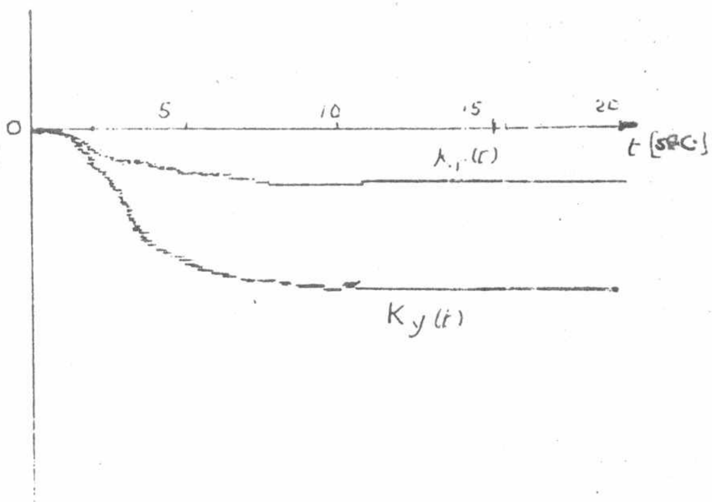

Fig.9.Controller parameters for bounded adaptive system.