ROBUST FLIGHT CONTROL FOR LINEAR SYSTEMS WITH TIME VARYING UNCERTAINTY

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ABSTRACT

This paper presents an alternative techniques for computing linear feedback control law which stablize uncertain linear systems . Specifically, flight control system of aircraft and missiles which contains uncertain parameters. These uncertain parameters may be time varying. Their values, however, are constrained to lie within known compact bounding sets. The main idea involves macking Lyapunov derivative negative via a one- dimensional parameter, leading to solve algebraic Riccati equation. Several examples are included to demonstrate the importance of these techniques.

KEY WORDS - Uncertain flight control system. Lyapunov methods

Robustness- Stability - State feedback- Control- Estimation.

1. INTRODUCTION

This paper deals with the problem of designing a controller or full state observer (estimator) when no accurate model is available for the process to be controlled. Specifically, the problem of stabilizing an uncertain linear system using state feedback control which usually appears in aircraft and missile models, these systems contain uncertain parameters whose values are known only to within a given compact bounding set. Further more, these uncertain parameters may be time varying. The fundamental idea of all methods used to establish asymptotic stability of the closed loop system including uncertain parameters, based on Lyapunov stability theory; involves constructing an upper bound for the Lyapunov derivative corresponding to the closed loop system, which leads to solve a sequence of Riccati-type equations parameterized by a scalar[1] - Petersen and Hollat[2] and W.E. Schmitendrof[3] ,[6] have given an alternative method for determining linear stabilizing controllers. Here we extend this technique to include problems with time varying uncertainty in all system matrices; A (system matrix), B(input matrix), C(Output matrix).

2. SYSTEM, ASSUMPTIONS, AND DEFINITION

The uncertain linear systems described by the state equations

$$\dot{x}(t) = \left[A_o + \Delta A(r(t))\right] x(t) + \left[B_o + \Delta B(s(t))\right] u(t) \qquad (1)$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control command, and (A_o, B_o) are the nominal system matrices. Uncertainty enters the system through the vectors $r(t) \in R^k$ and $s(t) \in R^\ell$ which ranges in the known compact sets

$$\mathcal{R} = \left\{ r : |r_i| \leqslant \bar{r} , \quad i = 1,2,\ldots,k \right\}$$
$$\mathcal{S} = \left\{ s : |s_i| \leqslant \bar{s}, \quad i = 1,2,\ldots,\ell \right\} \qquad (2)$$

Parameters uncertainty expressed as a sum of rank-1 matrices, 6

i.e.,

$$\Delta A(r(t)) = \sum_{i=1}^{k} A_i \, r_i(t)$$

$$\Delta B(s(t)) = \sum_{i=1}^{\ell} B_i \, s_i(t) \tag{3}$$

A_i , B_i can be written as

$$A_i = d_i \, e_i' \quad , \quad i=1,2,\ldots,k$$

$$B_i = f_i \, g_i' \quad , \quad i=1,2,\ldots,\ell \tag{4}$$

where d_i, e_i and f_i are n-vectors and g_i is an m vector.

Then we introduce the notations :-

The weighting matrices Q and R associated with the system (1), are positive definite symmetric matrices $Q \in R^{nxn}$, $R \in R^{mxm}$. Usually chosen by the designer, these matrices are analogous to the weighting matrices in classical LQG problems.

$$T = \bar{r} \sum_{i=1}^{k} d_i \, d_i' \quad , \quad U = \bar{r} \sum_{i=1}^{k} e_i \, e_i'$$

$$V = \bar{s} \sum_{i=1}^{\ell} g_i \, g_i' \quad , \quad W = \bar{s} \sum_{i=1}^{\ell} f_i \, f_i' \tag{5}$$

Considering the uncertain system (1), it is to determine a linear feedback control $u(t) = G\,x(t)$ such that $x(t) \longrightarrow 0$ for all admissible r(.) and s(.) and for all initial conditions $x_o \in R^n$. This control is robust in the sense that it guarantees asymptotic stability regardless of the disturbance.

