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MICROCOMPUTER GRID GENERATION FOR TWO-DIMENSIONAL AERODYNAMIC APPLICATIONS

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ABSTRACT

In recent years interest has been developed in generating numerical grids required for the solution of aerodynamic problems and other problems governed by partial differential equations ,on arbitrary regions. The importance of this subject stems from the fact that poor grids may cause results to be erroneous and that they may fail to reveal the critical aspects of the true solution. The acute shortage of proper computational resources in our educational institutions contributes to the lag in knowledge of this area in Egypt. This work outlines the state of the art in the two dimensional grid generation and presents the results of an effort made to provide a new low cost grid generator using a micro-computer. The algorithm presented belongs to the type of differential system and produces orthogonal grids. Examples from the program outputs are included. Also, a methodology for evaluating the output grid is briefly presented.

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I. INTRODUCTION

The use of computationally generated body fitted coordinate systems is now a well established practice in computational aerodynamics, computational fluid dynamics and in solution of partial differential equations in general. When curvilinear coordinates are employed in the numerical solution of boundary value problems, the bounding surfaces are constrained to be coordinate surfaces. This helps to resolve problems of incompatibility between the grid and the differenced flow equations. It also simplifies the application of boundary conditions, eliminates the need for interpolation, minimizes the complexity of the logic and the housekeeping involved and reduce other special treatments.

Particular point wise distributions may be needed near the boundaries. This presents a uniformity constraint and helps to resolve regions with high boundary curvature and/or large solution gradients. Also , internal constraints may make a particular type of coordinates advantageous and cause some simplifications in the formulation of problems.

Although the mathematical roots of using the body fitted coordinates are old, interest in numerical grid generation increased only during the last two decades. The termendous increase in power and capacity of computers was responsible for the development of this interest and the considerable progress that has been made especially during the last decade. Comprehensive reviews of this progress can be found in [10,17,19].

The state of the art in grid generation shows the existence of three primary categories of gridding techniques. These are , the classical complex variable techniques , the algebraic techniques and the differential systems techniques. The three types have been successfully used to generate many types of grids in two and three dimensions. This includes the two dimensional O-grids, C-grids and H-grids as seen in Fig. 1. In the O-grids ,coordinate lines are wrapped around each profile. The lines of the C-grids run around the profile and an infinite line issuing from the trailing edge. For the H-grids, the lines run from left to right , more or less as streamlines of the real flow would be.

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For the two dimensional grid generation , the state of the art is highly advanced. All the primary techniques have been successfully used for solving a variety of two dimensional problems with complex boundaries and topologies. Other shapes ,including rectangular and cascade types are easily implementable. Computer codes such as IN MESH [2], TOMCAT [14] and GRAPE [12]are now commercially available.

The present state of the art for the three dimensional grid generation specially for complete configurations is still somewhat restrictive and considerable work need to be done in this area. construction of a three dimensional grid for solving the Euler equations over a complete aircraft may take up to one man-year of work using a VAX-minicomputer[1]. The shortage of computational resources in the educational institutions in Egypt places such type of problems outside our interest, at least for the time being.

For two dimensional grid generation, conformal systems are usually the best. But, when an arbitrary boundary representation is used, conformal transformation may not be applicable for three dimensional cases. The orthogonal coordinates are the next preferable. They are generally applicable for two dimensional flows but may be severely restricted or even not applicable. Orthogonal or nearly orthogonal grids, especially near the boundary, make the application of boundary conditions more straight forward. Hoever, strict orthognality is not necessary, and the accuracy deteriorates if the departure from orthognality is too large.

More advanced topics in this area include the generation of time-varying adaptive grids and the analysis of the errors introduced by the grids on the solution of the partial differential equations.

II.GRID GENERATION SYSTEMS An Overview

The oldest classical technique for generating body fitted coordinates is the use of analytical conformal mappings. The possibility of mapping closed contours on circles was known by Riemann in year 1850. Names of Schwartz, Christofell, Kirchhoff, Helmholtz, treffetz, Jokowski, Theodorsen andmany others are associated with different transformations which reduce some arbitrary contour into a circle or a straight line segment. Such transformations allowed to solve for different incompressible potential flow situations using the Known solution of the flow past a circular cylinder.

The need for solving compressible and viscous flows have suggested the use of conformal mapping techniques to construct computational domains over which the equations are discretised. The closed form analytical mapping functions , and their first derivatives have the advantage of introducing the fewest terms in the transformed forms of the governing equations and produce minimum truncation errors in the computed solutions, [10]. The resulting grids are orthogonal. Hence the vector operations in the physical domain map directly in terms of their counterparts in the computational domain. However, in most cases it is hard to find the required analytical transformations, or sequences of transformations. The control of the line spacing is also a problem. The use of stretching, or intermediate transformations may produce nonconformal grids.

In algebraic grid generation techniques, the coordinates are determined using intermediate surfaces in the field. The methods are very fast, provide exact control over the mesh properties necessary to satisfy the given boundary, uniformity and internal constraints [7]. Generally, a multi-surface transformation is used [8]. Simple one dimensional stretching of coordinates and\or transfinite interpolation may be used. The interpolation function may specify the boundary values and specific derivatives of the variables. Values in the interior are determined using blending (interpolation) and tension spline functions. Algebraic techniques require more complex specification of the data to assemble the mesh complex [8]. Complex boundary topology may cause mesh construction difficulties.

Hyperbolic partial differential equations have been used successfully in producing grids for problems in which the outer boundary is not constrained [13]. A typical situation occurs in external flow problems in which the location of the outer boundary need only be far removed from the inner boundary. Two systems of nonlinear hyperbolic equations are in use. A system is recasted as a locally linearized system of first order partial differential equations by expanding the equations about a nearby known solution. The solution of the resulting system depends on the given initial data and whether or not the resulting system is well posed. The solution is obtained by marching along the outward from the inner boundary. Simple tri-diagonal solvers may be used. However, the discontinuations of boundary slopes will be propagated into the field . Similar conclusions hold for using parabolic partial differential equations [13].

The use of elliptic partial differential equations has definitive advantageous. Usually asystem of Poission equations is used as a transformation. The inherent smoothing nature of the Laplace operator tends to produce equal spacing in the field regardless of the point spacing on the boundaries. Another inherent property of this operator is that the grid mappings are second order differentiable which result in the interior grid smoothly connect the boundary. Special control functions may be used to help smooth progression in cell metrics and cell areas in all directions which allow efficient use of the resulting grid.

At the moment, no technique appears to be sufficiently superior to the others. The subject of numerical grid generation is still young. The area is rapidly advancing and several efforts are needed to cover the evaluation of grid introduced errors , the construction of adaptive grids and efficient production of three dimensional grids over complete configurations.

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III. PROBLEM FORMULATION

In principle , any region can be transformed into an empty rectangular block in a computational domain through the use of branch cuts, see Fig.2. All methods of grid generation involve three steps. First, a means for distributing points in the field in an orderly fashion so that proper data structures and defined logic can be used. Second, A means for representing continuous functions by discrete values with sufficient accuracy. Third , a means for evaluation of errors in the representation. Following Thompson concept of generation of body fitted coordinates, which is probably the most well known among the grid generation methods, An elliptic system of partial differential equations will be used to implement the mensioned requirements. The problem of generation of orthogonal grids, or almost orthogonal, will be addressed.

The physical domain is defined by a Cartesian coordinate system (X,Y). The computational domain is defined by a generalized coordinate system (ξ ,η). The objective is to find the general coordinates ;

$$\xi = \xi(X,Y)$$
$$\eta = \eta(X,Y) \qquad (1)$$

such that the boundaries of the domain are coordinate lines, surfaces for the three dimensional cases.