Determining a robust linear stabilizing control is equivalent to determining the matrix G(mxn)

3. THE DESIGN PROCEDURES

The main idea based on making Lyapunov derivative negative via a one dimensional parameters, which produce a matrix Riccati equation after decomposing the uncertain parameters as in (4) and forming the matrices mentioned in (5). If the produced matrix Riccati equation has a positive - definite symmetric solution, then the uncertain system (1) is stabilizable with a computable control gain. The necessary and sufficient condition for the existance of unique positive definite solutions and the asymptotically stable closed loop system is established [5] , the condition is stated in terms of the detectability and stabilizability of certain matrix pairs. We called this design method a"Riccati Approach". On other way, the robust linear stabilizing control gain constructed from two parts; one corresponds to the nominal system (A_o, B_o) in (1), second part which corresponds to the uncertain parameters ($\Delta A(r)$, $\Delta B(s)$) in (1), is derived from negative condition of Lyapunov derivative of Lyapunov quadratic function.

We called this way as "Lyapunov approach".

3.a) Riccati Approach

The required robust control is given interms of the solution of the matrix Riccati equation

$$A_o'P + PA_o - P\left[\frac{1}{\varepsilon}(B_oR^{-1}B_o - B_oR^{-1}VR^{-1}B_o - W)-T\right]P + U + \varepsilon Q = 0 \qquad (6)$$

where ε is a positive scalar, Q and R matrices are chosen to be identity matrices.

If the Riccati equation (6) has a positive - definite symmetric solution P, then the system (1) can has a stabilizable control,

$$u(t) = -\frac{1}{\varepsilon} R^{-1}B_oP\ x(t) \qquad (7)$$

ε... initialized by starting value 1, at case of unsuitable P, can be replaced by $\varepsilon/2$, until the prescribed computational accuracy ε_o,

3.b) Lyapunov Approach

i- Consider the nominal system of the uncertain system (1) as

$$\dot{x}(t) = A_o\, x(t) + B_o\, x(t) \tag{8}$$

Calculate the control gain G_o which stabilize the nominal system (8), by using linear quadratic control theory [1], or eigen value placement (G_o so that all eigenvalues of $A_o^* = \{A_o + B_o\, G_o\}$ have negative real part)

ii- Solve the Lyapunov equation of the nominal closed loop system

$$A_o^{*\prime} P + P A_o^* = -Q \tag{9}$$

where $A_o^* = \{A_o + B_o\, G_o\}$

Q is taken as identity matrix, using program in [1] for the continuous case, obtaining +ve definite matrix P

iii- Choose a scalar $\varepsilon > 0$ such that the derivative of the quadratic Lyapunov function ($V = x' Px$) is negative for all $x \neq 0$ (from standard Lyapunov theory, the origin is asymptotically stable)

iv- The required robust control is;

$$u(t) = (G_o - \varepsilon B_o P)\, x(t) \tag{10}$$

The closed loop system become,

$$\begin{aligned}\dot{x}(t) &= \left\{\left[A_o + \Delta A(r)\right] + \left[B_o + \Delta B(s)\right]\left[G_o - \varepsilon B_o' P\right]\right\} x(t)\\ \dot{x}(t) &= \left[A_o^* + \Delta A(r) + \Delta B(s) G_o - \varepsilon B_o B_o' P - \varepsilon \Delta B(s) B_o' P\right] x(t)\end{aligned} \tag{11}$$