The coordinates and are obtained by solving the system of Poisson equations ;

$$\nabla^2 \xi = P(\xi,\eta)$$
$$\nabla^2 \eta = Q(\xi,\eta) \qquad (2)$$

with proper boundary conditions ... !!
Here

∇^2 stands for the Laplace operator,and
P and Q are functions which control the grid line spacing and density

This type of linear transformation is used in this work because they produce grids of comparable quality to grids obtainable through the use of nonlinear systems atone order of magnitude less of cost ,[12].

The system of equations (2) is Then transformed with the dependent and the independent variables interchanged and solved in the computational plane. This facilitates the application of boundary conditions and the integration process . The transformed system of equations is ;

$$\alpha\, X_{\xi\xi} - 2\,\beta\, X_{\xi\eta} + \gamma\, X_{\eta\eta} = -\,J^2\,(\,P\,X_\xi + Q\,X_\eta\,)$$
$$\alpha\, Y_{\xi\xi} - 2\,\beta\, Y_{\xi\eta} + Y_{\eta\eta} = -\,J^2\,(\,P\,Y_\xi + Q\,Y_\eta\,) \qquad (3)$$

Where

α, β, γ and J are the fundamental metric coefficients and the Jacobian of the transformation.

$$\alpha = X_\xi^2 + Y_\xi^2 \quad , \quad \beta = X_\xi\, X_\eta + Y_\xi\, Y_\eta$$
$$\gamma = X_\eta^2 + Y_\eta^2 \quad \& \quad J = X_\xi\, Y_\eta - X_\eta\, Y_\xi \qquad (4)$$

Equations ,(3), and the definition of the metrics of the transformations,(4), are to be solved using proper boundary conditions. The boundary conditions may be specified by solving one dimensional grid generation problems along each boundary. This may be initialized by specifying Dirichlet conditions from the data of the problem. The functions P and Q can then be used to cluster the mesh at regions of high boundary curvature and to control the orthognality at the boundaries.

To assure the orthognality of the grid every where in the domain , it is possible to force the orthognality condition [3] ;

$$\beta = 0 \qquad (5)$$

Enforcement of this condition may produce overlapping grid lines if part of the boundary is concave with high curvature [9]. This problem may be solved by careful specification of some attraction function along the boundary opposite to the concavity.

VI. THE NUMERICAL METHOD

An iterative finite difference method is used to solve equations (3). In this method , a cut is introduced downstream of the trailing edge of of the two dimensional body. In the transformed plane,the lines of constant values of the transformed coordinates (ξ,η) are taken as straight lines. The spacing between these lines is arbitrarily set to be a constant. The value of the constant is taken as 1 to facilitate the differencing. Equations (3&4) are discretised using centeral differencing every where.

The following algorithm has beeen used in the solution process;

1-Boundary data are set, in accordance with Fig.1. and Fig. 2. Data are specified for the two dimensional object and both sides of the slit.

2-An initial solution is estimated using simple interpolation between the boundary data.

3-The metric coefficients are determined using second order differencing of the computed mesh solution.

4-A new solution is obtained using succssive line over relaxation solver (LSOR) ,[11].

5-Steps 3- and 4- are iterated untill convergence.

6-The grid error matrix , equation 7 of section V ,is examined and the boundary grid point distribution is suitably redistributed.

7- The whole process is repeated for an improved grid.

It should be noted that the second order differencing for the metric coefficients are used because this order is consistent with the order of the differencing used in the equations [17] . Also because its use is more economical than fourth order differencing previously used in this type of grid generation.

The control functions P and Q in equations (2) and (3) are set to zero. Automatic adjusting of boundary point distribution using pproper control functions is possible. Also , step 6- is currently done manually. A new version of the current grid generator featuring automatic clustering of boundary point distribution and control of the orthognality at the boundaries is in preparation.

V. ON THE ERRORS ASSOCIATED WITH NUMERICAL GRIDS

The grid errors resulting from discretization of the flow governing equations strongly affect the stability and the convergence properties of finite difference models. Examining such errors for a curvilinear coordinate system is vital to the success of constructing acceptable solutions to the governing equations. It is important to have tools which enable to examine the grid and the truncation errors so that possible inaccuracies in the solution be predictable.

Error analysis is usually obtained by considering the errors in terms of some power series expansion (e.g.Taylor Series). Usually the series is truncated after the second terms and third order derivatives are assumed to be bounded.

The errors can be classified into grid errors and truncation errors. It is usually possible to write the total truncation error ϵ as [4,16] ;

$$\epsilon + \delta = A\, d \qquad (6)$$

Where

δ^t is a vector containing difference approximations of Δ

$$\Delta^t = [\, \partial/\partial\xi \,, \partial/\partial\eta \,, \partial^2/\partial\xi^2 \,, \partial^2/\partial\xi\partial\eta \,, \partial^2/\partial\eta^2 \,]$$

$$\epsilon^t = [-1/6\ \partial^3/\partial\xi^3 \,, -1/6\ \partial^3/\partial\eta^3 \,, -1/12\ \partial^4/\partial\xi^4 \,,$$

$$1/6\ (\partial^4/\partial\xi^3\partial\eta + \partial^4/\partial\xi\partial\eta^3) \,, -1/12\ \partial^4/\partial\eta^4 \,]$$

$$d^t = [\, \partial/\partial X \,, \partial/\partial Y \,, \partial^2/\partial X^2 \,, \partial^2/\partial X\partial Y \,, \partial^2/\partial Y^2 \,] \qquad (7)$$

$$A = \begin{bmatrix} \partial X/\partial\xi & \partial Y/\partial\xi & 0 & 0 & 0 \\ \partial X/\partial\eta & \partial Y/\partial\eta & 0 & 0 & 0 \\ \partial^2 X/\partial\xi^2 & \partial^2 Y/\partial\xi^2 & (\partial X/\partial\xi)^2 & 2\frac{\partial x}{\partial\xi}\cdot\frac{\partial y}{\partial\eta} & (\partial Y/\partial\xi)^2 \\ \partial^2 X/\partial\xi\partial\eta & \partial^2 Y/\partial\xi\partial\eta & 2\frac{\partial x}{\partial\xi}\cdot\frac{\partial x}{\partial\eta} & a_{4,4} & 2\frac{\partial y}{\partial\xi}\cdot\frac{\partial y}{\partial\eta} \\ \partial^2 X/\partial\eta^2 & \partial^2 Y/\partial\eta^2 & (\partial X/\partial\eta)^2 & 2\frac{\partial x}{\partial\eta}\cdot\frac{\partial y}{\partial\eta} & (\partial Y/\partial\eta)^2 \end{bmatrix}$$

and

$$a_{4,4} = (\, \partial X/\partial\xi . \partial Y/\partial\eta + \partial X/\partial\eta . \partial Y/\partial\xi \,)$$

The matrix A deals only with the grid coordinate system. It's influence may be analyzed by considering its condition number[5]. It is clear that an ill-condition A-matrix will magnify the effect of the truncation error. Ill conditioned grid matrices results from extremely skewed coordinates. This causes additional errors due to non orthogonality . It was found , however , that reasonable departure from orthognality (<45°) is of little concern provided that the rate of change of the grid spacing is reasonable [19]. Therefore , grids should be made as orthogonal as practical especially near the boundaries.

The vector ,ϵ, is not known in advance. It envolves the solution of the flow governing equations. It can be estimated using differences of the solution.

The values of , $A^{-1}\epsilon$, can be used to distinguish regions of high grid errors from regions of lower grid errors. Hence, the truncation error in approximating the computational derivatives of the solution can be minimized by proper selection of the grid.

Nonlinear stability analysis for well posed problems and for which the truncation errors are small with respect to grid errors,showed that stability is guaranteed, and convergence implied, if the limit of the grid errors tend to vanish as the number of iterations are increased . This of course, depends on the iteration matrix.

For explicit systems, the effect of grid errors will be damped out only if the iteration matrix is a D-matrix [6]. For implicit systems,grid errors are damped out if the iteration matrix exists and is a D-matrix. A D-matrix here implies that the matrix has a particular norm that is less than unity. Note that, in order to check that the iteration matrix is a D-matrix , some information regarding the nature of the solution must be known in advance. However, this condition will be satisfied if no sharp discontinuities are present neither in the flow field nor on the boundaries.