Lyapunov trajectory derivative is ,

$$\begin{aligned}\dot{V}(x,t) &= 2x'P\left[A_o^* + \Delta A(r) + \Delta B(s) G_o - \varepsilon B_o B_o' P - \varepsilon B(s) B_o' P\right] x\\ &= x'\left[-Q + P\Delta A(r) + \Delta A'(r) P + P\Delta B(s) G_o + G_o' \Delta B'(s) P\right.\end{aligned}$$

$$-2\,\varepsilon\, B_o B_o' P - \varepsilon\,(P\Delta B(s) B_o' P + P\, B_o \Delta B'(s) P)\Big]\, x$$

$$= x' L(r,s)\, x$$

then

$$L(r,s) = P\Delta A(r) + \Delta A'(r) P + P\Delta B(s) G_o + G_o' \Delta B'(s) P$$

$$-\varepsilon (P\,\Delta B(s) B_o' P + P B_o \Delta B'(s) P) - 2\varepsilon P B_o B_o' P - Q \qquad (12)$$

thus step iii) prefly is:choose $\varepsilon > 0$ such that $L(r,s) < 0$ for all $r \in \mathcal{R}$, $s \in \mathcal{S}$

Simply we are in need to calculate real part of largest eigenvalue ($Re(\lambda^*)$) of $L(r,s)$, and if $Re(\lambda^*) < 0$, step iii) is satisfied,if not,increase ε and repeat the same step until obtaining suitable value of ε, then calculate u(t) using (10).

If the uncertainty is constant, the closed loop system defined by equation (11) become,

$$\dot{x}(t) = \bar{A}\,(r,s)\, x(t) \qquad (13)$$

where the closed loop matrix $\bar{A}\,(r,s)$ is idependant of time, therefore, it is no need to use Lyapunov theory, but eigenvalue analysis can be used. Step iii) becomes;choose ε such that all the eigenvalues of matrix $\bar{A}(r,s)$ have negative real part.

4. RICCATI APPROACH IN STATE OBSERVER DESIGN OF UNCERTAIN LINEAR SYSTEMS

We add to equations (1), (2), (3), (4), (5) the following equations respectively,

$$y(t) = \left[C_o + C(v(t))\right] x(t) \qquad (14)$$

where $y(t) \in R^p$ is the measurement output

$$\mathcal{V} = \left\{ v : \left| v_i \right| \leqslant \bar{v}\ ,\quad i=1,2,\ldots,q \right\} \qquad (15)$$

$$\Delta C(v(t)) = \sum_{i=1}^{q} C_i\, v_i(t) \qquad (16)$$

$$C_i = h_i \, w_i' \tag{17}$$

$$X \simeq \bar{v} \sum_{i=1}^{q} h_i \, h_i' \quad , \quad Y \simeq \bar{v} \sum_{i=1}^{q} w_i \, w_i' \tag{18}$$

For constructing a full state observer, we consider the following state equation

$$\dot{z}(t) = A_o z(t) + B_o u(t) - K(C_o z(t) - y(t)) \tag{19}$$

where $z(t) \in R^n$ is the observer state and K (nxp) observer gain matrix, and the control $u(t) = G\,z(t)$, where G(mxn) feedback gain matrix

To investigate the stability of the closed loop system, we study the dynamics of the error vector $e(t) \simeq x(t) - z(t)$, hence, we obtain the following state and error equations for the closed loop uncertain system.

$$\begin{aligned} \dot{x}(t) = & \left[A_o + \Delta A(r) + (B_o + \Delta B(s))G\right] x(t) \\ & - \left[B_o + \Delta B(s)\; G\right] e(t) \end{aligned} \tag{20}$$

$$\begin{aligned} \dot{e}(t) = & \left[A_o - K\,C_o - G \cdot \Delta B(s)\right] e(t) \\ & + \left[\Delta A(r) + G \cdot \Delta B(s) - K \cdot \Delta C(v)\right] x(t) \end{aligned} \tag{21}$$

Applying Lyapunov theory for the asymptotic stability of the system defined in (20), (21), using the quadratic function $V(x,e) = x' P_c x + e' P_o e$, where P_c, P_o are nxn +ve definite matrices of controller and observer respectively.

We obtain two algebraic Riccati equations with scalars ε_1, $\varepsilon_2 > 0$, Q_2 and R_2 are covariance matrices.

$$\begin{aligned} P_c A_o + A_o' P_c - P_c & \left[\frac{1}{\varepsilon_1} \left\{ B_o \left(R_1^{-1} - 2R_1^{-1} V\, R_1^{-1} \right) B_o' - 2W \right\} - T \right] P_c \\ & + 2U + \frac{1}{\varepsilon_2} Y + \varepsilon_1 Q_1 = 0 \end{aligned} \tag{22}$$

$$P_oA_o + A_o'P_o - \left[\frac{1}{\varepsilon_1}\left\{C_o'\left(R_2^{-1} - R_2^{-1}X\ R_2^{-1}\right)C_o\right\} - \frac{2}{\varepsilon_1}\ P_cB_oR_1^{-1}VR_1^{-1}B_o'P_c\right]$$

$$+ P_o\left[T + \frac{2}{\varepsilon_1}\ W + \varepsilon_2\ Q_2\right]P_o = 0 \qquad (23)$$

Following the technique in (3.a), the gain matrices G and K can be calculated as :

$$G = -\frac{1}{\varepsilon_1}\ R_1^{-1}\ B_o'\ P_c \quad , \qquad K = \frac{1}{\varepsilon_2}\ P_o^{-1}C_o'R_2^{-1} \qquad (24)$$

5. ILLUSTRATIVE EXAMPLES ON AIRCRAFT MODELS

The nominal linearized dynamic models of aircraft at certain flight conditions are usually represented by the state equation,

$$\dot{x}(t) = A_o\ x(t) + B_o\ u(t)$$

The two approachs will be examined for stablishing the longitudinal stability of several aircrafts.

example 1.

The states and controls of longitudinal dynamics of the aircraft A4D at flight conditions of 0,9 Mach and 15,000 ft altitude, [7] , are

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \begin{matrix} \text{.. forward velocity} & \text{(ft/sec)} \\ \text{.. angle of attack} & \text{(rad)} \\ \text{.. pitch rate} & \text{(rad/sec)} \\ \text{.. pitch angle} & \text{(rad)} \end{matrix}$$

$$u = \left[u_1\right] \text{ elevator deflection} \quad \text{(deg)}$$

and,

$$A_o = \begin{bmatrix} -0.0605 & 32,37 & 0 & 32,2 \\ -0,00014 & -1,475 & 1 & 0 \\ -0,0111 & -34,72 & -2,793 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} , \quad B_o = \begin{bmatrix} 0 \\ -0,1064 \\ -33,8 \\ 0 \end{bmatrix}$$

This nominal system is close to instability at zero control, hence, A_o eigenvalues are $(-2,125 \pm j5,851)$ and $(-0,039 \pm j0,0896)$.

The uncertainity in A_o in the term a_{32} which represents the change of pitching moment with angle of attack (longitudinal static stability derivative M_α), this parameter has a great effect in longitudinal aircraft dynamics. Furthermore, it is strongly influenced by aero- elastic distortions of wing, tail and fuselage, also by the changes of aircraft's center of gravity and aerodynamic centre (Roskam, P.5.12 and 6.73, McGrver et al. p. 272), i.e, a_{32} has a large degree of uncertainty.

This problem was solved by the two techniques mentioned in section 3.

1) Riccati Approach Technique

Since there is one uncertainty at a_{32}, i.e., k=1, and considering equ. (1-4), $r_1 \leqslant \bar{r}$.

$$\Delta A(r) = r_1 . A_1 = r_1 . d_1 . e_1' = r_1 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

From (5), matrices V = W = O and T , U were defined, while the weighting matrices Q = I and R = 1. and scalar $\varepsilon = 1$.