VI. RESULTS AND CONCLUSION

Fig.3 shows an O-grid around a NASA 0012 obtained by the current generator using an IBM-AT compatible micro computer running at 8 MHZ and equipped with a Hercules graphics card. The program was written in MicroSoft FORTRAN.

The results are for a grid of 33x17 points with 33 points taken along the airfoil an the slit. The boundary points are concentrated near the the leading and the trailing edges of the airfoil using simple trigonometric concentration function.

The rates of convergence for both the coordinate lines are showen in Fig.4. The run time was 677 sec. The number of points computed was 37.3 point/iteration/second , or 0.0267 sec/point/iteration, with no 80287 mathematical processor installed. Provision of a mathematical coprocessor should increase this rate by a facor of 3 to 4 at least.

No evidence of zero Jacobian was reported during the solution process.

This concludes that a new economical grid generator featuring the preferable advantageous of grid generation techniques has been constructed. The resulting grids are of acceptable quality. The cost of computing the grids are fairly low. Further modificatons of this grid generator are planned. It is hoped that the availability of this grid generator would help investigate more the nature ns of this area and the nature of the errors resulting from using different grid types with different applications.

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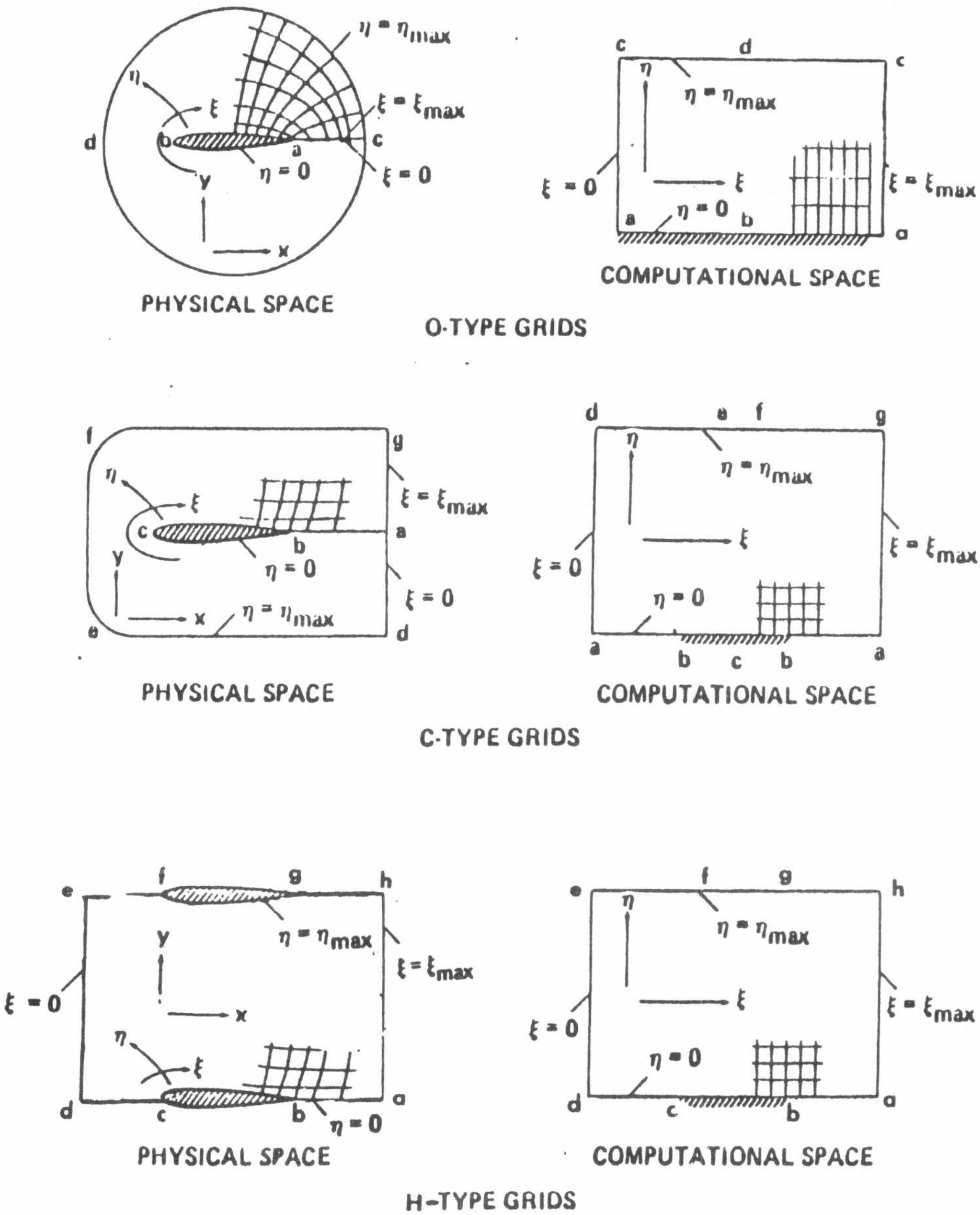