Substituting in the following Riccati equation (simplified from eq. (6)).

$$A_o'P+PA_o - P\left[\frac{1}{\varepsilon}(B_oR^{-1}B_o' - \bar{r}T\right]P+\bar{r}U+\varepsilon Q = 0$$

Using the efficient algorithm of [1], it yields a positive definite matrix P for $\bar{r} \leqslant 435$

Taking $\bar{r} = 50$ i.e, M_α varies from -85 up to 15, $\varepsilon=1$, the previous Riccati equation has the positive definite solution P.

$$P = \begin{bmatrix} 0,2269 & 0,4763 & 0,0283 & 0,8248 \\ 0,4763 & 13,6991 & 0,1278 & -5,7124 \\ 0,0283 & 0,1278 & 0,0370 & 0,2414 \\ 0,8248 & -5,7124 & 0,2414 & 14,5832 \end{bmatrix}$$

And from (7), the stabilizer controller u(t) was,

$$u(t) = \begin{bmatrix} 1,0058 & 5,7763 & 1,2650 & 7,5520 \end{bmatrix} x(t)$$

2) Lyapunov Approach Technique (section 3.b)

Step i-

Using the nominal system (A_o, B_o) with quadratic cost function $J = \int_0^\infty (x'Qx + u'Ru)\, dt$ with Q = I, R=1, the linear optimal control problem was solved, and nominal gain G_o is,

$$G_o = \begin{bmatrix} 0,9583 & -14,096 & 1,1112 & 19,6841 \end{bmatrix}$$

while the nominal closed loop eigenvalues at $\{-1,93 \pm 2,87j,\ -3,86,\ -32,69\}$

Step ii-

the solution of the Lyapunov equation (3) for obtained G_o is

$$P = \begin{bmatrix} 0,5862 & -4,6055 & 0,0279 & 5,7267 \\ -4,6055 & 48,9223 & -0,332 & -63,2628 \\ 0,0279 & -0,332 & 0,0166 & 0,4651 \\ 5,7267 & -63,2628 & 0,4651 & 84,4787 \end{bmatrix}$$

Step iii-

since B(s) = 0, the equation (12) was simplified as

$$L(r,s) = P \Delta A(r) + \Delta A(r) P - 2\varepsilon P B_o B_o' P - Q$$, which leads to

$$L(r,s) = \bar{r} \begin{bmatrix} 0 & 0,0279 & 0 & 0 \\ 0,0279 & -0,664 & 0,0166 & 0,4651 \\ 0 & 0,0166 & 0 & 0 \\ 0 & 0,4651 & 0 & 0 \end{bmatrix}$$

$$+\varepsilon \begin{bmatrix} -0,4639 & 6,0963 & -0,5098 & -9,0477 \\ 6,0963 & -80,1175 & 6,7001 & 118,9057 \\ -0,5098 & 6,7001 & -0,5603 & -9,9439 \\ -9,0477 & 118,9054 & -9,9439 & -176,4718 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

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Eigen values real part $R(\lambda^*)$ of $L(r,s)$ are negative for ; ε = 1,10,50 and $\bar{r}$ = 20,30,35 respectively.

At the case of constant uncertainty and $B(s) \simeq 0$ eq.(11) become,

$$\dot{x}(t) = \left[A_o^* + \Delta A(r) - \varepsilon B_o B_o' P \right] x(t) \text{ , and } \bar{A}(r,s) \text{ of eq.(13) is}$$

$$\bar{A}(r,s) = \bar{A}(r) = \left[A_o^* + \Delta A(r) - \varepsilon B_o B_o' P \right] =$$

$$= \begin{bmatrix} -0,061 & -32,4 & 0 & 32,2 \\ -0,096 & -0,0904 & 0,889 & -1,968 \\ -32,501 & 437,645 & -40,259 & -665,323 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \bar{r} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$+ \varepsilon \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0,048 & 0,633 & -0,053 & -0,939 \\ -16,278 & 213,927 & -17,89 & -317,497 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Here, we test directly the eigenvalues real parts of $\bar{A}(r)$, $R(\lambda^*)$ were negative for; ε = 0, 1 , 10 , 50 and $\bar{r}$ = 200, 320, 430, 0 respectively.

From results at varying uncertainty and constant one, on remark that for certain value of $\bar{r}$, a larger value of ε is required for constant uncertainty. Also it is shown that the parameter ε has a saturated value.