Fig.1 Topology of grid mappings.
(adapted from Ref. 12)

Ref.13

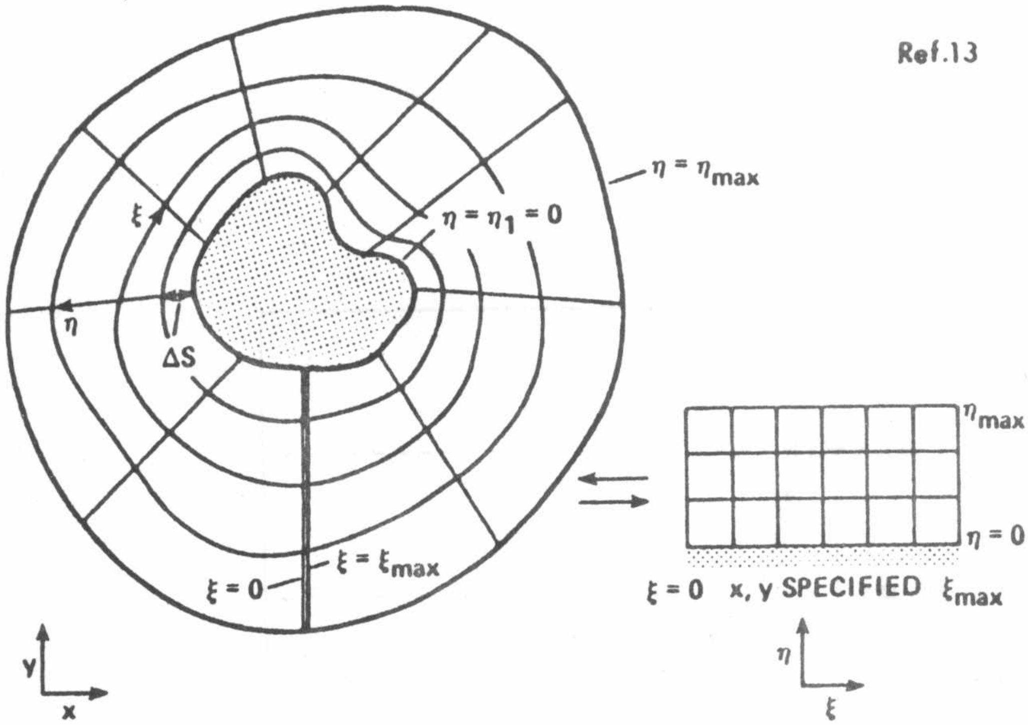

Fig.2 Sketch of physical and computational planes and the introduced cut.

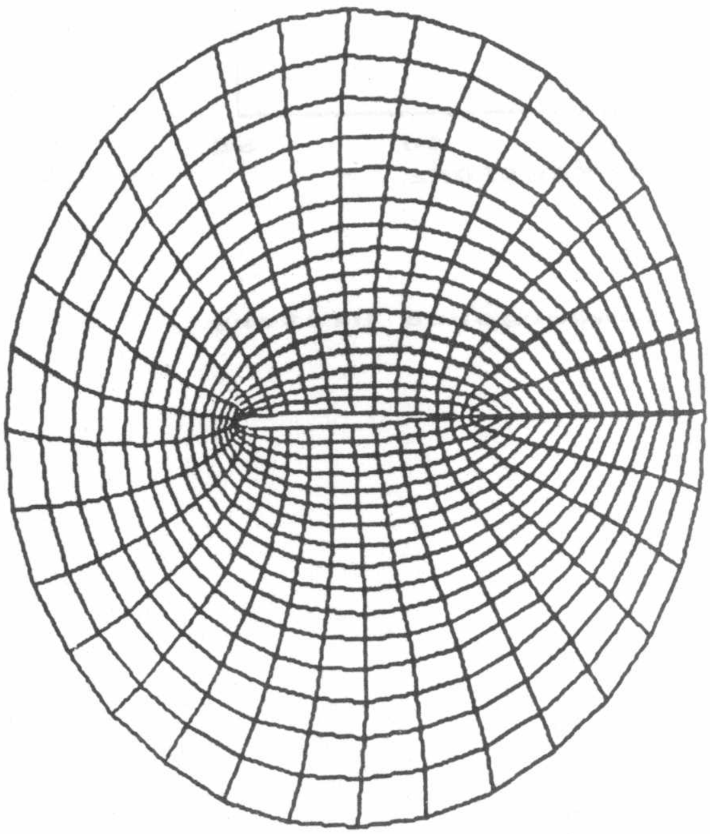

Fig.3 A Computed O-grid around NACA0012 airfoil

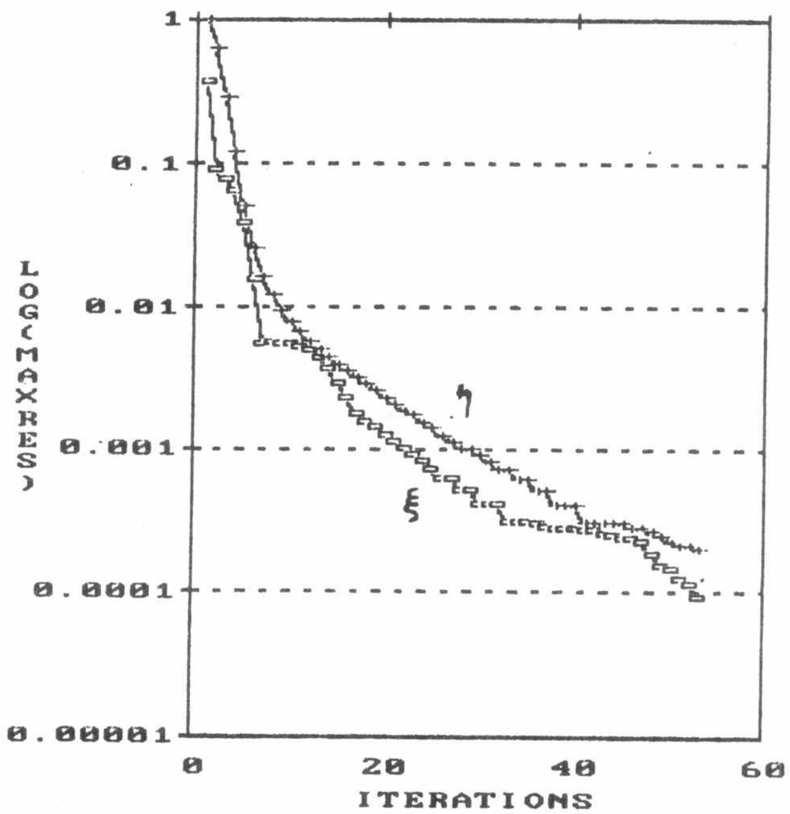

Fig.4 Convergence History