Example 2: The dynamics in the vertical plane for a helicopter are represented by the following model (4 states and 2 commands) which are,

x_1 ... horizontal velocity (u)
x_2 ... vertical velocity (v)
x_3 ... pitch rate (q)
x_4 ... pitch angle (θ)

$$A_o = \begin{bmatrix} -0,0366 & 0,0271 & 0,0188 & -0,4555 \\ 0,0482 & -1,01 & 0,0024 & -4,0208 \\ 0,1002 & 0,2855 & -0,707 & 1,3229 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

u_1 ... collective pitch

u_2 ... lonitudinal cyclic pitch control $B_o = \begin{bmatrix} 0,4422 & 0,1761 \\ 3,0447 & -7,5922 \\ -5,52 & 4,99 \\ 0 & 0 \end{bmatrix}$

If the airspeed changes from 60 Knots to 170 Knots, significant changes occur only in elements a_{32}, a_{34} at matrix A_o and b_{21} at matrix B_o. These uncertainties bounded as ;

$$a_{32} \leq 0,2192 \quad , \quad a_{34} \leq 1,2031 \quad , \quad b_{21} \leq 2,0673$$

which $\Delta A(r)$ and $\Delta B(s)$ are

$$\Delta A(r) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & a_{32} & 0 & a_{34} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} , \quad \Delta B(s) = \begin{bmatrix} 0 & 0 \\ b_{21} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Firstly by Riccati Approach :-

We put k=1, ℓ =1, $\bar{r}$=1 , $\bar{s}$=1, and let $\Delta A(r) = A_1 r_1$, $\Delta B(s) = B_1 s_1$

$$A_1 = d_1 e_1' = \begin{bmatrix} 0 \\ 0 \\ 1,2031 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0,1822 & 0 & 1 \end{bmatrix} , B_1 = f_1 g_1' = \begin{bmatrix} 0 \\ 2,0673 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} .$$

Putting Q = 0, R=I with ε = 0,9, the Riccati equation (6) has a positive definite solution P

$$P = \begin{bmatrix} 3,1118 & 0,2631 & 0,2284 & -1,3703 \\ 0,2631 & 0,2234 & 0,1879 & 0,0101 \\ 0,2284 & 0,1879 & 0,3033 & 0,1837 \\ -1,3703 & 0,0101 & 0,1837 & 2,4604 \end{bmatrix}$$

And from (7) the robust control $u(t) = G\,x(t)$ with

$$u(t)= \begin{bmatrix} -1,0181 & 0,2674 & 1,1123 & 1,7966 \\ 0,9531 & 0,8428 & -0,1412 & -0,7419 \end{bmatrix} x(t)$$

Secondly'By Lyapunov Approach ; and from step i),

$$G_o = \begin{bmatrix} -9247 & 0,0429 & 0,9364 & 1,3776 \\ 0,0433 & 0,8387 & -0,229 & -0,7567 \end{bmatrix}$$

From steps ii) and iii), the matrix $L(r,s)$ is a negative definite for all values of r and s lies in the bound sets with parameter $\varepsilon = 1$. Then finaly eq.(10) determine the robust control.

$$u(t)= \begin{bmatrix} -1,4082 & 0,0718 & 1,3898 & 2,0104 \\ 0,0372 & 1,3270 & -0,277 & -0,8931 \end{bmatrix} x(t)$$

With constant uncertainty $\bar{A}(r,s)$ is negative definite for $\varepsilon = 0$, therefore G_o is a stabilizable controller for these constant disturbances.

6. CONCLUSION

Both techniques are efficient, they are based on the same idea in making Lyapunov derivative negative via one dimensional parameter search. In Riccati approach technique the rank-one decompositions for the A_i and B_i matrices are not unique, i,e, there is no systematic method for choosing the best rank-one decompositions, this would be an important area for future research. Mentioned examples shows the great importance of these techniques in practical problems dealing with flying vehicles around nominal trajectories.

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